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Representation Distortions in Multiwinner Elections

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Abstract

The manifestation of voters' preferences with respect to representation considerations in the results of multiple-winners primary elections is studied, using both a real example and a simulation of a simple model. For the real example, results from primaries in Meretz party in Israel are analyzed with respect to three representation considerations ("new faces", ethnic origin and gender). For the simulation, a model of distribution of support between candidates in two representation groups is used, combining uniform distribution with bias, or partly based on empirical results from real elections. It is shown that large deviations between voters' preferences and the election results may exist, and in fact predicted as the most common result in typical scenarios.

Keywords

Primaries, Multiwinner, Representation, Voters, Preferences, Elections, Party

1. Introduction

1.1. Background and Motivation for the Study

In August 23, 2022, Meretz political party held primary elections for the list of candidates for Israel's parliament, the Knesset. The internal elections were divided to two parts—one part to determine the identity of the party's leader at the head of the list, and a second part to determine the order of candidates in the list following the party's leader. Out of the twenty two candidates in the second part, six were former members of parliament.

It turned out[1] that the top places in the list were occupied solely by the former

[1]Official results for Meretz Party primaries, 2022, can be found in the following link: https://meretz.org.il/wp-content/uploads/2022/08/%D7%AA%D7%95%D7%A6%D7%90%D7%95%D7%AA-%D7%9E%D7%90%D7%95%D7%A9%D7%A8%D7%95%D7%AA-%D7%9C%D7%90%D7%97%D7%A8-%D7%94%D7%A7%D7%A4%D7%A6%D7%95%D7%AA.pdf

members of parliament. As one journalist summarized (Zaken, 2022): "In practice, there is no real change in Meretz's list, except for the return of Galon. All the realistic positions chosen are members of Knesset except Galon that was in the past, and there are no new faces".

From the above summary and indeed from the resulting list, one may conclude that the voters (more than 15,000 members of Meterz party), preferred to have a list with no new faces in realistic places in the list. However, looking at the detailed election results, this conclusion appears to be incorrect. The primaries voting mechanism was quite simple: each voter chose exactly 4 candidates, without order between them and with equal weight. Of the total number of votes, 45% were given to "new faces". So how come there were no new faces in the top of the list? Is such a result expected or should it be considered a rare case? This is what we'll try to study here.

1.2. Definition of Problem

The representation of "new faces" in the list is a special case of a *representation consideration* having two options (say *option A* and *option B*). So we can frame a theoretical question as follows:

- Assume the existence of primaries in a political party to rank candidates in a list.
- Assume the candidates can be grouped according to the representation consideration options A and B.
- Assume each voter votes for a number of candidates (with equal weight between them), where the number of votes per voter is smaller than the number of candidates in each of the groups A and B.
- Assume that the top N places in the list are "realistic" places (i.e., good enough chances to have a place in the next parliament following general elections), desirably reflecting voters' preferences.
- Define the ratio between total votes to candidates from groups A and B to the total number of votes as the *preference of the voters* for the relative numbers of groups' A and B candidates in N first places in the list.

Under the above definition and assumptions, we can ask the following research question: "how well are voters' preferences expected to be manifested in the election results?".

2. Multiwinner Elections

In single-winner voting system (such as for a president, city mayor or political party leader) each candidate may be attributed with a hypothetical grade representing the cumulative satisfaction of voters if this candidate wins. A good electoral system will tend to result in a winner according to the highest grade or at least close to it. The challenges of a multiwinner electoral system involve additional complexities (Faliszewski, Skowron, Slinko, & Talmon, 2017). For example, if voters would like to see equal representation of different groups (e.g., men and women), then the inclusion of a winner from one group will increase the

preference for electing the next member from the other group. The performance of a simple electoral system in use in primaries in political parties, in fulfilling representation considerations, is the focus of this semi-empirical study.

3. Model and Simulation

In order to study the research question in a general way and not limited to specific assumption on distribution functions, a simulation was built[2].

The simulation gets as inputs the number of candidates in group A and group B and the number of places considered "important" for the public at the top of the list. Since the distribution of support between candidates affects the results, a distribution is composed as a simple model of a combination of uniformly distributed random support and a fixed, uniform support. The ratio between the random element and the fixed element is determined as additional input parameters separately for group A and group B. Alternatively, the support distribution can be replaced by fixed values, independently for each of the groups, for example for a simulation based on real data.

In each simulation, 500 random cases for each preference ratio with resolution of 1% are averaged. Each simulation thus provides the average number of candidates from group B in the "top of the list", for each simulated ratio, compared to the "expected" number according to the preference of the voters alone.

Figure 1 shows a simulation of equal number of candidates in groups A and B, 12, for "top of list" also of 12, with distribution of support in both groups randomized uniformly. In this case, the average number of candidates from group B at the top 12 places of the ranked list is similar to the voters' preference. As we'll show later, this is a special case, and deviations from it lead to deviation of the simulated results from the assumed public preference.

4. Variation of Control Factors

Now we can run the simulation, each time studying the effect of different input variable(s) or specific test scenario compared to the "base scenario" of Figure 1.

4.1. Number of Places in the Top of the List

Figure 2 shows the simulation results for identical parameters except for changing the number of places at the top of the list. While at 50% preference the simulation results do predict that half of the places at the top of list will be from group B, the results for different ratios differ from the expectations. For example, when the number of places at top of the list is defined as 6 (red line) and the preference for group B is 1/3, the voters' expectation is that 2 places will be occupied by candidates from group B, but the simulation predicts an average of less than 0.3. The deviation is opposite for preference of 2/3 for group B, where instead of 4 group B candidates out of 6, the simulation predicts an average of

[2]The simulation was built in Matlab®. The code is provided here.
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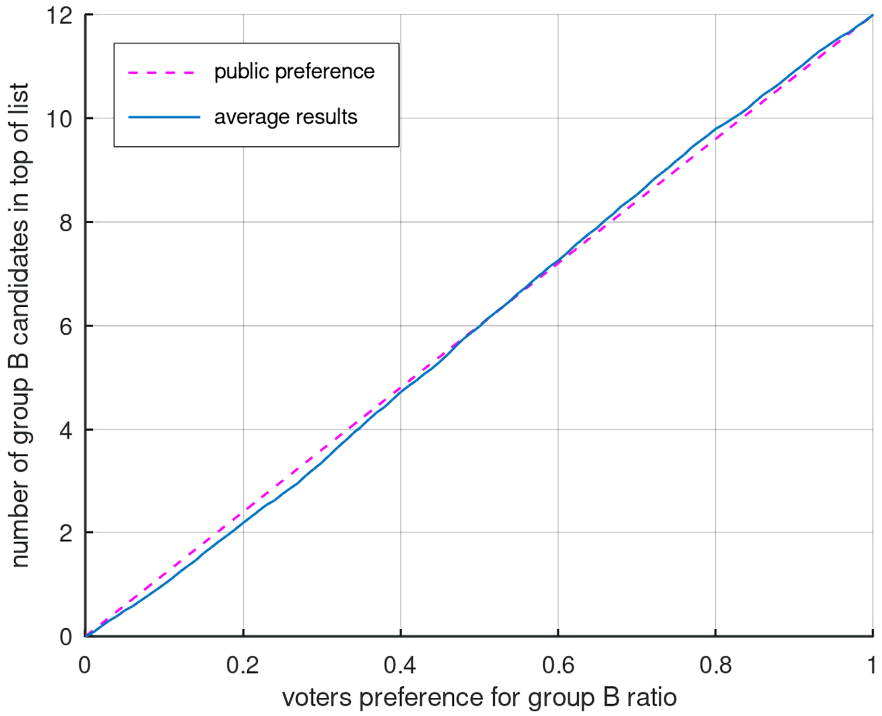

Figure 1. Simulation results, simple case with 12 candidates in each of the groups A and B and 12 places in the "top of the list", with uniform distribution of support in each group. The simulation for this scenario shows good agreement between the ratio of group B candidates in the "top of the list" and the ratio of votes to group B candidates.

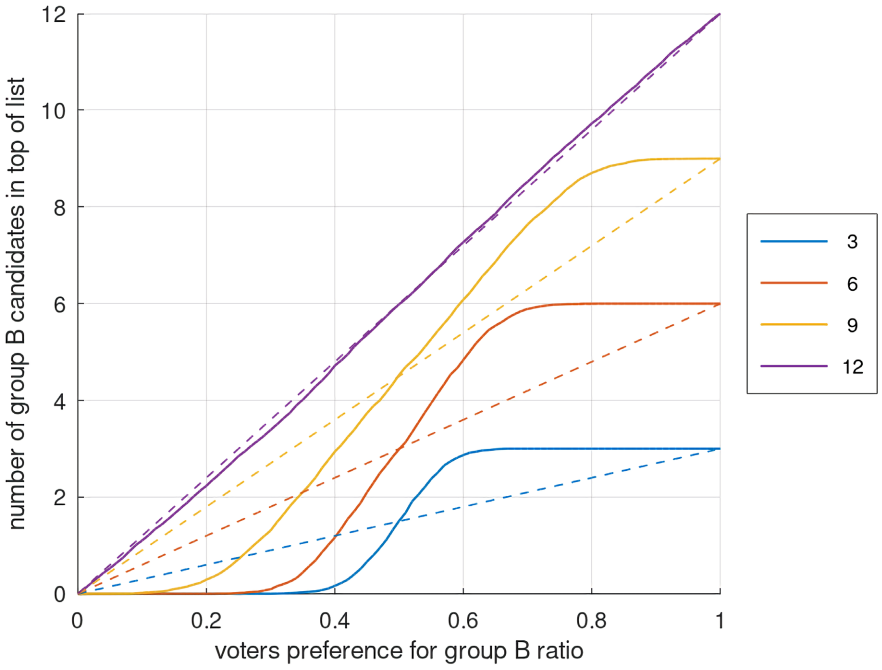

Figure 2. Changed number of places in top of the list. *Solid lines*: average simulation results for 3, 6, 9 and 12 places (see legend); *Dashed lines*: voters' preferences for 3, 6, 9 and 12 places (see legend). For scenarios with less places in the "top of the list" than the size of the candidate groups, simulations show significant deviations between the ratio of group B candidates in the "top of the list" and the ratio of votes to group B candidates.

about 5.7. The reason for the discrepancies is that when the relative representation preferences differ from the ratio between the sizes of groups, the votes are factorized accordingly. Then, the non-linear sort mechanism for ranking the candidates amplifies this effect (as in "vote splitting").

If the number of candidates in groups A and B is *smaller* than the number of places in the top of the list, then the latter will always be occupied by members of both groups, so the proportion cannot be zero or one irrespective of the voters' preferences. This creates another type of discrepancy between the preferences and the expected average election results, as shown in Figure 3. For example, for number of places of 18 in the top of the list (yellow line), say the voters prefer to have 1/3 of the list composed of group B. In such case the list should have been composed of 6 candidates from group B and 12 candidates from group A. However, the simulation results show that the average number of group B candidates in such case would be close to 8.

4.2. Randomness of Support Distribution within Each Group

In the former scenarios, the support for candidates in both groups was fully random with uniform distribution. How will a fixed bias affect the election results? By fixed bias, we mean that the support distribution is composed of a fixed part and a random part. Say the proportion of the random part in group A is α. Then, each of the candidates in group A will have an evenly distributed random

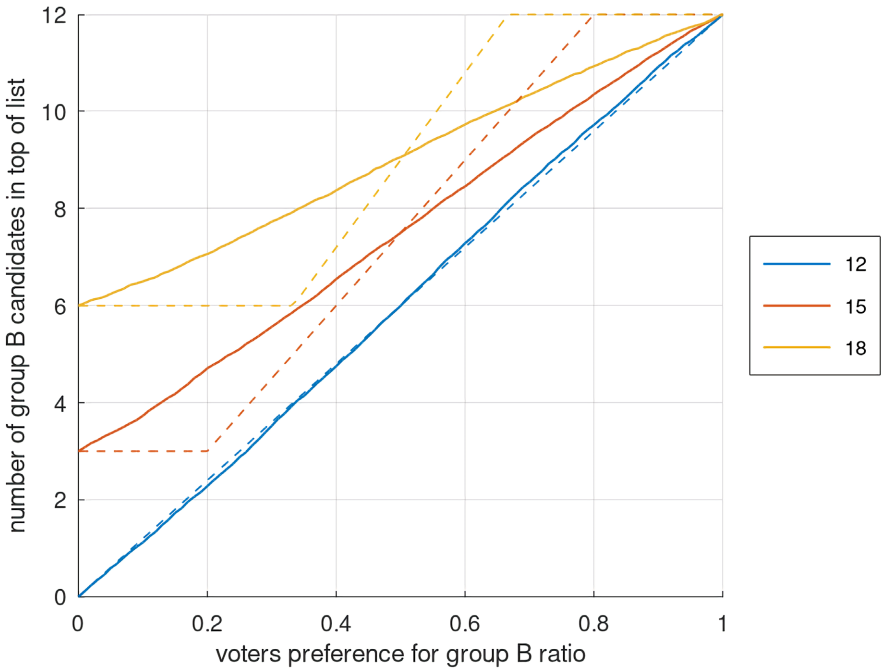

Figure 3. Changed number of places in top of the list. *Solid lines*: average simulation results for 12, 15 and 18 places (see legend); *Dashed lines*: voters' preferences for 12, 15 and 18 places (see legend). For scenarios with more places in the "top of the list" than the size of the candidate groups, simulations show moderate deviations between the ratio of group B candidates in the "top of the list" and the ratio of votes to group B candidates.

support on a scale between 1 − α and 1 calculated using the following formula: α·R + 1 − α, where R is a random number uniformly distributed between 0 and 1.

Figure 4 shows the simulation results when the proportion of randomness of support inside one of the groups is varied while the distribution of support inside the other group is random in the range [0, 1] with uniform distribution. It turns out that the less random the support inside the group is, the more its representation is augmented with respect to the expectation when it is favored (i.e. when voter would like to have more in the top of the list from this group than from the other group).

The deviation from voters' preference is augmented further if the amount of randomness of support is reduced in *both* of the groups, as shown in Figure 5. For example, for 50% randomness in both groups, for preference to have 1/3 of the placed in the top of the list occupied by members of group B, the simulation predicts a negligible average of less than 0.1 instead of the desired 4 candidates.

4.3. Sizes of Groups A and B

In the former scenarios, the support for candidates in both groups was fully random with uniform distribution. How will a fixed bias affect the election results? By fixed bias, we mean that the support distribution is composed of a fixed part and a random part. Say the proportion of the random part in group A is α. Then, each of the candidates in group A will have an evenly distributed random support on a scale between 1 − α and 1 calculated using the following formula: α·R + 1 − α, where R is a random number uniformly distributed between 0 and 1.

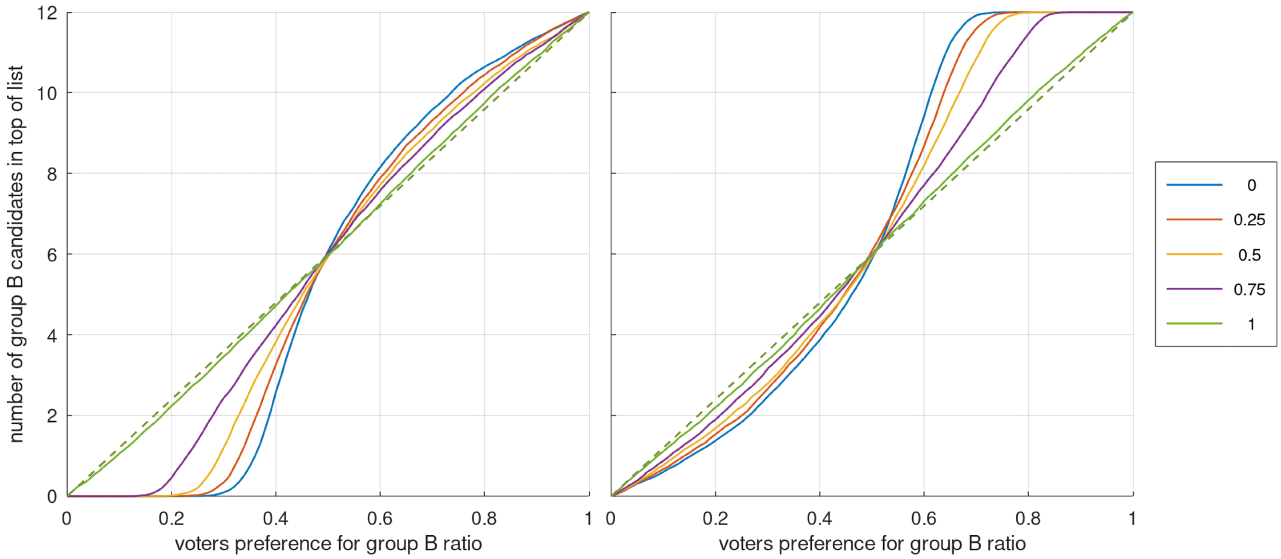

Figure 4. Changed amount of randomness of support in either one of the groups. *Left*: vary the amount of randomness of support inside group A (see legend; 1 = fully random); *Right*: vary the amount of randomness of support inside group B (see legend; 1 = fully random). For scenarios where the distribution of support in one of the groups deviates from uniformly random by adding a fixed bias, simulations predict that the representation of the less randomly distributed group in the "top of the list" will be larger than the ratio of votes to the candidates of this group when this group is the favored one.

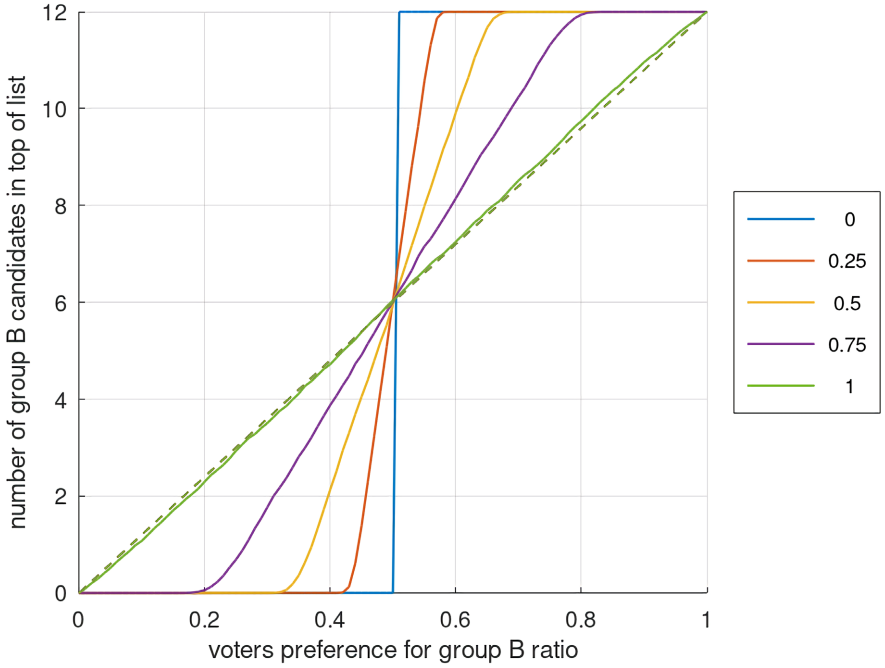

Figure 5. Change amount of randomness of support in both groups (see legend; 1 = fully random). For scenarios where the distribution of support in both of the groups deviates from uniformly random by adding a fixed bias, simulations predict that the representation of the favored group in the "top of the list" will be larger than the ratio of votes to the candidates of this group.

In typical elections, the number of candidates in each group differ. **Figure 6** presents the predictions of the simulation for changing divisions of a total fixed number of 24 candidates to group A and group B, from a minimum of 6 members to maximum of 18 members. The number of places at the top of the list in this simulation is selected to be 6 and both support distributions within the groups are fully uniformly random. As can be seen, all the expected election results differ from the voters' preferences, with "benefit" to the smaller group in most of the preference ratios (while a small preferred ratio for a group always results in under-representation).

For example, for 9 members in group A (red line), if the voters' preference is to have 40% percent of the places at the top of the list occupied by members of group B (i.e., 2.4 on the average), the simulation predicts an average of less than 0.5.

4.4. Combination of Factors

In the examples thus far, most of the factors were maintained as the original symmetric and fully random case, (where the elections expect to reflect the voters' preferences well as shown in **Figure 1**), and one or two factors were changed to demonstrate their effect relative to this "base case". Now we can combine a few factors to generate predictions for a few more elaborated cases of elections (and presumably more realistic or typical scenario).

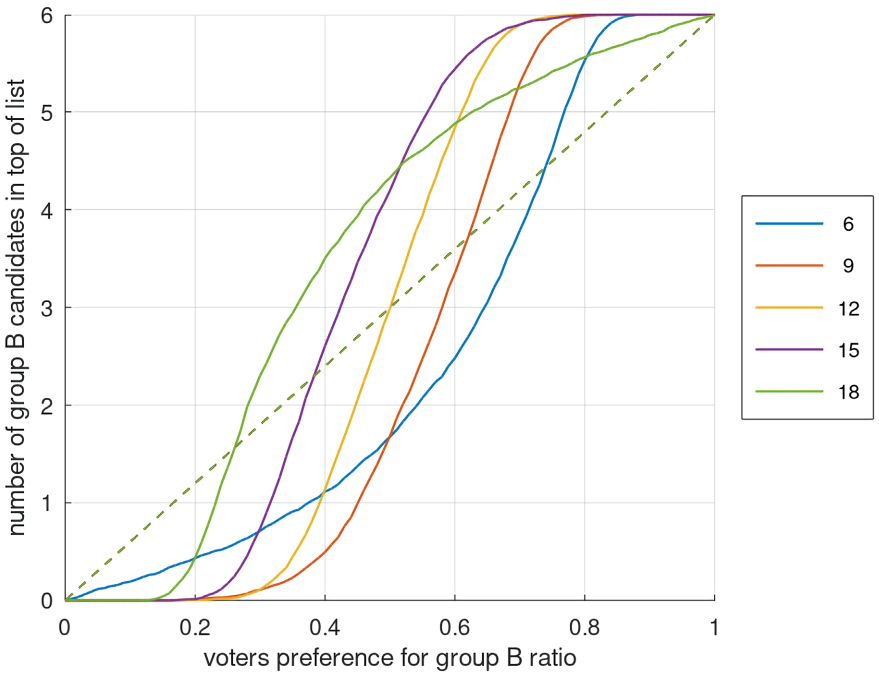

Figure 6. Change sizes of group A and B. *Group A size*: legend (i.e., 6, 9, 12, 15, 18); *Group B size*: 24-legend (i.e., 18, 15, 12, 9, 6). For scenarios with smaller number of places in the "top of the list" than the size of at least one of the groups, with uniformly random support distribution, simulations predict that the representations in the "top of the list" will always be distorted with respect to the expectation from the ratio of votes to the groups, and that the distortion will depend on the sizes of each of the groups.

Figure 7 shows a case of where 1/3 of the 24 candidates are in group A and the rest 2/3 of the candidates are in group B. The distribution of support in each of the groups is mostly random (0.7) but also has a substantial non-random bias (0.3). As can be seen, the average number of members of group B at the top of the list of 6 candidates is negligible below support level of about 50%. And this, if not understood correctly, may cause frustration and disappointment to the voters as they may be puzzled by the large discrepancy between their evaluation of the public preference and the results of the elections.

5. A More Detailed Look at the Results of the Primaries in Meretz Party

5.1. "New Faces" vs. Former Members of Knesset

The distribution of votes for former members of Knesset ("MK") and for "new faces" in the primaries in Meretz party on August 2022 (described in the introduction), are presented in **Figure 8**. Looking at the graph, we can evaluate that the distribution for MK's has a bias of 0.5 (10% minimum relative to maximum of 20%), and the distribution for new candidates is without bias (while not perfectly evenly distributed).

Fixing the distribution inside each group and changing only the relative number of votes between the groups, we can simulate the election results assuming that

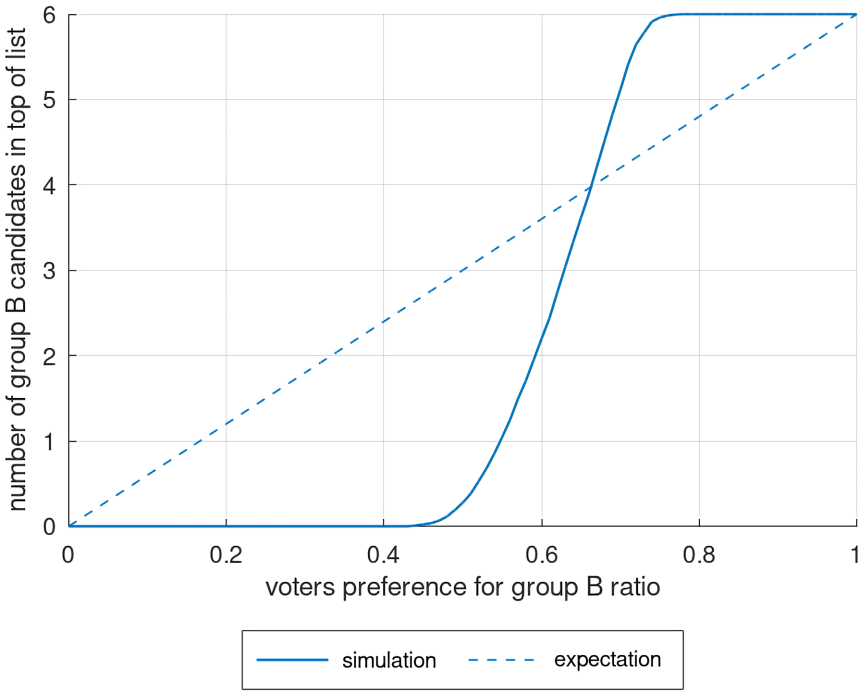

Figure 7. "Typical scenario" with combinations of deviations from ideally symmetric case.

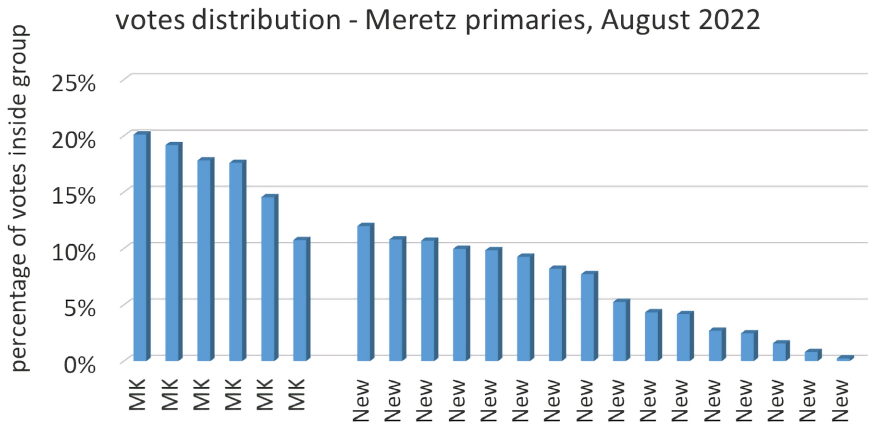

Figure 8. distribution of votes in Meretz primaries, "New Faces" and former Members of Knesset (MK), August 2022.

only the voters' preferences related to the proportion of "new faces" change from the actual proportion of 45% to other ratios. The results are presented in **Figure 9**. It so happens, that with small additional increase of support for new faces, at least one new candidate would have entered in the top 6 places. Nevertheless, "new faces" would still suffer significant under-representation even if 60% of the votes would have been allocated to members of this group. On the other hand, if the ratio of votes to new faces would have been 69% or more, all the top list would have been occupied by new faces.

Now we can compare the speculated dependence of the results shown in **Figure 9** with predictions of the simulation with approximations for distributions of

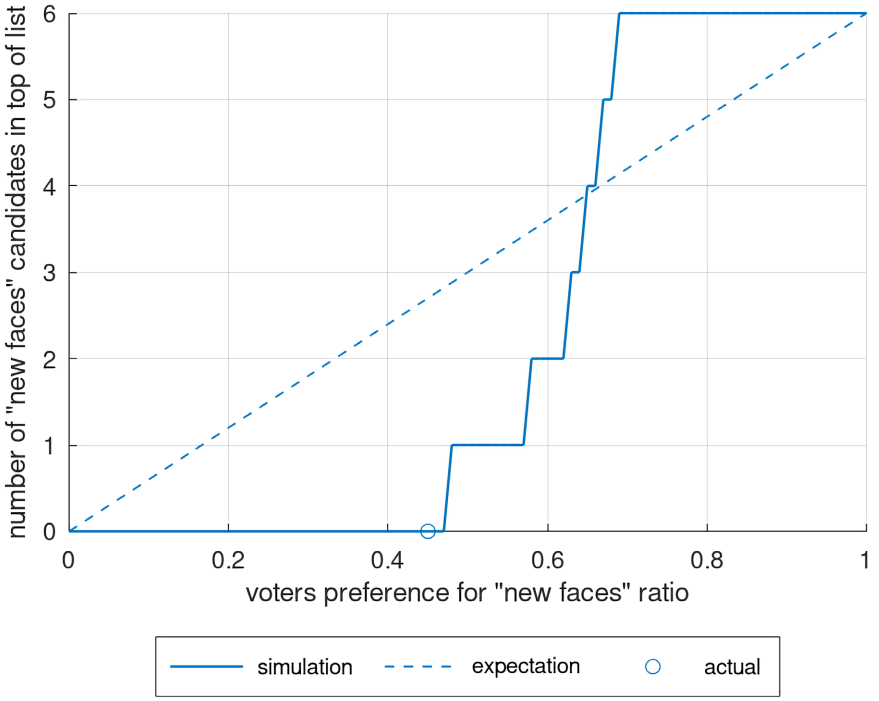

Figure 9. Simulated results for same internal distribution of support. Meterz primaries, August 2022, "new faces" and former Members of Knesset.

internal support in the groups—for example, using a fully random distribution for group B ("new faces") and random with bias of 0.5 for group A (former members of Knesset). The results of simulation with these inputs are presented in **Figure 10**, and show negligible average "new face" candidates when the votes ratio for "new faces" is up to 45% (as when the simulation is based on the actual election distributions), as well as a list fully occupied with "new faces" candidates when the ratio is 75% or more (compared to ~70% when based on the actual distributions in the elections).

5.2. Candidates of Arabic Origin

The representation of the Arabic minority in the list of candidates for the general elections is traditionally considered to be of high importance for Meterz voters. The 22 candidates can be grouped according to their ethnic origin: five of the candidates from Arabic ethnic origin (a somewhat higher ratio than in the general population) and the rest are of Jewish ethnic origin. The internal distributions of support within the groups, according to the number of votes for each candidate, are presented in **Figure 11**.

As can be seen, amongst the Arab candidates there was one dominant candidate as well as one candidate with markedly smaller support than others. Such distribution deviates from most of the distributions that are generated by the simple model of fixed base support plus uniform random distribution. Therefore, a simulation was carried out with the support for Arab candidates set according to the real election results, and the support for Jewish candidates modeled

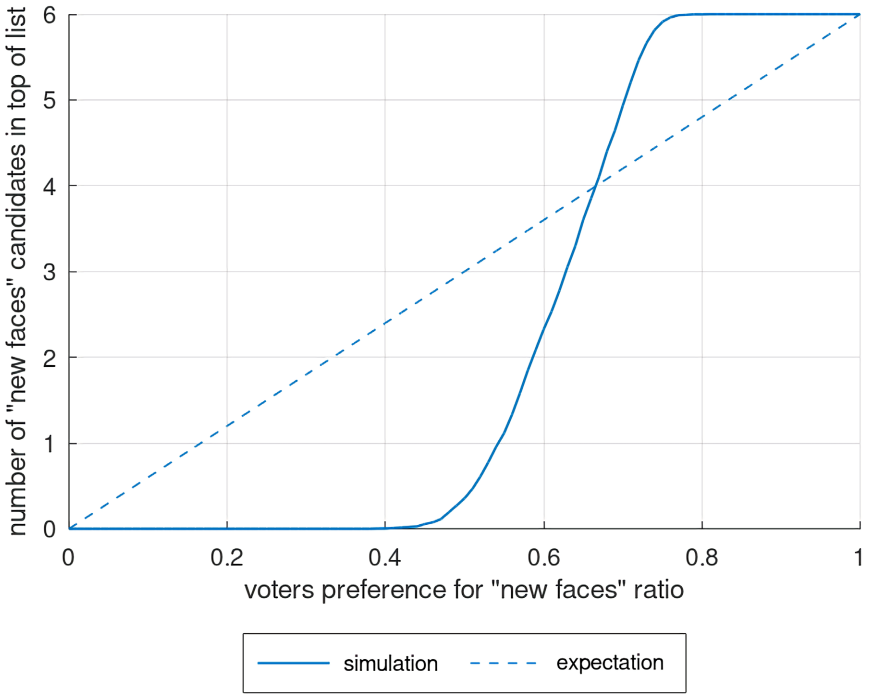

Figure 10. Simulation for Meretz primaries with respect to "new faces" at the top 6 places in the list.

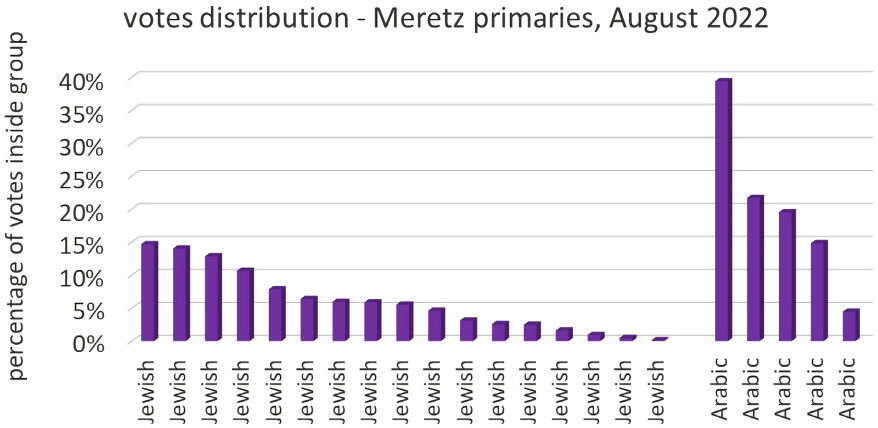

Figure 11. Distribution of votes in Meretz primaries, Arabic and Jewish ethnic origin, August 2022.

by fully random uniform distribution. The size of "top of the list" is chosen as 5 places, since there are only 5 Arab candidates. The results are presented in **Figure 12**. The actual ratio of votes for Arabic origin candidates was about 25%, and indeed one candidate entered the top of the list of 5 places (in reality won the third place). Interestingly, the results are predicated to be close to the expectations (shown in broken line) around this ratio, which is not so surprising as the proportion of Arab candidates is close to the preference (5/22 = ~23%), so the effect of vote splitting is small. The simulation predicts that if the voters' preference was smaller than 15% for Arabs then the representation was zero (although the expectation is an average of 0.75), while for voters' preference of ~35%

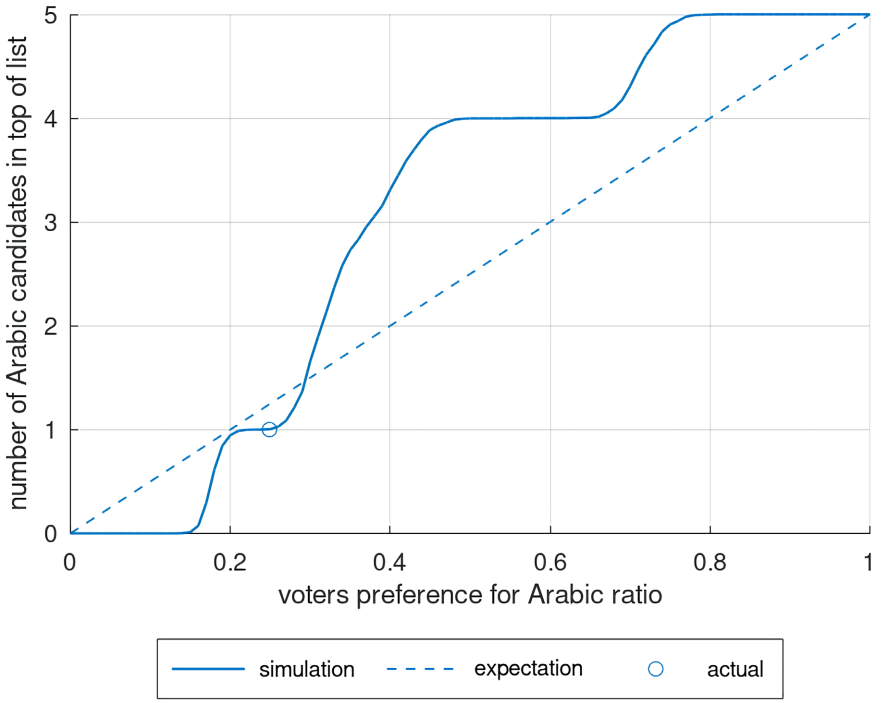

Figure 12. Simulation for Meretz primaries with respect to Arabic ethnic origin candidates at the top 5 places in the list.

or more, the representation of Arabs would have much exceeded the expected value.

5.3. Gender Considerations

A common representation consideration, also considered highly important in Meretz party, is gender representation. Meretz's constitution forces the representation of both genders to be at least 40% of any 5 consecutive places in the list[3]. This complicates the comparison between voters' preferences and election results, as voters may take the gender balance limitations into account in their allocation of votes[4]. However, for the sake of completeness of the study of Meretz primaries results, we will still assume that such considerations are either not highly prevalent, or are balanced between the genders.

The distributions of support inside the group of 13 men and group of 9 women candidates are shown in **Figure 13**. Here, we note that two candidates have significant dominance over other women. The distribution of support for men is divided mostly to two main levels. Therefore, we chose to base the predictions of the simulation fully on the actual support distributions within the groups, and only modify the ratio of total votes between the two groups.

About 32% of the votes in the elections were given to women, which constitute

[3]If the results do not fulfill the required proportions, candidates would be pushed up and down the list to meet them.

[4]For example, if a voter considers it highly important to have at least 40% women in the list but have no particular preference amongst the women candidates, the voter may decide to vote only for men, knowing that this choice will not jeopardize women representation and cause it to fall below 40%.

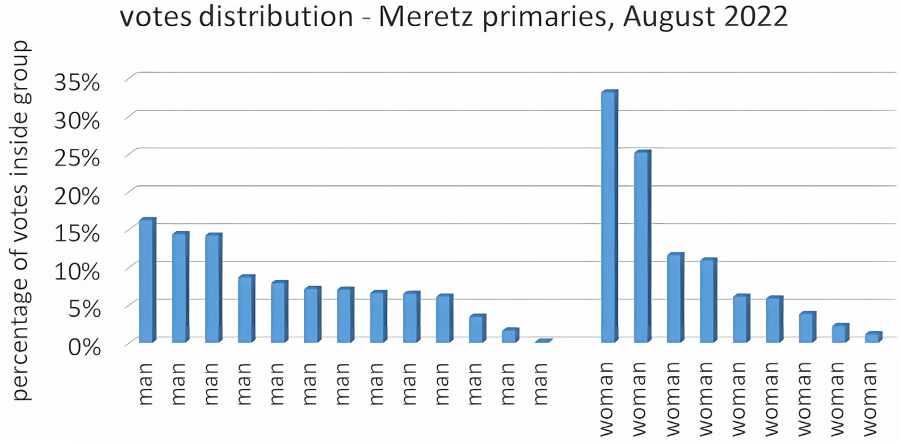

Figure 13. Distribution of votes in Meretz primaries, women and men, August 2022.

about 40% of the candidates. This, together with the specific distributions of support (in particular for women), results in relatively good correlation between the simulated predicted results and the expectations over most of the simulated ratios of votes for women, and an excellent matching for the actual case of 32% votes for women, as shown in **Figure 14**.

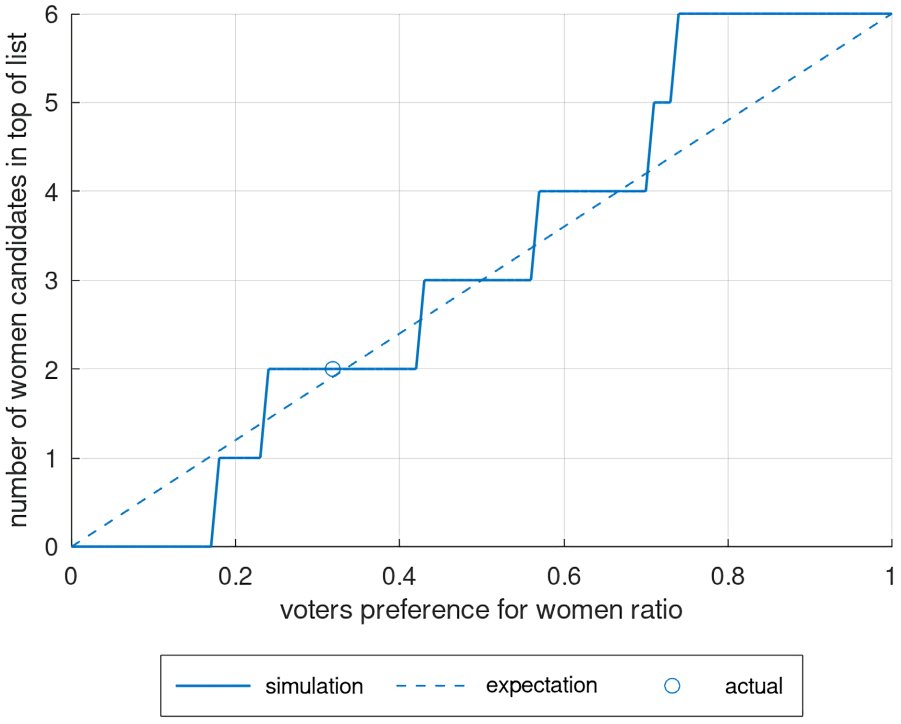

Figure 14. Prediction for Meretz primaries with respect to women representation at the top 6 places in the list (actual distributions).

These observations still hold even when the distribution of votes in the men group is replaced by uniform random distribution with 0.3 fixed bias (**Figure 15**), but does not hold if both distributions are replaced by the simple uniform random plus bias model (**Figure 16**).

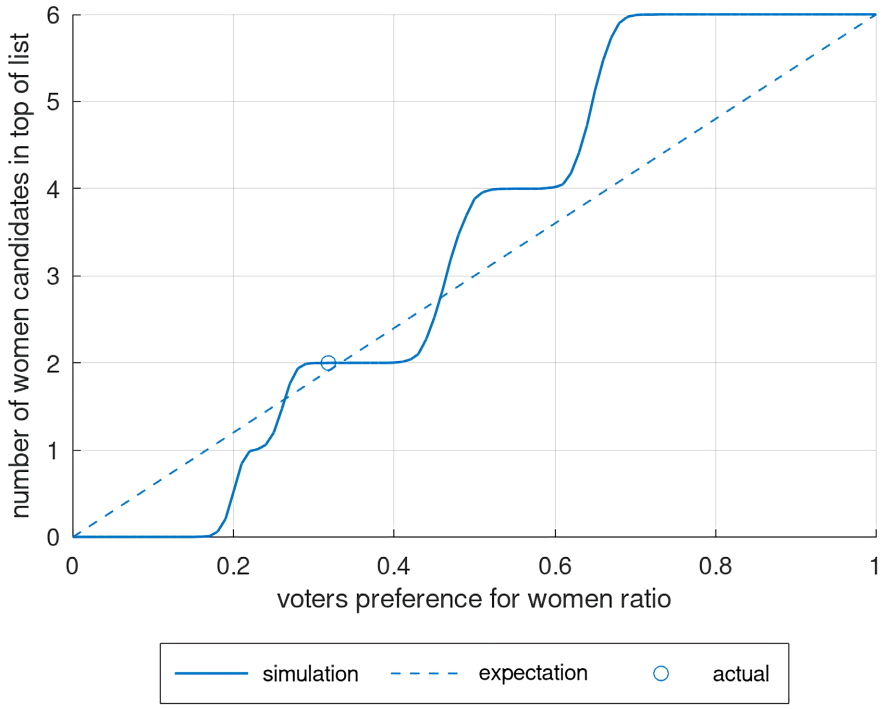

Figure 15. Prediction for Meretz primaries with respect to women representation at the top 6 places in the list (semi-actual distributions).

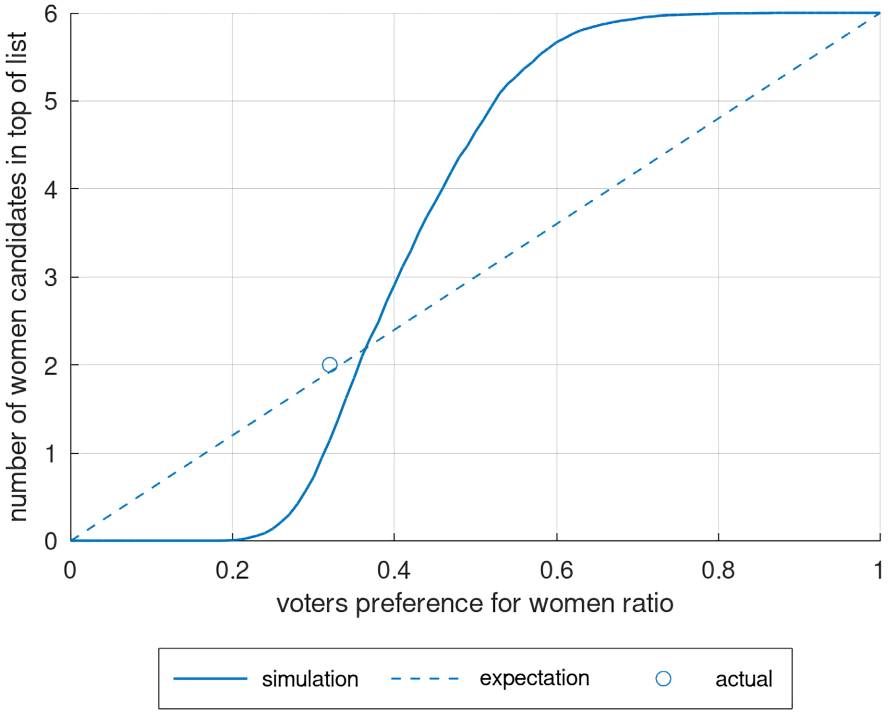

Figure 16. Simulation for Meretz primaries with respect to women representation at the top 6 places in the list (random model).

6. Summary

We have demonstrated that in multi-winner primary elections, the results of the

elections may deviate significantly from voters' preferences for representations of certain groups in realistic places in the list of candidates.

A few real cases of binary division of the candidates to two groups according to selected representation considerations were analyzed. From these real cases and by comparison with a simple statistical model for the distribution of support within the representation groups, it appears that while there is a monotonic non-decreasing dependence between the overall voters' preference of a representation consideration to the number of candidates from this group in realistic places in the list of candidates, the exact number of candidates may differ significantly from the expectations according to the ratio of votes.

Thus, an intuitive statement that using common primaries mechanism (like the one used in the examples from Meretz party) the public preferences are manifested reliably in the list of candidates may be highly inaccurate in some scenarios. These include real cases, like we have seen in the discrepancy between the zero number of "new faces" in realistic places in the list of Meretz in the primary elections of August 2022 and the high proportion of votes for "new faces" given by the public, 45% of all votes.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper. The author is a delegate of Meretz Party Convention.

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