Scientific Research Publishing

Journal of Quantum Information Science, 2023, 13, 131-137
https://www.scirp.org/journal/jqis
ISSN Online: 2162-576X
ISSN Print: 2162-5751

Bell's Theorem and Einstein's Worry about Quantum Mechanics

Han Geurdes

GDS Applied Mathematics BV, Gravenhage, Netherlands
Email: han.geurdes@gmail.com

How to cite this paper: Geurdes, H. (2023) Bell's Theorem and Einstein's Worry about Quantum Mechanics. *Journal of Quantum Information Science*, **13**, 131-137.
https://doi.org/10.4236/jqis.2023.133007

Received: May 30, 2023
Accepted: August 20, 2023
Published: August 23, 2023

Abstract

With the use of a local dependency on instrument setting parameters of the probability density of local hidden variables, it is demonstrated that a Kolmogorov formulation reproduces the quantum correlation. This is the novelty of the work. In a Bell experiment, one cannot distinguish between Bell's formula and the here presented local Kolmogorov formula. With the presented formula, no CHSH can be obtained. Therefore, the famous CHSH inequality has no excluding power concerning local extra Einstein parameter models. This result concurs with other previous research concerning difficulties with Bell's formula.

Keywords

Bell's Correlation Formula, Basic Measure Theory

1. Introduction

Einstein Podolsky and Rosen started a discussion about the foundation of quantum theory in 1935 [1]. Their work established entanglement of particles. Don Howard [2] wrote an interesting history of the discussion associated to the publication of what we now know as the EPR paradox [1]. The EPR paper wasn't Einstein's final formulation of his criticism. Here we will concentrate on Bell's approach to the problem.

In his famous paper, John Bell wrote down [3] a correlation that is based on (local) hidden variables. The experiment where Bell referred to is a spin-spin entanglement experiment. It was based on ideas of David Bohm [4]. Schematically one can formulate it thus

$$\left[A(\hat{a})\right]\longleftarrow\sim\cdots\sim\longleftarrow\sim\left[S\right]\sim\longrightarrow\sim\cdots\sim\longrightarrow\left[B\left(\hat{b}\right)\right] \tag{1}$$

Here, the $\left[A(\hat{a})\right]$ and $\left[B(\hat{b})\right]$ represent the two distant measuring instruments. The $\left[A(\hat{a})\right]$ is Alice's measurement instrument with parameter. The $\left[B(\hat{b})\right]$ is Bob's instrument with parameter. The $\hat{a}$ and $\hat{b}$ are the unitary vector setting parameters. The $[S]$ represents the source of an entangled pair of particles.

Einstein uncovered a correlation between distant measurements. Bell wrote a correlation formula between the setting parameters. It is presented in equation number (2) of Bell's paper. It is:

$$P(\hat{a},\hat{b}) = \int \rho(\lambda) A(\hat{a},\lambda) B(\hat{b},\lambda) \mathrm{d}\lambda \tag{2}$$

The λ represents hidden variables and $\rho(\lambda)$ represents the probability density of those variables. The $A(\hat{a},\lambda)$ represents the measurement at $\left[A(\hat{a})\right]$ in (1) given the setting $\hat{a}$. For spins, $A(\hat{a},\lambda) = \pm 1$ with 1 a spin-up and -1 a spin-down measurement. In Bell's article $|A(\hat{a},\lambda)| \leq 1$ and $|B(\hat{b},\lambda)| \leq 1$ are also discussed. From Equation (2), a number of inequalities were derived. The CHSH inequality is a very famous inequality and is turned into an experiment by Aspect [5].

2. Thoughts about Correlation and Locality

Despite the fact that people were awarded Nobelprizes for their work on the inequalities, which erroneously suggest a closure, we will argue that their research is incomplete. One cannot conclude from the Bell experiment research of the 2022 Nobelists that Einstein locality is ruled out in physical reality. This conclusion *cannot* be avoided by starting a metaphysical discussion about what is physical reality. The conclusion is mathematical.

Let us start with noting a published paper written together with Nagata and Nakamura, [6]. Here the mathematics of CHSH is inspected critically and a valid counter example is construed. It is remarkable that the Nobel committee chose to ignore it. One may wonder which (social) forces were at work to limit the committee's view. In [7] a statistical way is construed to locally violate the CHSH with probability nonzero. The criticism directed to [7] absolutely did not touch its conclusion. It is possible to locally violate the CHSH with probability nonzero. Other research such as [8] and [9] also rightfully voiced doubts about Bell's formula and experiment. Apparently, the committee thought that we all wrote nonsense. The present author nevertheless has sufficient reason to doubt the scope of search that such a committee has applied.

Furthermore and more importantly we can set up the following new form of analysis. Let us note that locality is not violated by allowing that the setting $\hat{a}$ influences a probability density at $\left[A(\hat{a})\right]$ only. Similarly for $\hat{b}$ at $\left[B(\hat{b})\right]$. This makes sense in an Einsteinian way when $\hat{a}$ does not influence $\left[B(\hat{b})\right]$ and $\hat{b}$ does not influence $\left[A(\hat{a})\right]$. The idea is to shift the dependence on the setting variables from the measurement functions to the respective densities and maintain locality in the sense of Einstein. The measurement instrument is considered a combination of a hidden variable density and a function that generates its output. We propose here that the density, influenced by the setting, regulates together with the influence of the hidden variables in the measurement function, what is registered as a measurement. It is not $\rho(\lambda)A(\hat{a},\lambda)$ but $\rho_{\hat{a}}(\lambda)A(\lambda)$. And we note that this can be done maintaining Einstein locality.

2.1. Preliminaries

In 3 dimensional Euclidian space three orthonormal base vectors are defined by, $\{\hat{e}_k\}_{k=1}^{3}$ with components, $(\hat{e}_k)_n = \delta_{k,n}$. Here $\delta_{k,n} = 1$, when $k = n$ and $\delta_{k,n} = 0$, when $k \neq n$ and $k,n = 1,2,3$. With this definition we are able to write down

$$\hat{\omega}(\varphi,\theta) = \sum_{j=1}^{3}\omega_j(\varphi,\theta)\hat{e}_j, \text{and,}$$
$$\omega_1 = \cos(\varphi)\sin(\theta), \omega_2 = \sin(\varphi)\sin(\theta), \omega_3 = \cos(\theta) \tag{3}$$

And, $\omega_j = \omega_j(\varphi,\theta)$. The ranges for the φ and θ are the sets $\Phi = \{x \in \mathbb{R}: 0 \le x \le 2\pi\}$ and $\Theta = \{x \in \mathbb{R}: 0 \le x \le \pi\}$. With $\|\cdot\|$ the Euclidean norm we have $\hat{\omega}^{\mathrm{T}} \cdot \hat{\omega} = \|\hat{\omega}\|^2 = 1$ for all $(\varphi,\theta) \in \Phi \times \Theta$. The upper T indicates the transpose of the vector.

Subsequently, with (3), we are able to define the setting parameters, $\hat{a} = \hat{\omega}(\varphi_{Aa},\theta_{Aa})$ and $\hat{b} = \hat{\omega}(\varphi_{Bb},\theta_{Bb})$. Both $(\varphi_{Aa},\theta_{Aa})$ and $(\varphi_{Bb},\theta_{Bb})$ in $\Phi \times \Theta$. The $[A(\hat{a})]$ associated hidden variables are denoted by $(\varphi_A,\theta_A) \in \Phi \times \Theta$. The $[B(\hat{b})]$ associated hidden variables are $(\varphi_B,\theta_B) \in \Phi \times \Theta$. Let us then, in the language of integration measure theory [10] write for the A side variables

$$\mu_{\hat{a}}(\mathrm{d}\varphi_A \mathrm{d}\theta_A) = \delta(\varphi_{Aa} - \varphi_A)\delta(\theta_{Aa} - \theta_A)\mathrm{d}\varphi_A \mathrm{d}\theta_A \tag{4}$$

While for the B side variables the measure is

$$\mu_{\hat{b}}(\mathrm{d}\varphi_B \mathrm{d}\theta_B) = \delta(\varphi_{Bb} - \varphi_B)\delta(\theta_{Bb} - \theta_B)\mathrm{d}\varphi_B \mathrm{d}\theta_B \tag{5}$$

The $\delta(y-x)$ is a Dirac delta function. This is a non-negative generalised function. It can be employed for probability density.

Subsequently, it follows that $\int_{\Phi\times\Theta}\mu_{\hat{a}}(\mathrm{d}\varphi_A \mathrm{d}\theta_A) = \int_{\Phi\times\Theta}\mu_{\hat{b}}(\mathrm{d}\varphi_B \mathrm{d}\theta_B) = 1$. Hence, the measures in (4) and (5) are valid short hands for a Bell-form correlation formula. Here, the influence of the setting is placed on the density and does not influence the measurement functions. There is no nonlocality. Again, $[A(\hat{a})]$ is not influenced by the setting $\hat{b}$ and vice versa, $[B(\hat{b})]$ is not influenced by $\hat{a}$. In addition, the values $(\varphi_{Aa},\theta_{Aa})$ are not influenced by $(\varphi_{Bb},\theta_{Bb})$ and vice versa.

Subsequently, let us per pair of entangled particles under investigation, here photons, define a $r_0 \in$ the interval $(0,1)$. The r_0 is randomly selected. Then a measure $\nu_0(\mathrm{d}r)$ is defined by

$$\nu_0(\mathrm{d}r) = \delta(r_0 - r)\mathrm{d}r \tag{6}$$

Here, δ, is again Dirac's delta function and the variable r is in the interval $(0,1)$ as well. Hence, $\nu_0(\mathrm{d}r) \geq 0$ and $\int_0^1 \nu_0(\mathrm{d}r) = 1$ and is allowed as density.

Let us then define two functions g_A and g_B with for short hand $\Omega_A \equiv (\varphi_A, \theta_A)$, $\Omega_B \equiv (\varphi_B, \theta_B)$. With

$$g_A(\Omega_A, \Omega_B, r_0) = \begin{cases} 1, & 0 < r_0 < \frac{1}{2} \\ \cos\left[\angle\left\{\hat{\omega}(\Omega_A), \hat{\omega}(\Omega_B)\right\}\right], & \frac{1}{2} \leq r_0 < 1 \end{cases} \tag{7}$$

The function g_B is defined as follows

$$g_B(\Omega_A, \Omega_B, r_0) = \begin{cases} 1, & \frac{1}{2} \leq r_0 < 1 \\ \cos\left[\angle\left\{\hat{\omega}(\Omega_A), \hat{\omega}(\Omega_B)\right\}\right], & 0 < r_0 < \frac{1}{2} \end{cases} \tag{8}$$

The $\angle\left\{\hat{\omega}(\Omega_A), \hat{\omega}(\Omega_B)\right\}$ is the angle between unit length vectors $\hat{\omega}(\Omega_A)$ and $\hat{\omega}(\Omega_B)$. No nonlocal "knowledge" is transported between Alice and Bob. Note that $|g_A| \leq 1$ and $|g_B| \leq 1$. Note also that if $\lambda = (\Omega_A, \Omega_B, r)$, the Bell correlation would then be equivalent to

$$P(\hat{a}, \hat{b}) = \int \rho_{\hat{a}}(\lambda) \rho_{\hat{b}}(\lambda) \rho_{r_0}(\lambda) A(\lambda) B(\lambda) \mathrm{d}\lambda \tag{9}$$

Please acknowledge that this expression of the correlation is a genuine Kolmogorov correlation formula. It differs from Bell's approach in (2). It is impossible to tell beforehand whether or not (2) exclusively occurs in nature while (9) does not.

We note that the dependence on the settings, which are local, is shifted to the densities. The effects are local as one can see from (4) and (5). If there are thoughts otherwise, proof of violation of Einstein locality is absolutely required. The present author thinks it obviously such a proof is not possible. The $g_A(\Omega_A, \Omega_B, r)$ is in fact (9) $A(\lambda)$ given in (7). The $g_B(\Omega_A, \Omega_B, r)$ is (9) $B(\lambda)$ from (8). Therefore, (4) and (5) are Einstein valid. In the next section the integration will be performed in our set of variables and notation.

2.2. Correlation

The λ of (9) is operationalised here with (Ω_A, Ω_B, r). To remind the reader: $\Omega_A = (\varphi_A, \theta_A)$ and similar $\Omega_B = (\varphi_B, \theta_B)$ are short-hands.

The following integral expression for $P(\hat{a}, \hat{b})$, with $\mathrm{d}^2\Omega_A = \mathrm{d}\varphi_A \mathrm{d}\theta_A$ similar B, can be obtained.

$$\begin{aligned} P(\hat{a}, \hat{b}) = &\int_{\Phi\times\Theta} \mu_{\hat{a}}(\mathrm{d}^2\Omega_A) \int_{\Phi\times\Theta} \mu_{\hat{b}}(\mathrm{d}^2\Omega_B) \\ &\times \int_0^1 \nu_0(\mathrm{d}r) g_A(\Omega_A, \Omega_B, r) g_B(\Omega_A, \Omega_B, r) \end{aligned} \tag{10}$$

From the definition of $\nu_0(\mathrm{d}r)$ it follows

$$P(\hat{a}, \hat{b}) = \int_{\Phi\times\Theta} \mu_{\hat{a}}(\mathrm{d}^2\Omega_A) \int_{\Phi\times\Theta} \mu_{\hat{b}}(\mathrm{d}^2\Omega_B) g_A(\Omega_A, \Omega_B, r_0) g_B(\Omega_A, \Omega_B, r_0) \tag{11}$$

and r_0 randomly from interval $(0,1)$ for each pair. Looking at the definition of g_A and g_B in (7) and (8), we arrive from the previous equation at

$$P\left(\hat{a},\hat{b}\right)=\int_{\Phi\times\Theta}\mu_{\hat{a}}\left(\mathrm{d}^2\Omega_A\right)\int_{\Phi\times\Theta}\mu_{\hat{b}}\left(\mathrm{d}^2\Omega_B\right)\cos\left[\angle\left\{\hat{\omega}\left(\Omega_A\right),\hat{\omega}\left(\Omega_B\right)\right\}\right] \tag{12}$$

The subsequent step is to observe that

$$\cos\left[\angle\left\{\hat{\omega}\left(\Omega_A\right),\hat{\omega}\left(\Omega_B\right)\right\}\right]=\hat{\omega}\left(\Omega_A\right)^{\mathrm{T}}\cdot\hat{\omega}\left(\Omega_B\right)$$

Therefore, the separation in the integration can be performed as

$$\begin{aligned}P\left(\hat{a},\hat{b}\right)&=\int_{\Phi\times\Theta}\mu_{\hat{a}}\left(\mathrm{d}^2\Omega_A\right)\int_{\Phi\times\Theta}\mu_{\hat{b}}\left(\mathrm{d}^2\Omega_B\right)\hat{\omega}\left(\Omega_A\right)^{\mathrm{T}}\cdot\hat{\omega}\left(\Omega_B\right)\\&=\left[\int_{\Phi\times\Theta}\mu_{\hat{a}}\left(\mathrm{d}^2\Omega_A\right)\hat{\omega}\left(\Omega_A\right)\right]^{\mathrm{T}}\cdot\left[\int_{\Phi\times\Theta}\mu_{\hat{b}}\left(\mathrm{d}^2\Omega_B\right)\hat{\omega}\left(\Omega_B\right)\right]\end{aligned} \tag{13}$$

Note that, $\Omega_A=\left(\varphi_A,\theta_A\right)$ hence by definition of $\mu_{\hat{a}}\left(\mathrm{d}^2\Omega_A\right)=\mu_{\hat{a}}\left(\mathrm{d}\varphi_A\mathrm{d}\theta_A\right)$ in (4) and of $\mu_{\hat{b}}\left(\mathrm{d}^2\Omega_B\right)=\mu_{\hat{b}}\left(\mathrm{d}\varphi_B\mathrm{d}\theta_B\right)$ in (5). And so,

$$\begin{aligned}&\int_{\Phi\times\Theta}\mu_{\hat{a}}\left(\mathrm{d}^2\Omega_A\right)\hat{\omega}\left(\Omega_A\right)\\&=\int_0^{2\pi}\int_0^{\pi}\delta\left(\varphi_{Aa}-\varphi_A\right)\delta\left(\theta_{Aa}-\theta_A\right)\hat{\omega}\left(\varphi_A,\theta_A\right)\mathrm{d}\varphi_A\mathrm{d}\theta_A\\&=\hat{\omega}\left(\varphi_{Aa},\theta_{Aa}\right)=\hat{a}\end{aligned} \tag{14}$$

and similar for $\hat{b}$ on B variables, *i.e.* $\int_{\Phi\times\Theta}\mu_{\hat{b}}\left(\mathrm{d}^2\Omega_B\right)\hat{\omega}\left(\Omega_B\right)=\hat{b}$. This implies from (13) that $P\left(\hat{a},\hat{b}\right)=\hat{a}^{\mathrm{T}}\cdot\hat{b}$. Or equivalently: $P\left(\hat{a},\hat{b}\right)=a_1b_1+a_2b_2+a_3b_3$. In other words, the quantum correlation has been reproduced from a Bell-type hidden variables model.

The way that the CHSH inequality is employed in experiment suggests that this is impossible. However, it is clear that our new approach with parameter dependence of densities allows it. Presently, only metaphysical, but not solid physical, reasons can eliminate the possibility of "only densities, but not measurement functions are locally influenced by setting parameters".

3. Conclusions

Because of the weight of the matter, one in the first place must acknowledge that the $P\left(\hat{a},\hat{b}\right)$ in (13), or more generally (9), is within the scope of what Bell intended with his correlation. To be more specific: Why would a formula with settings that only affect the density of local variables not be what Bell intended but forgot. This possibility may have been overlooked later. The most likely reason is researchers didn't want to acknowledge that Bell's formula could be an incomplete representation of what may occur in experiment.

Secondly, there is obviously no breach of locality as we have already argued in this paper, *i.e.* selection of $\hat{a}$ does not influence the B variables and vice versa. The settings are Einstein local and those settings influence the density of only local variables. Further, the A integration occurs encapsulated at $\left[A\left(\hat{a}\right)\right]$ while the B integration is encapsulated at $\left[B\left(\hat{b}\right)\right]$. Writing e.g. $\left[A\left(\hat{a}\right)\right]$ is in our sense deceptive. It should e.g. have been $\left[A\left(\lambda\right)\rho_{\hat{a}}\left(\lambda\right)\right]$. In addition, the $\nu_0\left(dr\right)$ integration occurs in $[S]$. The local hidden variables are the cause of the correlation.

Thirdly and quite importantly, there is no such thing as e.g. $A_1B_1 - A_2B_1 + A_2B_1 + A_2B_2$, with the indices referring to different $\hat{a}$ and $\hat{b}$ settings. No CHSH can be obtained from our formula. This is because the measurement functions do not depend on settings. Densities do. From (7) and (8) we see that measurement functions remain in the required interval $[-1,1]$.

Fourth and finally, therefore, the use of λ is similar to Bell's. It is however different from Bell's approach [3]. In addition, nobody knows how a measurement is brought about. Therefore, it is a genuine possibility of physics to have a correlation formula where only the density and not the measurement function, is locally influenced by the setting parameters.

Conflicts of Interest

The author has no conflict of interest. The work was not funded. There is no data associated.

References

[1] Einstein, A., Podolsky, B. and Rosen, N. (1935) Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? *Physical Review Journals Archive*, **47**, 777. https://doi.org/10.1103/PhysRev.47.777

[2] Howard, D. (1985) Einstein on Locality and Separability. *Studies in History and Philosophy of Science Part A*, **16**, 171-201. https://doi.org/10.1016/0039-3681(85)90001-9

[3] Bell, J.S. (1964) On the Einstein Podolsky Rosen Paradox. *Physics Physique Fizika,* **1**, 195. https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195

[4] Bohm, D. (1952) A Suggested Interpretation of the Quantum Theory in Terms of "Hidden" Variables. *Physical Review Journals Archive,* **85**, 166. https://doi.org/10.1103/PhysRev.85.166

[5] Aspect, A. (1976) Proposed Experiment to Test the Nonseparability of Quantum Mechanics. *Physical Review D,* **14**, 1944. https://doi.org/10.1103/PhysRevD.14.1944

[6] Geurdes, H., Nagata, K. and Nakamura, T. (2021) The CHSH Bell Inequality: A Critical Look at Its Mathematics and Some Consequences for Physical Chemistry. *Russian Journal of Physical Chemistry B*, **15**, S68-S80. https://doi.org/10.1134/S1990793121090050

[7] Geurdes, H. (2014) A Probability Loophole in the CHSH. *Results in Physics*, **4**, 81-82. https://doi.org/10.1016/j.rinp.2014.06.002

[8] Gu, Y. (2022) Conceptual Problems in Bell's Inequality and Quantum Entanglement. *Journal of Applied Mathematics and Physics*, **10**, 2216-2231. https://doi.org/10.4236/jamp.2022.107152

[9] Hess, K. (2020) Kolmogorov's Probability Spaces for "Entangled" Data-Subsets of EPRB Experiments: No Violation of Einstein's Separation Principle. *Journal of Modern Physics*, **11**, 683-702. https://doi.org/10.4236/jmp.2020.115044

[10] Masani, P. (1982) The Outer Regularization of Finitely-Additive Measures over

Normal Topological Spaces. In: Kolzow, D. and Maharam-Stone, D., Eds., *Measure Theory Oberwolfach* 1981, Springer-Verlag, Berlin, 116-144. https://doi.org/10.1007/BFb0096667