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Structured Financial Product Designing

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Abstract

Composed of fixed income securities and derivatives, structured financial products have received great popularity in China nowadays, and the income certificate issued by securities companies is one of them. This paper designs a 30-day floating capital protected income certificate with European shark fin option embedded, and prices it through martingale pricing as well as Monte Carlo simulation. Based on the pricing results, this product allows issuers to borrow at a rate of −0.08% and provides investors with an average annualized return of 1.99%, which is profitable for both issuers and investors. Additionally, this paper also talks about the potential risks through sensitivity analysis and gives a hedging strategy for the issuer.

Keywords

Income Certificate, Shark Fin Options, Martingale Pricing, Monte Carlo Simulation

1. Introduction

The structured financial product is a new type of financial product that combines fixed income securities and derivatives through financial engineering technology. Income certificate is an important variety of structured financial products, the principal and income of it are linked to the target. Income certificate is essentially a debt financing tool, and therefore, it is not affected by the new asset management regulations that forbid wealth management products to guarantee principle. In the future, income certificate may become the only structured product in the market that can guarantee the principal, which is quite attractive to risk-averse investors.

2. Literature Review

(Williams, 1938) proposed a dividend discount model, which holds that stock price equals to the present value of future cash flow. This idea can also be applied to price bonds and derivatives. Option is one of the most important derivatives. The core of option pricing is the risk-neutral and no-arbitrage pricing concepts. (Bachelier, 1900) derived the prototype of option pricing formula, (Sprenkle, 1961) and (Boness, 1964) improved the model by eliminating the possibility of negative stock prices and considering time value of money. (Samuleson, 1965) believed the expected return of options is higher than stocks because of the difference in risk.

(Black & Scholes, 1973) gave the B-S formula for pricing European call options, and deduced formula for pricing European put options based on option parity theorem proved by (Stoll, 1969). (Merton, 1973) proposed the B-S model almost at the same time. (Harrison & Kreps, 1979) proposed a martingale pricing model under risk-neutral conditions. With the optimization of computer performance, Monte Carlo simulation has been widely used. (Boyle, 1977) and (Tilley, 1993) talked about the effectiveness of pricing European options and American options by Monte Carlo method respectively.

(Chen & Sears, 1990) used B-S formula and cash flow discount formula to price the bond product SPIN linked to equity. (Li, 2019) gave the asymmetric shark fin option pricing formula based on the martingale pricing method. (Sun, 2015) gave the pricing formula of discrete double barrier knock-out options. (Zheng & Wei, 2018) discussed the hedging strategy of the barrier option.

3. Shark Fin Options

Income certificates are securities issued by securities companies to qualified investors in a private way. The repayment of the principal and income is related to the specific target. Common targets include interest rates, exchange rates, commodities, credit and stocks. Structured financial products can be divided into fixed income and floating income ones, and the latter can be divided into principal guaranteed and principal non-guaranteed ones. Embedded options usually include binary options, spread options and shark fin options.

Shark fin options are named according to their income structure. Compared with ordinary options, they have barrier prices. European shark fin options knock out when target price breaks through barrier price at the end of the period, while American shark fin option requires to be pegged to the market day by day. When the option is knocked out, the buyer can obtain the knock-out income, also known as the compensation income.

The unilateral shark fin option has only one barrier price, but the bilateral one has both an upper barrier limit $H1$ and a lower barrier limit $H2$. The initial price of the underlying is $S0$, the high exercise price is $K1$ and the low exercise price is $K2$. The relationships among these prices can be noted as $H1 > S0 > H2$, and $H1 > K1 > K2 > H2$. In a bilateral shark fin option, the option disappears automatically whenever any barrier level is touched.

4. Product Design

4.1. Target Customer

Golden Shark 30 days is a guaranteed income certificate, mainly designed for domestic investors with investment needs but have poor risk tolerance. The product has an investment period of 30 days. It is composed of fixed income security and option, which is linked to CSI 300 Index. Besides guarantee the principal and fixed return, investors can receive floating income according to the index trend. The characteristics of controllable risk and high potential income of this product can well meet the needs of investors.

4.2. Basic Clause

- The investment period is set to be 30 days, from September 13, 2022 to October 12, 2022. The investment term refers to the duration of the designed product, which is closely related to market trends and investor characteristics. Considering the target customers are investors with poor risk tolerance and high liquidity demand, we set the investment period to be 30 days.
- The investment currency is set to be RMB. The investment currency is generally determined by the flow of funds and the investment object. This product is mainly for domestic individual and institutional investors, and the funds are mainly invested in the domestic money market.
- The issuance scale is set to be 10 million yuan to ensure the issuance. Issuance scale refers to the amount of funds the issuer intends to raise. If the subscription doesn't reach the lower limit of issuance scale during the subscription period, the issuance will fail.
- The least subscription is 50,000 yuan and is increased by an integer multiple of 1000 yuan. The least subscription should fully consider the characteristics of investors. If the investment threshold is too high, potential investors may be turned away, resulting in financing failure. If the investment threshold is too low, it is not conducive to the organization and management of fundraisers.

4.3. Main Body

Structured products are usually composed of fixed income part and floating income part. The fixed income part is generally a fixed rate bond, and the floating income part is composed of financial derivatives. Most of the funds raised will be invested in fixed income securities to protect principal and fixed income, and the remaining funds are invested in derivatives for higher returns.

The investment period is from September 13 to October 12 in 2022, totally 30 days. The fixed income part of this product is designed as a fixed-rate bond, which principal and interest are paid at due. The floating income part of this product is a European bilateral shark fin option, which underlying index is the CSI 300 index.

4.4. Product Elements

The income certificate designed in this paper is a structured financial product embedded with European bilateral shark fin option. The specific elements in this product are set according to product SRJ173 by Guosen Securities.

- The guaranteed rate refers to the minimum proportion of principal that can be obtained at the end of the investment period, it is usually no more than 100%. Because this product is a guaranteed income certificate, the guaranteed rate is set to 100%.
- The fixed rate of return in this product is the guaranteed rate of return, which is the minimum return promised by the issuer. Setting fixed rate too high may increase the risk of payment, while setting it too low can't attract potential investors. The fixed return rate of this product is set to 1%.
- The participation rate measures the leverage of participation in option, it is mostly ranged from 50% to 200%. The participation rate is positively correlated with investors' demand for income and issuers' determination of future trend.
- The high exercise price, low exercise price, high touch price and low touch price of the European bilateral shark fin option embedded in this product are 102%, 98%, 107% and 93% of the initial price of the linked target.
- When option knocks out, investors can obtain a knock-out income, also known as compensation income. The compensation rate is set to be 1%, and the floating return rate equals to compensation rate × participation rate.

5. Income Analysis

5.1. Final Earnings

The price and volatility of CSI 300 index from September 13, 2021 to September 12, 2022 are drawn with MATLAB, as shown in **Figure 1**. The initial price is the closing price of the starting date. It can be seen that the CSI 300 index tends to be stable 50 days before the starting date, and the up and down of the price is basically between −2% and 2%. Therefore, it is reasonable to use the 2% deviation from the initial price as the option exercise price.

Figure 2 shows how the product yield changes with the index. The shape of the yield curve is similar to the shark fin, so this kind of option is named as shark fin option. The central axis is located at 100% of the CSI 300 index, that is, the initial price of the CSI 300 index is the center. We can also see that the highest yield of the product is 6%, and the lowest yield is 1%. **Table 1** illustrates the parameters in this paper. According to the price of the CSI 300 index from high to low at maturity date, the income of the product can be divided into the six cases. Situations A to F shown in **Table 2** indicate upward knock-out, execute call option, rise but not knock-in, fall but not knock-in, execute put option, and downward knock-out. ST is the price of CSI 300 index at the maturity date.

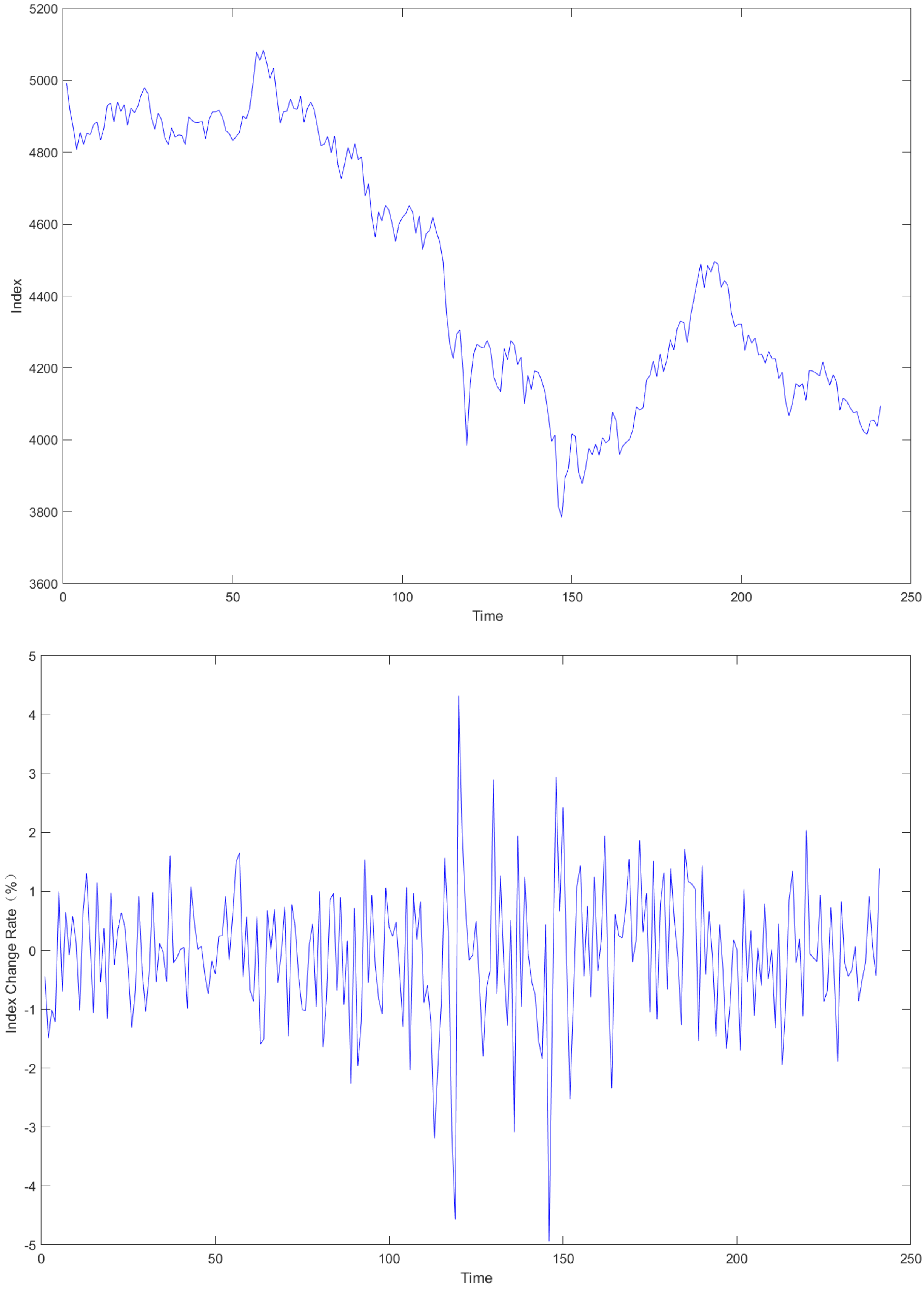


Figure 1. The price and volatility of CSI 300 index.

Table 1. Parameter setting instructions.

Variable	Sign	Value
risk-free rate	r	2.06%
volatility	σ	18.71%
principal	Y	50,000
fixed rate	$R0$	1%
compensation rate	RC	1%
initial price	$S0$	4093.79
high exercise price	$K1$	4175.67
low exercise price	$K2$	4011.91
high touch price	$H1$	4380.36
low touch price	$H2$	3807.22
investment horizon	T	30/365
join rate	J	100%

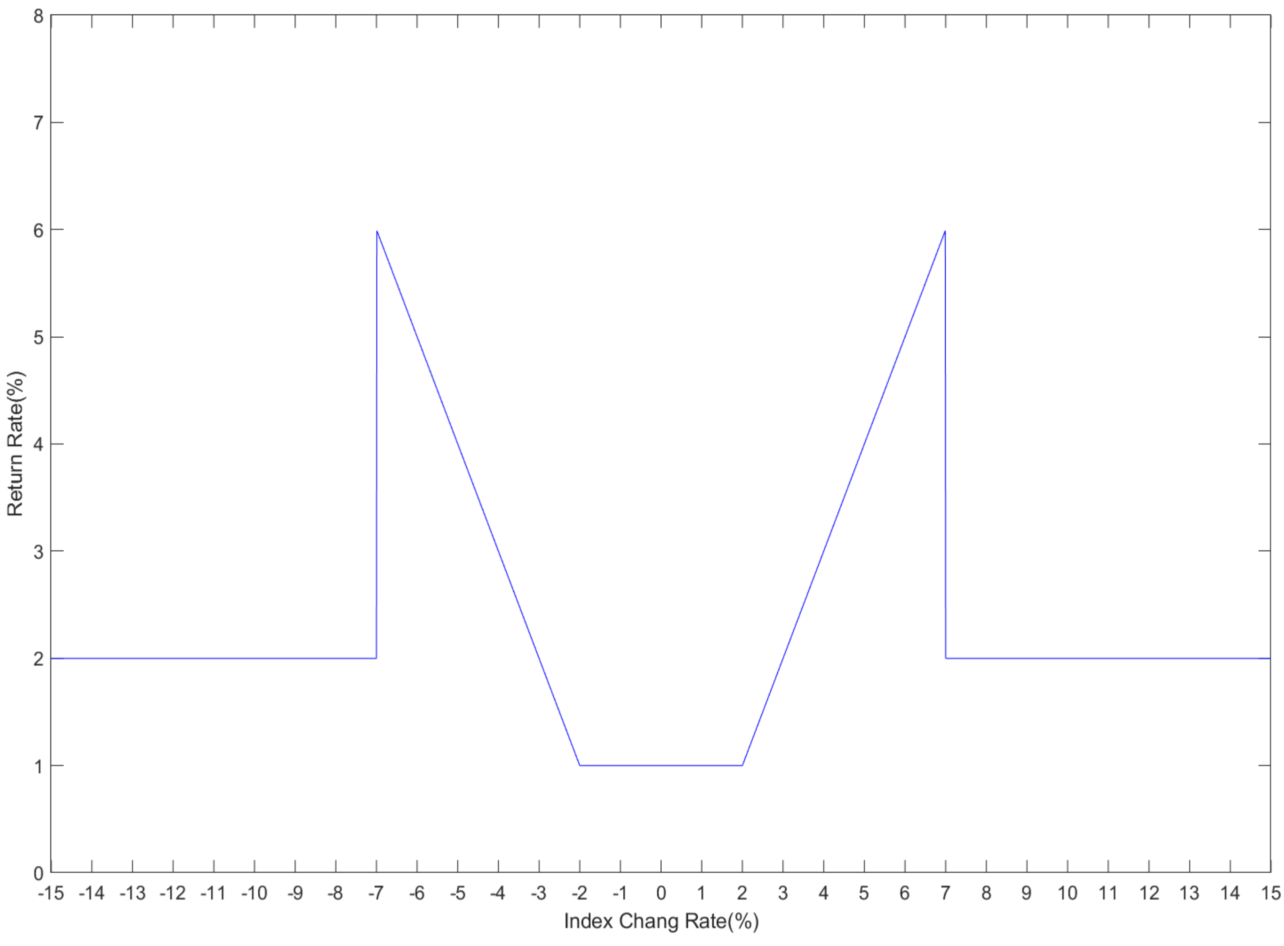


Figure 2. CSI 300 index and product yield.

Table 2. The maturity yield of income certificate.

Situation	CSI 300 index	Maturity yield
A	$ST > H1$	$Y \cdot T \cdot (1 + R0 + RC \cdot J)$
B	$K1 \le ST \le H1$	$Y \cdot T \cdot (1 + R0 + \frac{ST - K1}{S0} \cdot J)$
C	$S0 < ST < K1$	$Y \cdot T \cdot (1 + R0)$
D	$K2 < ST \le S0$	$Y \cdot T \cdot (1 + R0)$
E	$H2 \le ST \le K2$	$Y \cdot T \cdot (1 + R0 + \frac{K2 - ST}{S0} \cdot J)$
F	$ST < H2$	$Y \cdot T \cdot (1 + R0 + RC \cdot J)$

5.2. Pricing Analysis

One year before the issuance, the average rate of one-year treasury bonds is 2.06%, which can be used as a risk-free rate. Volatility uses historical volatility. Since the stock price approximately obeys the lognormal distribution, the annual volatility can be obtained by the standard deviation of the logarithmic return rate of the CSI 300 index.

The fixed income can be regarded as a bond with a face value of 50,000 yuan, an interest rate of 1%, and a term of 30 days. Considering the credit risk faced by the financing bond, it is not appropriate to discount the bond using the risk-free rate. To consider the credit risk of the issuer, the discount rate should be close to the coupon rate of short-term financing bonds of securities companies. As shown in **Table 3**, it is reasonable to set the discount rate to be 3%, and the fixed income $V1$ = 49917.86 yuan.

AS for the floating part, the B-S formula and martingale pricing method are analytical methods to price options, while the numerical methods include binary tree, Monte Carlo simulation, finite difference and so on. This paper uses martingale pricing method and Monte Carlo simulation to price the European bilateral shark fin option.

The martingale pricing method assumes that the stock price obeys the geometric Brownian motion, and the value of the bilateral option under the risk neutral measure in the six cases can be expressed as VA, VB, VC, VD, VE and VF. The floating value $V2 = VA + VB + VC + VD + VE + VF$. The pricing method refers to the formula given by (Li, 2019). Using MATLAB 2018a to calculate, the calculation results are as **Table 4**.

$$VA = T \times Y \times e^{-rT} \times RC \times J \times [1 - N(\frac{\ln(\frac{H1}{S0}) - (r - \frac{\sigma^2}{2}) \times T}{\sigma\sqrt{T}}) + e^{\frac{2\times(r-\frac{\sigma^2}{2})\times\ln(\frac{H1}{S0})}{\sigma^2}} \times N(\frac{\ln(\frac{S0}{H1}) - (r - \frac{\sigma^2}{2}) \times T}{\sigma\sqrt{T}})] \quad (1)$$

Table 3. The coupon rate of some short-term bond.

Issuer	Guosen Securities	Zhongtai Securities	Western Securities	AVIC Securities	TF Securities
Securities	22GUOSEN SECURITIES CP001	22ZHONGTAI SECURITIES CP001	22WESTERN SECURITIES CP010	22AVIC SECURITIES CP001	22TF SECURITIES CP001
Coupon rate	2.50%	2.60%	3.00%	3.00%	3.70%

Table 4. Floating income under martingale pricing method.

*V*2	*VA*	*VB*	*VC*	*VD*	*VE*	*VF*
40.74	8.55	11.59	0	0	13.42	7.18

$$\begin{aligned} VB = {} & -T\times Y\times e^{-rT}\times\frac{K1}{S0}\times J\times\Bigg[N\Bigg(\frac{\ln(\frac{H1}{S0})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg) \\ & -N\Bigg(\frac{\ln(\frac{S0}{H1})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)\times e^{\frac{2\times(r-\frac{\sigma^2}{2})\times\ln(\frac{H1}{S0})}{\sigma^2}}-N\Bigg(\frac{\ln(\frac{K1}{S0})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg) \\ & +N\Bigg(\frac{\ln(\frac{S0\times K1}{H1^2})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)\times(\frac{H1}{S0})^{\frac{2r}{\sigma^2}-1}\Bigg] \\ & +T\times Y\times J\times\Bigg[N\Bigg(\frac{\ln(\frac{H1}{S0})-(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg) \\ & -N\Bigg(\frac{\ln(\frac{S0}{H1})-(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)\times e^{\frac{2\times(r+\frac{\sigma^2}{2})\times\ln(\frac{H1}{S0})}{\sigma^2}}-N\Bigg(\frac{\ln(\frac{K1}{S0})-(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg) \\ & +N\Bigg(\frac{\ln(\frac{S0\times K1}{H1^2})-(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)\times(\frac{H1}{S0})^{\frac{2r}{\sigma^2}+1}\Bigg] \end{aligned} \tag{2}$$

$$\begin{aligned} VC = {} & T\times Y\times e^{-rT}\times 0\times\Bigg[N\Bigg(\frac{\ln(\frac{K1}{S0})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)-N\Bigg(\frac{-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg) \\ & -(\frac{H1}{S0})^{\frac{2r}{\sigma^2}-1}\times N\Bigg(\frac{\ln(\frac{S0\times K1}{H1^2})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)\Bigg] \end{aligned} \tag{3}$$

$$\begin{aligned} VD = {} & T\times Y\times e^{-rT}\times 0\times\Bigg[N\Bigg(\frac{-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)-N\Bigg(\frac{\ln(\frac{S0}{K2})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg) \\ & +(\frac{H2}{S0})^{\frac{2r}{\sigma^2}-1}\times N\Bigg(\frac{\ln(\frac{H2^2}{S0\times K2})+(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}}\Bigg)\Bigg] \end{aligned} \tag{4}$$

$$\begin{aligned}VE &= T\times Y\times e^{-rT}\times+\frac{K2}{S0}\times J\times[N(\frac{\ln(\frac{S0}{H2})-(-r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\\&-N(\frac{\ln(\frac{H2}{S0})-(-r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\times e^{\frac{-2\times(r-\frac{\sigma^2}{2})\times\ln(\frac{S0}{H2})}{\sigma^2}}-N(\frac{\ln(\frac{S0}{K2})-(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\\&+N(\frac{\ln(\frac{H2^2}{S0\times K2})+(r-\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\times(\frac{H2}{S0})^{\frac{2r}{\sigma^2}-1}]\\&-T\times Y\times J\times[N(\frac{\ln(\frac{S0}{H2})+(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\\&-N(\frac{\ln(\frac{H2}{S0})+(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\times e^{\frac{-2\times(r+\frac{\sigma^2}{2})\times\ln(\frac{S0}{H2})}{\sigma^2}}-N(\frac{\ln(\frac{S0}{K2})+(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\\&+N(\frac{\ln(\frac{H2^2}{S0\times K2})+(r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\times(\frac{H2}{S0})^{\frac{2r}{\sigma^2}+1}]\end{aligned} \quad (5)$$

$$\begin{aligned}VF &= T\times Y\times e^{-rT}\times RC\times J\times[1-N(\frac{\ln(\frac{S0}{H2})-(-r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})\\&+e^{\frac{-2\times(r-\frac{\sigma^2}{2})\times\ln(\frac{S0}{H2})}{\sigma^2}}\times N(\frac{\ln(\frac{H2}{S0})-(-r+\frac{\sigma^2}{2})\times T}{\sigma\sqrt{T}})]\end{aligned} \quad (6)$$

$$V2 = VA+VB+VC+VD+VE+VF \quad (7)$$

Under Monte Carlo simulation, suppose that stock prices follow a lognormal distribution, as shown in (8), where $\tau = T - t$.

$$S_T = S_t \times e^{(u-\frac{\sigma^2}{2})\tau+\sigma\varepsilon\sqrt{\tau}} \quad (8)$$

According to the initial price of 4093.79, we can generate random numbers through MATLAB to obtain the daily closing price of the CSI 300 index. μ is the expected return rate of stock, expressed as a risk-free interest rate of 2.06%, and the volatility is 18.71%. Repeated 20,000 times of the above process, we can simulate 20,000 price movements of the CSI 300 index. **Table 5** shows the number of simulations in each interval. The average value of the floating income in each simulation is the pricing of the floating income, which is 42.53 yuan.

The pricing results of the product are shown in **Table 6**. The difference between the floating income calculated by the two methods is not significant. In order to ensure the rationality of the calculation results, we finally choose the mean value of the results obtained by the two methods, 49959.49 yuan, as the value of the income certificate.

5.3. Income Analysis

According to the RMB deposit benchmark rate, the interest rates of three-month,

Table 5. Number of occurrences of different situations.

Situations	A	B	C	D	E	F
Times	2079	5106	2819	2930	5372	1694

Table 6. Income certificate pricing results.

Item	Martingale pricing		Monte Carlo simulation		Average	
Initial investment	1	50,000	1	50,000	1	50,000
Fixed income	0.9984	49917.8589	0.9984	49917.8589	0.9984	49917.8589
Floated income	0.0008	40.7423	0.0009	42.5279	0.0008	41.6351
Theoretical value	0.9992	49958.6012	0.9992	49960.3868	0.9992	49959.4940
Issue income	0.0008	41.3988	0.0008	39.6132	0.0008	40.5060
Premium rate	0.08%	0.08%	0.08%	0.08%	0.08%	0.08%

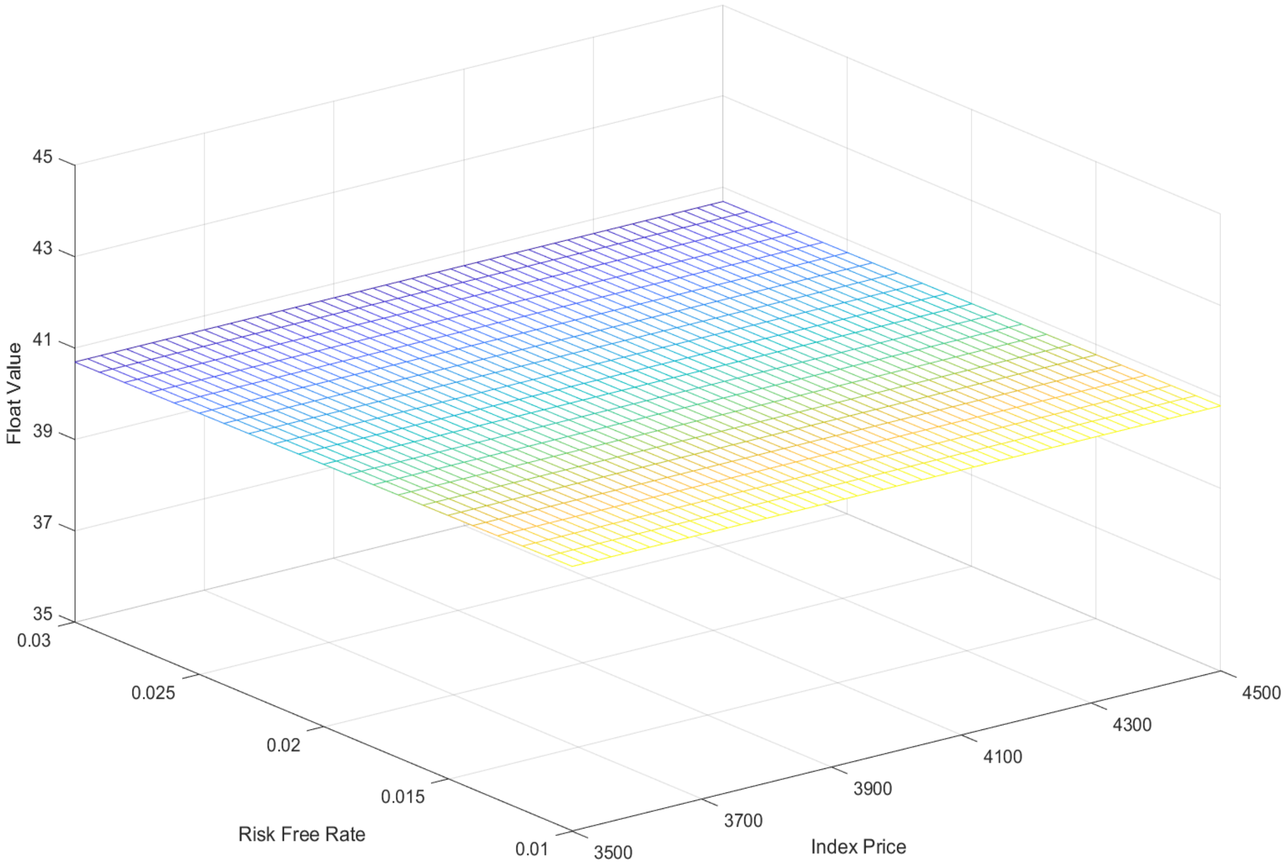
half-year, one-year, two-year and three-year term deposits of banks are 1.1%, 1.3%, 1.5%, 2.1% and 2.75% respectively. The highest yield of the income certificate is 6%, and the minimum yield is 1%, which is equivalent to the interest rate of 3-month term deposit. The yield to maturity obtained by Monte Carlo simulation is 1.99%, which is equivalent to the interest rate of 2-year fixed deposit, and the income is considerable. Additionally, the income certificate guarantees the minimum return, which is very attractive to risk-neutral and risk-averse investors.

Income certificate is a debt instrument. The securities companies borrow from investors with its own credit, and uses the funds raised income certificate to supplement their working capital. After the income certificate expires, the securities companies will repay the principal and interest. As can be seen from **Table 6**, the theoretical value of the income certificate is 49959.49 yuan, and the initial investment principal is 50,000 yuan, indicating that the product is issued at a premium rate of 0.08%, which means the issuer borrows from investors at an interest rate of −0.08%. Therefore, it is a good financing tool for the issuer as the financing cost is extremely low.

6. Risk Analysis

6.1. Sensitivity Analysis

Sensitivity analysis is not only the most important step in risk management, but also a common risk analysis method in actual investment. The sensitivity factor is a parameter that causes a large change in the dependent variable by a small change. If it cannot cause a large change, the parameter cannot be called a sensitivity factor. **Figure 3** shows the change of option value when the risk-free interest rate fluctuates from 1% to 3%, the term fluctuates from 0 to 60 days, the volatility fluctuates from 15% to 25%, and the initial price fluctuates from 3500 to 4500.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39
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0.02
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Risk Free Rate
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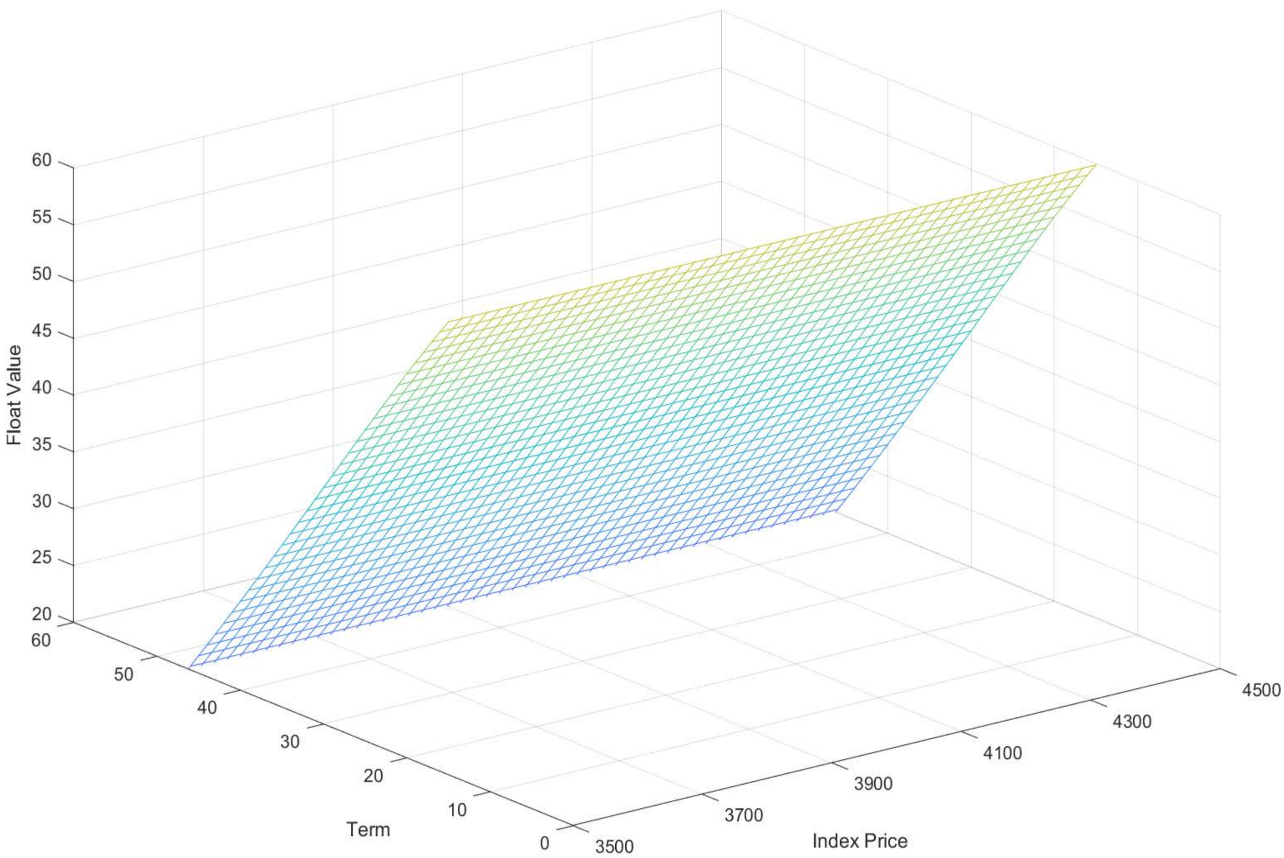
3700
3900
4100
4300
4500
Index Price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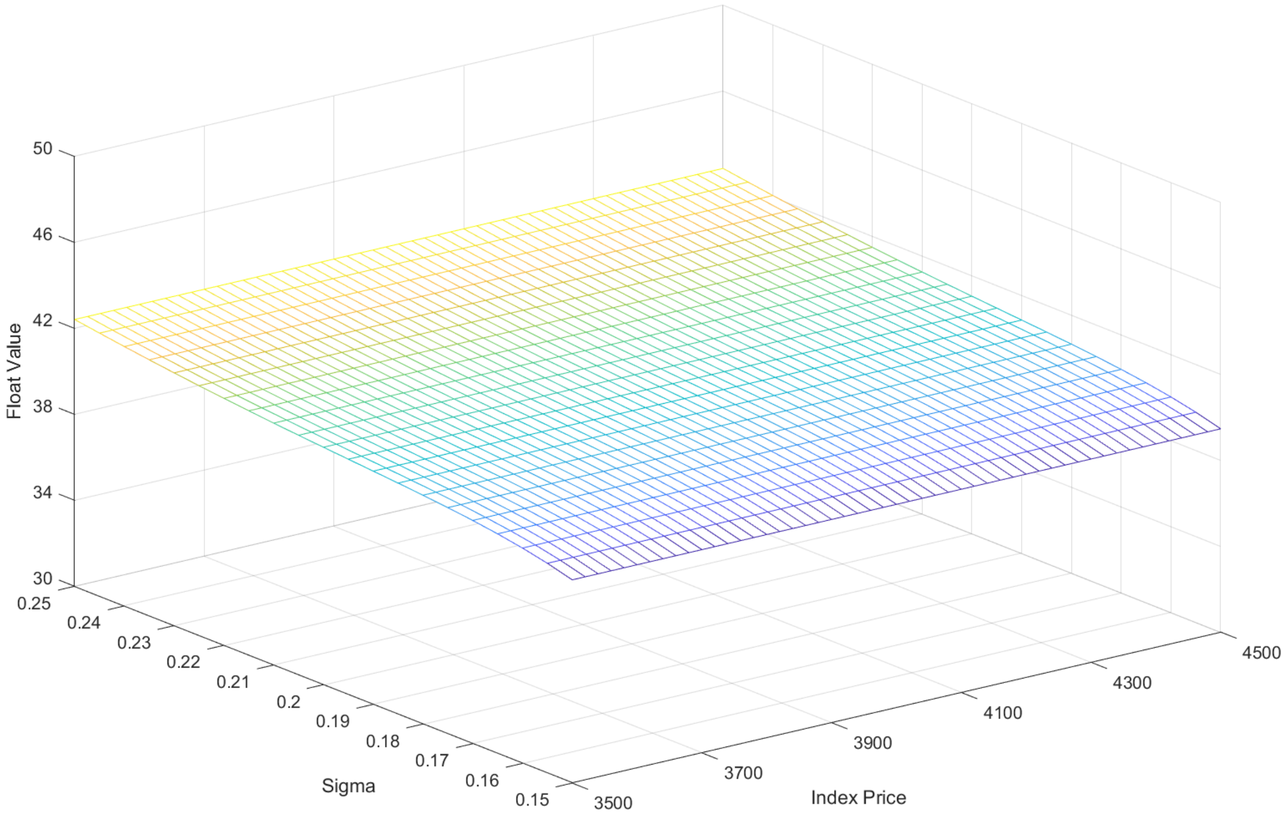

Figure 3. The influence of risk-free rate, investment term, volatility and initial price.

We find that the option value obtained after the change of the initial price is basically the same, which also shows that the change of initial price of the underlying has little effect on the option value. It is because the ratios of the touch price to the initial price, and the barrier price to the initial price are fixed, so the unit of the initial price change will not affect some expressions in the pricing formula.

Both the option value and the present value of fixed income decrease when the risk-free rate increase. Under the combined action of the two, the product value may exceed the issue price, so the risk-free rate selected cannot be too low, otherwise it will have a negative impact on the issuer. When the volatility changes slightly, the option value changes positively with it. However, when the volatility changes too much, the possibility of option knock-out increases and the option value decreases. Therefore, it is very important to set the volatility reasonably.

6.2. Hedging Strategy

According to the B-S formula, the option price mainly depends on the underlying asset market price, maturity time, volatility, risk-free rate and other factors. The influence of these factors on the option value can be expressed by formula. The risk of the option mainly refers to the influence of the above factors on the option price. These risk indicators are also called Greek values. Delta, Vega, Theta and Rho are the first-order partial derivatives of the option price to the underlying price, the underlying volatility, term and interest rate. Gamma is the second-order partial derivative of the option price to the underlying price.

$$\Delta V = Delta \times \Delta S + 0.5 \times Gamma \times \Delta S^2 + Vega \times \Delta\sigma + Theta \times \Delta t + Rho \times \Delta r \quad (9)$$

Option issuers can construct a portfolio based on several options and underlying assets, so that their value is not affected by changes in some of these factors, such a portfolio is called a risk-neutral portfolio. The common ones are Delta neutral portfolio, Delta-Gamma neutral portfolio and Delta-Gamma-Vega neutral portfolio. Due to the lack of non-linear tools in China, it is difficult for traders to effectively manage the risks of Gamma and Vega in practical operations. In this article, we focus on the hedging management of Delta.

Because the bilateral shark fin option has the dual attributes of call and put, it can hedge some risks within itself, but Delta changes greatly near the barrier price and execution price. The underlying asset is the CSI 300 index, which has no corresponding spot. Therefore, the CSI 300 index derivatives, such as futures and options, have almost become the only hedging product. $Delta'$ is the Delta value of the derivatives, for each option, the corresponding position of derivatives issuer should hold is shown as Formula (10), and the issuer should change the position in time when Delta changes.

$$position = \left| \frac{Delta}{Delta'} \right| \quad (10)$$

6.3. Other Risks

In addition, the income certificate also faces other risks, such as market risk, credit risk, liquidity risk and so on. The change of the price of the CSI 300 index is affected by the economic environment and the stock market. When the market environment changes greatly, the market interest rate or the price of the CSI 300 index may fluctuate sharply, which will bring potential losses to investors and issuers. In addition, if the market changes sharply, issuer may have difficulties in capital turnover and liquidity. When the issuer fails to pay, investors will face a certain risk of default.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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