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The Plausibility and Limits of the Closure Principle

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Abstract

The closure principle is a topic of great debate and much controversy for the last four decades. The closure principle has many different forms and variations depending on contexts, but the central problem is always the same: is knowledge closed under entailment? In this paper I examine the plausibility and limits of the closure principle on the basis of a detailed analysis of different epistemic routes to knowledge and different ways the actual world could be. The conclusion of this paper is that we can expand our knowledge deductively only when the antecedent is knowledge by acquaintance or a priori knowledge. Therefore the closure principle is plausible, but limited. This conclusion can provide us an explanation for our intuition that we can expand our knowledge base through deduction, while avoiding paradoxes and skeptical arguments related to closure.

Keywords

Closure Principle, A Priori Knowledge, Possible Worlds

1. The Concept and Main Forms of Closure Principle

In epistemology, closure refers to defining a set based on a certain concept of knowledge, where any element in this set, after undergoing a certain operation, still belongs to the same set. The closure principle is the conditional sentence or proposition that expresses this closure relationship, for example:

(CP1) If you know p, and p entails q, then you know q.

(CP1) is a typical principle of closure of knowledge, which asserts that knowledge is closed under the relation of entailment. This means that the property of knowledge will always be transferred from the antecedent to the consequent by the relation of entailment between two propositions. It is easy to see that (CP1)

is incorrect: in a mathematical or logical axiomatic system, several axioms imply all other truths. If (CP1) were true, it would mean that we only need to learn a little bit of logical or mathematical knowledge to know all the logical or mathematical truths. However, it is obvious that no one is omniscient in logic or mathematics.

If we change "p entails q" to "know that p entails q" in (CP1), we can avoid the problem of omniscience that (CP1) faces:

(CP2) If you know p and know that p entails q, then you know q.

Continuing to patch the problematic aspects of (CP2) and eliminate possible failures, we can arrive at a more complete form of the closure principle. As phrased as follows:

> (CP3) If one knows p and competently deduces q from p, thereby coming to believe q, while retaining one's knowledge that p and learning of no undefeated defeater for q in the process, one comes to know that q. (Kvanvig, 2006).

The phrase "competently deduce" indicates that proposition q comes from the cognitive agent's reasoning action rather than factors such as epistemic luck. "Believe q because of this inference" excludes the situation where the cognitive agent does not believe the conclusion of the inference, such as in Carroll's tortoise (Carroll, 1895). "Still have knowledge of p after this inference" excludes the situation where the cognitive agent loses knowledge of p during the reasoning process. Sometimes, realizing the logical consequences of existing knowledge will make us reassess or even abandon that knowledge, rather than believe its logical consequences. "Have not found any undefeated defeaters of q" is because if the cognitive agent finds any undefeated defeaters of q during the reasoning process, q cannot be the cognitive agent's knowledge. And this is unrelated to the truth or falsity of the defeaters, as even false defeaters will produce the same obstacle.

In the aforementioned excluded situations, the cognitive agent failed to gain new knowledge through deductive reasoning, and in some cases, these situations are even common. However, these situations are not related to the core issue of whether knowledge is closed under entailment. Therefore, I assume that (CP3) is satisfied.

2. The Plausibility of Closure Principle

We generally have the impression that deductive reasoning can expand knowledge, which can be said to be a common sense or even self-evident truth. The best explanation for this self-evident truth is that knowledge is closed under deduction. This impression provides two aspects of support for the closure principle of knowledge:

On the one hand, self-evident truths have strong resilience, and counterexamples of cognitive agents' failure to expand knowledge through deductive reason-

ing will not threaten the correctness of self-evident truths. Instead, the specific operations of cognitive agents will be questioned. For example, Nozick's denial of the closure principle based on the tracking theory has been refuted by many scholars as a powerful rebuttal of the tracking theory. For the closure principle, the specific form of the principle may be questioned, but not the correctness of the principle itself. For instance, when (CP1) was shown to be incorrect, scholars developed many different forms of the closure principle, and (CP3) can solve the various problems faced by (CP1).

On the other hand, if we only regard the closure principle of knowledge as an explanation of the common sense impression that deductive reasoning can expand knowledge, then we can deny the closure principle without denying common sense, but any attempt to deny the closure principle must reasonably explain why we have such impressions. If we regard the closure principle of knowledge as the underlying reason for common sense impression, denying the closure principle means simultaneously saying that our daily impressions are wrong, which will create absurd and difficult-to-accept pragmatic situations. In this case, denying the closure principle not only requires a reasonable explanation of common sense impression but also requires proving that the speaker in the absurd pragmatic situation is rational rather than absurd. Hawthorne provides an example of an absurd speech act scenario:[1]

> I ask S whether she agrees that P. She asserts that she does: "Yes," she says. I then ask S whether she realizes that Q follows from P. "Yes," she says. I then ask her whether she agrees that Q. "I'm not agreeing to that," she says. I ask her whether she now wishes to retract her earlier claims. "Oh no," she says. "I'm sticking by my claim that P and my claim that P entails Q. I'm just not willing to claim that Q." (Hawthorne, 2005).

According to Hawthorne, S is like Carroll's tortoise, both are irrational. S accepts the antecedent and the implication in material implication, but does not accept the consequent; the tortoise accepts a premise and the inference in a modus ponens argument, but not the conclusion. The behavior of S or the tortoise lacks conversational appropriateness and is unacceptable.

3. The Problem of Closure Principle

Some paradoxes are based on the principle of closure, such as the lottery paradox. Suppose that one million lottery tickets are issued, and only one ticket will win, so the probability of any given ticket winning is one in a million. A cognitive agent knows that "some lottery ticket will win". Due to the low probability of any one ticket winning, the cognitive agent can also say that they know "the first ticket will not win". For the same reasons, the cognitive agent knows that "the second ticket will not win", and so on, up to "the one millionth ticket will not win", which implies that the cognitive agent knows that "every ticket will not win". This leads to the paradox that the cognitive agent knows that "every ticket

[1]This example also depends on the norm of assertion of knowledge.

will not win and some ticket will win" at the same time.

Similarly using the closure principle, a slight modification of the lottery paradox can yield an example of skeptical reasoning:

1) S knows that they won't buy a mansion tomorrow.

2) S knows that if they won't buy a mansion tomorrow, then the lottery ticket they bought today won't win.

3) S doesn't know that the lottery ticket they bought today won't win.

4) Therefore, S doesn't know that they won't buy a mansion tomorrow.

Many views hold that in the above examples, the cognitive agent does not know propositions such as "the first lottery ticket won't win". For instance, according to Dretske (1970), the cognitive agent lacks decisive reasons for knowing the lottery propositions, and for a belief to count as knowledge, there must be decisive reasons. The only basis for the cognitive agent's belief in the lottery propositions is knowledge of the winning probability, which doesn't count as a decisive reason for knowing the propositions. Armstrong (1973) holds that knowledge requires belief to be secured as true, and probabilistic (and therefore less than 1) belief fails to meet this condition. Harman et al. (2004) hold that "someone knows that p only if the basis on which he holds the belief settles the truth value of p". The safety theory of knowledge holds that we know p only if our belief in p is based on a foundation that securely makes p true (Sosa, 2003).

Even when the starting point of reasoning is certain knowledge, the closure principle still leads to skepticism, as illustrated by the famous example of the zebra: a cognitive agent sees a zebra in the zoo and thus knows "That is a zebra", but it is difficult for the cognitive agent to say that they know "That is not a mule disguised to look like a zebra". "Is a zebra" implies "is not a mule", and if knowledge is closed, then by the modus tollens inference, the cognitive agent does not know "That is not a mule" and thus does not know "That is a zebra" either. In this example, the cognitive agent has sufficient evidence for their knowledge "That is a zebra", in addition to reliable visual perception, they know that they are in a zoo and the cage is labeled with "zebra". According to Dretske, who proposed this example, the cognitive agent knows "That is a zebra", but does not know "That is not a mule", and unlike skeptics, he believes that the closure principle on which skeptical arguments rely should be abandoned. Dretske's reason is that whether a cognitive agent can extend their knowledge through deductive reasoning depends on two factors: the fact expressed by the proposition and the relevant possibilities. Relevant possibilities refer to events that would occur under the same conditions if the actual state of affairs did not happen. Only when the conclusion and the premise are in the same relevant possibility network, the conclusion can become knowledge through reasoning alone. Therefore, Dretske calls "know" a semi-penetrating epistemic operator, and knowledge can only penetrate from the premise to the conclusion if both sentences in the implication belong to the same relevant possibility network. If the premise and the conclusion belong to different relevant possibility networks, such as "is a zebra" and "is a mule disguised to look like a zebra", "know" cannot penetrate from the pre-

mise to the conclusion. Whether the epistemic property of knowledge can be transmitted through implication depends on the context of the proposition, and is not relative to the closure of implication.

Similarly, contextualism also considers context as a condition for knowledge, but unlike Dretske, contextualism does not advocate abandoning the closure principle. Contextualism believes that the closure principle across contexts is not valid, but within the same context, the closure principle is effective. For example, "is a zebra" and "is not a donkey" belong to different contexts relative to the knower, so the knower cannot know "is not a donkey" through the closure principle.

In fact, Dretske and contextualists have key similarities in their views on the closure principle, namely that they both believe that there is no closure principle across related possibility networks/contexts. This solution to the paradoxes and skepticism caused by the closure principle also means that any discussion of knowledge must appeal to context, as if wearing the shackles of context. Therefore, this article attempts to examine the closure principle in a context-neutral manner and respond to the problems of paradoxes and skepticism.

4. The Limits of Closure Principle

In discussing the apriority[2] of propositions, Soames initially distinguishes between two cognitive paths, and on this basis, I further distinguish a third cognitive path, and then discuss the effective scope of the knowledge closure principle.

4.1. Three Cognitive Paths and How the World Presents Itself

Let P express proposition p. "@" strictly designates the real world.[3] "w" is a variable representing any possible world. The index operator "actually" directly denotes the possible world in which the speech act happens. Therefore, in possible world w, the sentence "actually P" expresses the proposition "p is true in w". For any sentence P, P(w) represents the possible world in which p is true, and P@ represents the property that @ has that makes p true. Soames believes that the propositions expressed by P and "actually P" are logically equivalent, that is:

(S1) For any proposition p, the proposition "p is true at @" is logically equivalent to p.

This can be proved by the following argument (Argument 1):

1a. P iff P (ass.)
1b. λw[(P iff P) w] @ (1a)
1c. λw[P(w) iff P(w)] @ (1b)
1d. λw[P(w) iff P(@)] @ (1c)
1e. λw[(P iff P@) w] @ (1d)

[2]A proposition p is knowable apriori iff there is some way of entertaining p such that, when one does so, it is possible to come to know p, with-out appeal to empirical evidence for justification.
[3]A possible world is a proposition or set of propositions describing what the world could be like; the actual world is a possible world that is instantiated.

1f. P iff P@ (1e)

1g. P iff actually P (1g)

(1a) is a self-evident, a priori truth. The reason for going from (1a) to (1b) is that in any possible world w, if a cognitive agent knows any proposition p, the cognitive agent needs no further evidence to know that w is a possible world where p is true. When w represents the actual world @, if the cognitive agent knows p, then the cognitive agent needs no further evidence to know that @ is a possible world where p is true. The reason for going from (1b) to (1c) is that if a cognitive agent knows that a possible world w makes a biconditional proposition true, the cognitive agent needs no further evidence to know that w makes the antecedent of the biconditional proposition true if and only if w makes the consequent of the biconditional proposition true. The reason for going from (1c) to (1d) is that, according to (1c), the actual world @, like any possible world w, can make "p is true in w if and only if p is true in w" true, so the constant @ is just like the variable w and can make "p is true in w if and only if p is true in @" true. The reason for going from (1d) to (1e) is a transformation of the biconditional expression, but in the opposite direction from (1b) to (1c). The reason for going from (1e) to (1f) is that for any proposition p and any possible world w, if a cognitive agent knows that w makes p true based on empirical evidence e (or without empirical evidence, that is, a priori), the cognitive agent can know that p is true based on the same evidence e (or without evidence). In other words, the cognitive agent knows firsthand that the actual world @ is true, and therefore can derive the truth of p from "@ makes p true/p is true in @". The cognitive agent knows that "@ makes P iff P@" is true, and therefore knows that "P iff P@" is true. Since (1f) and (1g) express the same proposition "p is true in @", (1g) is proven.

From the definitions of possible worlds and logical consequence, we can see that the following proposition is true:

(S2) For any possible world w and any logically consequent proposition p of w, the proposition "p is true in w" can be known a priori.

If we put (S1), (S2), and the following proposition together, it will lead to a contradiction:

(S3) All propositions that can be known a priori can also have their logical consequences known a priori.

Suppose that proposition p is a logical consequence of the actual world @ and can only be known a posteriori. According to (S2), "p is true in @" can be known a priori. According to (S1), p and "p is true in @" are logically equivalent and thus are each other's logical consequence. According to (S3), p is the logical consequence of the proposition "p is true in @", which can be known a priori, and thus p can also be known a priori. This conflicts with the initial assumption that p can only be known a posteriori. Since (S1) and (S2) have been proved to be true, only (S3) is false.

Soames's (2010) explanation for this is that the cognitive path from a priori knowledge "p is true at @ (the actual world)" to knowing p is different from the cognitive path to a priori knowledge of "p is true at @". To infer p from "p is true at @", a cognitive agent must know that @ is true, i.e., that @ is the actual world in which they find themselves; to know a priori that "p is true at @", a cognitive agent must grasp all the component propositions of @. The reason for the different cognitive paths is that the actual world presents itself to cognitive agents in different ways. Soames summarizes two ways in which the actual world can present itself:

The first way is to grasp all the constituent propositions of the actual world. When the actual world presents itself in this way, a cognitive agent entertaining the proposition "p is true in @" can have a priori knowledge of the proposition. However, the cognitive agent has no way of knowing whether @ is true, i.e., whether @ is instantiated. The cognitive agent does not know that the possible world they grasp is the actual world they are in, and the two possible worlds are both @, but this necessary truth cannot be known a priori.

The second way is to know by acquaintance that @ is true, i.e., that one is in the actual world @. With the acquisition of knowledge by acquaintance that p is true, the possible world where p is true is given to the cognitive agent as the actual world @ they are in. When the actual world presents itself in this way, a cognitive agent who knows that "p is true in @" can infer p. However, in this case, the cognitive agent does not know all the composing propositions of @, so a cognitive agent holding the proposition "p is true in @" cannot have a priori knowledge of the proposition.

In fact, there is a third way in which the actual world can present itself to a cognitive agent. The agent presupposes the actual world @, but does not know whether it is true or grasp all of its constituent propositions. Any proposition p is true relative to some possible world w. In any possible world w, the cognitive agent knows that p is true, i.e., commits to the proposition "p is true relative to the actual world (w)", but if the agent does not know by acquaintance the actual world where p is true, they cannot identify which actual world is w. For example, if a cognitive agent learns from a book that the capital of Hungary is Budapest, they commit to the proposition "The capital of Hungary is Budapest in the actual world". Since the agent's world is the actual world @, the modal phrase "actual world" and the indexical phrase "the world I am currently in" both refer to @. The statement "The actual world is the world I am currently in" is a posteriori necessary truth, but the cognitive agent reading the book does not know this. When the actual world presents itself in this way, the cognitive agent cannot infer p from "p is true in @", nor can they know a priori that "p is true in @".

Combining the three different ways in which the actual world can be presented to a cognitive agent, we can see that there are flaws in argument (1), and the truth of (S3) also needs to be reconsidered. First, the reasoning from (1a) to (1b) relies on the way in which the cognitive agent knows any proposition p, which must be limited. If the cognitive agent knows p experientially but not by

acquaintance, then the cognitive agent only commits to "p is true at @", the third way in which the actual world can be presented. In this case, the cognitive agent needs more empirical evidence to know that "p is true at @", and the reasoning from (1a) to (1b) does not hold. However, since the starting point of argument (1) is the logical truth "P iff P", argument (1) is valid, and (S1) is correct. Second, Soames claims that (S3) is invalid because the three propositions, taken together, produce a contradiction. In fact, a contradiction only arises in specific situations, such as the case where p can only be known a posteriori, which means that it is impossible to deduce a posteriori propositions from a priori knowledge. But if p is an a priori proposition, then even if @ is presented in the first way, the three propositions will not produce a contradiction.

4.2. The Scope of Closure Principle

Based on the three ways that the actual world can be presented to a cognitive agent, and the clarifications provided above, it can be proven that knowledge is closed for both a priori and a posteriori knowledge. "P ⇒ Q" means that in the natural language sense, P implies Q. The process of extending knowledge based on knowledge of implication can be demonstrated as follows (Argument 2):

2a. P (ass.)
2b. P ⇒ Q (ass.)
2c. λw [(P ⇒ Q) w] @ (2b)
2d. λw [P(w) ⇒ Q(w)] @ (2c)
2e. P@ ⇒ Q@ (2d)
2f. P@ (2a)
2g. Q@ (2e, 2f)
2h. Q (2g)

Suppose P is knowledge by acquaintance or a priori knowledge. (2a) and (2b) are the known conditions, indicating that the cognitive agent knows P and knows that P implies Q. The basis for the inference from (2b) to (2c) is that for any proposition P, if the cognitive agent knows P by acquaintance or a priori in any possible world w, then they do not need additional empirical evidence to know that the possible world they are currently in, denoted by @, is a possible world where P is true. Therefore, when @ is the actual world, the cognitive agent's a priori knowledge that "P implies Q" allows them to know a priori that the possible world they are currently in is the possible world where "P implies Q" is true.

The basis for the inference from (2c) to (2d) is that if the cognitive agent knows that any possible world w satisfies "P implies Q", then they do not need additional empirical evidence to know that w satisfies "if w makes P true, then w makes Q true (with the same values of w in the antecedent and consequent)". The basis for the inference from (2d) to (2e) is that if the cognitive agent knows that any possible world w satisfies "if w makes P true, then w makes Q true (with the same values of w in the antecedent and consequent)", then they can know a

priori that the value of @ also satisfies "if @ makes P true, then @ makes Q true". The basis for the inference from (2a) to (2f) is equivalent to the basis for the inference from (2b) to (2c).

From (2e) and (2f), we obtain (2g) as an affirmation of the antecedent. The basis for the inference from (2g) to (2h) is that the cognitive agent knows that the current actual world @ makes Q true, which means they naturally know Q. Therefore, when P is knowledge by acquaintance or a priori knowledge, the cognitive agent can know Q based solely on knowing P and knowing that P implies Q, and knowledge is closed under implication.

If P is empirical knowledge not by acquaintance, the cognitive agent does not know that the actual world @, in which they are now in, is the possible world where P is true. P being true in @ is truth necessary a posteriori, and the cognitive agent needs additional empirical evidence to verify this. Therefore, the cognitive agent cannot infer (2f) from (2a) a priori. For example, a cognitive agent learns from a book that "Budapest is the capital of Hungary", but they do not know whether the actual world where this proposition is true is the same as the world in which they are reading the book. The cognitive agent only assumes that the actual world where "Budapest is the capital of Hungary" is true is the same as their actual world, that is, they believe the truth necessary a posteriori that "the actual world is the world where I am now in". However, this belief alone does not suffice for it to be knowledge. Further empirical evidence is required. Additionally, the cognitive agent cannot infer (2h) from (2g) because they only know that Q is true in @ or that @ makes Q true, but they do not know whether @ itself is true, i.e., whether the actual world in which they are located is the one where Q is true. Therefore, the closure principle does not hold for empirical knowledge not by acquaintance or knowledge that is not a priori.

As per Argument 2, knowledge is closed relative to the entailment relationship only when the starting point of reasoning is knowledge by acquaintance or a priori knowledge. This conclusion can be verified through another form of argument (Argument 3):

3a. P (ass.)
3b. P ⇒ Q (ass.)
3c. λw [(P ⇒ Q) w] @ (3b)
3d. λw [P(w) ⇒ Q(w)] @ (3c)
3e. λw [(P(@) ⇒ Q(w) w] @ (3d)
3f. λw [(P@ ⇒ Q) w] @ (3e)
3g. P@ ⇒ Q (3f)
3h. P@ (3a)
3i. Q (3g, 3h)

From (3a) to (3d) use the same justification as from (2a) to (2d). The justification from (3d) to (3e) is that according to (3d), @ can satisfy "If w makes P true, then w makes Q true (where @ has the same value in the antecedent and the consequent)" like any possible world, so the cognitive agent can know a priori

that @ can also satisfy "If @ makes P true, then w makes Q true" like any possible world. The justification from (3e) to (3f) is that if the cognitive agent knows that any possible world w satisfies "If @ makes P true, then w makes Q true", then the cognitive agent can know a priori that w satisfies "@ makes P true implies Q" without additional empirical evidence. The justification from (3f) to (3g) is that the cognitive agent knows that the possible world @ in which they currently exist satisfies "@ makes P true implies Q", so the cognitive agent can know "P implies Q" without additional evidence. From (3a) to (3h) is the same as the case from (2a) to (2f): only when P is a priori knowledge or knowledge by acquaintance can the cognitive agent know that the possible world they now in is the one that makes P true.

In summary, knowledge is only closed under entailment when the antecedent is either a prior knowledge or knowledge by acquaintance; otherwise, the closure principle does not hold. Since a prior knowledge and knowledge by acquaintance are closed under entailment, the closure principle with the added restriction of knowledge (CP3) is also valid.

5. Conclusion

The conclusion of this article can solve the paradoxes and skepticism problems caused by the closure principle. Take the lottery paradox as an example. Even if lottery propositions can become knowledge, they are only acquired by cognitive agents through their understanding of probability, which is a type of non-privileged empirical knowledge. Therefore, closure inference cannot be applied on this basis, and the lottery paradox is not triggered in this case. As can be seen from arguments (2) or (3), whether the closure principle is valid depends on how the knowledge used as a premise in deductive reasoning is acquired, and is unrelated to the conclusion of the reasoning. Therefore, whether the premise and conclusion are in the same linguistic context is irrelevant to the validity of the closure principle. Even if the premise and conclusion are in the same linguistic context, the closure principle may still be invalid if the starting point of reasoning is non-privileged empirical knowledge. According to arguments (2) and (3), the premise P and conclusion Q of reasoning always belong to the same possible world w, so even if the premise and conclusion are in the same linguistic context, the closure principle may still be invalid.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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