Journal of Modern Physics, 2025, 16(7), 940-949
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Wave Generation by Bessel Polarized Laser Beam in Plasma Beat-Wave Accelerator

Bahaa F. Mohamed[1*], Amany M. Gouda[1], Rehab A. Albrulosy[2,3], Nermen M. Elbasha[4]

[1]Plasma Physics Department, Nuclear Research Center, Egyptian Atomic Energy Authority, Cairo, Egypt
[2]Department of Physics, Faculty of Science, Banha University, Banha, Egypt
[3]Physics Department, College of Science and Arts, Qassim University, Al-Badayea, Saudi Arabia
[4]Physics Department, Faculty of Science, Ain-Shams University, Cairo, Egypt
Email: *mohamedbahf@yahoo.co.uk, *amanygooda@gmail.com

How to cite this paper: Mohamed, B.F., Gouda, A.M., Albrulosy, R.A. and Elbasha, N.M. (2025) Wave Generation by Bessel Polarized Laser Beam in Plasma Beat-Wave Accelerator. *Journal of Modern Physics*, **16**, 940-949.
https://doi.org/10.4236/jmp.2025.167049

Received: May 13, 2025
Accepted: July 15, 2025
Published: July 18, 2025

Abstract

The plasma beat-wave accelerator (PBWA) scheme is one of a number of methods for producing relativistic electron plasma waves via the interaction of an intense laser pulse with an underdense plasma. The PBWA scheme involves the co-propagation through a plasma of two laser pulses of slightly differing frequencies, ω_1 and ω_2 such that ($\omega_1 - \omega_2 = \omega_b \ll \omega_1, \omega_2$). The superposition of these laser envelopes with which there is an associated ponderomotive force. If the frequency of the force is resonant with the electron plasma frequency ω_{pe}, a large-amplitude relativistic electron plasma wave (EPW) can be produced. In our work, we have studied the direct acceleration of electrons by using crossed linearly polarized Bessel beams with slightly different frequencies in underdense plasma. General wave equation which describes the beat magnetic field has been obtained and analytically solved for any plasma density and beam frequency. The electric field of a longitudinal electron plasma oscillation with plasma velocity (v_{ph}) near the speed of light (c) accelerates charged particle to high energies is presented. It is possible for radiation beat wave to resonantly drive large amplitude electron plasma waves. It is found that plasma waves are of particular interest can accelerate electrons efficiently to high energies with short distances. Such electrons may be used to address other therapies such as allowing for a handheld radiation therapy device that can directly target superficial skin cancers such as melanoma.

Keywords

Linearly Polarized Bessel Beams, Electron Acceleration, Plasma Beat Wave

1. Introduction

The charged particle acceleration is a subject of great interest to the research community due to its diverse applications in the field of nuclear physics, thermonuclear fusion research, coherent harmonic generation and high-energy particle physics. All over the world various efforts [1] have been made for achieving higher acceleration gradients for the particle acceleration, plasma beat wave acceleration, laser wake field acceleration etc.

The researchers have made theoretical as well experimental attempts for the particle acceleration [2]-[7]. Chan [3] showed that an abnormally large amount of energy can be transferred from the radiation to relativistic charged particle when it interacts with a laser beam moving almost together in same direction. In direct acceleration scheme, Mckinstrie and Startsev [5] have shown that the pre-accelerated electron can be accelerated significantly, but they neglected the effect of longitudinal field of the laser pulse. Lu *et al.* [7] have studied the electron motion in electromagnetic field of a short pulse high-intensity laser in the vacuum for the electron acceleration. Jawla *et al.* [8] have also studied the electron acceleration and evaluated the fields for fundamental mode in a waveguide filled with plasma under the effect of external magnetic field. Analytical and numerical study for electron dynamics in the fields associated with a transverse magnetic (TM) wave propagating inside a rectangular waveguide is investigated by Mohamed and Gouda [9]. It has been found that the acceleration gradient and deflection angle depend strongly on the parameters of the microwave (intensity, frequency, etc.) and the dimensions of the waveguide.

The plasma beat-wave accelerator (PBWA) scheme, first proposed by Tajima and Dawson [10], is one of a number of methods for producing relativistic electron plasma waves via the interaction of an intense laser pulse with an underdense plasma. The PBWA scheme involves the co-propagation through a plasma of two laser pulses of slightly differing frequencies, $\Delta\omega = \omega_1 - \omega_2 = \omega_{pe}$ and $\Delta k = k_1 - k_2$, where $(\omega_{1,2}, k_{1,2})$ are the frequencies and wave numbers of the two lasers, respectively, and ω_{pe}, k_p is the plasma frequency, wave number. The superposition of these laser envelope with which there is an associated ponderomotive force. If the frequency of the force is resonant with the electron plasma frequency ω_{pe} a large-amplitude relativistic electron plasma wave (EPW) can be produced. These plasma waves are of particular interest, since they can be used to accelerate electrons efficiently to high energies with short distances [11].

If $\omega_{1,2} \gg \omega_p$ then the phase velocity of the plasma waves $\upsilon_{ph} = \omega_p / k_p = (\omega_1 - \omega_2)/(k_1 - k_2) = \Delta\omega/\Delta k$ equals the group velocity of the laser beams $v_g = c\left(1 - \omega_p^2/\omega_{1,2}^2\right)^{1/2}$ which is almost equal to c in an under dense plasma. Particles that are injected into the beat wave region with a velocity comparable to the phase velocity of the electron plasma waves, can gain more energy from the longitudinal electric field. Since ω_1 is close to ω_2 and much larger than ω_p, the Lorentz factor δ_p associated with the beat waves is $\delta_p = \left(1 - \frac{v_{ph}^2}{c^2}\right)^{-1/2} = \frac{\omega_{1,2}}{\omega_p} \gg 1$.

The beat wave process is related to stimulate Raman forward scattering (SRFS) which investigate the scattered electromagnetic wave propagates in the same direction as the incident electromagnetic wave (as forward scattering). The general equations describing the beat wave and SRFS are similar. It is sufficient to analyze the problem of plasma wave growth and saturation using the relativistic fluid equations for electrons, and Maxwell and Poisson equations. Experiments have been conducted using microwaves, CO_2 lasers and glass lasers as the drive beams. In the experiments, the plasma wave was not driven to its limiting relativistic saturation level due to the growth of modulation instabilities which have a growth rate determined by the ion plasma frequency.

It was pointed out by Tang *et al.* (1984) that by deliberately allowing for the relativistic mass variation effect [12] and having a denser plasma such that the plasma frequency was initially larger than the laser frequency difference. The plasma wave would come into resonance as it grew, allowing a larger maximum saturation value to be attained. An increase of about 50% in the saturated wave amplitude can be achieved by this technique.

Previous work has shown that the beat-wave scheme is a reliable and reproducible method for generating plasma waves having relativistic phase velocities [13]-[16]. An important consideration in the beat wave scheme is to have sufficiently intense lasers such that the time to reach saturation is short compared to the ion plasma period. When the time-scale is longer than the latter, the ion dynamics becomes important and the electron plasma wave becomes modulationally unstable by coupling to low-frequency ion density perturbations (Amiranoff *et al.* 1995) [13].

In addition to electron acceleration applications, the long trains of periodic large-amplitude density oscillations achievable through autoresonant PBWA could serve as a controllable moving grating for manipulating light [17]-[19]. The PBWA scheme has also been proposed as a platform for accelerating heavier particles, such as muons [20]-[22]. To establish the realistic utility of the PBWA scheme in the applications mentioned, it is crucial to go beyond a simplified fluid description and to explore its parametric dependences.

Barraza-Valdez E. *et al.* [23] have considered high-density laser wakefield acceleration (LWFA) in the nonrelativistic regime of the laser. In place of an ultrashort laser pulse, they excited wakefields via the Laser Beat Wave (BW) that accesses this near-critical density regime. They used 1D Particle-in-Cell (PIC) simulations to study BW acceleration using two co-propagating lasers in a near-critical density material and allowed for acceleration of electrons to greater than keV energies at far smaller intensities, such as 10^{14} W/cm^2.

Also, the kinetic and nonlinear processes that come into play during autoresonant plasma beat-wave acceleration of electrons and on acceleration efficiency have recently been investigated by Luo M. *et al.* [24]. They used fully kinetic particle-in-cell (PIC) simulations to revisit autoresonance in the PBWA scheme.

2. Theoretical Model

The equations describing nonlinear waves in a cold, collisionless plasma with electron velocity (ν) electron density (n) (the ions are stationary) are:

$$\frac{\partial \mathbf{p}}{\partial t}+(\boldsymbol{\nu}\cdot\nabla)\mathbf{p}=-e\mathbf{E}-\frac{e}{c}(\boldsymbol{\nu}\times\mathbf{B}) \tag{1}$$

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\nu)=0\,, \tag{2}$$

$$\nabla\cdot\mathbf{E}=4\pi e(n_o-n)\,, \tag{3}$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}\,, \tag{4}$$

$$\nabla\times\mathbf{B}=\frac{1}{c}\frac{\partial\mathbf{E}}{\partial t}-\frac{4\pi e n\nu}{c} \tag{5}$$

where, $\mathbf{P}=m\gamma\boldsymbol{\nu}$ is the electron momentum, n and n_0 are the electron density and plasma density, e and m are the electron charge and mass respectively. E and B are the electric and magnetic fields. Here, neglecting ion motion and thermal effects are neglecting and relativistic mass variation is included only for electrons.

In a plasma beat-wave accelerator, the field of the two-frequency laser can be expressed as linearly polarized Bessel beams (x -polarized) and Langmuir wave are:

$$\mathbf{E}_j(r,\psi,z,t)=E_{oj}\mathrm{e}^{im\psi}J_m(k_g\rho)\mathrm{e}^{-i(\omega_j t-k_j z-\theta)}\mathbf{e}_x+c.c. \tag{6}$$

$$\mathbf{E}_3=\mathbf{E}_3(x)\mathrm{e}^{-i(\omega_3 t-k_3 z-\theta)}+c.c. \tag{7}$$

where, J_m is the m^{th} order of Bessel function, $E_{oj}\,(j=1,2)$ is the field amplitude, $\rho^2=x^2+y^2$, ψ is the azimuthal angle and θ is a constant.

The frequencies of the three waves and their transmitted modes are satisfied with the phase- matching conditions:

$$\omega_1-\omega_2=\omega_3\,,\quad k_1-k_2=k_3 \tag{8}$$

Substituting Equation (6) and Equation (7) into Equations (1)-(5) gives rise to following relations:

The first pump wave (ω_1 , k_1):

$$-i\omega_1 p_1+\nu_2\frac{\partial p_{3x}}{\partial x}+ik_2\nu_{3z}+\nu_{3x}\frac{\partial}{\partial x}p_{2x}=-eE_{1x}+\frac{e}{c}\nu_{3z}B_{2y}\,, \tag{9}$$

$$B_1=N_1E_1\,, \tag{10}$$

$$k_1B_y=\frac{\omega_1}{c}E_1-\frac{4\pi ie}{c}(n_o\nu_1+n_3\nu_2+n_2\nu_{3x})\,, \tag{11}$$

The Second pump wave (ω_2 , k_2):

$$-i\omega_2 p_2+\nu_1\frac{\partial p^*_{3x}}{\partial x}+ik_1\nu^*_{3z}p_1+\nu^*_{3x}\frac{\partial}{\partial x}p_1=-eE_2+\frac{e}{c}\nu^*_{3z}B_1\,, \tag{12}$$

$$B_2 = N_2 E_2 , \tag{13}$$

$$k_2 B_2 = \frac{\omega_2}{c} E_2 - \frac{4\pi ie}{c}\left(n_o v_2 + n_3^* v_1 + n_1 v_{3x}^*\right) \tag{14}$$

The generated beat wave (ω_3 , k_3):

$$-i\omega_3 p_{3x} + v_1 \frac{\partial p_2^*}{\partial x} + v_2^* \frac{\partial}{\partial x} p_1 = -eE_{3x} , \tag{15}$$

$$-i\omega_3 p_{3z} = -eE_{3z} + \frac{e}{c}\left[v_1 B_2^* + v_2^* B_1\right], \tag{16}$$

$$\frac{\partial E_{3x}}{\partial x} + ik_3 E_{3z} = 4\pi e n_3 , \tag{17}$$

$$-i\omega_3 n_3 + n_o\left[\frac{\partial v_{3x}}{\partial x} + ik_3 v_{3z}\right] - in_o \frac{\partial}{\partial x}\left(\frac{v^*}{\omega_1}\frac{\partial v_1}{\partial x} - \frac{v_1}{\omega_2}\frac{\partial v_2^*}{\partial x}\right) = 0 \tag{18}$$

where, $n_3 = n - n_0$ is the electron density fluctuation caused by the plasma wave. From Equation (9), Equation (11), Equation (12) and Equation (13), we have the momentum ratio $\alpha = p_1/p_2$ and $\sigma = \omega_1/\omega_2$ satisfies the following form:

$$\alpha = \frac{D_1 - iA_1 D_2/\omega_2}{D_2 - iA_2 D_1/\omega_1}$$

where,

$$D_1 = -e\left[E_1 - \frac{v_{3z}}{v_{2ph}} E_2\right], \quad D_2 = -e\left[E_2 - \frac{v_{3z}^*}{v_{1ph}} E_1\right],$$

$$A_1 = \left(\Delta_1 + \frac{\partial}{\partial x}\right) v_{3x} + ik_2 v_{3z}, \quad A_2 = \left(\Delta_2 + \frac{\partial}{\partial x}\right) v_{3x}^* + ik_1 v_{3z}^*,$$

$$\Delta_j = k_j \cos m\psi \frac{J_{m-1}}{J_m} - \frac{m}{\rho} \mathrm{e}^{im\psi}$$

The superscript (*) on different quantities represents the complex conjugate of the relevant quantity and ($j = 1,2$). Also, the phase velocities of the light waves are $v_{1,2ph} = \omega_{1,2}/k_{1,2}$, while $v_{3ph} = \omega_3/k_3$ is the phase velocity of the excited plasma wave.

The wave equations, for the electric fields of the pump waves, can be obtained from the Equations (10)-(13) with the Poisson equation as follows:

$$\left[1 - \left(c/v_{1ph}\right)^2\right] E_1 = \frac{4\pi ie n_o}{\omega_1}\left[v_1 + v_2\left(\frac{n_3}{n_o} - \frac{i\Delta_2}{\omega_2} v_{3x}\right)\right], \tag{19}$$

$$\left[1 - \left(c/v_{2ph}\right)^2\right] E_2 = \frac{4\pi ie n_o}{\omega_2}\left[v_2 + v_1\left(\frac{n_3^*}{n_o} - \frac{i\Delta_1}{\omega_1} v_{3x}^*\right)\right] \tag{20}$$

3. Beat-Wave and Electron Acceleration

From Equation (15) and Equation (16), we can derive the longitudinal and transverse currents due to the transfer of momentum flux as follows:

$$J_{3x} = \frac{i\omega_p^2 c}{4\pi\omega_3\gamma\eta v_{3ph}} B_{3y} + \frac{ien_o}{\omega_3\eta}\left(v_1 v_2^*\right)\left(\Delta_1 + \Delta_2^*\right) - \frac{i\omega_p^2}{\omega_3^2\gamma\eta}\left(n_1 v_2^* + n_2^* v_1\right) \quad (21)$$

$$J_{3z} = \frac{-\omega_p^2 c}{4\pi\omega_3^2\gamma\eta}\frac{\partial}{\partial x} B_{3y} + \frac{i\omega_p^2}{4\pi\gamma\omega_3\eta}\left(v_1 B_2^* + v_2^* B_1\right) \quad (22)$$

The electron density perturbation n_3 can be studied from the Equation (18) which depends on the transverse and longitudinal velocities due to the Bessel profile of the pump waves as follows:

$$\frac{n_3}{n_o} = \frac{v_{3z}}{v_{3ph}} - \frac{i}{\omega_3}\frac{\partial v_{3x}}{\partial x} - \frac{v_1 v_2^*}{\omega_3}\left\{\left(\frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right)\left(\Delta_1 + \Delta_2^*\right) + \frac{\partial}{\partial x}\left(\frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right)\right\} \quad (23)$$

Also, by using Equation (21) and Equation (22), it can be simplified into the following form depending on the pump waves:

$$\left(1 - \frac{\omega_p^2}{\gamma\omega_3^2}\right)\frac{n_3}{n_o} = \frac{v_1 v_2^*}{\omega_3}\left\{\left(\Delta_1 + \Delta_2^*\right)\left[\frac{\Delta_1 + \Delta_2^*}{\omega_3} + \frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right] + \frac{\partial}{\partial x}\left[\frac{\Delta_1 + \Delta_2^*}{\omega_3} + \frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right]\right.$$
$$\left. - \frac{ie}{m\gamma\omega_3 v_{3ph}}\left(\frac{v_1}{v_{2ph}} E_2^* - \frac{v_2^*}{v_{1ph}} E_1\right)\right\} \quad (24)$$

where, $\eta = 1 - \frac{\omega_p^2}{\gamma\omega_3^2}$,

$$\frac{\partial \Delta_j}{\partial x} = -k_j^2 \cos^2 m\psi\left[1 + \left(\frac{J_{m-1}}{J_m}\right)^2\left(1 - \frac{mJ_m}{k_j\rho J_{m-1}}\right) - \left(\frac{m-1}{k_j\rho}\right)\frac{J_{m-1}}{J_m^2}\right]$$
$$+ \frac{m}{\rho^2}\mathrm{e}^{im\psi}\left(\cos m\psi + im\sin m\psi\right) + \frac{mk_j\sin^2 m\psi}{\rho}\frac{J_{m-1}}{J_m}$$

A perfect resonance is impossible, for the maximum amplitude of plasma wave is limited by wave breaking, in which the electron density fluctuation n_3 becomes comparable to the plasma density n_o.

At the wave breaking limit (*i.e.*, $\omega_1 - \omega_2 = \omega_{pe}$) Equation (23) gives:

$$\frac{v_{3z}}{v_{3ph}} = 1 + \frac{ec}{m\gamma\eta\omega_3 v_{3ph}}\left(\frac{\partial B_3}{\partial x}\right) - \frac{v_1 v_2^*}{\omega_3\eta}\left\{\left(\frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right)\left(\Delta_1 + \Delta_2\right) + \frac{\partial}{\partial x}\left(\frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right)\right\} \quad (25)$$

In this case, it can be deduced that $v_{3z} \to v_{ph}$ when the magnetic field intensity of plasma wave satisfies the condition.

Through the equations of the beat current densities and maxwell's equations, the magnetic field component obeys to the following wave equation:

$$\frac{\partial^2 B_3}{\partial x^2} + \chi^2 B_3 = G \quad (26)$$

where, G is the source function due to nonlinear interaction of the pump waves.

$$G(x) = \frac{4\pi en_o k_3}{c}\left[\frac{\Delta_1 + \Delta_2^*}{\omega_3} - \frac{\Delta_1}{\omega_1} + \frac{\Delta_2^*}{\omega_2}\right]v_1 v_2^* + \frac{2i\omega_p^2}{c\gamma\omega_3}\left[\frac{\Delta_1 v_1}{v_{2ph}} E_2^* + \frac{\Delta_2^* v_2^*}{v_{1ph}} E_1\right] \quad (27)$$

and $\chi^2 = \frac{\omega_3^2}{c^2}\left[1 - \frac{\omega_p^2}{\gamma\omega_3^2} - \frac{c^2}{v_{3ph}^2}\right]$

However, it can be noticed that the electric and magnetic fields $\omega_3 \simeq \omega_p / \sqrt{\gamma}$ relativistic effect of the electron. The wave Equation (26) has the following solution:

$$B_3 = \frac{m\gamma\eta\omega_3 v_{3ph}}{ec}\int\left[\left(\Delta_1 + \Delta_2\right)\left(\frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right) + \frac{\partial}{\partial x}\left(\frac{\Delta_1}{\omega_1} - \frac{\Delta_2^*}{\omega_2}\right)\right] v_1 v_2^* \mathrm{d}x \tag{28}$$

The transverse and longitudinal electric field components of the beat—wave (E_{3z} and E_{3x}) can be obtained as follows:

$$E_{3z} = \frac{ic}{\omega_3\eta}\left(\frac{\partial}{\partial x}B_3\right) + \frac{\omega_p^2}{\gamma\omega_3^2\eta}\left(\frac{v_1}{v_{2ph}}E_2^* + \frac{v_2^*}{v_{1ph}}E_1\right) \tag{29}$$

$$E_{3x} = \frac{c}{v_{3ph}\eta}B_3 + \frac{4\pi e n_o}{\omega_3^2\eta}\left[\frac{\Delta_1 + \Delta_2^*}{\omega_3} - \frac{\Delta_1}{\omega_1} + \frac{\Delta_2^*}{\omega_2}\right] v_1 v_2^* \tag{30}$$

4. Conclusions and Results

In this work, the direct acceleration of electrons by using crossed linearly polarized Bessel beams with slightly different frequencies in underdense plasma is studied. The electric field of a longitudinal electron plasma oscillation with plasma velocity (v_{ph}) near the speed of light (*c*) accelerates charged particle to high energies is presented. It is possible for radiation beat wave to resonantly drive large amplitude electron plasma waves. An accelerating gradient on the order of GeV/cm is theoretically possible, where n is the electron number density in units of cm^{-3} the basic mechanism can be looked at in terms of the nonlinear, longitudinal ponderomotive force associated with the beat pattern. This force acts on plasma electrons to produce charge separation and hence plasma oscillations at the resonant frequency. It is found that when laser intensities, frequencies and plasma density are given, the current, perturbed density and the field components of the excited plasma wave are readily calculated.

The matching of the laser beat frequency to the plasma frequency (*i.e.*, $\Delta\omega = \omega_1 - \omega_2 = \omega_{pe}$) is generally regarded as the perfect resonance condition but in our case, the perfect resonance condition is $\omega_{pe}^2 = \gamma\left(\omega_1 - \omega_2\right)^2$. This mismatching of two frequencies is more favorable as the recent research proved that our results investigate that the amplitudes of the fields of the excited plasma wave have become large with high amplitude of laser beams. However, there is also a pondermotive force associated with the transverse spatial variation of the pump profile. Because the beat wave generation of plasma waves is resonant excitation, large amplitude plasma waves can be developed even though the laser beams are relatively weak. Besides, as we have seen in this article, by adjusting the laser duration or chirp rate, precise control over the electric field amplitude and electron beam energy can be achieved. It is also possible for electron acceleration applications that autoresonant PBWA could serve as a controllable moving grating for manipulating laser light to achieve the long trains of periodic large-amplitude density oscillations [23].

Ponomareva E. and Shevchenko A. [25] analyze the excitation and amplitude enhancement of a plasma wave by a beating optical beam. They compared between the generating highly energetic electrons with reduced transverse spread via a two-color Bessel-Gauss beam with those obtained when using a Gaussian PBWA. When a Bessel-Gauss beam is used, the beam diameter can be much smaller and the Rayleigh range is essentially unlimited. For example, the Bessel beam (with a radius of 2 μm only) can easily be made to remain propagation-invariant over a distance of 10 mm, making it possible to achieve much smaller electron spreads. So that, we have focused our work here on the study of plasma beat-wave acceleration driven by two co-propagating, linearly polarized Bessel laser beams of slightly different frequencies.

It can be seen that in the near-critical density regime, electron energies up to 10 keV can be obtained using intensities 10^{14} W/cm^2. These low laser intensities allow for the use of novel fiber technology described, along with many applications such as for radiation therapy treatment [23]. Accordingly, this introduces a new possible way to operate an endoscopic electron radiotherapy using fiber laser (that is, an endoscopic radiation therapy).

Authors' Note

It may be noted that only the abstract of the manuscript has been published in the preceding of "39th EPS Conference & 16th Int. Congress on Plasma Physics (2012)".

Conflicts of Interest

The authors declare that they have no conflict of interests.

References

[1] Zhao, Z. and Lü, B. (2007) Electron Acceleration Using Two Crossed Bessel Beams in Vacuum. *Optics & Laser Technology*, **39**, 1166-1169. https://doi.org/10.1016/j.optlastec.2006.09.005

[2] McMillan, E.M. (1950) The Origin of Cosmic Rays. *Physical Review*, **79**, 498-501. https://doi.org/10.1103/physrev.79.498

[3] Chan, Y.W. (1971) Ultra-Intense Laser Radiation as a Possible Energy Booster for Relativistic Charged Particle. *Physics Letters A*, **35**, 305-306. https://doi.org/10.1016/0375-9601(71)90397-5

[4] Esarey, E., Sprangle, P., Krall, J. and Ting, A. (1996) Overview of Plasma-Based Accelerator Concepts. *IEEE Transactions on Plasma Science*, **24**, 252-288. https://doi.org/10.1109/27.509991

[5] McKinstrie, C.J. and Startsev, E.A. (1996) Electron Acceleration by a Laser Pulse in a Plasma. *Physical Review E*, **54**, R1070-R1073. https://doi.org/10.1103/physreve.54.r1070

[6] Mohamed, B.F., Gouda, A.M. and Ismail, L.Z. (2011) Electron Dynamics in Presence of Static Helical Magnet Inside Circular Waveguide. *IEEE Transactions on Plasma*

Science, **39**, 842-846. https://doi.org/10.1109/tps.2011.2104369

[7] Lu, Q.M., Cheng, Y., Xu, Z.Z. and Wang, S. (1998) Electron Acceleration by a Laser Pulse in Vacuum. *Physics of Plasmas*, **5**, 825-827. https://doi.org/10.1063/1.872773

[8] Jawla, S.K., Kumar, S. and Malik, H.K. (2005) Evaluation of Mode Fields in a Magnetized Plasma Waveguide and Electron Acceleration. *Optics Communications*, **251**, 346-360. https://doi.org/10.1016/j.optcom.2005.03.019

[9] Mohamed, B.F. and Gouda, A.M. (2011) Electron Acceleration by Microwave Radiation Inside a Rectangular Waveguide. *Plasma Science and Technology*, **13**, 357-361. https://doi.org/10.1088/1009-0630/13/3/16

[10] Tajima, T. and Dawson, J.M. (1979) Laser Electron Accelerator. *Physical Review Letters*, **43**, 267-270. https://doi.org/10.1103/physrevlett.43.267

[11] Walton, B.R., Mangles, S.P.D., Najmudin, Z., Tatarakis, M., Wei, M.S., Gopal, A., *et al.* (2006) Measurements of Forward Scattered Laser Radiation from Intense Sub-Ps Laser Interactions with Underdense Plasmas. *Physics of Plasmas*, **13**, Article ID: 013103. https://doi.org/10.1063/1.2363170

[12] Tang, C.M., Sprangle, P. and Sudan, R.N. (1985) Dynamics of Space-Charge Waves in the Laser Beat Wave Accelerator. *The Physics of Fluids*, **28**, 1974-1983. https://doi.org/10.1063/1.865375

[13] Amiranoff, F., Bernard, D., Cros, B., Jacquet, F., Matthieussent, G., Marques, J.R., *et al.* (1996) A Summary of the Beatwave Experiments at Ecole Polytechnique. *IEEE Transactions on Plasma Science*, **24**, 296-300. https://doi.org/10.1109/27.509993

[14] Tochitsky, S.Y., Narang, R., Filip, C.V., Musumeci, P., Clayton, C.E., Yoder, R.B., *et al.* (2004) Experiments on Laser Driven Beatwave Acceleration in a Ponderomotively Formed Plasma Channel. *Physics of Plasmas*, **11**, 2875-2881. https://doi.org/10.1063/1.1651100

[15] Tochitsky, S.Y., Narang, R., Filip, C.V., Musumeci, P., Clayton, C.E., Yoder, R.B., *et al.* (2004) Enhanced Acceleration of Injected Electrons in a Laser-Beat-Wave-Induced Plasma Channel. *Physical Review Letters*, **92**, Article ID: 095004. https://doi.org/10.1103/physrevlett.92.095004

[16] Lindberg, R.R., Charman, A.E., Wurtele, J.S. and Friedland, L. (2004) Robust Autoresonant Excitation in the Plasma Beat-Wave Accelerator. *Physical Review Letters*, **93**. https://doi.org/10.1103/physrevlett.93.055001

[17] Lehmann, G. and Spatschek, K.H. (2016) Transient Plasma Photonic Crystals for High-Power Lasers. *Physical Review Letters*, **116**, Article ID: 225002. https://doi.org/10.1103/physrevlett.116.225002

[18] Riconda, C. and Weber, S. (2023) Plasma Optics: A Perspective for High-Power Coherent Light Generation and Manipulation. *Matter and Radiation at Extremes*, **8**, Article ID: 023001. https://doi.org/10.1063/5.0138996

[19] Wu, Z., Zuo, Y., Zeng, X., Li, Z., Zhang, Z., Wang, X., *et al.* (2022) Laser Compression via Fast-Extending Plasma Gratings. *Matter and Radiation at Extremes*, **7**, Article ID: 064402. https://doi.org/10.1063/5.0109574

[20] Peano, F., Vieira, J., Fonseca, R.A., Mulas, R., Coppa, G. and Silva, L.O. (2008) Direct Acceleration of Ions with Variable-Frequency Lasers. *IEEE Transactions on Plasma Science*, **36**, 1857-1865. https://doi.org/10.1109/tps.2008.926877

[21] Peano, F., Vieira, J., Silva, L.O., Mulas, R. and Coppa, G. (2008) All-Optical Trapping and Acceleration of Heavy Particles. *New Journal of Physics*, **10**, Article ID: 033028. https://doi.org/10.1088/1367-2630/10/3/033028

[22] Peano, F., Vieira, J., Mulas, R., Coppa, G., Bingham, R. and Silva, L.O. (2009) Pro-

spects for All-Optical Ultrafast Muon Acceleration. *Plasma Physics and Controlled Fusion*, **51**, Article ID: 024006. https://doi.org/10.1088/0741-3335/51/2/024006

[23] Barraza-Valdez, E., Tajima, T., Strickland, D. and Roa, D.E. (2022) Laser Beat-Wave Acceleration near Critical Density. *Photonics*, **9**, Article 476. https://doi.org/10.3390/photonics9070476

[24] Luo, M., Riconda, C., Pusztai, I., Grassi, A., Wurtele, J.S. and Fülöp, T. (2024) Control of Autoresonant Plasma Beat-Wave Wakefield Excitation. *Physical Review Research*, **6**, Article ID: 013338. https://doi.org/10.1103/physrevresearch.6.013338

[25] Ponomareva, E. and Shevchenko, A. (2023) Plasma-Wave Generation and Acceleration of Electrons by a Nondiverging Beating Optical Beam. *Physical Review Accelerators and Beams*, **26**, Article ID: 061301. https://doi.org/10.1103/physrevaccelbeams.26.061301