Journal of Modern Physics, 2025, 16(12), 1878-1912
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

The Quantum Gravity Wave Function

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How to cite this paper: Klingman, E.E. (2025) The Quantum Gravity Wave Function. *Journal of Modern Physics*, **16**, 1878-1912.
https://doi.org/10.4236/jmp.2025.1612086

Received: October 10, 2025
Accepted: December 21, 2025
Published: December 24, 2025

Abstract

The goal of this paper is to define the quantum gravity wave function, in a manner consistent with current experiments, and to discuss interpretation of the imaginary number, "i". A gauge field formulation is developed and an energy-density encoding used to derive a curved space formulation of gravity. A primordial field-based approach enables us to derive a formulation compatible with general relativity and quantum mechanics.

Keywords

Quantum Gravity, de Broglie, Schrödinger, Langevin, Heaviside, Primordial Field, Particle/Wave Dualism

1. Introduction

The topic of quantum gravity is currently of great interest. Theories range from "*atoms of geometry*" to "*causal sets*", "*string theory with Ads/CFT* etc.", "*causal dynamical triangulation*", and more, as treated in [1]. After a century of effort to provide such, the nature of the quantum mechanical wave function itself is still debated [2]. The goal of this paper is to provide a physical theory of the quantum gravity wave function that is intuitively comprehensible without the need to include "mystical" aspects that go far beyond the formalism in which theory is expressed. The century from 1926 to 2026 was filled with speculation on the nature of the wave function:

[1926] Heisenberg: "*the state of a system is given by its wave function, … the question* [is] *whether the latter should be seen as a 'spread-out' entity, a 'guiding field', a 'statistical state', or something else.*"

[2012] *Physical Review Letters*: [3] "*The wave function* Ψ *is at the heart of quantum mechanics, yet its nature has been debated since its inception.*"

A key question is "*What is the wave function 'waving' in*?" The wave concept originally involved a field, or medium defined at each point in space. Heisenberg's matrix mechanics did not use fields, and Schrödinger's wave mechanics did not specify the nature of the field in which the wave function propagates. Second-quantized quantum mechanics invokes a "*field per particle*", with "ladder-like operators" of the harmonic oscillator-type employed, but these *fields* have never been measured, and their ontological status is uncertain. The field of gravity, on the other hand, has been identified with "curvature of space-time" and the basic property of a real physical field, it's energy density, vanishes, according to the *Principle of Equivalence*, in a specific coordinate frame, leading to the paradoxical concept of *quasi-local mass* [4]. This is the background in which the quantum gravity wavefunction must be developed.

2. Primordial Field Theory of Quantum Gravity

Unlike Quantum Mechanics (QM) with unspecified field or Quantum Field Theory (QFT) with its "field per particle", Primordial Field Theory (PFT) assumes that only *one* field (and *no particles*) existed in the beginning. The Standard Model of Particle Physics (SM) assumes that *all* forces (gravity, electromagnetism, weak, strong) merge into *one* force at the Big Bang, but SM is unable to demonstrate this. If today's forces are considered to merge to one, then the converse must hold that the force field present at the beginning, the *primordial field*, should evolve into today's physical reality. The nature of the Primordial Field is unspecified, so we postulate it as follows:

The Primordial Principle of Self-Interaction

Postulate: In the beginning, a primordial field, *and nothing else*, existed; implying the *Self-Interaction Principle*, summarized as:

Axiom*: *If nothing else exists to interact with, the primordial field can only interact with itself.

A singular field, without particles, will have energy, hence equivalent inertial mass. The field is considered to be a perfect fluid, compatible with the 2006 discovery at the Large Hadron Collider that the highest energy collisions produce a perfect fluid rather than the expected "quark gas". Quantum field theory is based on the concept of multiple fields, one per particle type, with each particle type being instantiated as an excitation of the corresponding field, so QFT does not apply to the primordial universe. PFT, like Einstein, assumes that vacuum does not exist as pure space and time; space and time are essential qualities of a fundamental field, assumed to be gravity in general relativity. Our ontological foundation is thus a primordial field as the fundamental substance of the universe, and we formulate our theory in terms of such.

3. Geometric Calculus and Imaginary *i* in Quantum Mechanics

The goal is to create a physical model, or physical theory of reality, based on minimal semantic knowledge of physics at the time of creation and on the most effective syntax, mathematics. For a world with both logical relationships and shapes, the only case in which *every* mathematical term has both an algebraic *and* a geometric interpretation is Hestenes' *Geometric Calculus* [5], with its fundamental theorem on a smooth m-dimensional manifold M with boundary ∂M:

$$\int_M \mathrm{d}^m x \partial F = \oint_{\partial m} \mathrm{d}^{m-1} x F \tag{1}$$

This theorem is compatible with and contains Gauss's theorem, Stokes theorem, Green's theorem, and the Cauchy integral formula, in coordinate-free formalism. Types of geometric algebra entities in a (3 + 1)D universe are *scalar*, *vector*, *bivector*, and *trivector* or *pseudo-scalar*.

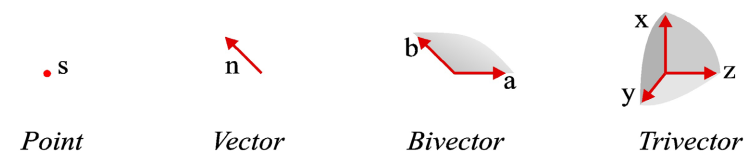


Figure 1. The fundamental objects in (3 + 1)D geometric algebra formulation are depicted, each of which has a corresponding algebraic term.

The fundamental geometric objects shown in Figure 1 have corresponding algebraic terms labelled in the figure and discussed below. An extraordinary aspect of geometric algebra is that the terms are additive; equations with scalars (points), vectors, bivectors and trivectors are valid and meaningful. Thus, geometric algebra partitions 3-space perfectly. One solves for these terms by separating them in the same manner that reals and imaginaries are kept separate in complex analysis. In this multi-element context, our physics model can represent a potential (point), a force or momentum (vector), angular momentum (bivector) and 3D volume (the trivector). Thus, *Geometric Algebra* is ideally suited to describe the physical universe; the fundamental theorem of calculus contains all of the key theorems and formulas of physics of fields and particles. It also introduces a fundamental geometric algebra operation: the *geometric product* of two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ is:

$$\boldsymbol{uv} = \boldsymbol{u} \cdot \boldsymbol{v} + \boldsymbol{u} \wedge \boldsymbol{v} . \tag{2}$$

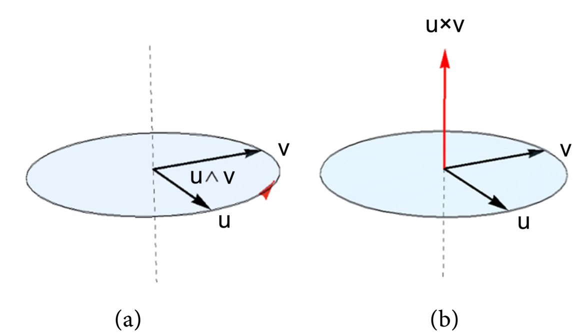


Figure 2. (a) Wedge product; (b) Cross product.

The geometric product of two vectors yields a multi-vector consisting of inner product $\boldsymbol{u} \cdot \boldsymbol{v}$, which is a scalar projecting one vector into the other, and outer product $\boldsymbol{u} \wedge \boldsymbol{v}$, which is a *bivector*, a directed area representing the rotation of $\boldsymbol{u}$

into $\boldsymbol{v}$. The scalar product is the vector dot product. The bivector can be related to the vector cross product as seen in Figure 2. Bivector $\boldsymbol{u} \wedge \boldsymbol{v}$ is a *directed area* with no defined shape, while $\boldsymbol{u} \times \boldsymbol{v}$ is an *axial vector*, which is *not* included as a vector in vector analysis, but which is required to represent magnetic fields.

Geometric Calculus allows another aspect of QM, that of complex numbers. A current paper [6] states: "*The role of complex quantities in quantum mechanics has been puzzling physicists since the beginnings*." They conclude that Real Quantum Mechanics reproduces the correlations observed in experiments only if some amount of entanglement exists before the experiment, an ontologically excessive requirement, noting however that others successfully reformulate complex QM as real. We interpret complex QM in terms of Hestene's duality operator, appearing in:

$$\boldsymbol{u} \wedge \boldsymbol{v} = i\boldsymbol{u} \times \boldsymbol{v} \tag{3}$$

The ***duality operator i*** transforms cross product into wedge, or vice versa: $\boldsymbol{u} \times \boldsymbol{v} = -i\boldsymbol{u} \wedge \boldsymbol{v}$. An axial vector or vector cross product's rotational aspect is a bivector. Interpretation of "i" as duality operator transforming one geometric algebra entity to its dual is different from identification with $\sqrt{-1}$ of complex analysis. Compare to a reviewer's statement: "*More importantly, the meaning of* 'i' *is that of a* ***generator of rotation*** (***Argand geometric*** *interpretation*), *which in Quantum Physics is* ***quantum phase*** $exp(i\omega t)$, *with* 't' *the* ***relativistic proper time***" and *quantum phase* considered as the proper, periodic **order parameter**, "*essential in the wave function or Feynman* ***Path Integral formalism*** *and hinted at by* ***de Broglie* '*pilot wave*'** *concept, as an* '***Einstein clock***' ***with one dial***, *carried by any* '*elementary particle*'". These eight *formal* terms (in bold) are <u>all</u> *abstract*, understood in the formalism of quantum physics. We will show that physical <u>*circulation*</u> of an inertial field related to its <u>*flow*</u> by "i" is intuitively comprehensible, leading to two real entities, *circulation* and *flow*, and one abstraction, *duality operator*, in comparison to the eight abstractions. This allows ontic interpretation of duality operator "i", in the QM formalism.

Also in geometric calculus, as in vector calculus, the *derivative operator* ∇ is viewed as a vector; the geometric product of ∇ with field $\boldsymbol{f}$ is as follows:

$$\nabla \boldsymbol{f} = \nabla \cdot \boldsymbol{f} + \nabla \wedge \boldsymbol{f} \qquad \textit{gradient} = \textit{divergence} + \textit{curl} \tag{4}$$

Relation *gradient* = *divergence* + *curl* is true only in Hestenes' ***Geometric Algebra*** and ***Calculus***. I believe the above aspects best suit physics and use them to analyze primordial field theory.

4. The Primordial Self-Interaction

The fundamental formal approach to physics assumes that change occurs due to physical interactions. *In PFT there is nothing to interact with other than the field* ψ *itself*, hence fundamental change in ψ, denoted by $\nabla\psi$ consists of the interaction of ψ with itself, written $\psi\psi$. The fundamental *dynamic* relation of the corresponding to the Self-interaction Principle of the primordial field is syntactically expressed as

$$\nabla \psi = \psi\psi \tag{5}$$

formulated in terms of a change operator $\nabla \sim \frac{\partial}{\partial \xi}$ with ξ a parametric aspect of the physical entity, denoted by $\psi = \psi(\xi)$, and change in ψ is written $\nabla \psi$. For parametric aspect ξ the equation has a scalar solution $\psi(\xi) = -\xi^{-1}$. If ξ is a vector, we can let $\psi\psi = \psi \cdot \psi$ and derive $\psi(\boldsymbol{\xi}) = \boldsymbol{\xi}^{-1}$. Physically, we interpret the scalar parameter as time *t* and vector parameter as position $\boldsymbol{r}$. If we apply normal field relations, the term $\psi\psi = \psi^2$ is interpreted as field energy density ρ yielding $\nabla \psi = \rho$. If field ψ is gravity $\boldsymbol{G} \sim \boldsymbol{r}^{-1}$, which has negative energy density, then $\boldsymbol{\nabla} \cdot \boldsymbol{G} = \boldsymbol{G} \cdot \boldsymbol{G}$ and

$$\boldsymbol{\nabla} \cdot \boldsymbol{G} = -\rho \tag{6}$$

reduces to Newton's equation, which recovers one of the forces from the primordial field. But Equation (5) was not expressed as inner product $u \cdot v$; the self-interaction equation is $\nabla \psi = \psi\psi$, where ∇ is the difference operator acting on the field ψ, assumed equivalent to the local field interacting with itself, so use geometric product $ab = a \cdot b + a \wedge b$ and duality operation $a \wedge b = -i(a \times b)$. Following electromagnetics ($\boldsymbol{E} + i\boldsymbol{B}$) we assume $\psi = \boldsymbol{G} + i\boldsymbol{C}$ and since the solutions are additive, $\nabla = \boldsymbol{\nabla} + \partial_t$, Equation (5) becomes

$$\nabla \boldsymbol{\psi} = \boldsymbol{\psi\psi} \Rightarrow (\boldsymbol{\nabla} + \partial_t)(\boldsymbol{G} + i\boldsymbol{C}) = (\boldsymbol{G} + i\boldsymbol{C})(\boldsymbol{G} + i\boldsymbol{C}) \tag{7}$$

Given that $\boldsymbol{\nabla \psi} = \boldsymbol{\nabla} \cdot \boldsymbol{\psi} + \boldsymbol{\nabla} \wedge \boldsymbol{\psi}$ Equation (7) can be multiplied out *term by term*, all geometric products expanded, and *like terms grouped*: (scalars, *i**scalars, vectors, *i**vectors). This is straight-forward; first, multiply out all terms, noting that duality operator i commutes with all vectors.

$$\boldsymbol{\nabla G} + \partial_t \boldsymbol{G} + i\boldsymbol{\nabla C} + i\partial_t \boldsymbol{C} = \boldsymbol{GG} + i\boldsymbol{GC} + i\boldsymbol{CG} - \boldsymbol{CC} \tag{8}$$

Next, expand the geometric products on both sides and then group like terms.

$$\begin{aligned} &\boldsymbol{\nabla} \cdot \boldsymbol{G} + i\boldsymbol{\nabla} \times \boldsymbol{G} + \partial_t \boldsymbol{G} + i\boldsymbol{\nabla} \cdot \boldsymbol{C} - \boldsymbol{\nabla} \times \boldsymbol{C} + i\partial_t \boldsymbol{C} \\ &= \boldsymbol{G} \cdot \boldsymbol{G} + i\boldsymbol{G} \times \boldsymbol{G} + i\boldsymbol{G} \cdot \boldsymbol{C} - \boldsymbol{G} \times \boldsymbol{C} + i\boldsymbol{C} \cdot \boldsymbol{G} - \boldsymbol{C} \times \boldsymbol{G} - \boldsymbol{C} \cdot \boldsymbol{C} - i\boldsymbol{C} \times \boldsymbol{C} \end{aligned} \tag{9}$$

The cross product of a vector with itself is identically zero so we delete terms $\boldsymbol{G} \times \boldsymbol{G}$ and $\boldsymbol{C} \times \boldsymbol{C}$ and note that $\boldsymbol{G} \times \boldsymbol{C} + \boldsymbol{C} \times \boldsymbol{G} = 0$. Remaining terms are grouped by like terms: first group scalars; next scalars multiplied by the duality operator, then vector terms, finally vectors multiplied by i. Expansion of the self-interaction equation yields four equations derived from the self-interaction of the primordial field according to our *Self-interaction Principle*:

$$\boldsymbol{\nabla} \cdot \boldsymbol{G} = \boldsymbol{G} \cdot \boldsymbol{G} - \boldsymbol{C} \cdot \boldsymbol{C} \tag{10a}$$

$$i\boldsymbol{\nabla} \cdot \boldsymbol{C} = i2\boldsymbol{G} \cdot \boldsymbol{C} \tag{10b}$$

$$\partial_t \boldsymbol{G} - \boldsymbol{\nabla} \times \boldsymbol{C} = 0 \tag{10c}$$

$$i\boldsymbol{\nabla} \times \boldsymbol{G} + i\partial_t \boldsymbol{C} = 0 \tag{10d}$$

These are quite explicit, yet to proceed further we need to make use of what more we know of physical reality, such as that physical fields are *real* and *have energy*. Ohanian and Ruffini state [7]: "*The gravitational field may be regarded as* *the material medium sought by Newton*; *the field is material because it possesses an energy density*." The energy-momentum density of electro-magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$ is given by $E^2 + B^2 + \boldsymbol{E} \times \boldsymbol{B}$, so assume that $\boldsymbol{G} \cdot \boldsymbol{G}$ and $\boldsymbol{C} \cdot \boldsymbol{C}$ represent energy density and $\boldsymbol{G} \times \boldsymbol{C}$ is a Poynting-like vector interpreted as momentum density: $\rho \boldsymbol{v} \sim \boldsymbol{G} \times \boldsymbol{C} = \boldsymbol{p}$. Energy has mass equivalence so $\rho_E = \rho_m c^2$ and when $c = 1$ then $\rho_E = \rho_m$.

Self-Interaction equations	**Heaviside equations**	
$\nabla \cdot \boldsymbol{G} = \boldsymbol{G} \cdot \boldsymbol{G} - \boldsymbol{C} \cdot \boldsymbol{C}$	$\nabla \cdot \boldsymbol{G} = -\rho$	(11a)
$i\nabla \cdot \boldsymbol{C} = i2\boldsymbol{G} \cdot \boldsymbol{C}$	$\nabla \cdot \boldsymbol{C} = 0$	(11b)
$\partial_t \boldsymbol{G} - \nabla \times \boldsymbol{C} = \boldsymbol{G} \times \boldsymbol{C} \pm \boldsymbol{C} \times \boldsymbol{G}$	$\nabla \times \boldsymbol{C} = -\rho \boldsymbol{v} + \partial_t \boldsymbol{G}$	(11c)
$i\nabla \times \boldsymbol{G} + i\partial_t \boldsymbol{C} = 0$	$\nabla \times \boldsymbol{G} = -\partial_t \boldsymbol{C}$	(11d)

Grouping like terms we re-express the equations in terms of mass density $\rho = \boldsymbol{G} \cdot \boldsymbol{G} + \boldsymbol{C} \cdot \boldsymbol{C}$. We interpret $\boldsymbol{G}$ as gravitational field and $\boldsymbol{G} \cdot \boldsymbol{G}$ as *self-energy density* of the field. The energy is *negative* since we must apply *positive* energy to a body captured in a gravitational field in order for it to escape the field, hence mass density $\rho \sim -\boldsymbol{G} \cdot \boldsymbol{G}$ leads to $\nabla \cdot \boldsymbol{G} = -\rho$. Rotational energy in molecules is equivalent to mass [8], therefore positive circulation energy of the C-field yields the appropriate sign for the term $-\boldsymbol{C} \cdot \boldsymbol{C}$, contributing correctly to Newton's equation. Thus, the fundamental self-interaction equation yields Heaviside's equations [9] for gravitomagnetism. The term $\nabla \cdot \boldsymbol{C} = 0$ analogous to $\nabla \cdot \boldsymbol{B} = 0$ implies that no gravitomagnetic "monopole" exists. The field equation of most significance for quantum gravity is Heaviside's Equation (11c), which, ignoring local change in gravity (*i.e.*, $\partial G / \partial t = 0$) is

$$\nabla \times \boldsymbol{C} = -\rho \boldsymbol{v} \tag{12}$$

and which, like Newton's equation for ***G***, is density-based. Axial vector $\nabla \times \boldsymbol{C}$ in the Heaviside equations in standard format could have been transformed into $-i\nabla \wedge \boldsymbol{C}$, in which case Equation (12) would appear $\nabla \wedge \boldsymbol{C} = -i\boldsymbol{p}$ with momentum density $\boldsymbol{p} = \rho \boldsymbol{v}$. The curl or circulation of the C-field, representing angular momentum, is the duality transform of the inducing momentum density vector. From the perspective of quantum theory Planck's constant $\hbar$ is often interpreted as "action", or amount of energy transported for a time period, but $\hbar$ also has dimensions of angular momentum, equivalent to a bivector. Thus, *when we encounter duality operator* "i" *paired with* $\hbar$, *the result implies a momentum density vector at right angles to the* $\hbar$ *angular momentum* (*spin*). This is to be taken literally in a physical sense, as opposed to the concept of *imaginary time*, *it*, or the group-based quantum formalism of a "generator of rotation". This coupling of circulation of the field to the orthogonal axial vector inducing the circulating field is the preferred explanation of "i" from our quantum gravity perspective.

In 1916, *Einstein* treated gravity ***as if*** it were geometry, in which context Heaviside's equation can be derived through linearizing his equation, leading to the mistaken belief that Heaviside was only the "*weak field approximation*" to general relativity, instead of being *formally equivalent to GR*. Einstein's general relativity field equations are simply differential geometry until they make contact with real physics in the form of Newton's law of gravity, whereas Newton's law of gravity actually falls out of the equations of primordial field theory. Per Helmholtz, any bounded continuous vector field is uniquely determined by its divergence, curl, and boundary conditions. Vector fields in Euclidean space are composed of an irrotational field, a gradient of scalar potential φ, and a vortex field, the curl of vector potential $\boldsymbol{A}$:

$$\boldsymbol{F}(\boldsymbol{r}) = -\nabla\varphi(\boldsymbol{r}) + \nabla\times\boldsymbol{A}(\boldsymbol{r}) \tag{13}$$

5. Gauge Formalism of Primordial Field Equations

The time-independent gravitational field is irrotational, as shown by Michaelson-Gale [10], and from $\nabla\cdot\boldsymbol{C}=0$, we can use vector identity $\nabla\cdot\nabla\times\boldsymbol{A}=0$ to replace $\boldsymbol{C}$ with potential vector $\nabla\times\boldsymbol{A}$. Derivable from Equation (5) are the gauge field equations

$$\boldsymbol{C} = \nabla\times\boldsymbol{A}, \quad \boldsymbol{G} = -\nabla\phi - \partial_t\boldsymbol{A}, \quad \partial_t\phi + \nabla\cdot\boldsymbol{A} = 0 \tag{14}$$

The first two equations define the fields in terms of four-potential A; the last equation specifies the Lorenz gauge condition, $\partial_\mu A^\mu = 0$, with scalar potential $\phi = -m/r$, and vector potential $\boldsymbol{A}=\boldsymbol{v}$. Gauge relations initially held no physical meaning; the electromagnetic $\boldsymbol{E}$ and $\boldsymbol{B}$ fields could be measured and exhibited, the gauge field, not. Maxwell used gauge conditions to simplify calculations [Coulomb gauge: $\nabla\cdot\boldsymbol{A}=0$, Lorenz gauge: $\partial_\mu A^\mu = 0$]. Although full unification of gravitation, electromagnetism, the strong and weak nuclear forces, has not yet been achieved in the Standard Model, the four fundamental interactions are generated by a single principle, the gauge principle, therefore we analyze the gauge aspects of Heaviside's equations. We formulate gauge field four-potential $A = \{\phi, \boldsymbol{A}\}$. Since $\boldsymbol{G} = -\nabla\phi + \partial_t\boldsymbol{A}$ if ϕ is constant then $\boldsymbol{G} = \partial_t\boldsymbol{A}$, and since $\boldsymbol{G}$ is the acceleration of gravity, then $\boldsymbol{G} = \mathrm{d}\boldsymbol{v}/\mathrm{d}t \Rightarrow \boldsymbol{A} = \boldsymbol{v}$. $\boldsymbol{C} = \nabla\times\boldsymbol{A}$ so $\boldsymbol{C} = \nabla\times\boldsymbol{v}$ is dimensionally correct: $|\boldsymbol{C}| \sim t^{-1}$. With gravitational potential $\phi = -M/r$ the $\boldsymbol{G}$-field has spatial dependence $|\boldsymbol{G}| \sim r^{-2}$, which is correct for Newtonian mass. For the primordial field, as explained in the references, $|\boldsymbol{G}| \sim r^{-1}$. Physically, Newtonian mass is treated as entirely within the sphere of radius *r*, whereas the mass of the primordial gravitational field is based only on the portion of the field within the sphere. For local mass density ρ the interaction energy density of the field is $\boldsymbol{j}\cdot\boldsymbol{A}$ where $\boldsymbol{j} = \rho\boldsymbol{v}$. Heaviside current density $\boldsymbol{j}$ is momentum density $\boldsymbol{p} = \rho\boldsymbol{v}$; the interaction density of the field is $\boldsymbol{p}\cdot\boldsymbol{A} = \boldsymbol{p}\cdot\boldsymbol{v} = \rho v^2$. Analogous with electrodynamics, momentum $\boldsymbol{\pi}$ is defined in terms of the Lagrangian $\mathcal{L}$ as

$$\boldsymbol{\pi} = \frac{\partial\mathcal{L}}{\partial\frac{\mathrm{d}x}{\mathrm{d}t}} = \boldsymbol{p} + \rho\boldsymbol{A}(x). \tag{15}$$

such that in $\boldsymbol{\pi}\partial\boldsymbol{A} = \partial\mathcal{L}(t) = \boldsymbol{p}\partial\boldsymbol{A} + \rho\boldsymbol{A}\partial\boldsymbol{A}$ every term has dimensions of kinetic energy density.

In gravito-statics, the final state of the particle with fixed mass, charge, and spin, will not change unless affected by external fields. Jefimenko [11] derives relevant expressions for the C-field:

$$\boldsymbol{C} = -\left(\frac{g}{c^2}\right)\int\left\{\frac{[\boldsymbol{p}]}{r^3} + \frac{1}{r^2 c}\frac{\partial[\boldsymbol{p}]}{\partial t}\right\}\times \boldsymbol{r}\,\mathrm{d}v' \tag{16}$$

where $[\boldsymbol{p}]$ is the retarded source current density distribution in volume element $\mathrm{d}v'$ and r is the distance to the field point at which the field is calculated. Thus, field $\boldsymbol{C}$ is produced by continuous mass distributions. For a mass point with velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ this yields: $\boldsymbol{C}_a = -\frac{\boldsymbol{G}\times[\boldsymbol{r}]}{c[r]}$ where $[r]$ is the retarded position of vector of the moving mass point. For a point mass without acceleration $\boldsymbol{C}_v = -\frac{\boldsymbol{G}\times\boldsymbol{v}}{c^2}$. Heuristically, since $\nabla\cdot\boldsymbol{G} = \boldsymbol{G}\cdot\boldsymbol{G}$ let $\boldsymbol{G}\approx\nabla$ and $c = 1$ and obtain for the unaccelerated mass the expression $\boldsymbol{C}_v = -\nabla\times\boldsymbol{v}$ or gauge field $\boldsymbol{C}_v = -\nabla\times\boldsymbol{A}$. Observe that

$$\boldsymbol{C}_v \sim \frac{\partial}{\partial t}\boldsymbol{C}_a \Rightarrow -\nabla\times\boldsymbol{A} \sim \frac{\partial}{\partial t}(\nabla\times\boldsymbol{r}). \tag{17}$$

The gravitomagnetic field presented in terms of the gauge velocity field, $\boldsymbol{A} = \boldsymbol{v}$ yields

$$\nabla\times\boldsymbol{A} = \boldsymbol{C} = \begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ \partial_x & \partial_y & \partial_z\\ v_x & v_y & v_z\end{bmatrix} = \begin{pmatrix}\hat{i}\left(\partial_y v_z - \partial_z v_y\right)\\ \hat{j}\left(\partial_z v_x - \partial_x v_z\right)\\ \hat{k}\left(\partial_x v_y - \partial_y v_x\right)\end{pmatrix} = \begin{pmatrix}\boldsymbol{C}_x\\ \boldsymbol{C}_y\\ \boldsymbol{C}_z\end{pmatrix}. \tag{18}$$

C-field circulation has physically real angular momentum analogous to the angular momentum contained in the magnetic field, experimentally proved in 1915 by Einstein & deHaas [12]. To demonstrate this for $g = c = 1$:
$\nabla\times\boldsymbol{C} = -\left(g/c^2\right)\rho\boldsymbol{v} = -\rho\boldsymbol{v}$ for $\dot{\boldsymbol{G}} = 0$. Next, multiply both sides by inverse curl operator [13] $(\nabla\times)^{-1} = (\boldsymbol{r}\times)$ to obtain $(\nabla\times)^{-1}(\nabla\times)\boldsymbol{C} = -(\boldsymbol{r}\times)\boldsymbol{p}$ where $\boldsymbol{p} = \rho\boldsymbol{v}$ is momentum density. Hence $\boldsymbol{C}\sim -\boldsymbol{r}\times\boldsymbol{p} = \boldsymbol{L}$ is angular momentum density. The angular momentum operator

$$\boldsymbol{C}\sim\begin{bmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ p_x & p_y & p_z\end{bmatrix} = \begin{pmatrix}-\hat{i}\left(zp_y - yp_z\right)\\ -\hat{j}\left(xp_z - zp_x\right)\\ -\hat{k}\left(yp_x - xp_y\right)\end{pmatrix}\overbrace{\hat{p} = -i\hbar\vec{\nabla}}^{\text{proportional to}}\begin{pmatrix}-\hat{i}\left(z\partial_y - y\partial_z\right)\\ -\hat{j}\left(x\partial_z - z\partial_x\right)\\ -\hat{k}\left(y\partial_x - x\partial_y\right)\end{pmatrix} \tag{19}$$

from the quantum correspondence principle, $\hat{\boldsymbol{p}} = -i\hbar\nabla$ with $\hbar = 1$.

We use relations (18) and (19) to interpret the Abelian form of the field strength:

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \tag{20}$$

Analogous to electrodynamics let $A = -\phi + \boldsymbol{A}$ and $\boldsymbol{A} = \boldsymbol{v}$ and evaluate $F_{\mu\nu}$.

$$F_{00} = -\frac{\partial\phi}{\partial t} - \left(-\frac{\partial\phi}{\partial t}\right) \equiv 0,\quad F_{01} = -\frac{\partial\phi}{\partial x} - \frac{\partial v_x}{\partial t},\quad F_{02} = -\frac{\partial\phi}{\partial y} - \frac{\partial v_y}{\partial t},\quad F_{03} = -\frac{\partial\phi}{\partial z} - \frac{\partial v_z}{\partial t}$$

$$F_{0\nu} = F_{00} + F_{0i} \Rightarrow -\nabla\phi - \frac{\partial \boldsymbol{v}}{\partial t} \Rightarrow \boldsymbol{G} = -\nabla\phi \tag{21}$$

$$F_{10} = \frac{\partial v_x}{\partial t} - \frac{\partial(-\phi)}{\partial x} \Rightarrow (\boldsymbol{G})_x + (\nabla\phi)_x$$

$$F_{11} = \frac{\partial v_x}{\partial x} - \frac{\partial v_x}{\partial x} = 0, \quad F_{12} = \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = -C_z, \quad F_{13} = \frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} = C_y$$

$$F_{20} = \frac{\partial v_y}{\partial t} - \frac{\partial(-\phi)}{\partial y} \Rightarrow (\boldsymbol{G})_y + (\nabla\phi)_y$$

$$F_{21} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = C_z, \quad F_{22} = \frac{\partial v_y}{\partial y} - \frac{\partial v_y}{\partial y} = 0, \quad F_{23} = \frac{\partial v_y}{\partial z} - \frac{\partial v_z}{\partial y} = -C_x$$

The $F_{3\nu}$ follows cyclically. The field strength matrix constructed from the above is shown:

$$F_{\mu\nu} = \begin{bmatrix} 0 & G_x & G_y & G_z \\ G_x & 0 & -C_z & C_y \\ G_y & C_z & 0 & -C_x \\ G_z & -C_y & C_x & 0 \end{bmatrix} \tag{22}$$

In classical electromagnetic theory the Lagrangian density in vacuum is $\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu}\right)$ which yields $\mathcal{L} = E^2 - B^2$, the energy density of the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields. For gravitomagnetism this yields:

$$F_{\mu\nu}F^{\mu\nu} = \underset{01}{G_x}\underset{10}{G_x} + \underset{02}{G_y}\underset{20}{G_y} + \underset{03}{G_z}\underset{30}{G_z} - \underset{23}{C_x}\underset{32}{C_x} - \underset{31}{C_y}\underset{13}{C_y} - \underset{12}{C_z}\underset{21}{C_z} \tag{23}$$

Thus, gravitomagnetic Lagrangian density is $\mathcal{L} = -\frac{1}{4}\left(F_{\mu\nu}F^{\mu\nu}\right) = \boldsymbol{G}^2 - \boldsymbol{C}^2$, whose terms appear as energy density of the primordial field supplying the field mass density for Newton's equation.

6. Angular Momentum in Primordial Gauge Formalism

Linking primordial field $\psi = \boldsymbol{G} + i\boldsymbol{C}$ to gravitomagnetic gauge field $\boldsymbol{v}$ corresponds to gauge field $\boldsymbol{A}$ for electromagnetic field $\boldsymbol{E} + i\boldsymbol{B}$. Today's focus on gauge field theory comes from Yang-Mills 1954 gauge treatment [14] of isotopic spin "angular momentum", in quotes because they are unsure what it means physically. Pauli's *SU*(2) spin matrices are applied to Heisenberg's *isospin*; a formalism applied to an abstract *internal* symmetry. The nature of spin, at least classically, is rotation, and rotation in 3D space entails angular momentum. Exactly what is entailed in the space of internal symmetry is unknown. But the nature of this gauge field is captured by the curl operation, so it must somehow entail an analog of angular momentum, as Einstein and deHaas showed to be possessed by the magnetic field. The Pauli spin matrix algebra is given by

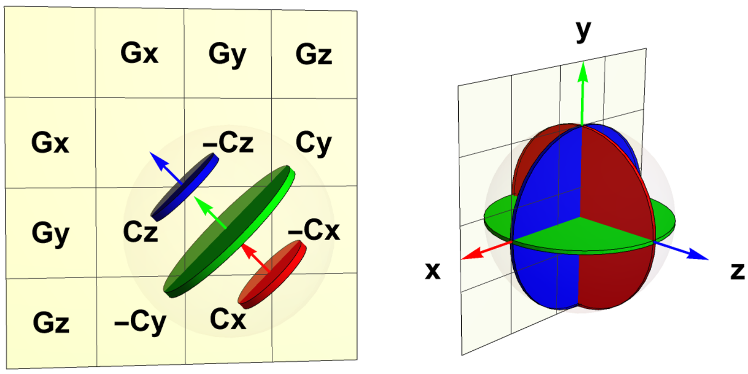
$$\sigma_j\sigma_k = -\delta_{jk} - i\varepsilon_{jkl}\sigma_l, \tag{24}$$

where $\{\sigma_x, \sigma_y, \sigma_z\}$ represent 2×2 Pauli spin matrices of quantum mechanics.

In 3D we can construct an orthonormal bivector basis from three orthogonal bivectors, $\{\beta_x, \beta_y, \beta_z\}$. The algebra of bivectors satisfies $\beta_x \beta_y = -i\beta_z$ specifically, and more generally the bivector algebra,

$$\beta_j \beta_k = -\delta_{jk} - i\varepsilon_{jkl}\beta_l \tag{25}$$

with Kronecker delta δ_{jk} and Levi-Civita alternating symbol ϵ_{jkl}: *bivector algebra is identical to Pauli spin matrix algebra, by inspection*. Identical algebras imply physics, syntactically, however there may be significant ontological differences. For example, there is no well-defined idea of *isotopic spin* "angular momentum", yet the gravitomagnetic field possesses well defined angular momentum; and, in fact, is proportional to angular momentum: $\boldsymbol{C} = (g/c^2)\boldsymbol{r} \times \boldsymbol{p}$ with dimension t^{-1}/l^3 as depicted for $F_{\mu\nu}$ in Figure 3.



(a) SU(2) (b) SU(3)

Figure 3. (a) The circulating C-field can be labeled by the (row, col) component or by the orthogonal axis about which the (row, col) component circulates. For example, the (*x*, *z*) element is labeled C_y and the (*z*, *x*) element is labeled $-C_y$ since both terms rotate about the y-axis—similarly for the other components. These rotations are shown abstractly as representing the field strength $F_{\mu\nu}$ matrix on the left. (b) The right-hand illustration maps the three bivector diagrams into 3-space. Colors are for visual convenience and suggest correlation with $SU(3) \times SU(2) \times U(1)$ symmetry.

In Figure 3, we pair C_y with $-C_y$, and cyclical iterations, with the index representing the axis about which these components of the field rotate. C_z components could be labelled as $-C_{xy}$ and C_{yx}, denoting their position in the (row, col) representation. In other words, the formalism contains the angular momentum aspect of the components. C-field components compatible with three bivectors are shown in the 3-space representation at the right, defined by the *x*, *y*, and *z* axes (vectors) and 100% compatible with bivector algebra (25), and identical to the *SU*(2) Pauli matrix algebra (24). The *Gravity Probe B* experiment [15] proved the existence of the C-field circulation induced by rotation of planet Earth's mass density. *The nature of C-field circulation, from every perspective, is angular momentum*. If this is to be quantized, we expect it to relate to a fixed minimum amount of angular momentum, which is exactly what Planck's constant $\hbar$ represents.

6.1. The C-Field "Weak Field Approximation"

In the common view gravitomagnetic effects are very small and difficult to detect, per *Gravity Probe B*. Yet, gravitomagnetic circulation induced by mass current density has energy density. If the matter that is inducing gravitomagnetic circulation is accelerated, the mass current density $\boldsymbol{p}$ changes, increasing induced gravitomagnetic field $\boldsymbol{C}$, altering energy density μ_C and equivalent mass density ρ_C, creating a non-linear feedback loop:

$$\boldsymbol{p} \to \boldsymbol{C} \to \mu_C \to \rho_C \to \boldsymbol{p} \tag{26}$$

Non-linearity based on self-interaction of the gravitational field distinguishes General Relativity from linear theories like Newtonian gravity, which should, *post-Heaviside* and *post-* $E = mc^2$, be recognized as self-interacting and non-linear. This non-linear feedback loop arising from gravito-magnetic energy density means that the weak field stipulation is insufficient to fully describe gravitomagnetism, which is inherently non-linear and consistent with non-linear GR. In reality, the gravitomagnetic formalism is applicable *at all field strengths*. The misleading and incorrect "weak field approximation" has obscured many aspects of physics associated with strong fields. Yet the term "weak" can be misleading, since [16] "*even at the surface of a dense object like a white dwarf…the weak field limit will be an excellent approximation*." The unexplained effectiveness of Heaviside's equations in stronger fields, such as those near Black Holes, has been noted [17]. The experimental proof of the existence of the C-field supports this misunderstanding: the field measured by *Gravity Probe B* is incredibly weak, and this field is based on the mass of the Earth! That *QuantumGravitoDynamics* (QGD) is not *mass-based* but *mass-density-based* implies that gravitomagnetism applies in high density, strong field situations. This means, to most physicists, physics near black holes, however, the densest known matter in the universe is probably not the black hole, but the elementary particle; the strongest fields are those at the sub-nuclear level of reality. Compare mass density of the Earth at the distance of the *Gravity Probe B* satellite in 400 mile orbit to the density of fermions at atomic and nuclear distances.

6.2. C-Field-Based Wave Equations

Next consider Heaviside Equations (11a…11d) with local density of matter $\rho \sim 0$. The Maxwell-like field equations invite the following procedure, based on the vector identity:

$$\nabla \times (\nabla \times \boldsymbol{V}) = \nabla(\nabla \cdot \boldsymbol{V}) - \nabla^2 \boldsymbol{V}, \tag{27}$$

where the last term can be written $-\nabla \cdot (\nabla \boldsymbol{V})$. The first term on the right vanishes at all times for $\boldsymbol{V} = \boldsymbol{C}$ since $\nabla \cdot \boldsymbol{C} \equiv 0$. For no mass density $\rho = 0$, and for minimal field density $\rho \approx 0$, we have $\nabla \cdot \boldsymbol{G} \approx 0$, leaving the relation $\nabla \times (\nabla \times \boldsymbol{V}) = -\nabla^2 \boldsymbol{V}$. Substituting $\boldsymbol{G}$ and then $\boldsymbol{C}$ into this identity:

$$\nabla \times (\nabla \times \boldsymbol{G}) = -\nabla^2 \boldsymbol{G} \quad \Rightarrow \quad \nabla \times \left(-\frac{\partial \boldsymbol{C}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \boldsymbol{C}) \quad \Rightarrow \quad -\frac{\partial^2 \boldsymbol{G}}{\partial t^2} \tag{28a}$$

$$\nabla\times(\nabla\times\boldsymbol{C})=-\nabla^2\boldsymbol{C}\quad\Rightarrow\quad\nabla\times\left(+\frac{\partial\boldsymbol{G}}{\partial t}\right)=+\frac{\partial}{\partial t}(\nabla\times\boldsymbol{G})\quad\Rightarrow\quad-\frac{\partial^2\boldsymbol{C}}{\partial t^2}\tag{28b}$$

Thus, we obtain wave-like equations:

$$-\nabla^2\boldsymbol{G}+\frac{\partial^2\boldsymbol{G}}{\partial t^2}=0\quad\text{and}\quad-\nabla^2\boldsymbol{C}+\frac{\partial^2\boldsymbol{C}}{\partial t^2}=0\tag{29}$$

Dimensional analysis shows that a velocity-squared term is needed, so we assume $v=1$ and include the symbolic speed in the equation, to obtain standard wave equations.

$$-\nabla^2\boldsymbol{G}+\frac{1}{v^2}\frac{\partial^2\boldsymbol{G}}{\partial t^2}=0\quad-\nabla^2\boldsymbol{C}+\frac{1}{v^2}\frac{\partial^2\boldsymbol{C}}{\partial t^2}=0\tag{30}$$

In 2017 [18] detection of inspiraling neutron stars established that the speed of light ($v=c=1$) in an absolute frame, defined by the Cosmic Microwave Background, is the same as propagation of gravity through the same frame which is pervaded by gravity. Will [19] analyzes the connection between gravity and speed of light by correlating electromagnetic parameters μ,ε with Newton's gravitational constant g in terms of $TH\varepsilon\mu$ formalism of Lightman and Lee. We have:

$$c=\frac{1}{\sqrt{\varepsilon_0\mu_0}}\quad\Rightarrow\quad\left[\varepsilon(g)\mu(g)\right]^{-1/2}=\left[\left(\frac{-1}{4\pi g}\right)_{\varepsilon}\left(\frac{-4\pi g}{c^2}\right)_{\mu}\right]^{-1/2}=\left(c^{-2}\right)^{-1/2}=c\tag{31}$$

Since velocity v in Equations (30) is equal to the speed of light we observe that the *Principle of Self-interaction* predicts gravitational waves.

7. Encoding Energy Density as Geometry

Many current problems with Quantum Gravity revolve around the treatment of gravity as "*curved space-time*" and ramifications of the Equivalence Principle. Every general relativity text contains some discussion of the fact that the energy of gravitating systems cannot be formulated in curved space-time. A Centennial paper [20] by Chen *et al.* begins by stating:

"*How to give a meaningful description of energy-momentum for gravitating systems…has been an outstanding fundamental issue since Einstein began his search for gravity theory.*"

The problem of how to describe energy-momentum and angular momentum in gravitating systems suffers from the fact that "*It is known that these quantities cannot be given a local density.*" Bart's 2019 PhD dissertation [21] summarizes the current situation: "*…there exists no general framework in which a definition of quasi-local energy is sufficiently understood.*"

Penrose attempted to address this problem by defining *quasi-local mass* (and angular momentum); others during the past forty years worked on this concept because "*many important statements in general relativity make sense only with the presence of a good definition of quasi-local mass.*"

The century-long failure to solve this fundamental problem indicates confusion; compounded by the fact that special relativity is based on pretending that gravity does not exist at all; and general relativity on pretending that gravitational energy does not exist "locally"—it can always be transformed away. Thus, gravitomagnetic energy density in quantum gravity requires clarification of the issue. In PFT gravity is not *curved space-time*, but a *field with energy density*.

Rather than viewing "information" as a real physical entity, we view information in its original *coding* perspective and observe that there is absolutely *no energy density information encoded in flat space coordinates*. One can translate the coordinate system with *absolutely no effect* on the field's energy density; one can change the energy distribution with absolutely no effect on the coordinate system. All of the information about the physical field is contained in the *energy density distribution*; *none in the flat space coordinates*.

In Table 1, flat space information is in the energy density; no information (other than scale) is associated with coordinate system units. To remove the energy density information, we normalize the energy by choosing local 3D cells to have units such that the cell contains *exactly* one unit of energy. But coordinates that vary according to local energy density yield curved space. Hence, in curved space, normalized energy represents no information; the relevant information is instead encoded in the geometry specified by the normalizing metric.

Table 1. Information matrix.

	Flat	Curved
energy density	1	0
metric length	0	1

What happens to the physics when we remove density information by enforcing unit local energy density? *The information must be replaced somehow*! If the only information we have is *energy density info* and *coordinate info*, then if we remove the density information, we must replace it with coordinate info, by *replacing constant length flat space coordinates with variable length* (*metric*) *curved space coordinates*. No information is lost; the real physical information encoded by the density distribution over flat space is replaced by abstract information in which *physical energy density is encoded as* "*geometry*". According to Poisson and Will [22]:

"*the metric…achieves two purposes*: *it encodes geometrical information about coordinate system*, *and it encodes physical information about the gravitational field*."

I say that ***the metric encodes physical information*** *about the gravitational field* ***as*** *geometrical information of a coordinate system*. Thus, clearly, physical field energy density is ontologically real and is mathematically equivalent to the description of the real physical density encoded in the abstract formulation of curved space time!

One best distinguishes *physical density-based reality* from *curved space-time-based reality* via inertia. In Euclidean space the special relativity γ-factor applies to *inertial mass*, in *curved space-time* real inertial force is replaced by the abstraction of geodesics. This is at the root of Feynman's, Weinberg's, Padmanabhan's, and others insistence that

"*Curved space-time is not a necessary conception of gravity.*"

We transform information from the energy density of the field ϕ, in constant (information-less) coordinates of flat space, to the information of the (variable) coordinate system with a constant (information-less) energy density in curved space-time. This is consistent with the fundamental requirement that *coordinate systems cannot affect physical reality*. They bring no information to reality; they only label physical reality such that we all agree upon the points under discussion.

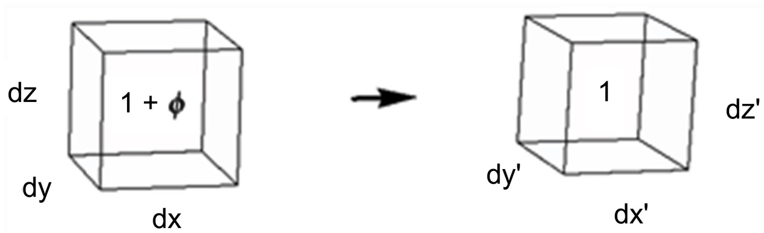


Figure 4. The primordial gravitational field alone has $\phi_0 = m/r = r/r = 1$ in flat space, $dx, dy, dz = 1$. For mass M at the origin, we add potential energy $\phi = M/r$. The goal is to transform to a unit density (no density information) in curved space-time as shown in the cube at right, $dx', dy', dz' \neq 1$.

One might ask why the value *one* is used for the apparent energy density in curved space instead of the value *zero* that seems to be implied by the equivalence principle. Because we do not require that real energy density vanish, only that *information about the energy density must vanish*, in accordance with Einstein's statement that "*gravitational energy is not localizable*". Our goal here is not to get rid of the physical energy density; only to get rid of the energy density *information*. We accomplish this by normalizing the energy density such that every local region in curved space contains exactly one unit of energy density in a region bounded by $dx', dy', dz' \neq 1$ as seen at right in **Figure 4**. We define dx'_i to transfer the physical density information in flat space to the curvature (metric) information of curved space. In flat space every differential has unit length and there is no information contained in the normalized unit of the coordinate system. In curved space, the metric intervals are defined to normalize the energy density of the field, removing the information from the energy density and transferring it to the curved space metric.

Now it *is* true that "gravitational energy is not localizable" because, in curved space-time, every local region contains a unit of energy indistinguishable from any other local region, whereas the local metric at any point *is* distinguishable from the metric at any neighboring point. This is captured by the expression of the generalized Pythagorean for curved space-time:

$$ds^2 = g_{ij} dx^i dx^j, \tag{32}$$

where g_{ij} represents the curvature on a manifold in four-space with tangent vectors *i* and *j*. The geometric meaning of g_{ij} is quite clear, however the physical meaning is debatable. Most treatments seem to identify the term with gravity, however MTW [23] state:

"*…nowhere has a precise definition of the term 'gravitational field' been given—nor will one be given. Many different mathematical entities are associated with gravitation: the metric, the Riemann curvature tensor, the Ricci curvature tensor, the curvature scalar, the covariant derivative, the connection coefficients, etc. … the terms 'gravitational field' and 'gravity' refer in a vague, collective sort of way to all of these entities.*"

This is backwards; a better statement of physics would be:

…all of these entities refer in a vague, collective sort of way to "gravitational field" and "gravity".

In our treatment the metric g_{ij} is key. Beckwith [24] states: "*In general relativity metric $g_{ij}(\mathbf{x},t)$ is a set of numbers associated with each point which gives the distance to neighboring points, i.e., general relativity is a classical theory.*"

Our goal is to calculate metric $g_{\mu\nu}$ necessary to transform energy density information contained in ϕ into the local coordinate information contained in $g_{\mu\nu}$. The $dx dy dz$ coordinate differentials are independent of ϕ, but the transformed coordinates are totally dependent on ϕ, *i.e.*,

$$dx'dy'dz' \equiv dx'(\phi)dy'(\phi)dz'(\phi). \tag{33}$$

For the most general solution we force each metric-based component to satisfy the relations

$$\begin{aligned}(1+\phi)dx_1'(\phi) &= dx_1\\ (1+\phi)dx_2'(\phi) &= dx_2\\ (1+\phi)dx_3'(\phi) &= dx_3\end{aligned} \tag{34}$$

In this case the key coordinate variable relation becomes

$$dx_i'(\phi) = \frac{dx_i}{1+\phi} \tag{35}$$

The consequent volume element becomes $dx_1'(\phi)dx_2'(\phi)dx_3'(\phi) \equiv \frac{dx_1}{1+\phi}\frac{dx_2}{1+\phi}\frac{dx_3}{1+\phi}$ and for any two of these dimensions, we obtain the (always positive) product:

$$dx_i'(\phi)dx_j'(\phi) \equiv \frac{dx_i}{1+\phi}\frac{dx_j}{1+\phi}. \tag{36}$$

This analysis, restricted to 3-space, produces a "time-slice" of Euclidian reality, $dt \equiv 0$. In order to touch base with general relativity, we observe that $\phi = -\frac{gM}{r}$ and rewrite Equations (32) as

$$ds^2 = -\left(1+\frac{2gM}{r}\right)\left(dx^2+dy^2+dz^2\right) \tag{37}$$

Compare this to Ohanian and Ruffini's Equation (12) p. 179 from the approximate metric tensor:

$$g_{\mu\nu} = \begin{bmatrix} 1-2gm/r & 0 & 0 & 0 \\ 0 & -(1+2gm/r) & 0 & 0 \\ 0 & 0 & -(1+2gm/r) & 0 \\ 0 & 0 & 0 & -(1+2gm/r) \end{bmatrix} \tag{38}$$

Then compare to their space-time interval [Equation (13) p. 179]:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = \left(1-\frac{2gm}{r}\right)dt^2 - \left(1+\frac{2gm}{r}\right)\left(dx^2+dy^2+dz^2\right) \tag{39a}$$

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (1+2\phi)dt^2 - (1-2\phi)\left(dx^2+dy^2+dz^2\right) \tag{39b}$$

If we use the alternative Minkowski metric $(-1,1,1,1)$ and let Newton's constant $g=1$ and speed of light. In this case $\phi = -(M/r)$ and we write

$$ds^2 = -\left(1+\frac{2M}{r}\right)\left(dx^2+dy^2+dz^2\right) + \left(1-\frac{2M}{r}\right)dt^2 \tag{40a}$$

$$ds^2 = -(1-2\phi)\left(dx^2+dy^2+dz^2\right) + (1+2\phi)dt^2 \tag{40b}$$

This is exactly the form of Tolman's [25] expression [Equation (82.15)] for the Schwarzschild line element. Many other general relativity books can be consulted for derivation of the Schwarzschild metric surrounding mass M at the origin. An exploded diagram depicting the way in which space is partitioned by the metric to normalize energy density is shown in **Figure 5**.

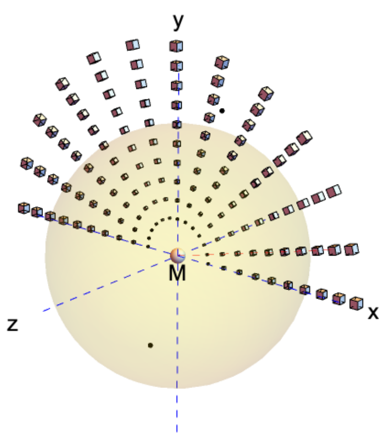


Figure 5. The graphical representation of quasi-local energy density based on the density information encoded geometrically. Every volume element in curved space holds exactly one unit of field energy. Here the curved space volume elements are spread in flat space to aid visualization. This approach yields *exactly* the Schwarzschild solution to Einstein's geometric formulation. One half of one slice through the spherical volume about mass M is shown; the full 3D image would be too "busy" and add nothing to our understanding.

Here we have used the encoding process to derive *exactly* the three-space metric representing the gravitational energy density equivalent information [26]. To state this clearly, we have derived Schwarzschild's metric, the first and one of the only *exact* solutions to Einstein's general relativity, from the *Self-Interaction Principle* of the primordial field, without his GR equation. This, and nonlinearity arguments, establish general relativity as derived from an energy density-based normalization encoding of the primordial field. The difference in these approaches to physics is that one is based on an ontological field known to be present at the Creation (the gravity of the immense energy) and experimentally detected in near earth orbit and at the heart of colliding nuclei, while the other is based on epistemological abstract "curvature of space" attributed to the "*Equivalence Principle*", which is known to be invalid in rotating frames and in tidal frameworks.

8. The Formal Approach to Quantum Gravity

The following uses a recent paper [27] to demonstrate the *formal* approach to quantum gravity, an attempt to combine the gauge theory of particle fields with the geometrical description of gravity, without addressing ontological paradoxes such as quasi-local mass, the meaning of the imaginary *i*, or the nature of the wave function. We do not analyze the paper; we use it only for comparison.

A gravitational spinor (GS), $\hat{\psi}_{ABCD}$ is Qiao's fundamental quantum field describing gravitational fluctuations. He notes that the spinors can be induced from other quantized gauge fields (e.g., electromagnetic, weak, and strong) and *suggests a unification of the four fundamental interactions*. His gravitational interactions are mediated by virtual spinor exchange, with the Newtonian limit recovered at low energies.

Spinors are essentially quantized C-field circulation, the primary phenomenon for understanding quantum gravity, and "*At high energies and strong fields, the GS nonlinear self-interaction,* $\mathcal{L}_{int} = \frac{1}{4}\left(\hat{\psi}_{ABCD}\hat{\psi}^{ABCD}\right)^2$ *generates gravitational solitons, which act as condensed states of quantum gravity.*" This is somewhat compatible with primordial field theory of particle production. Also, "*the linearized gravity field can be expressed in terms of spinors* $h_{\mu\nu} = \epsilon_{\mu\nu}^{CD}\psi_{ABCD}$ *where* $\epsilon_{\mu\nu}^{CD}$ *is the spinor-tensor mapping factor, involving the Pauli matrix.*" The spin-2 aspect of the gravitational field leads many to believe that the field cannot be expressed in terms of vectors, and that rank-2 tensors are required. But the primordial description in terms of *Geometric Calculus* is not vector-based but is based on vectors *and* dual bivectors, which relate two vectors (as do rank-2 tensors) and which we have shown to be algebraically identical to Pauli matrices.

"*Quantum gravity aims to unify GR and QM*", which typically means to unify the *formalisms*. This leads to UV-divergence, *i.e.*, infinite integrals at high energies, and lacks "background independence", *i.e.*, the dynamic nature of space-time does not provide a fixed background. Qiao prefers (as do I) to avoid the extra dimensions or discrete space-time structures currently associated with many *Loop Quantum Gravity* approaches.

His paper provides an excellent comparison between syntactic and ontic approaches. In essence, we have both settled on the same physical phenomenon, quantized gravitational circulation, as the fundamental description of unified theory. The syntactic approach employs the formulation of GR and of QM and attempts to bridge the syntactic gap, that is to bring the formalisms into congruence, without ontological qualification. The ontic approach, outlined in this paper, attempts to derive a physical theory of reality compatible with the current dominant formalisms of physics. In the following I do not develop his theory but compare and contrast these approaches.

Consider background independence, a syntactic artifact, inherent in a curved space-time approach to GR. As seen, local energy density information in flat space can be encoded as uniform energy density in curved space, yielding the Schwarzschild metric—one of the few exact solutions of Einstein's equation. This is significant and we take it as an axiom that

"*Multiple syntactical formalisms can describe ontological reality.*"

The key point: ontology is singular—there is *only one physical reality*, a fundamental postulate. This distinguishes the two approaches to quantum gravity. The syntactical approach attempts to unite formalisms which implicitly yield different physical realities. The ontic approach attempts to identify the nature of physical reality and derive a formal description thereof. The outcomes differ significantly. Since QM often "*promotes fields to operators*" Qiao formally "promotes Einstein's equation to an operator": $\hat{G}_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G \hat{T}_{\mu\nu}$. One consequence of believing that curved space-time is physically real is seen in Qiao's final sentence: "*future exploration will focus on verifying GS solitons and* [*gauge*] *-induced effects*, *potentially paving the way for realization of curvature drives* [for spaceships] *or time machines*…" Primordial field ontology implies a uniform coordinate system that spans the universe with local distributions of gravitational energy without wormholes or time loops, as opposed to a quasi-local distribution of gravitational energy subject to dynamic space-time curvature. Note: the Schwarzschild metric is the solution to a *One-body problem*, a mass at the origin of the overlaid coordinate systems. No *N-body solutions* of general relativity exist; all such approaches are approximate.

9. The Nature of Quantum Gravity

We have derived a theory of the primordial field as gravity. It is known that Heaviside's equations can be derived from GR as a weak-field approximation, but we have seen how general relativity can be derived from the primordial field via *conservation of energy density information across coordinate systems*. We wish to develop a theory of *Quantum Gravity* from this and begin with Okino's [28] list of current problems of quantum mechanics (QM) in terms of *micro particles*:

- Newtonian mechanics is not applicable to the behavior of a micro particle.
- Such a particle corresponds to a matter wave of wavelength λ.
- The existence of a particle is obtained only as a probability.
- The interference effect is shown in the behavior of a particle itself.
- The energy E and momentum $\boldsymbol{p}$ of a particle are treated not as original

quantities in physics, but imaginary operators $\hat{E} = i\hbar\partial_t$ and $\hat{p} = -i\hbar\nabla$ in accordance with the *correspondence principle*.

- The maximum size of a particle having wave nature is not understood.
- Whether de Broglie's relation $p = h/\lambda$ is valid.

Okino re-interprets time parameter as imaginary time, $t \to it$ and applies this to Fick's diffusion equation of 1855

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho \tag{41}$$

with ρ the concentration of diffusing particles and D the diffusivity in units of area per unit time, l^2/t. Anticipating Planck, we multiply and divide D by mass m to obtain dimensionality $\left(\frac{1}{m}\right)\left(\frac{ml^2}{t}\right)$. The second term has dimensionality of angular momentum or Planck's constant of action, $\hbar$, so we rewrite Equation (41) as $\hbar\frac{\partial \rho}{\partial t} = \frac{\hbar^2}{m}\nabla^2 \rho$ with concentration ρ having dimension of substance per unit volume, *i.e.*, density, now reinterpreted as probability amplitude ψ. Okino applies *imaginary time* to obtain Schrödinger's equation, the formalism of QM

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{m}\nabla^2 \psi \tag{42}$$

"derived" from the almost two-century-old diffusion equation for micro particles. However, of the many attempts to interpret quantum mechanics over the last century, [29] "*all…introduced, at some point, interpretation elements* (*semantics*) *that find no correlate in the formalism* (*syntactics*)." This applies to the Copenhagen interpretation, and to Bohm's, Everett's, Ballantine's and other interpretations, including *Consistent Histories*, *Qbism*, and *Relational* theories of QM. "*All these older proposals continue to be developed with some complement or another, and one is faced with a forest of ontological entities that hardly converse with one another.*" Introducing ontological constructs that have no relation to the underlying formalism is, effectively, *pulling a rabbit out of a hat*. Such *wildcards* can interpret almost any phenomenon but need not have any relation to physical reality.

A major question is whether Schrödinger-based QM applies to particles, or only to ensembles. At this point there is no trace of randomness in the formal structure of the derivation. Olavo, based on two axioms, uses Boltzmann's entropy to formally derive Schrödinger's equation:

$$\frac{-\hbar^2}{2m}\frac{\partial^2 \psi(q;t)}{\partial q^2} + V(q)\psi(q;t) = i\hbar\frac{\partial \psi(q;t)}{\partial t} \tag{43}$$

From whence do random dispersions come? "*They come from the kind of separation we make in the physical system, by looking at the particle subsystem in detail, while assuming only a fixed expression for the field subsystem that furnishes the potential V*(*q*)…". This is seen, for example, *in wave packet collisions with the potential barrier*: *the wave packet deforms, but the barrier is unchanged, as if nothing is happening*. Thus, QM is a theory with stochastic support, the basis of which is the separation of the field and particle subsystems, with the field playing the role of a heat reservoir. The Langevin equation of quantum mechanics provides concrete values for the random variable p_k, leading to an interpretation of Bohm as a mean-field approach to which the Langevin equation reduces when averages are taken. Langevin's equation differs from the Newtonian equation by introducing a dissipation and a fluctuation term, thus we can go continuously from classical mechanics to quantum mechanics in the Newtonian limit as fluctuation and dissipation terms are made to vanish.

QM thus relies upon two equations; Schrödinger's represents the statistical behavior of ensembles, while the Langevin equation represents the individual behavior of particles, randomly moving in phase-space. This focuses attention on two questions: the behavior of the system, given by probability amplitude ($\rightarrow$ probability density) and the nature of the particles, the constituents of the ensembles. So far, departure from Newtonian classical physics is more of the nature of formal convenience than ontological necessity. This bears on a major issue: *is QM objective or subjective*? An objective theory refers to actual particles moving with actual phase-space trajectories and having, at any instant of time, definite position and momentum; the notion of an observer is irrelevant. Here, randomness is based on assuming asymmetry between particle and field, with the field acting on the particle, but the field treated as a reservoir, keeping its energy constant in time. Thus, randomness is an artifice imposed by QM's formal construction.

The major unanswered question: "*Does the wave function correspond directly to some kind of physical wave*?" Schrödinger's equation describes ensemble behaviors, but, if nondestructive measurements of single systems can be made, it can refer to single systems. As it turns out, in 2011 experiments [30] [31] based on Aharonov *weak measurement* techniques measured the wave function *directly* by performing first a weak measurement on one conjugate variable, *momentum* or *position*, that does not disturb the particle, then a strong measurement on the other variable that does—finally averaging over weak measurements. The conclusion: *the wave function is real*—since particles exhibit de Broglie-Bohm type *trajectories* [32], **Figure 6**, incompatible with Copenhagen *collapse* interpretations. These experimental results argue against Schrödinger's fictitious wave packet—a Fourier super-position of abstract plane wave solutions to his equation.

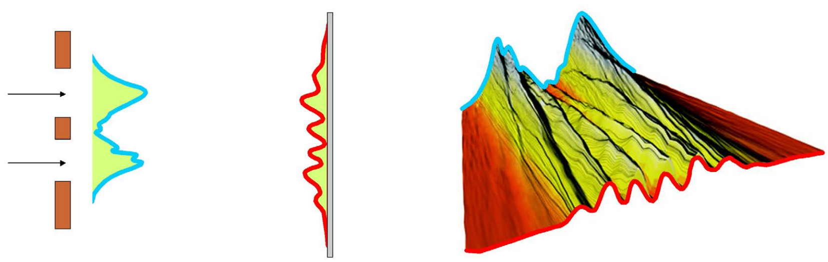


Figure 6. Results of Aharanov "weak" measurement-based experiments.

Schrödinger derived his equation based on de Broglie's *particle-plus-wave*, then he simply deleted the particle in favor of "pure waves". His wave packets always disperse yet experiments [33] show that a non-dispersing Rydberg electron in a Bohr orbit can be maintained *indefinitely* via a circulating polarized μ-wave beam, unlike always-dispersing *wave packets*. The electron *persists*; it does *not* disperse. As for *wave packets in atoms*, in 1927 Lorentz noted the mathematical difficulties of constructing packets in the atom: "… *we do not have at our disposal wavelengths sufficiently small or sufficiently close together… The frequencies of stable waves in the atom* (*eigenvalues*) *are more or less separate from each other* [*but*] *to construct a packet, one must superpose waves of slightly different wavelengths: now one can use only eigenfunctions* ψ_n, *which are sharply different from one another. In atoms then, one cannot have wave packets.*" That is, the problem isn't *dispersing wave packets* in the atom, but the impossibility of such wave packets even existing in the atom.

10. The Nature of the Quantum Gravity Wave Function

Planck introduced the quantum of action, $\hbar$, in 1905, and the physics of *particle-plus-wave* was introduced by de Broglie's relation in 1923. Although he failed to specify just what the wave *is*, he related its wavelength λ to momentum P via

$$P = h/\lambda . \tag{44}$$

We begin with a *general relativity equation* [34] that we derived from Heaviside's equations

$$\nabla \times \boldsymbol{C} = -\frac{g}{c^2}\boldsymbol{p} \tag{45}$$

where g is Newton's gravitational constant, ρ is (local) mass density and $\boldsymbol{v}$ is local velocity. The physical field, C, induced by mass current density $\boldsymbol{p} = \rho\boldsymbol{v}$ is the basis of our wave function

$$\psi = \exp\left[i\left(\boldsymbol{P}\cdot\boldsymbol{r} - \hbar C t\right)/\hbar\right] \tag{46}$$

We use equation $\nabla \times \boldsymbol{C} = -\boldsymbol{p}$ with $g = c = 1$. *Gravity Probe B* measured the *decoherent* C-field of a thermal body (the Earth)—and in 2011 reported agreement with general relativity. But in 2006 Tajmar measured [35] a C-field *coherency* factor 10^{31} higher than expected from general relativity, because the density dependence of the C-field has been generally ignored. Electrons, arguably the highest matter density ρ in the Universe, maximize local C-fields induced by mass current density $\rho\boldsymbol{v}$ or momentum density $\boldsymbol{p}$.

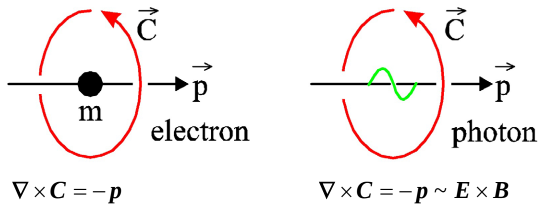


Figure 7. Gravitomagnetic C-field circulation is induced by momentum density.

Momentum density $\boldsymbol{p}$ induces a left-handed *gravito-magnetic* circulation analogous to *electro-magnetic charge-current-density*-based circulation induced in the B-field equation $\nabla \times \boldsymbol{B} = \boldsymbol{j}$. This C-field circulation constitutes the *wave* of de Broglie's *particle-plus-wave* basis of quantum mechanics. Somewhat analogous to a *3D bow wave* with the electron as boat analog, every moving particle is accompanied by its induced C-field wave. *Mass current* is *particle momentum*, so photons, with non-zero momentum, also induce C-field circulation $\nabla \times \boldsymbol{C} \propto -\boldsymbol{E} \times \boldsymbol{B}$ as shown in Figure 7. The *mass-based* Lorentz force equation $\boldsymbol{F} = m(\boldsymbol{G} + \boldsymbol{v} \times \boldsymbol{C})$ implies that *gravitomagnetic* C-field circulation has units of frequency $1/t$, corresponding to *gravitoelectric* G-field units of acceleration l/t^2. The C-field circulation is thus described by exponential $\exp[iCt]$, which, via energy equation $E = \hbar C$ we can rewrite as $\exp[iEt/\hbar]$, showing C-field-based wave functions to be plane wave solutions to Schrödinger's equation. But how is Schrödinger's equation obtained from the C-field to connect with quantum mechanics?

We have seen several derivations of Schrödinger's equation, but to present a formal theory of the quantum gravity wavefunction, we review a formal derivation of quantum theory, in the textbook "*Modern Quantum Mechanics*" [36]. Sakurai describes a neutron interference experiment and concludes that "*gravity is not purely geometric at the quantum level because the effect depends on* $(m/\hbar)^2$." In his development of quantum mechanics, a state ket $|\alpha\rangle$ formally represents the state of a physical system. Sakurai *postulates* that the probability of a system described by $|\alpha\rangle$ jumping into some particular $|\alpha'\rangle$ is $\alpha' = |\langle \alpha' | \alpha \rangle|^2$. This postulate is where probability enters quantum mechanics. Much of his effort is devoted to enforcing a formal system compatible with this postulate. As he states, "*this is one of the fundamental postulates of quantum mechanics*, *so it cannot be proven*." In a similar sense, he states that the use of complex numbers is an *essential* feature of quantum mechanics, with no explanation even attempted.

He continues the formal development by defining an infinitesimal translation by dx' in terms of the operator $\mathcal{T}(dx')|x'\rangle = |x' + dx'\rangle$. Since ket $|\alpha\rangle$ is normalized to preserve the unitary nature of probability conservation, so must translated ket $\mathcal{T}(dx')|x'\rangle$ and $\langle \alpha | \alpha \rangle = \langle \alpha | \mathcal{T}^\dagger(dx)\mathcal{T}(dx)|\alpha\rangle$ be normalized to unity, which is true if $\mathcal{T}^\dagger(dx)\mathcal{T}(dx) = 1$. Also, successive translations must be additive: $\mathcal{T}(dx'')\mathcal{T}(dx') = \mathcal{T}(dx' + dx'')$. Finally inverse relation $\mathcal{T}(-dx') = \mathcal{T}^{-1}(dx')$ and $\lim_{dx' \to 0} \mathcal{T} = 1$ must hold. Given these stipulations he then demonstrates that $\mathcal{T}(dx') = 1 - i\boldsymbol{K} \cdot \boldsymbol{dx}$ where $\boldsymbol{K}$ is the Hermitian operator. In classical mechanics momentum generates infinitesimal translations so it is natural to attempt to identify the operator with momentum, but $\boldsymbol{K} \cdot \boldsymbol{dx}$ must be dimensionless, requiring a scale factor $\hbar \boldsymbol{K} = \boldsymbol{P}$ with $\hbar$ having dimension of *action*. Sakurai notes that $\hbar$ appears in de Broglie's $\frac{2\pi}{\lambda} = \frac{P}{\hbar}$ where λ is the wavelength of a "particle wave". Hence,

$$\mathcal{T}(dx') = 1 - \left(\frac{i}{\hbar}\right)\boldsymbol{P} \cdot \boldsymbol{dx}. \tag{47}$$

After formalizing translation in space, he asks how a state changes with time and postulates that a state ket $|\alpha\rangle$ evolves with time according to a unitary time evolution operator $U(t,t_0)$ via equation $|\alpha(t)\rangle = U(t,t_0)|\alpha(t_0)\rangle$. For infinitesimal time change dt Sakurai *asserts* that a satisfactory time-evolution operator is

$$U(t_0+dt,t_0)=1-i\Omega dt \tag{48}$$

where Ω is an operator having dimensions of frequency, then asks: "*Is there a familiar observable with dimensions of frequency*?" He recalls the "old" quantum theory Einstein's relation $E=\hbar\omega$, hence "it is natural" to relate Ω to the Hamiltonian operator $\mathcal{H}$, $\Omega=\mathcal{H}/\hbar$ such that

$$U(t_0+dt,t_0)=1-\left(\frac{i}{\hbar}\right)\mathcal{H}dt \tag{49}$$

from which he immediately derives Schrödinger's equation for the time evolution operator:

$$i\hbar\frac{\partial}{\partial t}U(t,t_0)=\mathcal{H}U(t,t_0) \tag{50}$$

which can be multiplied on the right by state ket $|\alpha\rangle$. The above is based on the electromagnetic circular frequency ω Einstein treated in the photoelectric effect. The formalism includes two cases; the Hamiltonian operator is independent of time, $U(t,t_0)=\mathrm{e}^{-\left(\frac{i}{\hbar}\right)\mathcal{H}(t-t_0)}$, and is a function of time, $U(t,t_0)=\mathrm{e}^{-\left(\frac{i}{\hbar}\right)\int_{t_0}^{t}\mathrm{d}t'\mathcal{H}(t')}$.

Thus, a major textbook formulation of the formalism of QM is based on hypotheses, postulates, assertions, and extension of classical ideas. The quantum formalism is taken seriously only because it works so well. The accounting procedure, based on requiring unitary compliance with conservation of probability, yields probabilities that match experiment. But the cost is considerable: Sakurai uses quotes around "particle wave" because he does not know what a particle wave is. In fact, Leifer and Maroney [37] state:

"*The nature of the quantum state has been debated since the early days of quantum theory. Is it a state of knowledge or information* (*an epistemic state*) *or is it a state of physical reality* (*an ontic state*)?"

The answer today is that no one knows. In this paper we define and derive an ontic state, a quantum gravity wavefunction based on physical reality. We begin at the point where Sakurai asks: "*Is there a familiar observable with dimensions of frequency*?" Recall that the first solution we derived from the Primordial Field equation $\nabla\psi=\psi\psi$ was scalar solution $\psi(\xi)=-\xi^{-1}$ which we interpret as inverse time, followed by identification of $\psi(t)$ as gravitomagnetic field, $C(t)$. Hence, we replace Ω by C and derive the differential equation $i\,\partial U(t,t_0)/\partial t = CU(t,t_0)$ then multiply both sides by Planck's $\hbar$ to obtain Schrödinger's equation for the time evolution operator,

$$i\hbar\frac{\partial}{\partial t}U(t,t_0)=\hbar CU(t,t_0). \tag{51}$$

where, in analogy with Einstein, $E=\hbar C$. Of fundamental Equation (51) Sakurai states:

"*Everything that has to do with time development follows from* [*it*]."

Since (51) is an operator equation, we multiply by a state ket and obtain

$$i\hbar\frac{\partial}{\partial t}\left|\alpha(t)\right\rangle = \hbar C\left|\alpha(t)\right\rangle. \tag{52}$$

So, an equation of general relativity yields a *coherent* circulating wave—induced in *density dependent* manner by the densest material in the universe, the electron—leading to Schrödinger's wave equation. If the state ket depends upon momentum, then duality operator *i* relates change in momentum with time to circulation of the induced C-field. All of this falls out of *the self-interaction principle* based on *primordial field theory* expressed in terms of *Geometric Calculus*.

11. Relation of Quantum Gravity Wave Function to Probability

With experimental evidence of *particle plus real wave*, and a path to Schrödinger's equation of quantum mechanics, we now ask how *real physical waves* provide abstract *probability amplitudes*? The partition function is well known from statistical theory:

$$Z = \mathrm{e}^{-E/kT} \tag{53}$$

The partition function [38] and *Fermi-Dirac*, *Bose-Einstein*, and *Maxwell-Boltzmann* versions, tells us that the probability of finding a microstate of a given energy is inversely related to the energy—the greater the energy, the smaller the probability of occurrence and vice versa. A free particle with momentum $P = h/\lambda$ has energy:

$$E = \frac{p^2}{2m} \Rightarrow \frac{-h^2}{2m}\left(\frac{1}{\lambda^2}\right) \tag{54}$$

This basic relation between energy and wavelength combined with the partition function yields probability $P(E) \sim f(1/E) \sim f(\lambda^2)$. The quantum mechanical expectation value of any observable is $\langle O\rangle = \int \Psi^* O \Psi \,\mathrm{d}\tau \big/ \int \Psi^*\Psi \,\mathrm{d}\tau$ hence the expectation energy is

$$\langle E\rangle = \frac{\int \Psi^* E \Psi \,\mathrm{d}\tau}{\int \Psi^*\Psi \,\mathrm{d}\tau} \sim \int \Psi^*\left(\frac{p^2}{2m}\right)\Psi \,\mathrm{d}\tau \sim f\left(\frac{1}{\lambda^2}\right) \tag{55}$$

Thus, particle energy is *always* a function of wavelength, and hence of the C-field induced by the particle's momentum density. Non-normalizable free particles can have any energy, but quantum systems of interest are *bound* states, with discrete energy eigenvalues E_n that correspond to discrete wavelengths with $P(E_n) \sim P(\lambda_n^2)$. Physical wave functions [39]

$$\Psi(\boldsymbol{r},t) = C_0 \exp\left[i\left(\boldsymbol{P}\cdot\boldsymbol{r} - \hbar C t\right)/\hbar\right] \tag{56}$$

are functions of wavelength. A physical wave is non-normalizable—one must sum the square of all possible waves and normalize by the root of the sum to obtain a *probability amplitude* or specific measure of probability in an energy basis. But this wave function is based on a *real local particle* and a *real physical field*, not a superposition of fictitious waves that *collapse* upon measurement. Quantum systems are always found in real ("pure") states, with probability implicit in the physical wave by virtue of the partition function and of de Broglie's relation, $P = h/\lambda$.

Thus, quantum gravity is based on *real local particle-plus-induced-wave*, not on *mystical non-local superposition of non-real wavefunctions* of the kind Bohr, Feynman and others insist "no one can understand." Recall that John Bell was inspired by de Broglie's theory and noted that *the wave is just as real as Maxwell's fields*, stating [40]:

"*No one can understand this theory until he is willing to think of* Ψ *as a real objective field rather than just a 'probability amplitude'*."

Bell also noted "…*two particles interact at short range and strong spin correlations are induced which persist when the particles move far apart*." This is now known as *entanglement*.

But just how far can one particle interfere with another or with itself? **Figure 7** for $\nabla \times \boldsymbol{C} = -\boldsymbol{p}$ simply shows a circle around momentum density $\boldsymbol{p}$, but from orbital dynamics, we know that the wave *must* extend over several wavelengths in order to support self-interference. De Broglie's $\lambda = h/P$ defines a wavelength and thus a minimum extent of the wave function, but maximum extent could range from one wavelength to infinity, since Schrödinger's wave packet is conceptually built of monochromatic plane waves of infinite extent. We can safely ignore wave functions of infinite extent, but treatments of atomic orbits assume an integral number of wavelengths—the link that connects wavefunction to both energy and probability. How many wavelengths? No maximum number is found in Schrödinger's equation.

12. The Extent of the Quantum Gravity Wave Function

Electron orbits can extend over trajectories hundreds of wavelengths long. Underlying the quantum mechanical idea of discrete energies is interference of an extended wave with itself, so the wave itself must extend at least over the length of an orbit. Vortices in Bose-Einstein condensates; generally, "pinned" in a 2D framework, do not involve considerable lengths. Vortex lengths exist many multiples of the size of the inducing "particle". A real experience: flying into New York—suddenly "*Bam*! *Bam*!" sounded and it felt like a car crossing a railroad track at high speed—we had flown through the *trailing vortices* of another aircraft. FAA-required separation between aircraft implies the vortex extended at least a mile behind that aircraft. Considering the aircraft as a *point*, its trailing wave was hundreds to thousands of times larger in extent than the point generating the wave. An aircraft vortex may not represent a coherent quantum effect; nevertheless, examples of real vortices are informative. Pilots of small aircraft are told to avoid following too closely behind large jets taking off or landing, as induced vortices can turn small planes upside down! **Figure 8** shows the scale of real vortices.

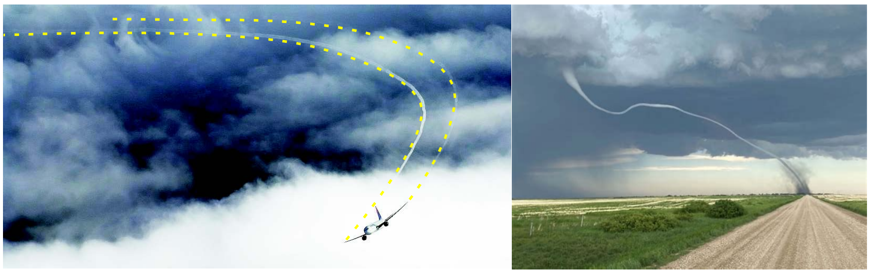


Figure 8. (a) Wingtip vortices trail an airplane for long distances. (b) Tornado vortices can be very long.

We begin by introducing equation $\nabla \times \boldsymbol{C} = -\boldsymbol{p}$ and use de Broglie's $P = h/\lambda$ to represent momentum $\boldsymbol{P}$ as a volume integral over the momentum density:

$$\boldsymbol{P} = \int \boldsymbol{p} \mathrm{d}V \tag{57}$$

For constant momentum we write this as $\boldsymbol{p} = \frac{\boldsymbol{P}}{\int \mathrm{d}V}$ or $\boldsymbol{P} = \boldsymbol{p} \int \mathrm{d}V$ and apply de Broglie:

$$\boldsymbol{p} \int \mathrm{d}V = \frac{h}{\lambda} \tag{58}$$

To convert Heaviside's C-field equation for constant momentum we can write:

$$(\nabla \times \boldsymbol{C}) \int \mathrm{d}V = \frac{h}{\lambda} \tag{59}$$

where we drop the minus sign that signifies the left-handed circulation, since it has no meaning here—C-field circulation is *always* left-handed. Next, multiply both sides of this equation by wavelength λ. Since the wavelength of the C-field circulation is in the direction of the momentum, and perpendicular to the C-field circulation bivector, we simply denote this fact by use of directed wavelength $\boldsymbol{\lambda}$ and perform the multiplication via dot product to obtain:

$$\boldsymbol{\lambda} \cdot (\nabla \times \boldsymbol{C}) \int \mathrm{d}V = h \tag{60}$$

Finally, we note that a vector dotted into a bivector is a trivector, which is an unspecified volume of space, therefore we choose a measure of space based on the unit volume: $\int \mathrm{d}V = 1$, in which case out expression becomes:

$$\boldsymbol{\lambda} \cdot \nabla \times \boldsymbol{C} = h \tag{61}$$

which is the key new relation defining the quantum gravity wave function. Thus $\boldsymbol{\lambda} \cdot \nabla \times \boldsymbol{C} \sim h$ is a *volume* shown as a *trivector*, a cylinder with cross section depicting C-field circulation $\nabla \times \boldsymbol{C}$ with wavelength λ and volume proportional to Planck's constant of action h as seen in **Figure 9**.

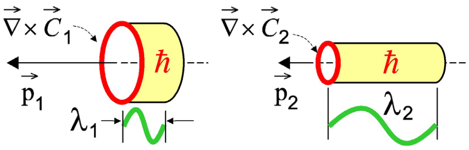


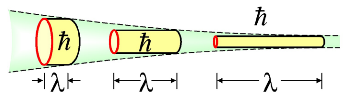
Figure 9. Wavelength associated volumes.

This *wave function conservation* relation gives a simple physical picture that fits perfectly with the equation $\oint p\,\mathrm{d}q = nh$ describing orbital angular momentum as an integral number of Planck actions. For a circular orbit:

$$\sum_{j=1}^{j=n} \boldsymbol{\lambda} \cdot \nabla \times \boldsymbol{C} = nh \tag{62}$$

Schrödinger, acknowledging that de Broglie's approach led him to his own theory of the wave function in configuration space, said, "*I have tried in vain to make for myself a picture of the phase wave of the electron in the Kepler orbit.*" We, however, view an elliptical orbit as a thin cylinder near apogee, that smoothly thickens near perigee as the momentum increases, wavelength decreases, and C-field circulation grows.

$$\sum_j \boldsymbol{\lambda}_j \cdot \nabla \times \boldsymbol{C}_j = nh \quad \Leftrightarrow \tag{63}$$



Intuitively, input $n-1$ units of action to drive ground state ($n=1$) to high orbital states ($n>1$). The *extent* of the wave function then depends on the number of units of orbital angular momentum of the system as shown in **Figure 10**, providing the picture Schrödinger sought.

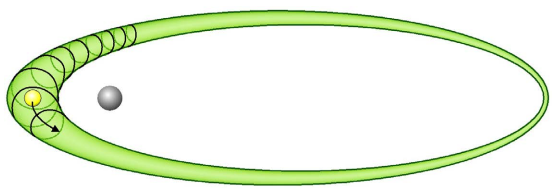


Figure 10. C-field atomic Kepler orbit.

Thus, quantum gravity theory, derived from the primordial field theory, compatible with general relativity and consistent with recent experiments, possessing intuitive correlation between physical wave and probability amplitude, is *real* and *local*. While *the Equivalence Principle* often serves as the basis of general relativity, it is invalid in a rotational framework, while accelerating systems *inherently* produce gravitomagnetic rotation of field. We have set Newton's gravitational constant g to 1 and the speed of light c to 1, which simplifies equations visibly, but suppresses the semantics. In the following we restore these terms, and generalize to non-constant momentum:

$$\int \boldsymbol{\lambda} \cdot (\nabla \times \boldsymbol{C})\,\mathrm{d}V = \frac{gh}{c^2} \tag{64}$$

Dimensional analysis is informative; begin with $g = \left[\frac{l^3}{mt^2}\right]$ and $c = \left[\frac{l}{t}\right]$ where m, l, and t represent mass, length, and time respectively, hence $\frac{g}{c^2} = \left[\frac{l}{m}\right]$, $\lambda = [l]$, $\nabla \sim \left[\frac{1}{l}\right]$ and $\int \mathrm{d}V \sim [l^3]$, so the quantum gravity equation dimensionally reduces to

$$C\int \mathrm{d}V \sim \frac{gh}{c^2} \tag{65}$$

with $C \sim \left[\frac{l^3}{mt^2}\right]\left[\frac{t^2}{l^2}\right]\left[\frac{ml^2}{t}\right]\left[\frac{1}{l^3}\right] = \left[\frac{1}{t}\right]$ having dimensions of frequency, expressed in terms of the fundamental constants for a given volume, with trivector $\boldsymbol{\lambda}\cdot(\nabla\times\boldsymbol{C})$ representing the volume. Relation (64) expresses the fundamental quantum nature of gravity.

13. Implications of Quantum Gravity

The basic Equation (11c) can be derived from general relativity. However, a more fundamental view shows that energy density can be encoded as geometry by transferring the information from field density to coordinate system in a straightforward manner. Thus, Einstein's general relativity can be derived as abstraction from energy density normalization of the primordial field as proved by our derivation of the Schwarzschild metric without Einstein's equation. Suppression of physical aspects of the field and transformation to a purely abstract geometric "curved space" formalism has yielded century-old paradoxes such as that of "quasi-local mass" that contribute little but confusion to physics.

A similar confusion about non-local aspects derives from replacing the reality of the rotation of the objective particle and of its associated field with a two-state abstraction based on forcing the field into one of two final states via "preparation" in an electromagnetic $\boldsymbol{B}$-field. In his formulation of the problem of measurements made by Stern-Gerlach magnets on spins, John Bell claimed: "…*quantum mechanically* $E(a,b)$ *is given by the expectation value*… $\left\langle \boldsymbol{\sigma}_1\cdot\hat{a}, \boldsymbol{\sigma}_2\cdot\hat{b}\right\rangle = -\hat{a}\cdot\hat{b}$. [*This*] *result cannot be reproduced by a hidden-variable theory which is local*." Here $\boldsymbol{\sigma}$'s are Pauli spin matrices and $\hat{a}$ and $\hat{b}$ are filter angles chosen respectively by generic experimenters "Alice" and "Bob", and *expectation value* refers to correlated measurements over a *series* of runs. Crucially, Bell states in his first equation that quantum measurements *must* yield values ±1:

$$A(\boldsymbol{a},\lambda) = \pm 1,\ B(\boldsymbol{b},\lambda) = \pm 1 \tag{66}$$

where $\boldsymbol{a}$ and $\boldsymbol{b}$ represent the measurement field and λ is a classical parameter that is "hidden" from the quantum mechanical formulation. The probability distribution

$$P(\boldsymbol{a},\boldsymbol{b}) = \int \mathrm{d}\lambda\, \rho(\lambda) A(\boldsymbol{a},\lambda) B(\boldsymbol{b},\lambda) \tag{67}$$

should equal the quantum mechanical expectation value, which for the singlet state is

$$\left\langle \boldsymbol{\sigma}_1\cdot\boldsymbol{a}, \boldsymbol{\sigma}_2\cdot\boldsymbol{b}\right\rangle = -\boldsymbol{a}\cdot\boldsymbol{b}\,. \tag{68}$$

Bell claims this is impossible, concluding that no classical model can produce quantum results. And countless analyses and numerous experiments on photons (*not* Stern-Gerlach experiments) have shown the specified result $-\boldsymbol{a}\cdot\boldsymbol{b}$. The problem, as it often does, resides in the initial premise, that Stern-Gerlach must yield ±1 values. This is false, as immediately obvious from examination of the Stern-Gerlach results shown on the famous "Bohr postcard", which produces a spectrum of measurements, not two discrete measurements. An analysis of this problem, using computed classical results of measurements, shows the postcard-like distribution and gives the result $-\boldsymbol{a}\cdot\boldsymbol{b}$. Bell eliminated ontologically real measurements in favor of abstract formal values, ±1, and concluded that classical physics does not match the quantum formulation. He was correct, but this is merely *formal* with no implications for physical reality. His nonlocal entanglement is classical conservation of momentum over the entire trajectory of both particles, a trajectory rejected by consensus quantum mechanics. That measurement of one particle determines instantaneously the value of another an arbitrary distance away is quantum mysticism, from the perspective of PFT.

14. A Prediction of the Quantum Gravity Wave Function

The physical reality of the quantum gravity wave function should be demonstrable. It *has been measured* using Aharonov "weak measurements" experiments. It is understood in de Broglie-Bohm physics as a "guiding field" that produces a "quantum potential" or generalized interaction of the wave functions, leading to interpretation of Bohm as a mean-field approach, to which the Langevin equation reduces when averages are taken. In this context, Bohm's equation is for averaged values; the "quantum potential" does not appear as a true random force, but only an averaged one, based on a thermal distribution of particles and velocities.

Consider an idealized situation: two particles exist side by side with identical momentum. Since both left-handed circulating fields add at every point the point exactly between the particles cancels exactly, in analogy with electromagnetic field between parallel wires. Equation (16) implies that we can calculate the C-field at distance $\boldsymbol{r}$ from momentum $\boldsymbol{p}$. This is true for any point in 3-space, and, if we sum the fields, for any number of particles.

Figure 11 shows the energy density calculated along a straight dashed green line, with the blue curve representing C-field energy summed over all particles, represented as momentum vectors out of the page (the red dots) with local C-field circulation circles around each particle.

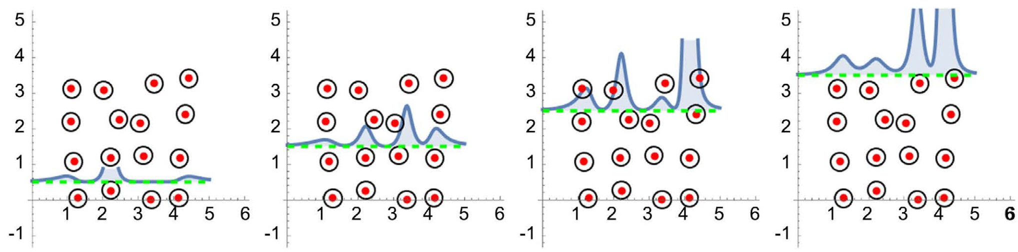


Figure 11. A 4 × 4-array with random displacements from the 4 × 4 lattice points. The C-field energy density is sampled on lines midway between rows of lattice points. Randomly arranged momenta yield random C-field energy density distributions across the array, that is, the C-field energy varies within the material based on thermal fluctuations.

For a given thermal arrangement of particles (shown in an x-y plane) we compute the C-field energy density along four green lines midway between rows of particles. The resultant energy distribution appears to be random, as expected.

How does one obtain particles with parallel momentum vectors? Clearly a crystal or orderly arrangement of particles in motion is approximated by such a scheme, with thermal distribution accounting for irregular positions in the x-y plane. Cooling the system should minimize the irregularities, ideally bringing it to a more orderly arrangement than shown in **Figure 11**.

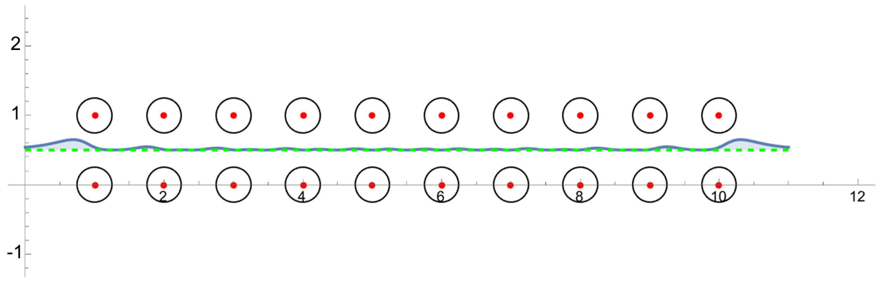
In fact, Tajmar and others spun rings of various materials in a cryogenically cooled container and used accelerometers and later laser gyroscopes to detect the gravitomagnetic field generated when the angular velocity of the spinning ring (and hence the inducing momentum density vectors) is varied. Detectors are positioned inside the ring and above the rings with detectors in mirror pairs to cancel mechanical signals. They varied materials, temperatures, and velocity profiles. As expected, the gravitomagnetic field varies directly proportional to the applied angular acceleration of the ring, and the direction of the peak signal changes with the sense of rotation. Niobium gave the strongest acceleration in the tangential direction when angularly accelerated, with gravitational peaks observed when the superconductor passed its critical temperature while rotating. In 2006 Tajmar, Plesescu, Marhold, and de Matos, reported that the niobium ring:

"*reached* 30 *orders of magnitude higher than what general relativity predicts classically*!"

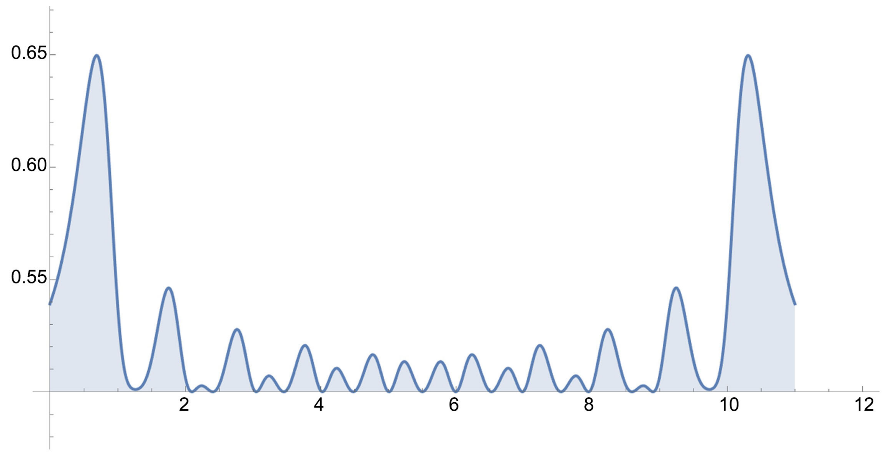
In 2008, Tajmar, Plesescu, and Siefert [41] concluded that the gyro signal follows the rotating ring velocity with high correlation. "*Compared to classical frame-dragging spin-coupling predictions*, *our signals are up to 18 orders of magnitude larger*." Tajmar and de Matos explored gravito-magnetism in conjunction with the London moment of superconductivity. BCS Theory related "Cooper pairs" of electrons whose anomalous mass was not understood but this failed to explain the extraordinary results. Calculations based on quantum gravity are represented in **Figure 12**.

In **Figure 12(a)** C-field energy density is calculated exactly halfway between two rows of ten atoms (the green line), with nuclei assumed to have little thermal energy to disrupt the orderly (x, y) arrangement. Symmetry causes C-field circulation to cancel at the midpoint between the rows. Infinitely long rows would exhibit such behavior along the length of the rows, but finite length rows experience end-effects, as shown. **Figure 12(b)** scales up the energy curve to enhance the effects of the finite row length. It is seen that energy is "excluded" from the symmetrical interior of the material, and "piles up" outside the "momentum crystal" which has been produced by supercooling.

This predicts a *quantum gravity Meissner effect* [42]. Instead of two rows as shown in **Figure 12(a)**, Tajmar's rotating plate consisted of approximately 10^{18} rows of particles, all moving with close to identical momentum density. Thus, our quantum gravity wave function interpretation predicts the surprising measured giant C-field leap when the supercooling threshold is passed.



(a)



(a)

Figure 12. (a) C-field sampled online midway between rows of momenta shows cancellation of circulation. (b) The sampled energy density diagram is scaled up, demonstrating that much of the C-field energy density has been excluded from the material.

15. Conclusions

The Science special issue on 100 years of the Schrödinger equation states [43]: "*One of the biggest unsolved mysteries is quantum gravity.*" The quantum gravity wave function developed herein is based on primordial field theory and the corresponding Self-Interaction Principle. We began by contrasting primordial field theory, with no particles present at the creation, with quantum field theory, in which particles are excitations of particle specific fields. While the Standard Model assumes all forces converge to one, the converse would be true, that one field produces everything in our current universe.

In this paper we have not explained how particles arise, or electric charge, or the strong force. Instead, we have focused on the simplest aspects, those flowing from the primordial field including Heaviside's equations, involving only partial differential equations. The wave function follows from the existence of dense particles with momentum. It is known that particles appeared almost immediately after the Big Bang, hence particle formation occurs at extremely high energy densities and turbulent conditions. We cannot, in general, solve such problems, but the introduction of topology (very abstract) and fractional lattice dynamics (also abstract) allow us to derive all fermions from the primordial field with mass, charge, and spin 1/2. The derivations from primordial field theory [44] [45] are ignored here because topology and fractional lattice dynamics would significantly distract from developing the theory of the quantum gravity wave function.

We derived Heaviside's gravitomagnetism and extended Newtonian gravity to make sense of *de Broglie's relation* in terms of the circulation induced by momentum density of a particle, noting that particle density far exceeds the density of materials constructed of particles, such as the Earth. We thermodynamically linked the momentum-based wavelength to probability and also explained the appearance of "i" as *Hestenes' geometric algebra duality operator* that transforms vectors into bivectors, appropriate for linking momentum vector to circulating field. We then showed that the length of the circulation vortex is contextual, but may be many wavelengths long, all of which confirm the validity of de Broglie's relation, $P = h/\lambda$ in the context of Okino's list of QM issues. The most comprehensive treatment of quantum gravity theories, the first reference mentioned, *Conversations on Quantum Gravity*, asks: "*What do we expect from a theory of quantum gravity*?", and provides a list including:

- A controlled classical limit
- Internal consistency and compatibility with standard physics
- Unification of all interactions
- A non-perturbative definition of the theory
- First principle derivation of the dynamics
- A background-independent formulation
- A solution space that includes our universe
- An understanding of the underlying fundamental degrees of freedom
- Well-defined physical observables
- The ability to perform computations
- A finite number of external parameters is input
- Suggestions of phenomenology and dialogue with experiment
- Potential experimental tests and compatibility with observations

Primordial field theory addresses every issue on this list, successfully, I believe. Here the quantum gravity wave function is fundamental based on Heaviside's theory of gravitomagnetism (1883) interpreted as nonlinearly self-interacting via $E = mc^2$ (1905), combined with de Broglie's quantum relation $P = h/\lambda$ (1923) and shown to be compatible with Schrödinger's equation (1925) and with Born's probability interpretation (1927):

$$\int \boldsymbol{\lambda} \cdot (\nabla \times \boldsymbol{C}) \mathrm{d}^3 x = \frac{gh}{c^2} \tag{69}$$

Circulation of field $\boldsymbol{C}$ induced by momentum $\boldsymbol{P}$ with characteristic wavelength λ is related to fundamental quantum constant h of angular momentum and fundamental gravitational constant g and the speed of disturbances in the gravitational field, c. The relation is new, but *terms involved in the relation are not new*; Heaviside equations are almost 150 years old; Hestenes' math over 50 years. Misconceptions of the past century simply prevented realization of this relation. It should be noted that this relation includes *all* of the fundamental constants, whose arrangements produce dimensions of mass, length, and time.

The same *Science* article ends with: "…*at the beginning of the 21st century, modern physics encompasses many fundamental and applied problems that are waiting to be unraveled*." This fundamental quantum gravity relation is based only on a reappraisal of century-old misconceptions, such as the "*Equivalence Principle*", "*weak-field approximation*", "*collapse of the wave function*" and "*superposition*" of quantum mechanics, as well as Bell's ignoring the actual distribution of measurements on angular momentum in favor of bipolar (±1) values, with his subsequent claim of "non-locality". We introduce no semantic *wildcards* which have no correlate in the formalism, we use the syntactic formalism of Hestenes' mathematics, applied to the *one field* that the *Standard Model of Particle Physics* assumes existed at the Creation, and we derive field equations equivalent to Einstein's nonlinear GR. *All of our semantic interpretations have correlates in the syntactics*. No new physical phenomena are proposed, no new terms introduced to physics, no new "mystery" is required in order to accept this quantum gravity relation, only a re-appraisal of assumptions that were built-in at the beginning of relativity and quantum theory, when things were still very much "up in the air". The universally acknowledged confusions of the past century are simply re-interpreted according to primordial field theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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