

# A Physical Core-Loss Model for Laminated Magnetic Sheet Steels

Kuofeng Chen

Metal Forming Technology Section, Metal Industries Research & Development Centre, Kaohsiung  
Email: bill\_chen@zohomail.com

**How to cite this paper:** Chen, K.F. (2024) A Physical Core-Loss Model for Laminated Magnetic Sheet Steels. *Journal of Power and Energy Engineering*, 12, 115-123.  
<https://doi.org/10.4236/jpee.2024.123008>

**Received:** March 15, 2024

**Accepted:** March 26, 2024

**Published:** March 29, 2024

## Abstract

A full-frequency instant core-loss equation built from the induction physical model of magnetic materials, where the iron loss, eddy loss, and hysteresis loss no longer have an integral term, and this new equation provides high simulation accuracy and performs dynamic core loss analysis on non-sinusoidal or pulse magnetic fields. The simulation examples use a high-grade electrical steel sheet 65CS400 by Epstein experimental data covering magnetic field 0.1 - 1.8 T and frequency 50 - 5000 Hz, and the average error of the simulated core loss is less than 4%. Since the simulation is converged by magnetic physical parameters, so the physical relevance of the similar laminated materials can be compared with the coefficient results.

## Keywords

Core Loss, Hysteresis Loss, Electrical Steel Sheet

## 1. Introduction

The models for the power loss of magnetic materials in the changing magnetic field have been developed for decades from the earliest Steinmetz equation [1].

$$\bar{P} = kf^\alpha \hat{B}^\beta \quad (1)$$

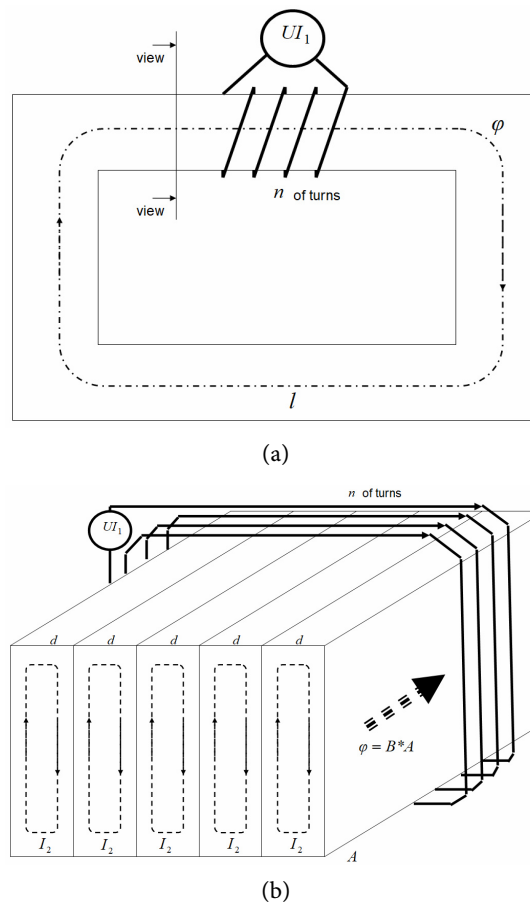
$\bar{P}$  denotes the average of core loss,  $k, \alpha, \beta$  are coefficients. Steinmetz equation only locates on frequency domain to present limited accuracy for most of magnetic materials unless ferrites. Based on frequency-domain Steinmetz equation, a series of improved time-domain equations [2] [3] [4] [5] are developed but still have not good accuracy for electrical steel sheets.

$$P = k \left| \frac{dB}{dt} \right|^\alpha |B|^{\beta-\alpha} \quad (2)$$

Time-domain equations can simulate the complex power loss in non-sinusoidal or non-periodic alternating magnetic field, including high-frequency trapezoidal transformers, PWM drivers, variable-speed motors and so on.

[8] [9] [10] give some accuracy solution for limited ranges of applications, and the necessary integral terms increase the calculation process and deviate from the instantaneous accuracy. [6] [7] use multiple of indexes and coefficients to improve the weakness and provide high accuracy for very wide range of frequencies and magnetic densities. Following the researches, a new equation is deduced from the physical magnetic induction equation to avoid the integral calculation and eliminate complex coefficients. This physical core-loss equation even demonstrates each parts of eddy loss, hysteresis loss and excess loss. Considering a simple electrical-magnetic induction model as the following core and coils:

**Figure 1(a)** shows a simple magnetic inductance system with a core and coils, the magnetic flux locates inside of the core and electrical current locates on the coil. **Figure 1(b)** of the section view shows the eddy currents of the laminated structure of the core. If the laminated layers are uniform, then a induction equation can be expressed as



**Figure 1.** (a) The magnetic induction system of a laminated core and turning coils; (b) The section view with induction eddy currents.

$$\varphi = \frac{nA}{l} \mu I_1 + k_1 \frac{A}{l} \mu I_2 \quad (3)$$

$k_1$  is a constant coefficient.

The relationship between eddy current and magnetic flux can be expressed as an approximate equation of [11],

$$I_2 = -k_2 \frac{l}{A} \frac{d^2 \varphi}{\sigma dt} \quad (4)$$

$k_2$  is a constant coefficient. The independent relation between the coil current and the magnetic flux can be obtained by substituting Equation (4) into Equation (3)

$$I_1 = \left( \frac{l}{nA\mu} \right) \left( \varphi + k_1 k_2 \mu \frac{d^2 \varphi}{\sigma dt} \right) \quad (5)$$

Therefore, the input power per unit volume is

$$P_{in} = \frac{UI_1}{Al} = \frac{nI_1 d\varphi}{Al dt} = \left( \frac{1}{A^2} \right) \left[ \frac{\varphi d\varphi}{\mu dt} + k_1 k_2 \frac{d^2 \left( \frac{d\varphi}{dt} \right)^2}{\sigma} \right] \quad (6)$$

where

$$\begin{aligned} \frac{\varphi d\varphi}{\mu dt} &= \left( \frac{\varphi}{\mu} \right) \frac{d(\mu\varphi/\mu)}{dt} = \left[ \left( \frac{\varphi}{\mu} \right) \frac{d\mu}{dt} + \mu \frac{d(\varphi/\mu)}{dt} \right] \left( \frac{\varphi}{\mu} \right) \\ &= \frac{1}{2} \left( \frac{\varphi}{\mu} \right)^2 \frac{d\mu}{dt} + \frac{1}{2} \frac{d(\varphi^2/\mu)}{dt} \end{aligned} \quad (7)$$

## 2. Physical Core-Loss Equations

The primary part of Equation (7) denotes the inner power of magnetic flux and the secondary part is usually ignored. Actually the permeability should be dependent by time in real cases, the increased permeability presents a loss power in the equation by an irreversible transformation of heat. This phenomenon is similar to the resistivity of the core which produces the eddy-current loss, here the called permeability loss is one kind of core loss and consists of hysteresis loss and excess loss as usual definitions. To integrate the permeability loss and eddy loss, the core loss shows as Equation (8).

$$\begin{aligned} P_{core} &= \frac{1}{2} \left( \frac{1}{A^2} \right) \left( \frac{\varphi}{\mu} \right)^2 \frac{d\mu}{dt} + \left( \frac{1}{A^2} \right) k_1 k_2 \frac{d^2 \left( \frac{d\varphi}{dt} \right)^2}{\sigma} \\ &= -\frac{1}{2} B^2 \frac{d(1/\mu)}{dt} + k_1 k_2 \frac{d^2 \left( \frac{dB}{dt} \right)^2}{\sigma} \end{aligned} \quad (8)$$

Under a periodic magnetic field, It can be reasonably predicted that the variation of  $(1/\mu)$  should be periodic and synchronized with the magnetic flux density. The core losses for most of magnetic materials are positive, therefore the increasing of  $(1/\mu)$  will happens on the ranges of large magnetic flux density, and the decreasing of  $(1/\mu)$  will happens on the ranges of magnetic flux density close to zero. By the above hypothesis, a simplified permeability loss equa-

tion is suggested as following:

$$P_{\text{perm}} = c_{\text{perm}} \left| \frac{d^2 B}{dt^2} \right|^{n_1} |B|^{n_2} \quad (9)$$

$c_{\text{perm}}$  is a constant coefficient of the simplified permeability loss equation,  $n_1$ ,  $n_2$  are constant index coefficients.

However, the hypothesis simplification of Equation (9) will not approach accurately enough to the real measuring data by curve fitting, especially on the range of lower frequencies and lower magnetic densities. The phenomenon is similar to hysteresis loss, which exists a saturation limitation and can be approached accurately by the following frequency-domain equation:

$$E_{\text{hyst}} = k_3 \left[ 1 - \exp \left( - \left| \frac{\widehat{B}}{B_0} \right|^m \right) \right] \quad (10)$$

$k_3, m$  is the constant coefficients.  $B_0$  presents a magnetic point for the saturation of hysteresis loss. The frequency-domain loss energy can be transferred into the time-domain power by differential:

$$P_{\text{hyst}} = \frac{dE_{\text{hyst}}}{dt} \approx c_{\text{hyst}} \exp \left( - \left| \frac{B}{B_0} \right|^m \right) \left( \left| \frac{B}{B_0} \right|^{m-1} \right) \left| \frac{dB}{dt} \right| \quad (11)$$

$c_{\text{hyst}}$  is the constant coefficients of hysteresis loss equation.

Finally, the tolerance of the theoretic eddy-loss equation can be improved as adding extra one parameter of an exponential magnetic density term to compensate the nonlinear effects:

$$P_{\text{eddy}} = c_{\text{eddy}} \frac{d^2}{\sigma} \left( \frac{dB}{dt} \right)^{r_1} B^{r_2} \quad (12)$$

$c_{\text{eddy}}$  is the constant coefficients of eddy loss equation.  $r_1, r_2$  are constant index coefficients.

The core loss equation becomes to three parts including eddy loss, permeability loss and hysteresis loss,

$$P_{\text{core}} = P_{\text{eddy}} + P_{\text{perm}} + P_{\text{hyst}} \quad (13)$$

The simplified permeability loss  $P_{\text{perm}}$  replaces the excess loss term of the traditional core loss equation with the physical and mathematic interpretations.

### 3. Simulation Results

65CS400 of China Steel Corporation is a high-grade magnetic steel sheet, composed of Si-Fe-Al alloy and 0.65 mm thickness with good efficiency and permeability properties to be applied at high efficient motors and transformers. Compared to normal grade productions, 65CS400 provides more complete loss data measured in laminated structure. **Table 1** shows the measured core loss by Epstein test in the range of frequency 50 - 5000 Hz and magnetic flux density of 0.1 - 1.8 T.

**Table 1.** The core-loss data of 65CS400 measured by Epstein test.

50 Hz		60 Hz		100 Hz		200 Hz		400 Hz		1000 Hz		2500 Hz	
T	W/kg	T	W/kg	T	W/kg	T	W/kg	T	W/kg	T	W/kg	T	W/kg
0.14	0.04	0.133	0.05	0.1	0.06	0.1	0.16	0.093	0.43	0.105	2.17	0.109	9.87
0.25	0.13	0.234	0.15	0.2	0.22	0.2	0.61	0.222	2.11	0.213	7.82	0.209	31.67
0.384	0.27	0.36	0.31	0.3	0.45	0.3	1.24	0.327	4.2	0.295	14.19	0.314	68.75
0.521	0.46	0.492	0.53	0.4	0.75	0.4	2.07	0.424	6.72	0.412	26.89	0.403	116.36
0.65	0.66	0.62	0.79	0.5	1.11	0.5	3.09	0.508	9.47	0.49	38.35		
0.753	0.85	0.732	1.05	0.6	1.54	0.6	4.33	0.618	13.97	0.592	57.94		
0.851	1.05	0.824	1.29	0.7	2.02	0.7	5.82	0.717	18.99	0.695	83.44		
0.937	1.25	0.918	1.56	0.8	2.58	0.8	7.58	0.805	24.36	0.82	123.86		
1.002	1.41	0.986	1.78	0.9	3.22	0.9	9.64	0.91	32.05	0.928	168.95		
1.097	1.67	1.09	2.15	1.0	3.95	0.999	12.02	1.006	40.35	1.026	219.04		
1.206	2.0	1.205	2.61	1.1	4.78	1.1	14.78	1.115	51.51	1.162	302.33		
1.303	2.33	1.301	3.05	1.2	5.71	1.199	17.95	1.212	63.14				
1.403	2.74	1.403	3.59	1.3	6.76	1.3	21.56	1.301	75.32				5000 Hz
1.503	3.22	1.502	4.22	1.4	7.94	1.4	25.47	1.421	93.89				T W/kg
1.605	3.73	1.606	4.91	1.5	9.33	1.5	29.92	1.487	105.25			0.107	28.2
1.7	4.18	1.7	5.5									0.209	92.27
1.794	4.6	1.794	6.09									0.29	182.02

There are 8 coefficient variables needed to be converged in Equation (13). The convergence error  $\delta$  is calculated by using the root mean square error method,

$$\delta^2 = \sum \left( \frac{P_{\text{cal}} - P_{\text{meas}}}{P_{\text{meas}}} \right)^2 \quad (14)$$

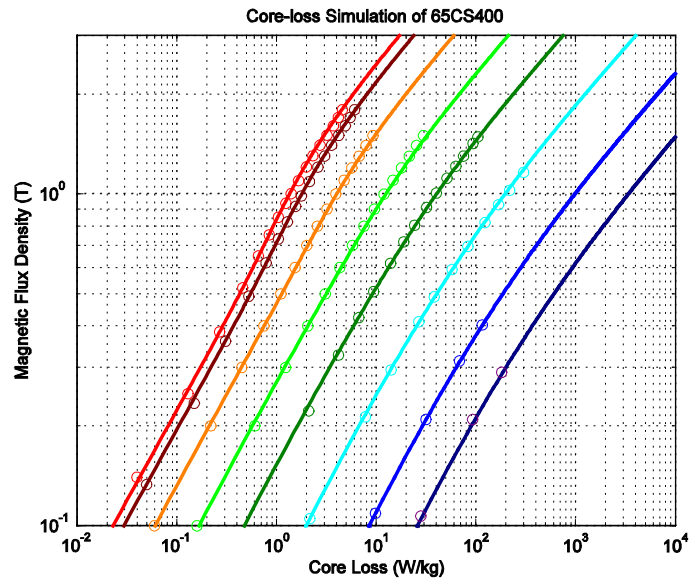
$P_{\text{cal}}$  is the calculated value of core-loss equation,  $P_{\text{meas}}$  is the measured core-loss data of 65CS400. The converging process is to adjust the 8 coefficient variables step by step to make error  $\delta$  minimum. Here sheet thickness  $d$  is 0.65 mm, the resistivity  $\sigma$  of 65CS400 is assumed as  $55e-8 \Omega \cdot m$ , the density is  $7800 \text{ kg/m}^3$ . The data of 50, 100, 200, 400, 1000, 2500 and 5000 Hz is applied in the converging process, the final optimum coefficients are converged to  $c_{\text{eddy}} = 0.84073$ ,  $r_1 = 1.62$ ,  $r_2 = -0.029$ ,  $c_{\text{perm}} = 0.198$ ,  $n_1 = 0.93$ ,  $n_2 = 2.15$ ,  $c_{\text{hyst}} = 55.5$ ,  $B_0 = 0.58$ ,  $m = 2.11$ , and the average error  $\delta$  is 3.38%.

**Figure 2** shows the simulation results of physical core-loss equation in frequency domain, the line curves preset the periodic average core losses of 50, 60, 100, 200, 400, 1000, 2500 and 5000 Hz from left to right, and the circles present the measured core losses.

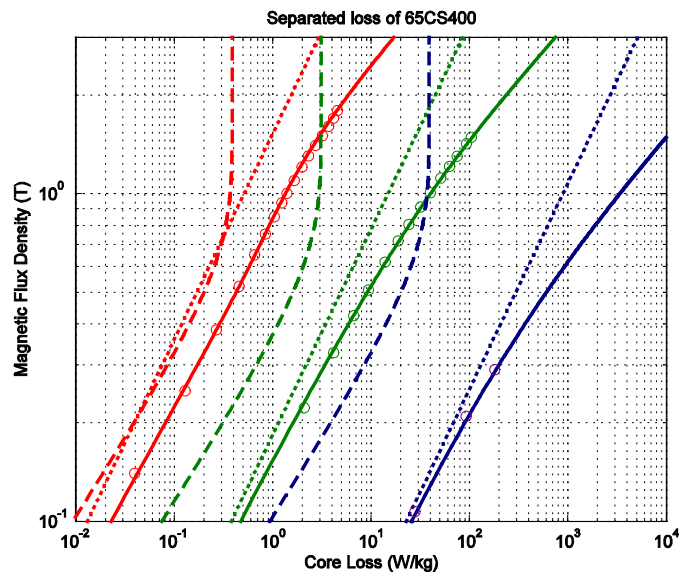
**Figure 3** shows the each parts of total core loss, hysteresis loss and eddy loss in frequency 50, 400 and 5000 Hz from left to right, the dash lines present the

hysteresis losses for each frequency, the dot lines present the eddy losses for each frequency.

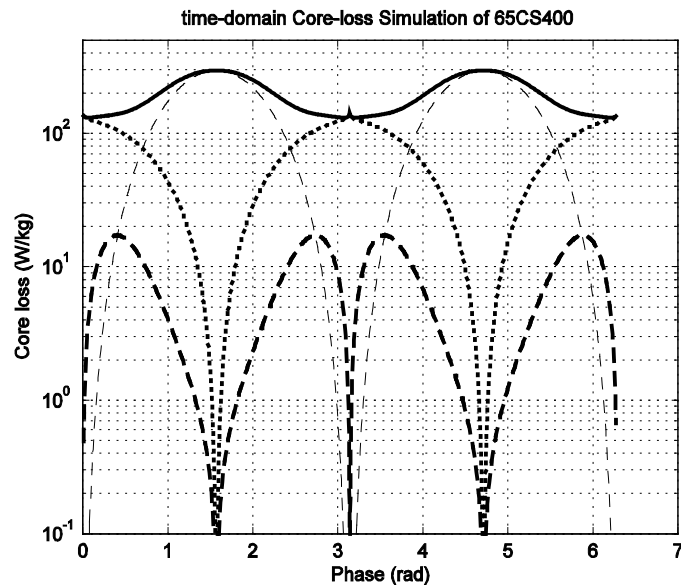
**Figure 4** presents the time-series core loss of 1000Hz-1.0T for one period, the solid line shows the total core loss, the dot line shows the eddy loss, the dash line shows the hysteresis loss and the fine line shows the permeability loss, the maximum loss exists at the peak area of the magnetic density.



**Figure 2.** Simulation of physical core-loss equation in frequency domain, the line curves preset the periodic average core losses of 50, 60, 100, 200, 400, 1000, 2500 and 5000 Hz from left to right, and the circles present the measured core losses.



**Figure 3.** The total core loss, hysteresis loss and eddy loss in frequency 50, 400 and 5000 Hz from left to right, the dash lines present the hysteresis losses for each frequency, the dot lines present the eddy losses for each frequency.



**Figure 4.** Time-series core loss of 1000 Hz - 1.0 T for one period, the solid line shows the total core loss, the dot line shows the eddy loss, the dash line shows the hysteresis loss and the fine line shows the permeability loss.

#### 4. Conclusions

A interesting phenomenon is found when we apply the physical core-loss equation to several electrical steel sheets of China Steel Corporation, the optimum value of coefficient  $m$  of the hysteresis-loss equation is equal to 2 precisely, and the others approach a very small difference from the optimum when  $m$  is fixed at 2. For example of 65CS400, the coefficients of fixed  $m = 2$  are converged to  $c_{\text{eddy}} = 0.82953$ ,  $r_1 = 1.63$ ,  $r_2 = -0.007$ ,  $c_{\text{perm}} = 0.192$ ,  $n_1 = 0.93$ ,  $n_2 = 2.15$ ,  $c_{\text{hyst}} = 53$ ,  $B_0 = 0.62$ , and the average error  $\delta$  of 3.41% is very closed to the optimum 3.38%.  $m$  fixing at 2 which means the hysteresis loss is only related to magnetic energy in physical explanation. To reduce one variable makes the converging process easier and faster.

The physical core-loss equation can serve many kinds of machines from the low-speed motors to high-frequency pulse transformers. It performs so good predictability for wide range of frequencies and magnetic densities, therefore we can consider that each coefficient of the equation might be associated with the physical properties, such as alloy composition, grain size or sheet thickness and so on. To study the influence of sheet thickness on the 8 coefficients will be our next subject.

#### Acknowledgements

I would like to acknowledge Metal Industries Research & Development Centre which collectively funded this project.

#### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

## References

- [1] Steinmetz, C.P. (1984) On the Law of Hysteresis. *Proc IEEE*, **72**, 196-221. <https://doi.org/10.1109/PROC.1984.12842>
- [2] Reinert, J., Brockmeyer, A. and Donker, R.W.D. (1999) Calculation of Losses in Ferro- and Ferromagnetic Materials Based on the Modified Steinmetz Equation. *Proceedings of 34th Annual Meeting of the IEEE Industry Applications Society*, **3**, 2087-2092. <https://doi.org/10.1109/IAS.1999.806023>
- [3] Li, J.L., Abdallah, T. and Sullivan, C.R. (2001) Improved Calculation of Core Loss with Nonsinusoidal Waveforms. *Conference Record of the 2001 IEEE Industry Applications Conference. 36th IAS Annual Meeting*, 2203-2210. <https://doi.org/10.1109/IAS.2001.955931>
- [4] Venkatachalam, K., Charles, R.S., Abdallah, T., *et al.* (2002) Accurate Prediction of Ferrite Core Loss with Nonsinusoidal Waveforms Using Only Steinmetz Parameters. *COMPEL 2002: 8th IEEE Workshop on Computers in Power Electronics*.
- [5] Bossche, A.P., Syde, D.M. and Valchev, V.C. (2005) Ferrite Loss Measurement and Models in Half Bridge and Full Bridge Waveforms. *Power Electronics Specialists Conference*, 1535-1539.
- [6] Chen, K.F. (2009) Iron-Loss Simulation of Laminated Steels Based on Expanded Generalized Steinmetz Equation. *Asia-Pacific Power and Energy Engineering Conference*, 1-3.
- [7] Chen, K.F. (2010) The Core Loss Prediction for Laminated Steels. *4th International Conference Magnetism and Metallurgy WMM10*.
- [8] Mühlethaler, J., Biela, J., Kolar, J.W., *et al.* (2012) Improved Core-Loss Calculation for Magnetic Components Employed in Power Electronic Systems. *IEEE Transactions on Power Electronics*, **27**, 964-973. <https://doi.org/10.1109/TPEL.2011.2162252>
- [9] Hiroaki, M., Toshihisa, S., Takashi, K., *et al.* (2020) Core Loss Calculation for Power Electronics Converter Excitation from a Sinusoidal Excited Core Loss Data. *Magnetism and Magnetic Materials*.
- [10] Sobhi, B. and Kent, B. (2021) Core Loss Calculation of Symmetric Trapezoidal Magnetic Flux Density Waveform. *IEEE Open Journal of Power Electronics*, 627-635. <https://doi.org/10.1109/OJPEL.2021.3133929>
- [11] Fiorillo, F. (2004) *Measurement and Characterization of Magnetic Materials*. Elsevier Academic Press.



---

## Symbol Definition

$\phi$  : The magnetic flux inside the magnetic core

$B$  : The magnetic flux density

$\hat{B}$  : The peak value of magnetic flux density for frequency domain

$f$  : The frequency of periodic magnetic field

$\mu$  : The magnetic permeability of the magnetic core

$\sigma$  : The electrical resistivity of the magnetic core

$A$  : The section area of the magnetic core

$l$  : The loop length of the magnetic core

$d$  : The sheet thickness of the magnetic core

$n$  : The turn number of the coil

$U$  : The coil voltage

$I_1$  : The coil current

$I_2$  : The eddy current of the magnetic core

$P_{in}$  : The input power of unit volume

$P_{inner}$  : The magnetic inner power of unit volume

$P_{eddy}$  : The eddy loss of unit volume

$P_{perm}$  : The simplified magnetic permeability loss of unit volume

$P_{hyst}$  : The magnetic hysteresis loss of unit volume

$P_{core}$  : The dynamic core loss of unit volume

$E_{hyst}$  : The hysteresis loss energy