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Selenu's Phase through Statistical Density Matrix

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Abstract

In this article, a statistical approach to the calculus of quantum Selenu's phase on a measured data set of expectation values of quantum operators is reported, to calculate the density matrix of a quantum system. A new mathematical model, based on the calculus of groupal derivatives [1] of expectation values shows density matrix to be the integrated correlation Wigner density function of quantum states.

Keywords

Electron Field, Theoretical Physics

1. Introduction

Statistical mechanics is founded on the density matrix [2], which is used to calculate the mean values of observables. These mean values, interpreted as the expectation values of measured data sets [3]-[5], constitute a generalization of the calculus of mean values to arbitrary operators. On a quantum framework [6]-[8] of the description of quantum correlated systems, a mathematical model is developed to calculate quantum Selenu's phase [1] [9] [10] of a quantum field system, at constant number of particles when a statistical frequency of states $|\Psi\rangle$, $\langle\Psi|\Psi\rangle$ is equal to a constant. When electron field propagation is concerned on a statistic scattering process, a new method is developed based on the use of groupal derivatives. Groupal derivatives of expectation values are admissible to an integrated density matrix [3] [5], allowing to the avoidmnet of classical quantum gauge field theory [11]-[13], due to the developing in the system of a momentum linear connection [9] correlating quantum states, making variations of the quantum field phase, extending Aharonov-Bohm effect to correlated matter systems. Conclusions of the article will be reported in the last section and let the next section to the writing of the expression of quantum Selenu's phase as a functional [2] of the Wigner correlation density function.

2. Groupal Derivative as a Density Matrix

In this section is reported the expression of the Wigner correlation density matrix of quantum states to perform the calculus of expectation values of arbitrary quantum operators $\hat{A}$. A statistical data modeling, of measured data sets of expectation values of quantum operators, is studied on a quantum dephasing of quantum field states in space. Calculated Selenu's quantum phase, of the statistical scattering process on a data sampling, is written the latter as a functional of the density matrix. Latter operators are not always commuting with the Hamiltonian operator of the quantum system, and expectation values of field states are calculated, initially varying on a discrete indexing of quantum states then generalized to a continuous indexing. Representing space coordinates on variations of the quantum phase, expectation values of quantum operators are written with respect to the continuous indexing, being the latter a random value of an n dimensional coordinate space. Mean values are then calculated as linear combinations of expectation values on a tensorial form:

$$\langle i|\hat{A}|j\rangle = \Pi_{i,j} \otimes \mathbf{A} \tag{1}$$

being written towards expected values on a matrix operator $\mathbf{A} = \{A_{n,l}\}$ [14]-[16] of a measured data set and $\Pi_{i,j} = \{\rho_{i,j,n,l}\}$ written as a generalized density matrix, will be shown in the continuous limit of space coordinates to be the Wigner correlation density function. Expectation values between quantum states $|i\rangle$ and $|j\rangle$ are calculated in the following equation:

$$\langle i|\hat{A}|j\rangle = \sum_{m,k} \langle i|m\rangle \langle m|\hat{A}|k\rangle \langle k|j\rangle = \sum_{m,k} \rho_{i,j,m,k} A_{m,k} \tag{2}$$

showing possible to perform groupal derivatives [10] of mean values and calculate the discrete correated density matrix between quantum states $|m\rangle$ and $|k\rangle$, from the two reference frame states [17] $|i\rangle$, $|j\rangle$:

$$D\langle i|\hat{A}|j\rangle = \sum_{m',l} \sum_{m,k} \rho_{i,j,m,k} \frac{\mathrm{d}A_{m,k}}{\mathrm{d}A_{m',l}} = \rho_{i,j,m,k} \tag{3}$$

During the statistical scattering process is determined a correlation density matrix, when quantum states acquire a quantal phase factor, between scattered states and reference frame states:

$$\begin{aligned} D\langle i|\hat{A}|j\rangle &= \langle i|m\rangle \langle k|j\rangle \\ &= \langle m|\mathrm{e}^{-i\phi_{i,m}}|m\rangle \langle k|\mathrm{e}^{i\phi_{j,k}}|k\rangle \\ &= \Delta N \mathrm{e}^{i[\phi_{j,k} - \phi_{i,m}]} \end{aligned} \tag{4}$$

on an observation time T of the scattering process itself, being also $\Delta N = \langle m|m\rangle \langle k|k\rangle = 1$ for normalized states $\langle k|k\rangle = 1$. Relative change of quantum Selenu's phase is then calculated and reported in the following equation:

$$\mathbf{Im} \ln D\langle i|\hat{A}|j\rangle = [\phi_{j,k} - \phi_{i,m}] \tag{5}$$

correlating quantum states m,k and i,j. The study of quantum statistical processes on real space makes quantum states then dependent on space coordinates generalizing quantum field theory approach, on the calculus of mean values [9], as considered to refer to a discrete indexing of a data sampling of space coordinates, extended then to a successive the limit of continuous data sets of space coordinates:

$$\mathbf{Im}\ln D\left\langle(\mathbf{x}_0-\mathbf{x})\left|\hat{A}\right|(\mathbf{x}_0-\mathbf{x}')\right\rangle=\left[\phi_{\mathbf{x}_0,\mathbf{x}'}-\phi_{\mathbf{x}_0,\mathbf{x}}\right] \tag{6}$$

When is the case of the same reference state $\left|(\mathbf{x}_0)\right\rangle$ being chosen in order to evaluate mean values of quantum operators $\hat{A}$, the continuous limit in space coordinates of the quantum phase variation is calculated as the quantum Selenu's phase, reported in the following equation:

$$\Delta\Phi_s=\mathbf{Im}\ln D\left\langle(\mathbf{x}_0-\mathbf{x})\left|\hat{A}\right|(\mathbf{x}_0-\mathbf{x}')\right\rangle \tag{7}$$

being the quantum phase equal to zero only for not correlated states evaluated at points $\mathbf{x}_0=\mathbf{x}$, in fact:

$$\Delta\Phi_s=\mathbf{Im}\ln\lim_{\mathbf{x}'\to\mathbf{x}}\left[\rho_{\mathbf{x}_0,\mathbf{x},\mathbf{x}'}\right]=0 \tag{8}$$

showing instead to be a functional of the correlated density [2] matrix by Equation (7):

$$\rho=\left[\rho_{\mathbf{x}_0,\mathbf{x},\mathbf{x}'}\right] \tag{9}$$

The scattering process is then written on a new statistical quantum field theory [6]-[8] directly related to Selenu's quantum phase written as a functional [2] of the Wigner correlation density [2] function,

$$\rho(\mathbf{x}_0-\mathbf{x},\mathbf{x}_0-\mathbf{x}')=D\left\langle(\mathbf{x}_0-\mathbf{x})\left|\hat{A}\right|(\mathbf{x}_0-\mathbf{x}')\right\rangle \tag{10}$$

making expectation values evaluatable through Wigner correlation function. Groupal derivatives of mean values $\left\langle\hat{A}\right\rangle$, between correlated states evaluated on points in space along the scattering process through integration of the groupal derivative are also written,

$$\Delta\Phi_s=\mathbf{Im}\ln\left[\rho(\mathbf{x}_0-\mathbf{x},\mathbf{x}_0-\mathbf{x}')\right] \tag{11}$$

making able to calculate quantum phases $\Delta\Phi_s$, on new gauge transformations of correlated states $\left|\Psi\right\rangle'=\mathrm{e}^{i\Delta\Phi_s}\left|\Psi\right\rangle$, generalizing then Aharonov-Bohm effect [9], let to conclude the article in the next section.

3. Conclusion

The article is concluded having shown the existence on a scattering process of the quantum Selenu's phase written as a functional of the quantum Wigner correlation density function, on generalizing gauge transformations to correlated quantum states.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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