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The "PAD + BOPPPS" Hybrid Mode of Large Class Teaching

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Abstract

China's higher education has entered a new era of comprehensive quality improvement and innovation. The mode of university mathematics public courses mainly based on teacher's teaching is facing more and more severe challenges in the innovation era. This paper proposes a new model of "PAD + BOPPPS", which is suitable for college mathematics teaching in China and aims at solving the common key teaching problems in domestic universities at present. Taking the section of "Total Probability Formula" in "Probability Theory and Mathematical Statistics" as an example, this paper discusses the ideas and methods of innovative teaching design. In this case, through the data collected by classroom interaction tools, it can be seen that the method proposed in this paper has achieved good results in improving students' interest in learning and classroom interaction. The method proposed in this paper can effectively improve the classroom teaching of college mathematics and has high promotion value.

Keywords

Teaching Design, PAD Class, BOPPPS Mode, Probability Theory and Mathematical Statistics

1. Introduction

On April 1, 2021, Director Wu Yan of the Higher Education Department of the Ministry of Education pointed out that the National Conference of Higher Education Directors: China's higher education has entered a new era of comprehensive quality improvement and innovation. In the era of innovation, the teacher-based teaching mode of university mathematics public courses is facing more

and more severe challenges. This paper will explore how to break the "indoctrination" classroom teaching and reshape the new class-teaching mode of university mathematics public courses. In this way, the efficiency of students' learning in class is maximized, and talents with mathematical thinking and ability are cultivated in response to the innovation requirements of the times.

This paper combines PAD teaching mode and BOPPPS teaching mode, organically integrates, adjusts and improves the two methods, puts forward a new "PAD + BOPPPS" mode, and applies it to the classroom teaching of college mathematics, effectively improves the "teaching-oriented" college mathematics classroom atmosphere, and can stimulate students' interest in learning, mutual communication and active inquiry. In view of the problems existing in the traditional university mathematics large class teaching system, a solution model is proposed. Taking the section of "Total probability formula" in Probability theory and Mathematical Statistics as an example, this paper expounds on how to use "PAD + BOPPPS" mode to design classroom teaching.

2. Background and Current Situation

The teaching status of mathematics public courses in university

1) Now the teachers "full hall" teaching method is dominating the majority of college mathematics public classes. Students passively accept the knowledge (Gu & Du, 2021). This kind of class makes it difficult for students to participate in studying and to experience the fun of learning. The lack of interaction and thinking collisions between teachers and students and among the students is not conducive to the cultivation of students' distinguishing and cooperative spirit.

2) At present, the teaching content of mathematics public courses learns according to the content of the textbooks. The application of knowledge or the background of the examples is both outdated and uninspired. The knowledge that students receive in class is almost unrelated to their actual life experience and information obtained from all sides. Such classes actually stifle students' interest in learning.

3) In traditional college mathematics public classes, most teachers teach in accordance with the routine of "definition-theorem-proof-inference-example", and the teaching mode is single and rigid. This kind of teaching routine is difficult for students to concentrate for a long time (Jia, 2019). The lack of well-designed classroom teaching is not conducive to mobilizing students' enthusiasm for learning.

4) Many universities have made great achievements in the field of teaching mode exploration of flipped classroom, and have accumulated a lot of experience (Yuan, 2016). However, flipped classroom is not fully applicable to the teaching of public mathematics course. On the one hand, due to the particularity of mathematics courses, the theory is obscure and logical, but the students' narration usually cannot touch the depth of teachers' intention. On the other hand, public courses often teach one or two hundred students at the same time, so the flipped format cannot cater for all students in the class.

5) This is a highly information age (Gong, 2020). However, textbooks, PPT and blackboard writing are still the main teaching tools in the public courses of mathematics in universities, lacking the use of information means, which is not in line with the requirements of the modernization of education in the new era.

In short, the present classroom teaching of college mathematics takes teacher's lectures as the main method of knowledge transmission, textbooks, PPT, blackboard writing as teaching tools, and definitions, theorems, proofs, inferences, examples, exercises as paradigms. This is contrary to the "student-centered" educational reform concept proposed by us in the era of teaching innovation (Hou, 2020). It not only hinders the improvement of students' interest in learning, but also is not conducive to the cultivation of students' independent thinking habit, critical thinking custom, innovative spirit and collaborative ability.

3. PAD and BOPPPS Model Elaboration

The PAD (Presentation, Assimilation, and Discussion) teaching model has achieved good results in China. BOPPPS (Bridge-in, Objective/Outcome, Pre-assessment, Participatory Learning, Post-assessment, and Summary) teaching model has been a great success abroad. Both of the above modes try to strengthen the interaction between students and teachers, increase students' active participation in classroom, to improve the teaching effect.

1) The so-called "PAD" teaching mode is a teaching concept first put forward by Zhang Xuexin, professor of psychology in Fudan University. The key to putting forward the concept of "PAD class" is to stimulate students' subjective consciousness of learning in class (Bao, 2019). In short, half of the classes will give to the teacher and half to the students. This is different from the traditional teaching and indoctrination class. In the course of classroom teaching and studying activities, students take the initiative to participate in learning, which can reflect students' subjective learning consciousness. In the process of participation, the way of impartation of knowledge has changed. Students no longer receive information passively as in the past, but gain a certain initiative to explore, talk and debate. They are no longer passive "listening" and "remembering", but exploring and expressing opinions independently.

The status of teachers and students has also changed. In traditional class teacher is the protagonist, and can decide the completion of the classroom-teaching task alone. In PAD class, students occupy the main position in learning. Teachers changed from "actors" to "directors", and students changed from "audiences" to "actors". "PAD class" helps to ignite students' enthusiasm for learning in class, fully mobilize their brains to think in class, and make full use of the classroom environment in the process of mutual discussion. The effective communication and thinking collision achieve real learning and build a community of teachers and students. "PAD class" can effectively increase students' interaction and promote team cooperation in class. At the same time, the interaction between teachers and students is also increased. Teachers can join the discussion

between student groups at any time. The interaction between teachers and students has been greatly improved than before (Jin, 2018). "PAD Class" does not abandon traditional teaching, but on the premise of ensuring that knowledge can be transferred systematically, accurately and effectively, combined with the characteristics of discussion-based teaching, highlighting the status of students in classroom teaching.

2) The BOPPPS teaching model originated in Canada. It breaks the classroom into: Bridge-in, Objective/Outcome, Pre-assessment, Participatory Learning, post-assessment and Summary (Niu & Lou, 2017).

Bridge-in: Teachers can use pictures, videos, stories, questions, hot topics and other methods to start a lesson. The introduction must be lively and interesting to attract students' attention and stimulate their curiosity and interest in learning.

Objective/Outcome: Use blackboard writing, PPT and other methods to present students with specific learning objectives. Learning objectives should include three aspects: cognition, emotion and skills. Learning objectives should be set according to the analysis of learning conditions, which is clear and in line with the actual situation of students. At the same time, learning goals should be attainable and verifiable. This part is intended to allow students to clarify what goals they want to achieve in the lesson, so they can learn in a targeted manner.

Pre-assessment: Before explaining the new knowledge of this lesson, teachers can use questions and answers, quizzes, anonymous voting, group discussions and other methods to understand students' interest in this topic and prior knowledge. Then teachers can adjust the depth and progress of the subsequent teaching content to make the learning objectives and classroom topics more focused.

After the main knowledge including concepts, key points, and difficulties is clarified, students can participate in learning activities in a plentiful and interesting way such as personal reports, group discussions, case studies, role-playing, experiments, storytelling, and solitaire. Through participatory learning activities, students can improve their understanding of knowledge, deepen their impression of knowledge mastery and improve their interest in learning. Practice shows that the effect of student active participatory learning is far better than passive learning. Furthermore, through the design of this part, students can exercise their expression, communication and cooperation skills in activities.

Post-assessment: The purpose of post-assessment is to verify and test learning results. After the participatory learning, check the students' mastery in this lesson by answering questions, quizzes, exercises, operating demonstrations and reporting. Then teachers can check if the learning objectives achieve and how the teaching effects are (Du, Fu, & Su, 2020), which calls post-testing.

Summary: Finally, teachers should make a summary of what we have learned in this lesson to strengthen students' impression. The teacher can highlight the knowledge points, emphasis, difficulty and learning objectives of this lesson. Let

the students summarize first, the rest people supplement, then the teacher emphasize the key and difficult points. Then the effects are better. At last, teachers can extend and consolidate the effects of this lesson by assigning homework.

3) The application of PAD model is more suitable for small class, but the university mathematics public courses often have hundreds of students in a classroom at the same time. How to apply the advanced teaching mode effectively to the large class of mathematics public courses is the main problem we focus on. BOPPPS mode has strict teaching regulations, making it lack of flexibility. This is actually disadvantageous to the promotion and application of the mode.

Therefore, in practical application, it needs to be improved according to the characteristics of the course. This paper proposes a new "PAD + BOPPPS" teaching mode, which organically combines them to complement each other's disadvantages, and thus proposes a new teaching mode. At the same time, we should change the current situation of "teaching-oriented" in university mathematics public courses, give full play to students' subjective initiative, and truly realize the "student-centered" and "learning-oriented" innovative classroom.

Drawing on the idea of PAD teaching method, we will also design the teaching of mathematics public class, reserving no more than half of teachers' explanation, and leaving the initiative of learning to students. Combined with the process of BOPPPS, the classroom teaching link is redesigned and adjusted, the "Bridge-in" link is retained, the "Participatory Learning" is enriched, and the "Summary" is changed to the independent induction of students. Thus fully mobilize the enthusiasm of students to learn. The teaching process will be described in detail in the next section.

4. Case Study of PAD + BOPPPS Model in Class

This article takes the course "Total Probability Formula" in the public course "Probability Theory and Mathematical Statistics" in science and engineering mathematics as an example to discuss our innovative teaching design practice using the new "PAD + BOPPPS" model.

Every lesson has formulated three levels of teaching goals in this course, namely Knowledge objective, Ability objective, and Value objective. In the section of "Total Probability Formula", the three levels of goals are as follows:

Knowledge objective: Deeply understand and master the total probability formula and Bayes formula. Keep in mind the basic steps of applying the total probability formula and Bayesian formula, and be able to calculate probability of random events according to the formulas.

Ability objective: Correctly calculate the probability according to the model with the total probability formula and Bayes formula, and solve practical problems with the above two formulas. Accumulate experience in mathematics activities during the learning process, train students to analyze and solve problems systematically. Cultivate their habit and spirit of questioning and independent thinking, and help them gradually establish right random concepts. The students

can consciously use the knowledge to observe their life, and solve practical problems through mathematical models.

Value objective: Let students realize "knowledge changes destiny" with fable stories, and teach students to be honest with cheating rate survey. The examples around us illustrate that "probability knowledge can reveal the truth in life", and use a posteriori probability to guide students to think that they should not only rely on experience in study and work, but further dig out more information to help them make judgments.

Then we will show our teaching activities and practices designed according to the above teaching objectives.

1) **Import**: In this lesson, we do not choose the import case in traditional textbooks, but use a self-made Flash animation to tell the students a short story:

There was a young boy offended a local powerful man, so the man was very angry. He took out two cans, one with 50 *black balls, and the other with* 50 *white balls. He ordered the young boy pick a can with eyes covered, and then took out a ball. If the boy got a white ball, he would send the boy home. If the boy got a black one, he would put him to death. The young man was frightened, while the powerful man was very proud, he said:* "*Before you are blindfolded, you can mix the* 100 *balls and put them into two cans. The chance of your survival is only* 50%. *The rest is up to god!*" *Hearing this, the young man smiled and said,* "*That would greatly improve my chances of survival.*" *The young man left only one white ball in the first jar and poured the remaining* 49 *balls into the black one.* "*Then my chance of survival is greatly improved*", *he said.*

How dare this young man say that? Did he survive in the end? (Qi, 2015)

After the story has been told, the students discuss in groups and analyze whether the young man's chances of survival is changed. Students can present the results of group discussions through the App "rain class", and then group representatives share their group ideas.

In the import part of this lesson, teacher will tell stories to the students with Flash as the carrier, and then deliver the class to the students. Students discuss in groups and then share their results. In this case, we were surprised and pleased to see that the graph the student submitted through the "rain-class" was a "probability tree" (Dawn, 2018). This shows that students are able to make inferences independently and are aware of the meaning of the Total Probability Formula, but they have not reached the theoretical level.
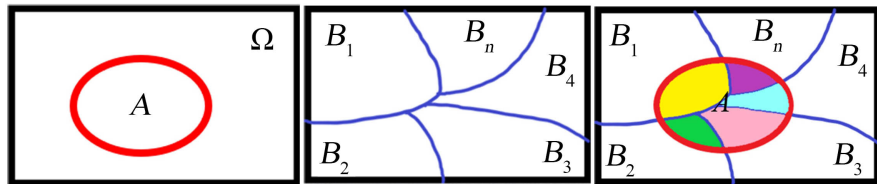
2) **New knowledge exploration**: This part of this class is to learn the full probability formula.

Total Probability Theorem: *Consider a complete set of disjoint events* $\{B_1, B_2, \cdots B_n\}$, *which means that:* $P(B_i) > 0, i = 1, 2, \cdots, n, B_i \bigcap B_j = \varnothing$ *for* $1 \le i \ne j \le n$; *and* $\bigcup_{k=1}^{n} B_k = \Omega$. *For any event* $A \subset \Omega$, *we have that*

$$P(A) = \sum_{i=1}^{n} P(B_i) P(A|B_i).$$

Proof: Step 1: Decompose A

$$A = A \bigcap \Omega = A \bigcap \left(B_1 \bigcup B_2 \bigcup \cdots \bigcup B_n \right) = AB_1 + AB_2 + \cdots + AB_n.$$

Step 2: Addition theorem and multiplication theorem

$$P(A) = P(AB_1) + P(AB_2) + \cdots + P(AB_n) = \sum_{i=1}^{n} P(AB_i) = \sum_{i=1}^{n} P(B_i) P(A|B_i)$$

Teachers' comments and an in-depth interpretation of the total probability formula:

a) $\left\{ B_1, B_2, \cdots B_n \right\}$ can be viewed as the causes or conditions that causes A to occur. The probability of A is just the sum of the probabilities of all of these things happening. That is where the total probability formula comes from.

b) In general, the probability of A is not easy to calculate. If you know the probability of a occurring under some conditions, you can use the total probability formula. Therefore, the total probability formula is a process of "from cause to effect".

3) **Participative learning**: The participative learning link combines the post-test link, designing group discussion, sharing and communication to achieve the purpose of practice and interaction, which is not only the application of new knowledge, but also a test of their mastery.

Let's go back to the story at the beginning of this lesson. After group discussion, students fulfill the solution independently, and submit it in class. The group representatives will share the discussion results:

The young man's chances of survival have increased to about 75%! So this is a typical "knowledge changes destiny" story!

4) **New knowledge exploration**: In this part of the class, students will explore the Bayes formula.

Firstly, teachers should help students recall the conditional probability theorem, and guide students to derive Bayes formulas on their own. After student discussion, the student representatives deduce the Bayes formula for everyone, which cultivate students' mathematical thinking ability from the known to the unknown.

Teachers' comments and an in-depth interpretation of the Bayes formula:

a) Bayes formula: It is known that result A has occurred, and then go back to see what causes A to occur, so it is a conditional probability. Therefore, the Bayes formula is a process of "holding the effect and finding the cause".

b) When explaining the posterior probability, take the example of doctor's therapy, and finally carry out ideological and political education. It is the same

in our life and study, we always need to dig out more information to help us make a choice or judgment, and we cannot do anything only by experience.

5) **Participatory learning**: This link combines the post-test link with group discussion, group confrontation, communication and sharing, which is both interactive and testing.

There is a typical application of Bayes formula in the medical field. Please see the following scenario:

A patient goes to the doctor and says, "Doctor, my test result is positive." Doctor: "You needn't worry too much, positive may not represent sick." Patient: "I checked it on the Internet. The accuracy rate of this test is 95%, *so I have a* 95% *chance of getting sick." Doctor: "You miss that the prevalence of this disease is extremely low."*

What does the doctor mean? If the probability of a patient getting sick is really 95%?

Example: The prevalence of a certain disease in a city is 4‰. A general survey conducts on the residents of the city. It assumes that 95% of the patients with the disease tests positive, and 96% of the patients without the disease are tested negative. Then: a) When the test result is positive, what is the probability that the tested person is really sick? b) When the test result is negative, what is the probability that the tested person does not have the disease?

Solution 1: The teacher explains the first question, and the students discuss the second question in groups. Then the students can submit the answers through the "rain-class". The group representatives can share the discussion results.

Solution 2: There are two questions. In this case, we divided the students into two teams. Each team completed the questions respectively and then explained them separately. The two teams can check with each other, pick out mistakes, which activates the classroom atmosphere and improves the initiative of students in learning.

This example also can replace with the background of nucleic acid testing, which is both practical and easy to improve students' interest in learning. In fact, this is a real case where the teacher introduces the background to the students:

In a 1978 *article published in the New England Journal of Medicine,* 60 *doctors at four hospitals affiliated with Harvard Medical School asked a similar question. Only* 11 *of them answered correctly, and almost half-answering* 95 *percent.*

6) **Expand learning**

As one of the most basic formulas in Probability theory, the formula of Total Probability has a long history of application and has many names. Kolmogorov has named it the Theorem of Total Probability in an article in 1956. Many articles do not even mention the name of the formula and just use it as a common sense formula. Today, total probability formulas use for speech recognition, spam filtering, drilling oil Wells, FDA new drugs approval, and Xbox games scores.

7) Independent summary

In this session, the students will discuss in groups, check and supplement each other. Then the group representatives summarize for everyone, which intends to cultivate the high-level ability of students' independent induction (Cui & Yang, 2019).

Homework will assign after the course summary. After this class, students are required to do a math activity: sensitivity survey.

5. Conclusion

In this case, we combine PAD and BOPPPS teaching mode, which fully embodies the "student-centered" teaching concept.

1) In the import process, we combine the lead-in and pre-test steps of BOPPPS teaching method flexibly. In this link, we used the original Flash as the carrier to introduce a new lesson with an interesting story, which reflects the wisdom of the protagonist and easily arouses students' empathy. The effect of stimulating students' interest must be very good.

At the same time, in this story, we use the way that students first discuss and then learn new knowledge. Students analyze how the protagonist's actions in this story change the probability of the event. Take the above design as a pre-test.

Learning from the PAD teaching method, half of the time is given to teachers to tell stories and make comments, and half of the time is given to students to discuss, answer, share and exchange.

2) In the part of participatory learning, we combined the participatory learning and post-test steps in the BOPPPS teaching method. In the part of the total probability formula, we will give half of the time to the lecturer to analyze the application steps, key points and background of the total probability formula. The other half of the time gives to student groups to discuss, answer, share and exchange. The Bayes formula proves by the students and analyzed by the teacher's comments to sublimate the students' understanding of the theory. It designs that the students will divide into two teams to complete the test problems and find faults with each other.

This link not only can test the students' learning effect but also can strengthen the communication between students, improve the ability of students' expression and collaboration.

3) In the expansion of learning, we not only introduce the origin and development of knowledge, and more importantly, but also introduce students to the application of the knowledge learned in real life.

4) In the summary part, students can summarize the lesson by themselves. Students discuss in groups and check each other's deficiencies. Then, the group representatives will summarize and encourage students to share their learning experience and tips. This link exercises students' advanced abilities.

Through the practice and analysis of this case, we believe that teaching innovation should always carry out the concept of "student-centered". The new teach-

ing method of "PAD + BOPPPS" not only exercises students' ability to analyze and solve problems, but also strengthens the interaction among students. It cultivates students' responsibility and communication skills in a team, strengthens their ability to solve practical problems with mathematical knowledge, and stimulates their interest in active learning.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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