

# The Use of Imitation Models at Developing and Introducing Information-Control Systems

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## Abstract

Imitation models for computing the environmental water pollution level depending on the intensity of pollution sources created by the author over the years are presented. For this purpose, an additive model of a non-stationary random process is considered. For the modeling of its components, models that consider only dilution and self-purification processes are proposed for waste water and three-dimensional turbulent diffusion equations for river waters, and multidimensional Gaussian Markov series are proposed for modeling the random component. The purpose, the capabilities and the peculiarities of such imitation models are discussed taking into account the peculiarities of the water objects. The modular principle of creating imitation models is proposed to facilitate their development and use.

## Keywords

Environmental Water Pollution, Imitation Model, Pollution Source, Pollution Condition, Deterministic Model, Stochastic Model.

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## 1. Introduction

The formation of the quality of natural waters is a very complex process. Among the numerous factors that determine their condition, industrial waste water should be singled out. In order to protect natural waters from pollution by industrial effluents, expensive treatment facilities and wastewater control systems are being built. The efficiency of investments depends on the optimality of the

adopted design decisions, in particular, on the calculation of the capacities of treatment facilities, the rational choice of control equipment, as well as on the quality of decisions made during the operation of treatment facilities, and on the maximum use of their capabilities. To successfully solve these problems, it is of great importance to know the process of formation of wastewater quality, the ability to predict its development depending on the time, taking into account all kinds of critical situations that arise in the process of the operation of the enterprise [1].

## 2. Imitation Models of the Formation of the Quality of the Environmental Water

Imitation models of wastewater quality formation make it possible to predict the quality of wastewater in dynamics, depending on the given mode of operation of the enterprise, without disturbing the normal mode of its operation. To develop them, the following initial information is required: a detailed diagram of the relative position of pollution sources connected by the sewerage network of a given enterprise; water consumption for each source of pollution and the concentration of discharged ingredients in all possible technological modes of operation; working models of the dissemination of polluting ingredients in the considered section of the sewer network; type and nature of the random component of the pollution process for sources of discharges.

The development of simulation models for the dissemination of polluting ingredients into natural water objects (for example, rivers, lakes, etc.) is also of great importance for solving the problems of environmental protection and utilization. They allow us to optimize the planning, implementation and operation of various industrial, environmental and economic activities related to the use of natural water objects. Simulation models for the formation of the quality of natural and/or waste waters, in addition to the abovementioned, make it possible to: indirectly control the operation of water quality auto-analyzers by comparing the simulation results and measured values of the same parameters; in case of temporary failure of any measurement channel in the auto-analyzer, fill in the gaps in the measurements of this parameter; calculate the concentrations of controlled ingredients in an uncontrolled point of a water object in accordance with the conditions of wastewater discharge by pollution sources (this allows to minimize the required number of auto-analyzers necessary to control a water object with a given reliability); compute maximum allowable discharges (MPD) for polluted objects in order to maintain the concentration of controlled ingredients within the maximum allowable concentration (MAC), predict the concentration of controlled ingredients at a given point in a water object depending on the conditions of discharge of wastewater from pollution sources; detect emergency pollution sources [2] [3]; testing, coordinating, optimizing the technical, information-software and mathematical support of the automated water quality control system being developed, which significantly increases the efficiency of such

developments and reduces the time for their implementation at a real object to a minimum.

In order to unify algorithms and programs, simulation models should be developed on a block-modular basis with optimal division of functions among blocks, allowing simulating various pollution processes by rearranging the execution order and minimally replacing the developed blocks.

It seems appropriate to have the following main blocks in simulation models: generation of technological modes of operation of pollution sources, *i.e.* the block of control; implementation of mathematical models for the dissemination of pollutants in water; generation of multidimensional random processes having a given character; generating random numbers according to a given probability distribution law.

Under the influence of natural hydrological and hydro-biological conditions, runoff formation factors and anthropogenic impacts, the physicochemical parameters characterizing the state of a water object continuously change over time. The non-stationary random process of these changes can be represented as [4]

$$S_p(t) = m_S^p(t) + x_p(t), \quad p = 1, \dots, m, \quad (1)$$

where  $S_p(t)$  is the value of the  $p$ -th controlled parameter of the water object;  $m_S^p(t)$  is deterministic component of the process;  $x_p(t)$  is stochastic component;  $m$  is the number of controlled parameters.

Therefore, the task of determining mathematical models for the pollutants dissemination in water objects can be divided into two parts: the development of models that describe the deterministic and stochastic components, respectively.

### 3. Deterministic Component of Simulation Models

As a deterministic part of mathematical models for the formation of industrial wastewater quality, models that take into account only dilution and self-purification processes can be used [4]:

$$y_p^k(t) = \begin{cases} \sum_{j=1}^{q_k} y_{p,j}(t - \tau_j) + x_p(t) & \text{at } p = 1, \\ \frac{1}{\sum_{j=1}^{q_k} y_{1,j}} \left[ \sum_{j=1}^{q_k} y_{1,j}(t - \tau_j) \cdot y_{p,j}(t - \tau_j) \right] + x_p(t) & \text{at } p = 2, \dots, m, \end{cases} \quad (2)$$

where  $y_1^k(t)$  is the volume of water at the  $k$ -th node;  $y_p^k(t)$ ,  $p = 2, \dots, m$  concentration of the  $p$ -th ingredient in the  $k$ -th node;  $q_k$  - number of sources of discharges involved in the formation of water quality in the  $k$ -th node;  $y_{p,j}(t)$  - concentration of the  $p$ -th ingredient discharged by the  $j$ -th object of discharges;  $\tau_j$  - time of water running from the  $j$ -th discharge object to the  $k$ -th node;  $x_p(t)$  - stochastic component of the concentration of the  $p$ -th ingredient.

As a deterministic part of the mathematical models for the formation of the quality of natural water objects, one, two and three-dimensional equations of turbulent diffusion of non-conservative substances can be used, depending on the specific character of the modeled water object. The equation of 3D-turbulent diffusion of non-conservative substances is the following [4] [5] [6]:

$$\begin{aligned} \frac{\partial \Phi}{\partial t} = & \frac{\partial}{\partial x} \left( K_x \frac{\partial \Phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial \Phi}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial \Phi}{\partial z} \right) \\ & - V_x \frac{\partial \Phi}{\partial x} - V_y \frac{\partial \Phi}{\partial y} - V_z \frac{\partial \Phi}{\partial z} - u \frac{\partial \Phi}{\partial y} - K(\Phi) + f(x, y, z, t), \end{aligned} \quad (3)$$

where  $\Phi$  is the concentration of the non-conservative dissolved substance averaged over time;  $t$  is time;  $x, y, z$  are the spatial co-ordinates (the axis  $x$  is horizontal and its direction coincides with the direction of averaged current of all stream, the axis  $y$  is perpendicular to the free surface and it is directed downwards, the axis  $z$  is directed across to the stream);  $K_x, K_y, K_z$  are the coefficients of the turbulent diffusion in the direction of axes  $x, y, z$ ;  $V_x, V_y$  and  $V_z$  are components of speeds on axes  $x, y, z$  averaged over time;  $u$  represents the largest hydraulic particles;  $K(\Phi)$  is the a parameter characterizing the non-conservativeness of pollutant (one often uses simple approximation of this dependence  $K(\Phi) \equiv K \cdot \Phi$ , where  $K$  is the coefficient of non-conservativeness);  $f(x, y, z, t)$  is the total intensity of external sources of pollution. In general, the coefficients  $K_x, K_y, K_z, V_x, V_y, V_z$  and  $K(\Phi)$  are the functions of a point of space and time.

Initial and boundary conditions of solving the equation of diffusion (3) are set in the form

$$\Phi(0, r) = S_0; \quad \Phi(t, r)|_{x=0} = \sigma \quad (4)$$

( $S_0, \sigma = const$ ). Boundary conditions in the lower end of the section can be classical or non-classical. Classical condition looks like

$$\left. \frac{\partial}{\partial x} \Phi(t, r) \right|_{x=\mathfrak{S}} = 0; \quad (\text{condition of full mixing}) \quad (5)$$

non-classical condition

$$\Phi(t, r)|_{x=\mathfrak{S}} = q \cdot \Phi(t, r)|_{x=\mathfrak{S}-l}. \quad (\text{not local boundary condition}), \quad (6)$$

where  $q$  is the coefficient of self-purification of the river on the considered section;  $\omega$  is the concentration of the pollutant dropped by pollution source in the point  $x = \mathfrak{S}$  [7].

At  $m \geq 2$ , boundary conditions on the other part of the line or on the surface  $\partial G$ -Neumann's conditions, are set also

$$(\nu \cdot \nabla) \Phi(t, r)|_{r \in \partial G} = 0, \quad (7)$$

where  $\nu$  is unit vector of external normal to the border  $\partial G$ . In particular, at  $m = 3$ , it should be

$$\left. \frac{\partial}{\partial z} \Phi(t, r) \right|_{z=0} = 0. \quad (8)$$

The methods of the solution of Equation (3) and their realization at different initial and boundary conditions are given in [5] [6]. These methods were widely used in automated water quality control systems at solving many different ecological problems [4].

Many other deterministic models of pollutants transport in the environmental

water are considered in literature (see, for example [1] [8]-[14]). The choice of the concrete model depends on the specificity of the water object, the pollution of which is considered, on the condition of the pollution and the peculiarity of the pollution substance under consideration, on the aim for which the model is used and so on. Application of these models in concrete cases must be made depending on the reasons named above.

#### 4. Stochastic Component of Simulation Models

The Markov model is known to be the best for hydrological data [15]. It is also expected to be suitable for natural water pollution data. Our field studies confirmed this assumption [16]. Therefore, as a stochastic component of the concentration of pollution substances,  $m$ -dimensional Gaussian Markov series  $X(t) = (x_1(t), x_2(t), \dots, x_m(t))$ , with the depth of connection equal to  $N$ , given by formula

$$x_p(t) = \sum_{l=1}^{p-1} b_l^p x_l(t) + \sum_{i=1}^m \sum_{j=1}^N a_{ij}^p x_i(t-j) + \sigma_p \xi_p(t), \tag{9}$$

is used, where coefficients  $b_l^p$ ,  $a_{ij}^p$  depend on the auto- and inter-covariance functions of  $m$ -dimensional random series  $x(t) = (x_1(t), x_2(t), \dots, x_m(t))$ ;  $\sigma_p^2$  is a residual variance of a random series  $x_p(t)$ ;  $\xi_p(t)$  is normally distributed standard random variable. The method of computation of the coefficients of the model (9) which determines the number of observations, necessary for modeling series (9) with given accuracy is given in the work [4] [17].

The unknown coefficients and the residual variance in Equation (9) are found by means of least-squares technique. With the following designations:

$$R'_{k,i}(|h-j|) = \begin{cases} R_{k,i}(|h-j|) & \text{at } h \geq j, \\ R_{i,k}(|h-j|) & \text{at } h < j, \end{cases}$$

$$k, i = 1, \dots, m; \quad j, n = 1, \dots, N;$$

$$A_p^T = (b^p, a^p)_{1 \times [m \cdot N + (p-1)]}; \quad C_p^T = (R_{k,i}(h))_{1 \times m \cdot N} \tag{10}$$

$$B_p = (R'_{k,i}(|h-j|))_{m \cdot N \times [m \cdot N + (p-1)]}, \quad p = 1, \dots, m,$$

where  $R_{k,i}(|h-j|)$  are the corresponding covariances, the expression for the unknown coefficients assumes the following form:

$$A_p = B_p^+ \cdot C_p, \tag{11}$$

where  $B_p^+$  is pseudoinverse matrix; expression for the residual variance is

$$\sigma_p^2 = R_p(0) - \sum_{l=1}^{p-1} \sum_{k=1}^{p-1} b_l^p b_k^p R_{l,k}(0) - \sum_{i=1}^m \sum_{j=1}^N \sum_{k=1}^m \sum_{l=1}^N a_{ki}^p a_{lj}^p R'_{l,k}(|l-j|) - 2 \sum_{l=1}^{p-1} \sum_{j=1}^N \sum_{i=1}^m b_l^p a_{ij}^p R_{i,l}(j), \tag{12}$$

where  $R_p(0)$  is variance of the  $p$ -th random process.

Let's introduce the following designations:  $\gamma_p$  required accuracy of Marko-

vian series generation;  $\Delta_R$  -maximum absolute error of calculation of covariant function one value.

If

$$\Delta_R \leq \frac{\gamma_p}{N^{1/2} \left( \sum_{i=1}^m \beta_i^2 \right)^{1/2} \|A_{p0}\| \left\{ N^{1/2} \left[ \sum_{i=1}^m \left( \text{cond}^+ B_i \|A_{i0}\| D_i \right)^2 \right]^{1/2} + \text{cond}^+ B_p D_p \right\}}; \tag{13}$$

$$D_i = \frac{\left[ m^2 N^2 + mN(i-1) \right]^{1/2} \|C_i\| + (mN)^{1/2} \|B_i\|}{\|B_i\| + \|C_i\|}, i = 1, \dots, m,$$

holds for all  $p = 1, \dots, m$ , the multidimensional Gaussian Markovian series is generated to the given accuracy with probability equal to or greater than  $(1 - \alpha)$ . Here:  $\|\cdot\|$  is Euclidean norm of the corresponding matrix;  $\text{cond}^+ B_p = \|B_p\| \cdot \|B_p^+\|$  is conditionality number of matrix  $B_p$ ;  $\beta_i$  is the value for which  $P(|\hat{x}_i(t-j)| \leq \beta_i) = 1 - \alpha$  holds.

Sample size  $n$ , that ensures computation of covariant function values with absolute error not exceeding  $\Delta_R$ , is determined from the following relation [18]:

$$n = \max_{\{i\}} \left\{ i + \frac{1}{\gamma_i \sqrt{\alpha}} \left[ (n^* + 1) R^2(0) + (n^* + 1 - i) R^2(i) + 2 \cdot \sum_{j=1}^{n^*} (n^* + 1 - j) R^2(j) + 2 \cdot \sum_{j=1}^{n^*} (n^* + 1 - i - j) R(j+i) R(j-i) \right]^{1/2} \right\}, i = 0, 1, \dots, \max\{n, n^*\} \tag{14}$$

Input information for the computation is:  $R_{i,k}(j)$ ,  $i, k = 1, \dots, m$ ;  $j = 1, \dots, N$ ;  $\gamma_p$ ;  $\Delta_R$ ;  $\alpha$ .

The above results are obtained provided that the system of linear equations

$$B_p A_p = C_p \tag{15}$$

is compatible. System (15) compatibility means that

$$B_p^+ B_p - E = 0, \tag{16}$$

where  $E$  is a unit matrix.

The described models were used to model the pollution processes of different rivers and wastewaters. For example, models (3) were used to model the pollution processes of the Khobistskali River basin (western Georgia). The comparison of modeling results with the results of measurements of pollutant concentrations in rivers has clearly shown the high quality of the created models and the great potential for their use [5] [16].

The described models of wastewater pollution levels were used for modeling the wastewater quality of the Nitrogen Plant in Odessa (Ukraine) in the automated wastewater pollution control and management systems of the same plant, both in the design and testing of these systems and in the conditions of its operation [1].

## 5. Conclusion

The described models were widely used in automated systems for controlling and managing the pollution of water objects, both in their creation and in operation, to solve many different problems, such as: making optimal decisions in the creation, testing and operation of automatic monitoring systems; when calculating the values of the pollution parameters at the uncontrolled points of the control environment object; to calculate the maximum allowable discharge for pollution sources in a dynamic mode taking into account the condition of the environmental object in the period under consideration; taking into account the pollution conditions and the possible variability of the condition of the control facility for the forecast of possible variability of the condition of the environmental object; for automatic detection of emergency pollutants in the conditions of their existence, etc.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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