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Masses of Up and Down Quarks

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Abstract

Quarks are one of the most abundant and stable fundamental particles of the universe. However, measuring quark masses is challenging, and the use of quantum chromodynamics in solving for the masses of up and down quarks is a difficult and tedious task, with no simple mathematical theory. This manuscript uses a combination of the chemical equation for down quark decay as well as time-independent Schrödinger equation to formulate a theoretical calculation for the values of the masses of up and down quarks as a function of the masses of neutron, proton, electron, and binding energy. The calculated values for up and down quarks yield 2.058028 MeV and 4.793593 MeV respectively.

Keywords

Fundamental Particles, Neutron Decay, Quarks

1. Introduction

Neutrons and protons found inside the nucleus of every atom have internal quarks which are distributed differently than free neutrons or protons. This may be due to environmental differences between the inside and outside of atoms. The nuclear environment is important to the quark structure of nucleons as the distribution of down quarks is more modified by the internal nuclei environment versus up quarks. In 1983, the European Muon Collaboration observed what is now known as the EMC effect, concluding that quark momentum distributions in nucleons bound inside nuclei are different from those of free nucleons [1]-[3]. These environmental differences have made neutrons relatively stable within the nucleus of an atom. On the other hand, a free neutron is unusable and decays into a proton, an electron, and an electron anti-neutrino, $\bar{\nu}_e$, with a half-life of about 10 minutes. According to the conservation of energy, neutron decay can be expressed as

$$m_n c^2 = m_p c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2 + E_Q \tag{1}$$

where c is the speed of light in vacuum, m_n, m_p, m_e, and $m_{\bar{\nu}_e}$ are the rest mass of neutron, proton, electron, and electron anti-neutrino respectively, E_Q is the decay energy of this reaction and is the difference in the rest mass of the neutron and the sum of the rest masses of the products. Due to the relative size of protons and electrons, almost all this energy is taken away as kinetic energy by the electron.

2. Down Quark Decay

It is already understood that a free neutron decays into a proton and an electron. However, both neutrons and protons are baryons with smaller constituents known as up and down quarks. Both up and down quarks are known to be the fundamental particles with no smaller constituency, and thus far there is no evidence of free quarks [4]. In quantum field theory, a fundamental particle, such as a down quark, can decay into another fundamental particle, an up quark, within a boundary of a composite particle, such as a neutron, if the conservation laws are not violated [5].

Hadrons such as nucleons are composite subatomic particles that are made up of quarks held together by the strong force. According to the Standard Model of Particle Physics, a neutron is composed of three valence quarks, with two down quarks each having a charge of −1/3 e and one up quark with a charge of +2/3 e. Similarly, a proton is composed of three valence quarks, with one down and two up quarks with charges of −1/3 e and +2/3 e respectively, making neutrons and protons neutral and +e charge accordingly [6].

In a neutron decay, one of the down quarks decays to an up quark, and as a result, an energy equivalent to the binding energy of down quark, E_d, is released. According to the conservation of energy, this decay can be expressed as

$$m_d c^2 = m_u c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2 + E_d \tag{2}$$

where m_u and m_d are masses of up and down quarks respectively. Most of the mass-energy of a nucleon is due to Quantum chromodynamics (QCD) binding energy that holds the nucleon together, and only a fraction of this mass is due to the valence quarks [7].

To calculate binding energy, E_d, let us consider the deuterium atom. Deuteron (the nucleus of deuterium) consists of one neutron and one proton. The total mass of deuteron $m_D c^2$ is less than the total masses of a neutron and proton combined [8]. This is due to the binding energy of a down quark, and can be expressed as

$$E_d = m_n c^2 + m_p c^2 - m_D c^2 \tag{3}$$

However, if the neutron in deuteron could decay, it would decay into a proton, an electron, and an electron anti-neutrino. After the decay, the total mass-energy of the particles would be $2m_p c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2$. But this time, the difference, E_u, is due to binding energy of an up quark, and can be expressed as

$$E_u = \left(2m_p c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2\right) - \left(m_D c^2\right) \tag{4}$$

That is, in the decay of a down quark a binding energy, E_d, is released. However, part of this energy, E_u, is used to bind the newly created up quark within the proton, and the rest is released outside of the nucleon as E_Q, which is mostly picked up by the newly created electron as kinetic energy. Combining Equation (2) and Equation (3), we can express the difference in mass-energies of quarks as

$$\varepsilon = m_d c^2 - m_u c^2 = m_n c^2 + m_p c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2 - m_D c^2 \tag{5}$$

Hence, the nuclear binding energy is directly proportional to the difference in the current masses of up and down quarks.

3. Schrödinger-Like Equation

Consider time-independent Schrödinger equation

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = E\psi \tag{6}$$

where ∇^2 is the Laplacian operator, $\hbar$ is the reduced Planck constants, m has a unit of mass, V and E have unit of energy, and ψ is a wave function.

If we consider the nucleus of deuterium, there are 6 valance quarks, 3 up and 3 down. Hence, the average rest mass energy per quark is $\bar{m}_{du}c^2 = \frac{m_d c^2 + m_u c^2}{2}$. Similarly, there are two nucleons, a neutron and a proton. Therefore, the average rest mass-energy per nucleon is, $\bar{m}_{np} = \frac{m_n + m_p}{2}$.

Since any particle with energy and momentum is a de Broglie wave, then we can assume that the quarks in the nucleons behave more like a wave than a particle. Such particles can have a wavelength that is bent into a circular shape of radius r, forming a standing wave within the nucleon (much like an electron in a hydrogen atom). The total energy associated with this wave can then be expressed in terms of the radius of the circle, that is $E = h\nu = \frac{\hbar c}{r}$.

If we change some of the notations in Equation (6), such that $m \to \alpha\bar{m}_{np}$ (where α is fine structure constant), $V \to \bar{m}_{du}c^2$, and $E \to \frac{\hbar c}{r}$. Then using Laplacian in spherical coordinates, Equation (4) becomes

$$\begin{aligned} &-\frac{\hbar^2}{2\alpha\bar{m}_{np}}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\varphi^2}\right] \\ &+ \bar{m}_{du}c^2\psi = \frac{\hbar c}{r}\psi \end{aligned} \tag{7}$$

where $\psi = \psi(r,\theta,\varphi)$.

We can simplify the above equation by expressing the wave function as two directionally independent variables, radial $R(r)$, and angular $F(\theta,\varphi)$, such that $\psi(r,\theta,\varphi) = R(r)F(\theta,\varphi)$, then Equation (7) becomes

$$\begin{aligned} &\frac{1}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2\alpha\bar{m}_{np}r^2}{\hbar^2}\left(\frac{\hbar c}{r} - \bar{m}_{du}c^2\right) \\ &= -\frac{1}{F(\theta,\varphi)\sin^2\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial F}{\partial\theta}\right) + \frac{\partial^2 F}{\partial\varphi^2}\right] \end{aligned} \tag{8}$$

Since the two sides of the above equation are independent of each other, we can set them equal to zero, that is $-\frac{1}{F(\theta,\varphi)\sin^2\theta}\left[\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial F}{\partial\theta}\right)+\frac{\partial^2 F}{\partial\varphi^2}\right]=0$, and $\frac{1}{R(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right)+\frac{2\alpha\bar{m}_{np}r^2}{\hbar^2}\left(\frac{\hbar c}{r}-\bar{m}_{du}c^2\right)=0$. In this manuscript we are interested in the radial equation. By taking the derivative of the radial equation, rearranging and simplifying yields

$$\left(\frac{\mathrm{d}^2R}{\mathrm{d}r^2}-\frac{2\alpha\bar{m}_{np}\bar{m}_{du}c^2}{\hbar^2}R\right)+\frac{2}{r}\left(\frac{\mathrm{d}R}{\mathrm{d}r}+\frac{\alpha\bar{m}_{np}c}{\hbar}R\right)=0 \tag{9}$$

If we try a solution in the form of $R(r)=A\mathrm{e}^{-\frac{r}{a}}$, Where a has a dimension of length, then, we arrive to a simple expression $\left(\frac{1}{a^2}-\frac{2\alpha\bar{m}_{np}\bar{m}_{du}c^2}{\hbar^2}\right)+\frac{2}{r}\left(-\frac{1}{a}+\frac{\alpha\bar{m}_{np}c}{\hbar}\right)=0$. To satisfy this expression, we let each term in the parenthesis equal zero. That is $\frac{1}{a^2}-\frac{2\alpha\bar{m}_{np}}{\hbar^2}\bar{m}_{du}c^2=0$, and $-\frac{1}{a}+\frac{\alpha\bar{m}_{np}c}{\hbar}=0$. Solving for $\bar{m}_{du}c^2$ and a in these equations' yields $\bar{m}_{du}c^2=\frac{\hbar^2}{2\alpha\bar{m}_{np}a^2}$, and $a=\frac{\hbar}{\alpha\bar{m}_{np}c}$ respectively. Combining these results will give us an expression for the average mass of an up and down quark in terms of the average mass of a neutron and proton, that is $\bar{m}_{du}=\frac{\alpha}{2}\bar{m}_{np}$, or

$$m_d+m_u=\alpha\bar{m}_{np} \tag{10}$$

The expression above relates the average mass of up and down quarks in a nucleon to the average mass of the nucleon through the electromagnetic coupling constant α.

4. Results

To find the mass-energies of up and down quarks, we simply combine Equation (5) and Equation (10) and solve for m_u and m_d respectively, that is

$$\begin{pmatrix} m_u c^2 \\ m_d c^2 \end{pmatrix}=\frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} \alpha\bar{m}_{npc^2} \\ \varepsilon \end{pmatrix} \tag{11}$$

Table 1 shows the numerical results of these energies.

It is worth noting that the reduced mass of up and down quarks, μ, is approximately equal to the binding energy, E_u, that is

$$\mu=\frac{m_d m_u}{m_d+m_u}\approx E_u \tag{12}$$

Table 1. Calculated values of up and down quarks.

Mass-energy		Value (MeV)
Neutron [8]	$m_n c^2$	939.56542194

Continued

Proton [8]	$m_p c^2$	938.27208943
Electron [8]	$m_e c^2$	0.51099895069
neutrino	$m_{\bar{\nu}_e} c^2$	~0
Deuteron [8]	$m_D c^2$	1875.612945
Ave. nucleon	$\bar{m}_{np} c^2 = \left(\frac{m_n + m_p}{2}\right) c^2$	938.91875569
Decay energy	$\varepsilon = m_n c^2 + m_p c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2 - m_D c^2$	2.7355653
down quark	$m_d c^2 = \frac{1}{2}\left[\alpha \bar{m}_{np} c^2 + \varepsilon\right]$	4.79359326
up quark	$m_u c^2 = \frac{1}{2}\left[\alpha \bar{m}_{np} c^2 - \epsilon\right]$	2.05802793
Binding energy of an up quark	$E_u = \left(2m_p c^2 + m_e c^2 + m_{\bar{\nu}_e} c^2\right) - \left(m_D c^2\right)$	1.44223281
Reduced mass of quarks	$\mu = \frac{m_d c^2 m_u c^2}{m_d c^2 + m_u c^2}$	1.43985613

α = 0.0072973525643 is the fine structure constant.

5. Discussion

Neutrons are normally bound through the strong force in the nuclide of atoms. However, they may get released as a free neutron because of nuclear reaction. Once free, they live a short life before decaying into a stable proton, an electron, and an electron antineutrino. To satisfy the conservation of energy, the left-over mass is released in the form of energy that is mostly picked up by the electron as kinetic energy. On a sub-nucleon level, the decay of a neutron to proton happens when one of the down quarks within a neutron, decays into an up quark forming a proton, as well as an electron and electron antineutrino. Similarly, to satisfy the conservation of energy, the leftover mass, equivalent to the binding energy is released. However, this binding energy is greater than the energy released outside the nucleons, suggesting that part of this energy is used in the internal conversion from a neutron to proton. Moreover, it suggests that the nuclear binding energy is directly related to current mass of up and down quarks. This binding energy can be calculated as the difference between the sum of the mass-energies of the two nucleons and the mass-energy of the nucleus of deuterium. Hence, the difference in mass-energies of up and down quarks can be calculated. Next, a Schrodinger's time-independent wave equation-like can be formulated to express the sum of up and down quarks in terms of the average masses of the nucleons. Knowing both the sum and difference of the mass-energies of up and down quarks, we can then calculate the mass-energies of up and down quarks as $m_u c^2 = 2.058028\ \text{MeV}$ and $m_d c^2 = 4.793593\ \text{MeV}$, respectively. These results are well within the values reported experimentally by various groups. The value published by various groups ranges from $m_u c^2 = 1.7\ \text{MeV}$ and $m_d c^2 = 3.68\ \text{MeV}$ to $m_u c^2 = 3.02\ \text{MeV}$ and $m_d c^2 = 5.49\ \text{MeV}$ using lattice computations, with the average reported by Particle Data Group $m_u c^2 = 2.16\ \text{MeV}$ and $m_d c^2 = 4.70\ \text{MeV}$ [9]-[11].

The non-relativistic time independent approach for solving the masses of up and down quarks suggests that the obtained values are the "naked" or current rest mass of quarks and hence neglecting the quantum chromodynamic effects. The result in Equation (10) indicates that the average current mass of quarks to the average mass of nucleons is proportional to electromagnetic filed coupling constant α, that is $\dfrac{\bar{m}_{du}}{\bar{m}_{np}} = \dfrac{\alpha}{2}$.

6. Conclusion

The use of quantum chromodynamics in solving for the masses of up and down quarks is a difficult and tedious task, as the solutions exhibit themselves in an infinite series. It is nearly impossible to measure the masses of quarks directly as they are confined within hadrons. One method of finding the masses of quarks is the lattice simulation using very high-performance computers to find the solution of large linear systems [12] [13]. There are some theoretical models to reshape the estimated quark masses, however, to date, there are no simple mathematical theories to derive these masses [14]. This manuscript formulates a theoretical calculation for the values of the masses of up and down quarks that yields 2.058028 MeV and 4.793593 MeV respectively.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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