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# Physical Space Was Not Expanding

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## Abstract

Plurality of characteristic peaks observed in number density distribution of galaxy redshift reveals that extent of physical space has been finite. Significant portion of observed celestial objects is found pair-wise associated, *i.e.*, the observed lights were emitted from one and same luminescent source but seen at different sky directions of observer, which is a unique phenomenon that can occur but only in finite space. Cosmic microwave radiation has always been interpreted as afterglow of Big Bang event. However, such radiation is shown unobservable to current observer if Hubble-Lemaître Correlation is interpreted as caused by receding motion of celestial objects. On the other hand, cosmic radiation can be understood as a common and ordinary phenomenon due to space lens, a unique property only of finite space. From Sloan Digital Sky Survey data, internal diameter of physical space is measured as 2.0 billion light years. If celestial objects were receding, hence physical space was expanding, then characteristic peaks of finite physical space should not appear evenly in number density distribution of redshift of the objects but more sparsely with respect to redshift increase. However, as revealed by the data, locations of the characteristic peaks in the distributions are rather even that do not match the locations as required by receding motion of object. Therefore, as evidenced by the data, physical space was not expanding, at least during the recent 18 billion years. In addition, considerable portion of observed quasars is found sharing a common factor of ~1/2 for their respective gravitation redshifts.

## Keywords

Geometry of Physical Space, Big Bang Model, Astrophysics

## 1. Introduction

In eras of Galileo and Newton, physical space (PHS) was perceived as of infinite extent [1]. Special relativity theory [2] inherited the relativity principle of Newton [3] but did not alter the assumption thereunder, *i.e.*, PHS being of infinite extent. On the other hand, general relativity theory (GRT) [4] implied PHS being finite that was manifested to an expandable and/or contractible PHS [5]. On such ground, there arose the Big Bang Model [6] that has evolved to the mainstream theory in modern astrophysics. However, implication/ramification of finite PHS was and is not well understood or even aware of, let alone observation consequences thereof, that caused perplexity on, e.g., findings from observations via James Webb Space Telescope (JWST) [7] [8]. It has recently been shown by metrological analysis that, under the law of mass-energy conservation (LME) [9], PHS must be finite [10]. This work is to analyze celestial observation data to show that PHS is indeed finite. Several immediate consequences of finiteness of space are also derived that leads to nonexpanding PHS via measurement to the data.

## 2. Internal Radius of Physical Space

PHS is known or believed to have three dimensions, *i.e.*, of three and only three orthogonal directions of motion at any point of the space. Therefore, if PHS is a finite space, then PHS is categorized as $\mathbf{S}^3$, *i.e.*, a continuous, homogeneous, isotropic, and finite space without boundary except three-dimensions. Then, total area of a two-dimensional sphere in PHS is [10],

$$A_2 = \frac{16R_i^2}{\pi}\sin^2\left[\frac{\pi}{2}\frac{s}{R_i}\right] \rightarrow dV_3 = A_2 ds \tag{1}$$

$A_2$: Total area of two-dimensional sphere in $\mathbf{S}^3$. $R_i$: Internal radius of $\mathbf{S}^3$. *s*: Internal radius of two-dimensional sphere in $\mathbf{S}^3$. $dV_3$: Volume of observation sphere centered at observer in $\mathbf{S}^3$. *ds*: Thickness of observation sphere.

That is, volume of an observation sphere is a function of internal distance between observer and the sphere as in Equation (1). If PHS was not finite then $dV_3 = 4\pi s^2 ds$.

**Assuming** spacial distribution of light emitting objects (LEOs) in PHS was uniform at large scale (uniform distribution assumption, UDA) then, in finite PHS at any moment of Rest Time (RT) [10], number of LEOs residing in an observation sphere shall be proportional to volume of the sphere,

$$N_s \equiv N_{s,\mathrm{LEO}} \propto dV_3 \propto \sin^2\left[\frac{\pi}{2}\frac{s}{R_i}\right]ds,\ 0 \le s \le 2R_i. \tag{2}$$

$N_s$: Number of LEOs residing in an observation sphere of internal radius *s* and thickness *ds*.

Therefore, under UDA, $N_s$ shall exhibit a symmetric peak centered at $R_i$, if plotted against *s*, and profile of the peak, which is characteristic but only of finite space, shall be congruent to form of Expression (2). In contrast, if PHS was not finite, *i.e.*, spacial extent of PHS was infinite, then the plot shall be congruent to $s^2 ds$ and no peak shall show up in the plot. Even if UDA were not valid therein, this would still hold true, since no center should exist in physical space.

In two-dimensional analogy (TDA), a two-dimensional finite space $\mathbf{S}^2$ can be visualized as surface of a three-dimensional ball, similar to surface of Earth but perfect in geometry, wherein all matter and radiation exists. Then, a one-dimensional sphere in $\mathbf{S}^2$ is a circle at the surface. Total area of a one-dimensional sphere, *i.e.*, length of a circle, $A_1$ is $4R_i \sin\left[(\pi/2)(s/R_i)\right]$, wherein, $R_i$ is internal radius of the surface, defined as half of the distance between North and South Poles of the surface measured internally, *i.e.*, at and along the surface, and $s$ is internal radius of the circle. Therefore, two-dimensional volume, *i.e.*, area, of an observation sphere $dV_2$ is $A_1 ds$, wherein, $s$ is internal distance between observer and the circle and $ds$ thickness of the sphere internal to the surface and perpendicular to the circle. Therefore, under UDA, $N_s$ plot shall be congruent to form of $\sin\left[(\pi/2)(s/R_i)\right]ds$. If PHS was infinite then the surface would become an Euclid plane and $N_s$ plot shall be congruent to $s\mathrm{d}s$ with no peak shown up in the plot.

Universe is generally understood as comprised of *space*, *time*, and *matter* (including radiation and vacuum). Therefore, in the context of expanding universe, PHS resembles a simply connected volume contained/occupied by *matter* at any single moment of, e.g., cosmic time.

In LEO observation in PHS, existence of foreground LEO and/or other object may interfere with light from LEO behind. Let $p_L$ be the probability of presence of light obstructing entity in light path of length $L$, e.g., average distance between galaxies, which is invariant under UDA to location of light path and RT, then

$$\begin{matrix} p_s = (1-p_L)^{s/L} = e^{-b_L s} \\ b_L \equiv -\ln[1-p_L]/L \end{matrix} \quad \rightarrow N_{s,\text{obs.}} \propto e^{-b_L s} \sin^2\left[\frac{\pi}{2}\frac{s}{R_i}\right] ds . \tag{3}$$

$p_s$: Probability of receiving photon from LEO having light path of length $s$. $p_L$: Probability of presence of light interfering object in light path of length $L$ anywhere/when in PHS. $N_{s,\text{obs.}}$: Observed number of LEOs residing in observation sphere of radius $s$ and thickness $ds$.

Therefore, if observer looks into sky in all accessible directions and counts for LEOs (for purpose of the counting, each galaxy is counted as one LEO) then plot of $N_{s,\text{obs.}}$ versus $s$ shall be congruent to form of Expression (3), under UDA. If PHS was infinite then $N_{s,\text{obs.}}$ plot shall be congruent to form $s^2 \exp\left[-b_L s\right]ds$, $0 \le s < \infty$.

To most celestial objects in PHS, distance between LEO and observer are not measurable geometrically, except those in close proximity of observer such that their distances may be determined via, e.g., triangulation, leveraging orbital motion of Earth around Sun. During 1920s, Lemaître [6] and Hubble [11] discovered independently that correlation exists between redshift of a LEO and distance of the LEO to observer, known now as Hubble-Lemaître Correlation (HLC),

$$z \equiv (\lambda_p - \lambda_0)/\lambda_0 , \; z_{\text{HLC}} \propto s . \tag{4}$$

*z*: Redshift of photon from a LEO as received/measured by observer. *s*: Path length of the photon from the LEO to observer. $\lambda_p$ : Wavelength of the photon as measured by observer at observer site. $\lambda_0$ : Wavelength of a reference photon created by same species, e.g., hydrogen atom, at observer site via same process.

That is, HLC redshift of a LEO, *i.e.*, redshift of photons from a LEO that is not due to relative motion between the LEO and observer nor absolute motion of the LEO and/or light emitting particles thereof nor caused by presence of gravitation field or some other causes, is proportional to path length of the photons from the LEO to observer. Therefore, if HLC is universal and persistent, *i.e.*, applicable to any LEO exists anywhere/when in PHS, then observed LEOs can be sorted out according to their respective HLC redshifts and distances of the LEOs to observer computed via HLC,

$$N_z \equiv N_{z_{\mathrm{HLC}},\mathrm{LEO}} \propto A_2 \frac{ds}{dz_{\mathrm{HLC}}} dz_{\mathrm{HLC}} . \tag{5}$$

$z_{\mathrm{HLC}}$ : Redshift of LEO due to HLC. $N_z$ : Number of LEOs having HLC redshift in range of $z_{\mathrm{HLC}} - dz_{\mathrm{HLC}}/2 \sim z_{\mathrm{HLC}} + dz_{\mathrm{HLC}}/2$ .

Therefore,

$$N_{z,\mathrm{obs.}} \propto e^{-b_L s} \sin^2\left[\frac{\pi}{2}\frac{s}{R_i}\right]\frac{ds}{dz_{\mathrm{HLC}}},\ s = s\left[z_{\mathrm{HLC}}\right]. \tag{6}$$

$N_{z,\mathrm{obs.}}$ : Number of observed LEOs as a function of HLC redshift.

The *s* herein refers to length of photon path from LEO to observer. In infinite space, this *s* is identical to distance between LEO and observer. In finite space, *s* is also identical to distance between LEO and observer but only if $s \le 2R_i$ . In any space, distance between a place sometime ago and the same place sometime later is and is always zero, no matter how long ago it was and/or how later it may be, whether the space was/is expanding or not. On the other hand, in finite space, a photon emitted from a place shall come back to the same place time and again, unless/until interfered by others. Therefore, in finite space, the *s* can be very large while distance between emitter and receiver being zero.

Lemaître interpreted HLC redshift as caused by expansion of PHS [6], in such, any and all objects in the space were/are moving away from each other even if they were stationary. Therefore, from perspective of any object therein, all other objects were/are receding away from it, due to expansion of the space, and the farther away an object from an observer the faster the receding velocity of the object with respect to the observer will be while none is in motion. Thus,

$$v_{\mathrm{HLC}} = H_0 s . \tag{7}$$

$v_{\mathrm{HLC}}$ : Receding velocity of LEO according to Lemaître. $H_0$ : Hubble constant, 70 km s$^{-1}$ Mpc$^{-1}$ [12].

Then, as a Doppler Effect due to LEO receding from observer per Lemaître,

$$z_{\mathrm{HLC}} = \frac{1}{1 - H_0 s / c_i} - 1 \ \rightarrow\ \frac{s}{R_i} = \frac{1}{a}\frac{z_{\mathrm{HLC}}}{1 + z_{\mathrm{HLC}}},\ a \equiv \frac{H_0 R_i}{c_i} . \tag{8}$$

$c_i$ : Speed of light in vacuo (SLV) as measured/defined in Rest Frame (RF) [10].

Substitute into Expression (6),

$$N_{z,\text{obs.}} \propto \exp\left[-\frac{b}{a}\frac{z_{\text{HLC}}}{1+z_{\text{HLC}}}\right]\sin^2\left[\frac{\pi}{2a}\frac{z_{\text{HLC}}}{1+z_{\text{HLC}}}\right]\frac{1}{\left(1+z_{\text{HLC}}\right)^2},\ b \equiv b_L R_i. \quad (9)$$

For sufficiently small $z_{\text{HLC}}$,

$$z_{\text{HLC}} \ll 1 \rightarrow N_{z,\text{obs.}} \propto \exp\left[-b\frac{z_{\text{HLC}}}{a}\right]\sin^2\left[\frac{\pi}{2}\frac{z_{\text{HLC}}}{a}\right]. \quad (10)$$

Therefore, plurality of characteristic peaks of finite space shall show up in number density distribution of observed LEO versus redshift, and centers of the peaks shall be around $z_{\text{HLC}}/a = 2k-1$, $k = 1,2,3,\cdots$, if $a$ is not too large.

In TDA, Lemaître expansion of PHS corresponds to expansion of the ball along its radius direction in forward time. Radius of the ball is external to surface of the ball, *i.e.*, inaccessible from within the surface, therefore referred to as external radius of the surface. Accordingly, surface of the ball is also expanding in forward time hence internal distance between any pair of points of the surface is increasing with the time. Therefore, as viewed from any point of the surface, any other point of the surface is receding from it, and the farther the other point is the faster the receding velocity of that point is. Therefore, characteristic peaks of finite space shall show up in redshift distribution of number density of observed LEO at around $z_{\text{HLC}}/a = 2k-1$, $k = 1,2,3,\cdots$, if $a$ is not too large. Assign location of observer at the surface as North Pole, then $k = 1$ corresponds to the case that lights from LEOs at Equator travel along the surface towards North Pole directly; $k = 2$ corresponds to the case that lights from LEOs at Equator travel towards and passing through South Pole then to North Pole; and $k = 3$ corresponds to the case that lights from LEOs at Equator travel towards and passing through North Pole and to North Pole again; and so on. Therefore, if $a$ is not too large, plurality of peaks in number density distribution is inevitable whether the surface is expanding or contracting. Such plurality of the peaks is characteristic but only of finite space.

If a photon were emitted from North Pole $H_0^{-1}$ ago and received now by observer at North Pole, then path length of the photon is $s = c_i/H_0$. Therefore, per Equation (7), $v_{\text{HLC}} = c_i$. That is, receding velocity between North Pole $H_0^{-1}$ ago and the same place now has reached speed limit for photon. Therefore, any photon emitted from anywhere else of the surface $H_0^{-1}$ ago and any photon emitted from anywhere of the surface before $H_0^{-1}$ shall not reach North Pole hence cannot be observed by observer thereat. Therefore, to observer at North Pole now, the point at North Pole is commonly referred to as edge of the observable universe to the observer. Further, if the surface is still expanding, even *the* photon can no longer be observable by the observer after "now," since photon can but only travels along the surface at speed of $c_i$. Accordingly, only photons emitted from certain latitude circle of the surface $H_0^{-1}$ ago that may be observable, with $z_{\text{HLC}} \rightarrow \infty$, by observer at North Pole after "now". Therefore, to observer at North Pole after "now", the circle at the latitude of the surface is the edge of the observable universe to the observer. If PHS were infinite then the surface is a Euclidean plane, which is not expandable nor contractible in uniform manner. Therefore, only one asymmetric peak shall show up in $z_{s,\text{obs}}$ plot, in form $se^{-b_L s}$.

Large-scale sky survey of galaxy redshift has been ongoing efforts since about a half century ago [13] and enormous data have been accumulated [14]. From such data, number of observed galaxies can be plotted versus their observed redshifts, as shown in **Figure 1**. It can be seen from the plot that LEO distribution in PHS was not quite uniform with respect to sky directions, presumably due in partial to instrument limitations and/or survey strategies [15] [16]. It is nevertheless evident that multiplicity of characteristic peaks exists in galaxy distribution versus observed redshift in association, a unique feature only of finite space.

From **Figure 1(a)**, the first major peak is found centered at $z_{\text{obs}} \approx 0.076$ upon smoothing the distribution. In comparison with Expression (10),

$$a \approx 0.076 \;\rightarrow\; z_D \approx 0.15,\; R_i \approx 1.1\ \text{GLY},\; T_D \approx 2.1\ \text{GJY}\,. \tag{11}$$

$R_i$: Internal radius of $\mathbf{S}^3$. $z_D$: Redshift of LEO at internal distance of $D_i \equiv 2R_i$ from observer. $T_D$: RT taken for photon to travel in vacuo through internal distance $D_i$. GLY: Billion light years. GJY: Billion Julian years.

Therefore, internal diameter of PHS one billion years ago is estimated as ~2 GLY.

In measuring distances of LEOs in determining Hubble constant, any viable techniques in astronomy may be applicable except those involving luminosity of object via the law of inverse square attenuation of light (LISA), which was derived from flux conservation theorem (FCT) of Gauss for infinite space. From Expression (1),

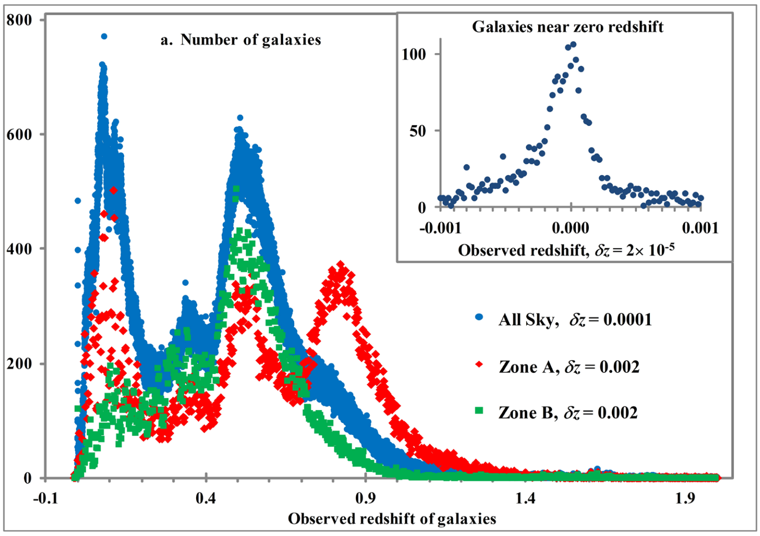
$$S_p = \frac{16R_i^2}{\pi}\sin^2 x = 4\pi s^2\left(1-\frac{1}{3}x^2+\cdots\right),\; x \equiv \frac{\pi}{2}\frac{s}{R_i}\,. \tag{12}$$

$S_p$: Area of propagation sphere of light of LEO in $\mathbf{S}^3$.

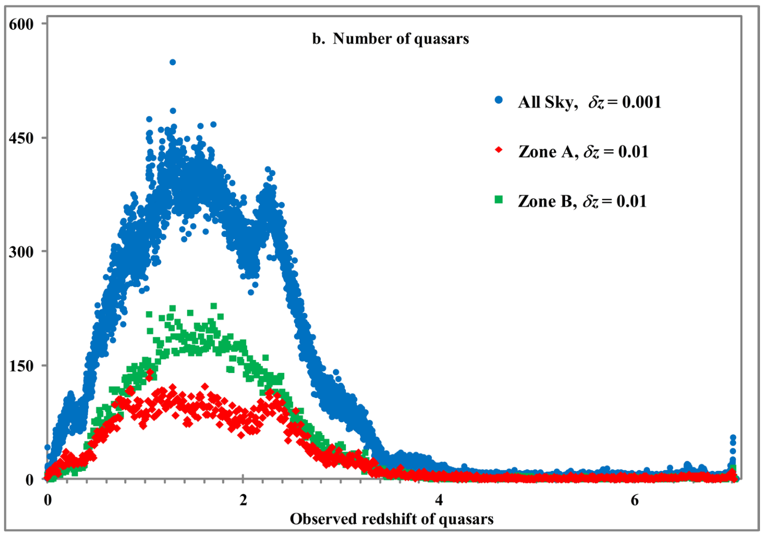
That is, area of propagation sphere of internal radius $s$ is only approximately $4\pi s^2$ for $s \ll R_i$. Therefore, LISA is suitable only to those LEOs in vicinity of observer, as an approximation. In other words, distance measurement of remote object via, e.g., standard candle, should incorporate effect of finite PHS instead of using LISA as is. Therefore, current estimation of Hubble constant is likely an upper bound due to involvement of LISA in calibration measurement of distances of LEOs, which would lead to underestimation of actual distances of the LEOs in determination of $H_0$. Therefore, the estimation of internal radius/diameter of PHS in Expression (11) is likely a lower bound.

From Equation (12),

$$S_p\Big|_{s=R_i+\delta} = S_p\Big|_{s=R_i-\delta}\,. \tag{13}$$




(a)




(b)

**Figure 1.** Distribution of LEOs as function of observed redshifts. (a) Distribution of galaxies with respect to observed redshift in association (from Sloan Digital Sky Survey IV [14] Data Release 17 [17] (SDSS-DR17), 2 733 615 LEOs, classified as Galaxy, are selected that are of relative uncertainty < 10% for $y \equiv 1 + z_{\text{obs}}$ and spectral magnitudes of all the five bands [18] as well to ensure quality of the data, referred to hereinafter as Gdata). (b) Distribution of quasars with respect to observed redshift in association (833 717 LEOs, classified as quasi-stellar object (QSO), are selected that meet the same uncertainty criteria, referred to hereinafter as Qdata). Zone A: 135˚ < RA < 165˚, 20˚ < DEC < 30˚. Zone B: 330˚ < RA < 360˚, 15˚ < DEC < 30˚. RA: Right ascension with respect to vernal equinox of J2000. DEC: Declination.

That is, identical LEOs at internal distance $R_i - \delta$ and $R_i + \delta$ to observer shall have identical apparent luminosity if depletion of light of the latter is negligible through the extra distance $2\delta$. On the other hand, HLC redshifts of the LEOs shall be different due to monotonicity of HLC on path length of light. Further, LEO at $R_i$ shall appear as the dimmest if interstellar extinction (ISE) is ignored.

## 3. Space Lens

Without interference of others in path of light (clear path, clear space), e.g., reflection, refraction, scattering, absorption, etc., light intensity (number of photons per area per time) of LEO at propagation sphere is inversely proportional to area of propagation sphere, due to FCT. Such area is dependent on radius of propagation sphere. In finite space, such dependency is not monotonic but with extremes. From Equation (12),

$$S_{p,\max} = S_p\big|_{s=R_i} = \frac{16}{\pi}R_i^2,\ S_{p,\min} = S_p\big|_{s=D_i} = 0\,. \tag{14}$$

That is, at internal distance $R_i$ from a LEO, area of propagation sphere of the LEO is maximal. Therefore, light intensity of the LEO at the distance is minimal. Beyond $R_i$ (from perspective of LEO), area of propagation sphere shall shrink monotonically with increasing distance and approach zero at antipode (diametrically opposite point in $\mathbf{S}^m$, $m$-dimensional finite space) of where light was emitted from.

In TDA, lights emitted local simultaneously from a LEO at North Pole shall propagate along the surface towards Equator. Therefore, the propagation sphere is a circle of same latitude. Total length of the circle, *i.e.*, one-dimensional area of the sphere, is increasing with propagation of the light that shall reach maxima at Equator. Therefore, intensity of the light at Equator is minimal. Thereafter, the light shall continue to propagate towards South Pole, which is antipode of North Pole, and area of the propagation sphere is shrinking with the propagation that shall reach minima at the antipode. Therefore, intensity of the light at South Pole is maximal.

Therefore, in clear space, light emitted from point source at same local moment shall diverge before length of the light path reaching $R_i$, converge after passing $R_i$, and eventually focus onto antipode of source. Thereafter, the light shall be defocused and its intensity reduced to minimum again at $R_i$ and eventually focused back onto where it was emitted from originally. If original emitter is no longer there, journey of the light shall repeat itself as if re-emitted from the original spot, and so on and so forth, until total attenuation of the light by others. Such is a unique property of finite space, referred to as space lens. In other words, finite space itself acts as an optical lens to photon travel therein. Characteristics of space lens include:

$$\begin{array}{ll} \text{Magnification} = 1 & \text{Depth of Field} \subset \text{Invariant} \\ \text{Focal Length} = R_i & \text{Orientation of Image} \subset \text{Invariant} \end{array}. \tag{15}$$

Therefore, if a LEO is an extended object then size, shape, optical feature, etc., of the LEO shall be identical to that of its image at focal surface (FCS) of the LEO. While time delayed by $T_D$, image and source of image reside at diametrically opposite locations of PHS, respectively. Orientation of any pair of points of light emitting surface (LES) of a LEO and that of its image are identical with respect to any geodesic containing the two pairs. If LES of a LEO is transparent to its inner LES then image of inners of the LEO shall also be enclosed by outer LES in original order. If FCS of a LEO happens to be in region of suitable display media, e.g., some cloud of dusts, then image of the LEO may become visible to optical observer and the observed image shall be identical to the LEO geometrically.

In TDA, light emitted from any point of the surface shall be focused onto antipode of the point one $T_D$ later, since light can propagate but only along the surface. Therefore, a luminescent ring of any shape at the surface around South Pole shall have its image of exactly the same size and shape around North Pole, if the surface is not expanding. If the surface is of moderate radius and expanding with moderate rate then shape of the image will be identical to that of the original but size of the image become larger. If expansion rate of the surface is high and radius of the surface large such that receding velocity of antipode exceeds SLV, then no image can be formed, since light from any point of the surface shall never reach antipode of the point. If the ring is transparent and has inner LEO then image of the LEO shall be enclosed by image of the ring around North Pole. Image is generally invisible unless immersed in display media.

Therefore, while $T_D$ delayed, image of LEO is 1:1 optical replica of the LEO with depths, if any, and located at antipode of where light was emitted from originally. In clear space, image of LEO shall create its own image one $T_D$ later, and image of image shall create its own image one $T_D$ later, and so on. Nevertheless, PHS is not quite clear and plenty of stars and dusts exist therein. Therefore, image shall always be dimmer than source even if display media were of the highest efficiency. If HLC is universal, persistent, and monotonic then image shall always be redder than source. However, such differences are insufficient in distinguishing a LEO from its image, if visible, since redder and dimmer object may be interpreted as unrelated LEO remote to observer even if the object may actually be in neighborhood of observer. Therefore, in general, differentiation between image and LEO may have to resort to investigation on display media and conditions thereof. On the other hand, if observer is in vicinity of FCS of a LEO revealed by display media thereat then it is possible to recognize the observed object as image of a LEO in opposite sky direction, because both objects shall have near identical redshifts and spectral profiles except opposite Doppler Effects and differences in brightness due to property/efficiency of the display media. In TDA, skirt of Antarctica would be imaged around North Pole. If visible, observer in Alaska may realize that section of the image nearby corresponds to section of the skirt south of Sydney, Australia.

If observer is inside FCS of a LEO then observer is observing the LEO at vantage position of space lens and shall have the unique experience that LES of the LEO is seen in every sky direction of observer, if enclosure of the LES was complete. If the FCS happens to be immersed in suitable display media such that image of the LEO becomes visible to observer, then the image shall be observed as foreground LES over entire sky of observer and superimposed on LES of the LEO in background. If there is visible feature on LES of the LEO then similar feature shall be observed on the image as well but in opposite sky direction. That is, pattern of LEO/image is of inverse symmetry if observed from inside of FCS of LEO. If the LEO is of inner LES, then inner LES shall be observed by observer as behind outer LES of same. However, image of inner LES shall be observed by observer as on front of image of outer LES as if observer is observing from inside of the LEO. While redshift and spectral profile of entities in background and foreground are nearly identical (brightness pending on property/efficiency of display media), distance of observer to image is much shorter than that to the LEO. Therefore, it is possible to measure distances to various parts of the image here/now hence determining size/shape of the LEO there/then.

In TDA, if observer is inside image of the skirt then observer shall see the skirt in every direction of observer, if not blocked by entities in between, as if observer is observing from inside the skirt around South Pole but seeing outside face of the skirt. If the image is visible then it is seen at foreground of observer and the skirt seen at background. Similarly, image of mountains of Antarctica is enclosed by image of the skirt around North Pole. If observer is inside the image then observer shall see the mountains in every direction, if not blocked by others, as if observer is observing from inside the mountains but seeing outside face of the mountains behind the skirt. If image of the mountains is also visible then it is seen as foreground image with respect to image of the skirt. If there is recognizable feature of the skirt/mountain in longitude 45˚ East of observer then same feature should be found in image of the skirt/mountain but at the direction of longitude 45˚ West. Since the images are much closer to the observer, it is therefore possible to measure internal distances of observer to various parts of the images to determining size/shape of the original LEO.

Focused by space lens, focal point/surface of a LEO is region of space with relatively higher intensity of light from the LEO. If an object steps into such region by chance then the object may behave in unusual manner due to interaction with the high intensity light in the region. For instance, if the object is a LEO or becomes one due to interaction with high-density photons available thereat then the system may be observed as having two sets of distinctly different redshifts that may be interpreted as, e.g., two LEOs overlapped in light of sight of observer. On the other hand, such event, if happens, is chance event and such phenomenon is transient in nature due to motion of LEO hence that of its focal region and motion of the object as well. Nevertheless, each and every LEO in $\mathbf{S}^m$ shall have its own focal region at corresponding antipode of space. Therefore, volume of space swept through by trajectories of focal regions of LEOs may not be as small as it might seem to be, further considering that light emitted from LEO shall not fade away until total attenuation by others. Therefore, chance of observing such unusual event in sky may not be as slim as it might be thought of otherwise.

## 4. Space Expansion

Space is a geometric construct comprising geometric points. Points of space are identified, recognized, distinguished by their respective labels/identifiers. Therefore, a point is the same or not the same as another point if and only if identifier of the point is the same or not the same as that of the other. Identifier of point of space is assigned by coordination system in and for space. Geometric relationships among points of space and distance information between points are encoded in identifiers of points regulated by norm defined for space. Therefore, if a label is assigned to a point of space then the point of the space is assigned with the label. Accordingly, distance between points of space is assigned/defined by assignment/definition hence immutable upon assignment/definition.

Space expansion refers to such scenario that distances among points of space are increasing in time, for cause/reason. However, alteration of distance of points of space is synonymous with reassignment/redefinition of points of space since distances among points are not set by anything else but only assignment/definition. Therefore, space expansion can only be due to artificial intervention. Therefore, in preserving logical consistency, space cannot be expanded unless/until agreed upon or ordered by otherwise.

Upon embedding object in space, points occupied by/coincided with object are regarded as representing, carrying geometric information of object. Accordingly, distance of points of object is decoded from identifiers of corresponding points of space by norm defined for space. If object is found occupying different sets of points of space from time to time such that distance of corresponding points of space are not the same then it is said that object is expanded/contracted or expanding/contracting and cause/reason for causing such is subject of discipline of science. Regardless of cause/reason, it is with respect to the invariant space that longer/shorter, bigger/smaller, expansion/contraction, etc., are defined and meaning of the words provided. In other words, it is the immutability of space that makes variation/invariation of size/shape of entity measurable and meaningful. Accordingly, in preserving metrological integrity, space cannot be regarded as expanding nor contracting.

Suppose a pair of end marks on a metallic bar in physical world is defined/assigned/appointed as unit of length of PHS. Accordingly, size of any object is measured with respect to the unit object, e.g., by mapping an object with plurality of replica of the unit object and/or fraction of same. If size of an object varies from measurement to measurement then the object is regarded as expanding/contracting and cause/reason for such is subject of discipline of science. Regardless of outcomes of the measurements and cause/reason thereof, length of the unit object is and must be regarded as invariant, by definition of unit object. In other words, length of unit of length is, by definition, one unit length, regardless.

There may be situations wherein length between end marks of the unit object varies due to, e.g., temperature variation of the entity, evaporation loss of the material, variation of interaction among substance comprising the unit object, etc. As a consequence, size of an object measured by the unit object may appear to be expanding. However, such situation is generally regarded as challenges of metrology in maintaining/preserving unit object instead of regarding object being measured as truly, really expanding. In other words, unit object in such situation is not regarded as the true one as defined in metrology but compromised version of it. In situation wherein plurality of variety of objects that are generally believed to have been stable in size/shape are found expanding if measured with the unit object then it is the unit object instead of the objects being measured that is considered as being contracting, for known/unknown causes. Such situation is regarded in metrology as revelation of defect in the unit object used in the measurement hence the unit object is not true representation of the one as defined in metrology, and finding, defining true unit object of length is a subject of metrology. However, in taking stand as such, sizes/shapes of the objects measured are regarded as invariant hence become multiplicities of unit object of length *de facto*. It is with respect to this invariance that the assigned unit object is disqualified as true unit object of length. In other words, taking such stand itself is *de facto* act of redefinition/reassignment of unit object of length, and the *de facto* unit objects involved in such action are not regarded as expanding/contracting regardless of nature of the situation.

In most popular scenario [19], every point of space is expanding, *i.e.*, distance between any pair of points of the space is increasing in time. Such scenario can be divided to two logical cases. In one such case, objects in space are coexpanding with the space. However, unit object of length of space is also object in space. Therefore, by the specification, unit object of length of the space must also be coexpanding with the space. Therefore, as measured by unit object of length of the space, size of the space must be invariant, *i.e.*, measurements of distance between any pair of points of the space from time to time must yield one and same result. Therefore, by metrological test, the space is not expanding nor objects therein. This includes the scenario of nonuniform expansion of space, wherein, some region of the space may be expanding faster/slower than others of same. By specification, however, unit object of length of the space in such region must also be coexpanding in same manner as that of points of the region. Therefore, measurements of size/shape of the region and distance between any pair of points of the region must yield one and same result regardless of where/when such measurement is conducted therein. Therefore, by metrological test, size/shape of the region is not expanding nor objects therein. Therefore, there can be no faster/slower expansion of any region of the space.

In the other case, objects in space are not expanding with the space, *i.e.*, distance between any pair of points of space is increasing in time while that of object is not. Consequently, as measured by unit object of length of the space, size of object in the space is not expanding in time while that of the space is. In other words, spacial points are being created or becoming accessible in time while objects are withdrawing from sets of spacial points and accessing to other sets of spacial points congruently in same time, for cause/reason. Cause/reason aside, such scenario touches upon some most fundamental aspects of the concept *space* itself hence may deserve further analysis (cf. **Appendix**).

In short, the case of the scenario in consideration is identical/equivalent/indifferent to the claim of existence or creation of something not-none inbetween directly connected nodes of space that, in correlation with some recurring events [10] occurring in space, is becoming accessible/available to the space. Therefore, in preserving logical and metrological integrity, space cannot be expanding. In addition, space is continuous, homogeneous, and isotropic with no boundary other than dimension.

Therefore, the only rational outlet to the space expansion scenario is to assume *a prior* the existence of a space of higher dimension (HDS) that PHS is but a subset therein. Under the context of expanding universe, PHS, *i.e.*, volume contained by *matter* at any moment of *time* has to be finite, or otherwise homogeneous and isotropic expansion of the space is impossible. Since PHS is finite, the subset is therefore a finite subspace (FSB) in HDS. Accordingly, if PHS is found expanding/contracting by observation/measurement in PHS then that shall prove the HDS does exit. On the other hand, if PHS is found not expanding/contracting by observation/measurement, that does not rule out the possibility of the existence of HDS. Hence, other means are needed in verifying/falsifying the HDS hypothesis, e.g., by observing openness of PHS or checking mass balance therein. Further, if HDS does exist then it may or may not be finite space, which might be evaluable by, e.g., testing inertial motion of atomic clock in PHS. In addition, if PHS is genuine space then it must be continuous, homogeneous, and isotropic by definition of space; if PHS is a FSB then it does not have to be continuous, homogeneous, or isotropic. Therefore, measurement of isotropy of PHS may indicate whether the HDS exists or not. All versions of Big Bang theories have assumed, *a prior* but implicitly, that PHS was/is finite, expansion homogeneous and isotropic, *i.e.*, PHS was/is finite and expanding while homogeneity and isotropy of PHS is intact during the expansion. However, homogeneous and isotropic expansion of FSB is possible but can only be along external radius of FSB or otherwise homogeneity/isotropy of FSB compromised.

## 5. Finite Subspace Expansion

Since homogeneous and isotropic expansion of a FSB can but only be along external radius of the FSB, centroid angle between any pair of points of the FSB is therefore invariant under such expansion,

$$l = R_e\theta_e,\ \frac{d\theta_e}{dt} = 0 \ \rightarrow\ \upsilon_R \equiv \frac{dl}{dt} = \theta_e \frac{dR_e}{dt}\,. \tag{16}$$

$l$ : Internal distance between any points of FSB. $\theta_e$ : Centroid angle between the points of FSB. *t*: RT.

$\upsilon_R$ : Receding velocity between the points of FSB. $R_e$ : External radius of FSB.

In TDA, angle between, e.g., Paris-Earth Center-London is invariant with respect to expansion/contraction of the ball/surface. Therefore, from Equation (7),

$$v_R = H_0 s \;\rightarrow\; \frac{dR_e}{dt} = \frac{H_0 s}{\theta_e}. \tag{17}$$

$H_0$ : Hubble constant. *s*: Path length of photon trajectory in FSB.

By definition of photon trajectory,

$$\left(\frac{ds}{dt}\right)^2 = \left(\frac{dR_e}{dt}\right)^2 + R_e^2\left(\frac{d\theta_e}{dt}\right)^2 = c_i^2. \tag{18}$$

$c_i$ : SLV as measured/defined in RF.

Therefore,

$$\frac{d\theta_e}{ds} = \frac{1}{R_e}\sqrt{1-\left(\frac{dR_e}{ds}\right)^2},\; \frac{dR_e}{ds} = \frac{1}{\theta_e}\frac{s}{s_c},\; s_c \equiv \frac{c_i}{H_0},\; \begin{matrix} \theta_e\big|_{s=0} = 0 \\ R_e\big|_{s=0} = R_0 \end{matrix}. \tag{19}$$

$s_c$ : Critical path length of photon trajectory. $R_0$ : External radius of FSB at current moment assigned as $t = 0$.

Numerical solution of Equation (19) is obtainable assuming Hubble constant is RT invariant. From the solution,

$$\lim_{R_e \to 0} s = 2s_c - \varepsilon_0,\; \varepsilon_0 \sim 0. \tag{20}$$

That is, at photon path length of $\sim 2s_c$ from current observer, external radius of PHS is zero. In other words, at backward RT of $\sim 2/H_0$, radius of PHS was shrunk to zero. The limit is stable with respect to variations/uncertainties of $H_0$ and $R_0$ (other than minor alterations to $\varepsilon_0$). Therefore, if PHS was expanding then age of the FSB should be $\sim 2/H_0$, *i.e.*, ~28 billion years, instead of ½ of that as commonly believed currently, unless expansion rate during the beginning period was much faster, e.g., Big Bang. In other words, in Big Bang model, Hubble constant during the beginning period was drastically different from what is measured now.

Further,

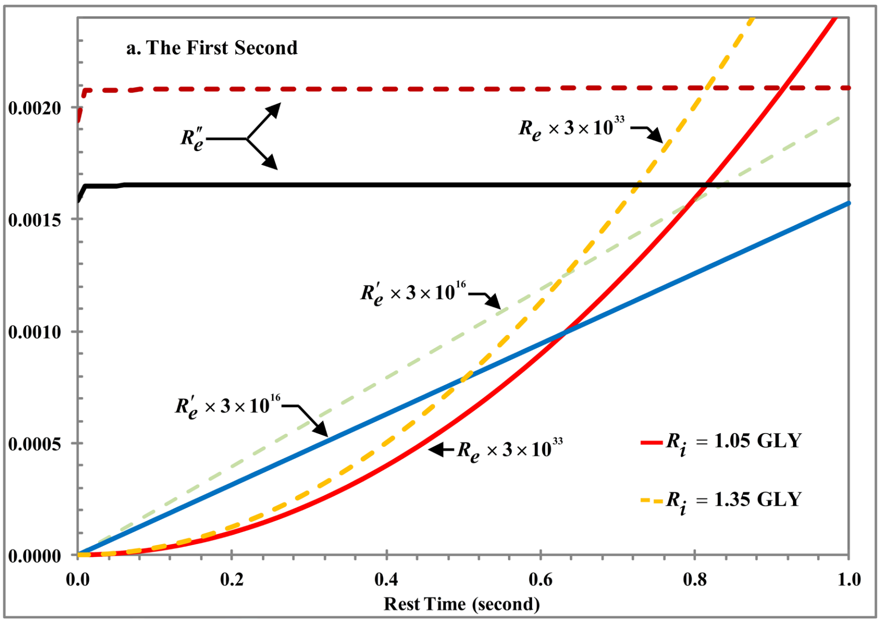
$$\lim_{R_e \to 0} \frac{dR_e}{dt} \equiv R'_e\big|_{R_e \to 0} = 0. \tag{21}$$

Therefore, if Hubble constant was RT invariant then there was no bang of any kind, *i.e.*, fast(er) expansion of FSB in short(er) duration, nor in need of such at the beginning. Instead, rate of the expansion was zero at the beginning and has since been accelerating with forward RT almost linearly and has reached 5%~6% of SLV by now, as can be seen in **Figure 2**.

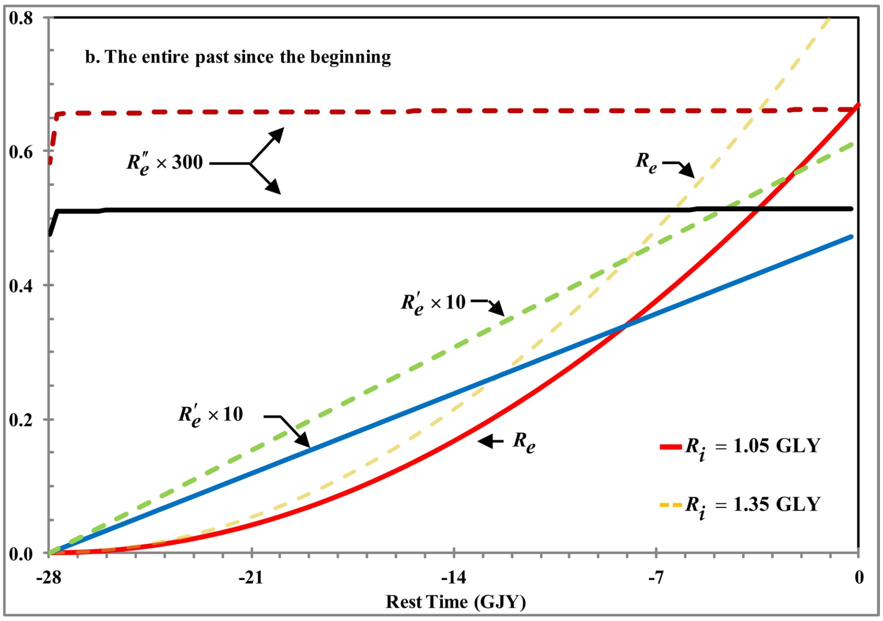
In addition,

$$R''_e\big|_{R_e = 0} = 0,\; R''_e\big|_{R_e \to +0} > 0. \tag{22}$$

That is, acceleration of the expansion was discontinuous at the beginning but become near constant since then. In other words, driving force or pressure for




(a)




(b)

Figure 2. External radius of expanding FSB. (a) During the first second of the expansion. (b) During the entire past since the beginning. Length is in unit of GLY and time in unit of GJY. $s_c \approx 14\,\text{GLY}$ .

the expansion was endogenous in nature, *i.e.*, caused/generated by the expansion itself, and has been essentially invariant since the beginning.

Redshift caused by receding of LEO is, from Expression (8),

$$z_L = \frac{1}{1 - \upsilon_R / c_i} - 1 \rightarrow z_L = \frac{s}{s_c - s} \rightarrow \lim_{s \to s_c} z_L = \infty . \tag{23}$$

$z_L$ : Lemaître redshift.

Therefore, photon emitted from $s_c$ and beyond shall never reach current observer. If a bang of some sort happened before the critical time $t_c \equiv s_c/c_i = H_0^{-1}$ then any residue radiation from such event should not be observable to current observer, since, with respect to any point of PHS then, receding velocity of current observer is exceeding SLV. Therefore, the currently observed cosmic microwave radiation (CMR) cannot be due to a bang happened before $t_c$. If a bang would have happened at $\sim t_c$ ago then timing of the event would be rather coincidental, since there is nothing special about the $t_c$, which merely indicates that light emitted from current location of observer $t_c$ ago shall never catch the location again nor the current observer, due to expansion of the FSB.

From solution of Equation (19),

$$\lim_{s\to s_c} \frac{R_e}{R_0} = \frac{1}{4} - \varepsilon_1,\ \varepsilon_1 \sim 0\,. \tag{24}$$

This limit is also stable with respect to variations/uncertainties of $H_0$ and $R_0$ (other than minor alterations to $\varepsilon_1$). Therefore, if PHS was expanding then size of the FSB at $z_L \to \infty$ would have been ~1/4 of that of the current, *i.e.*, internal radius of PHS should be ~250 million light years ~14 billion years ago, which is not small by any means. Therefore, if there was a bang happened $\sim t_c$ ago then the event would have been rather dramatic and artificial as well, considering that even if expansion of the FSB during the beginning period were at maximum rate permissible, *i.e.*, SLV, it would still take ~160 million years for the FSB to grow from a single point to the size at $t_c$.

Solution of Equation (19), $\{\theta_e[s \| t], R_e[s \| t]\}$, can be conveniently presented in polar plot as parametric curve of *s* or *t*. If PHS does not expand nor contract then the curve is a perfect circle, otherwise it resembles helix. Assign location of current observer in PHS as $\theta_e = 0$. Then, $\theta_e = \pi/2$ corresponds to equatorial sphere of observer. In TDA, this corresponds to Equator if latitude of observer at, e.g., North Pole is assigned as zero. Similarly, $\theta_e = 3\pi/2$ also corresponds to equatorial sphere but seen over antipode by observer. Likewise, $\theta_e = 5\pi/2$ corresponds to same sphere but seen again by observer; and so on. Therefore, characteristic peaks of finite space in galaxy redshift distribution should be expected to appear at $\theta_e = \pi/2 + k\pi$, $k = 0,1,\cdots$. If PHS was expanding then the peaks should become more sparse with respect to increasing redshift hence *s*. If PHS was contracting then the peaks should becoming denser with redshift increase. However, as can be seen from **Figure 3**, the peaks in the distribution are located rather evenly and do not match that per Lemaître scenario (the first peak in the distribution was used to determine $R_i$).

Receding motion of celestial objects is a sufficient and necessary condition for expanding PHS. If any object in PHS is found receding from any other object in same then PHS must be expanding. Conversely, if PHS is expanding then any object in PHS must be found receding from any other object in same. Thus far, HLC is the only decisive evidence from observation that supports existence of

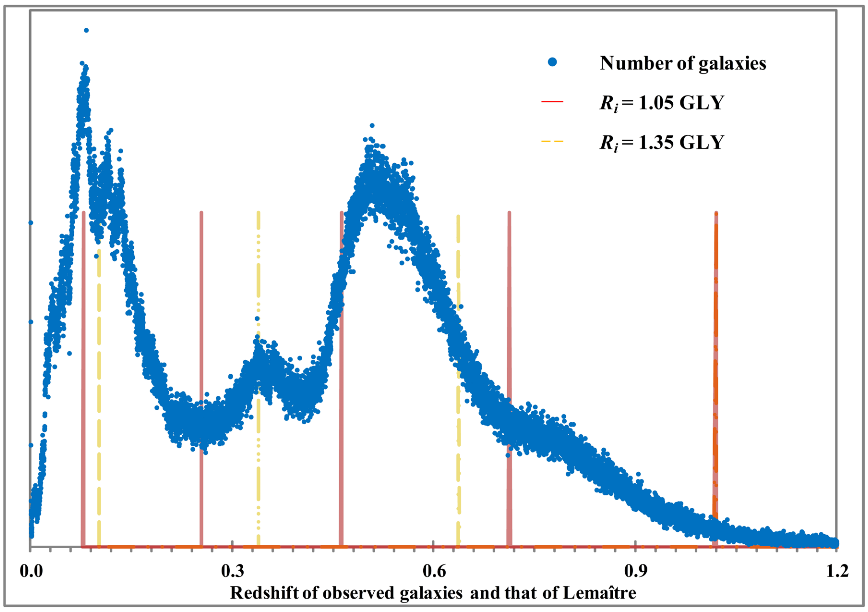



**Figure 3.** Comparison of locations of characteristic peaks of finite space in redshift distribution of observed galaxies and that of Lemaître. First major peak in galaxy redshift distribution is measured as centered at $z_{\text{obs}} \approx 0.076$, corresponding to $R_i = 1.05\,\text{GLY}$. However, it may be argued that fluctuations in the distribution around $z_{\text{obs}} = 0.1$ were due to clustering motions of celestial objects. Therefore, center of the first characteristic peak of finite space could be at $z_{\text{obs}} \approx 0.1$, corresponding to $R_i = 1.35\,\text{GLY}$. Overlapping of location of the fourth peak with $R_i = 1.35\,\text{GLY}$ and that of the fifth peak with $R_i = 1.05\,\text{GLY}$ in Lemaître redshift distribution is mere numerical coincidence.

receding motion of celestial objects, but only if HLC is interpreted as caused by receding motion of celestial objects. Observation on location distribution of characteristic peaks of finite space in redshift distribution of celestial objects provides a direct and decisive way to probe receding motion of celestial objects. If PHS was expanding, hence celestial objects were receding, then the peaks in the distributions should be located unevenly, even if Hubble constant were RT dependent. Therefore, the mismatch between the locations as observed in the data and that as predicted per Lemaître scenario shows that receding velocity interpretation of HLC does not comply with the observation data. Accordingly, PHS has to be finite space or FSB with no expansion.

## 6. Cosmic Microwave Radiation

When interpreted as afterglow of Big Bang, existence of CMR is one of the major observation evidences supporting expanding PHS. However, as analyzed above, afterglow of Big Bang is unobservable to current observer, and PHS is finite space or nonexpanding FSB. Therefore, current interpretation on origin of the observed CMR is questionable for the least. In this regard, space lens offers an alternative explanation to origin of CMR. If a microwave radiation source was existing at location of current observer or antipode thereof in certain time of past then it shall be observed as CMR by current observer. Therefore, the observed

CMR is likely due to light from a LEO at antipode of current location of Solar System odd multiples of $T_D$ ago or LEO at location of current observer even multiples of $T_D$ ago. In other words, Earth observer is currently inside FCS of a LEO and space lens is providing an opportunity for observing such ancient and remote (if from antipode) object in close proximity.

It is generally believed that the CMR has long been in existence before its discovery [20] and has since been under observation for more than a half century. It is also believed that the CMR is unlikely to disappear any time soon. Therefore, in consideration of motion of observer in Milky Way, that of the galaxy in Virgo Supercluster, etc., the LEO causing the CMR must be an EO with extensive spacial extent, at least on order of light years. Therefore, the LEO was an object with significant spacial extent, likely an ancient cloud of dusts.

Chance is slim that an ancient object would have had been in motion along same geodesic in same direction with same velocity as that of current observer. Therefore, relative motion between observer and the LEO, hence Doppler Effect thereof, should exist, as has been observed/measured [21]. Further, anisotropy of the effect should exist, that may be utilized to determine velocity and direction of absolute motion of the observer. Further, the EO was not rigid body hence inline velocity of the LES is unlikely to be uniform and such is in principle mapdable.

Chance is slim that EO of light-year scale would have had uniform LES. As cloud of dusts, gravitation interaction among the dusts themselves would have caused internal motions of the LEO. Such motion would have convoluted with motion caused by centrifugal force of the EO in absolute motion in PHS. Therefore, it is impossible that dusts of the cloud would have had all been in RS. Further, photons emitted from the LEO have traveled through entire PHS, which is not entirely clear, to have arrived here/now and being observed. Therefore, temporal and spacial variation of intensity of the CMR is inevitable due to interactions of the lights with entities encountered during their journey even if original LEO was uniform and in RS. Therefore, spacial variations of intensity of CMR across the sky should exist, as has been observed/studied [22]. Similarly, temporal variations of intensity of the CMR with respect to single sky direction should also exist.

If the LEO was at the antipode and Solar System is currently immersed in some microwave scattering media suitable for display of the image then it is plausible that the observed CMR could be superposition of the remote LEO and its image nearby. If indeed so then the image should be observed as CMR in foreground (CMFR) and the LEO as CMR in background (CMBR) with respect to observer, as has been observed/studied [23].

Since CMBR does have recognizable/distinguishable nonuniformity relative to its near uniform background, CMFR should exhibit similar patterns correlating to that of CMBR with inverse symmetry. Further, patterns of CMFR or that of CMR by LEO from even multiples of $T_D$ ago may be utilized to measure their respective distances to Earth observer, leveraging motion of Earth against sky background and relatively shorter distance between observer and the image or the source. Therefore, size/shape of the image/source, hence that of the LEO, may be measurable if the observed CMR is indeed superposition of a LEO at antipode and its image or source at current location. It is also plausible that the LEO was of multiple layers of LES. In such case, correlated patterns of CMFR and CMBR, if any, should be seen in same sky direction of observer and geometric measurement of the CMR would be fruitful only to the LEO at current location of observer even multiples of $T_D$ ago.

Temperature of CMR has been measured as 2.726 ± 0.001 K [24]. From Expression (11), HLC redshift of LEO one $T_D$ ago is ~0.15. That is, as observed here/now, light from LEO there/then was redshifted by $z_D$. Therefore, if the LEO was from one $T_D$ ago then temperature of the LEO was ~3.13 K. Likewise, if the LEO was $2T_D$ ago then temperature of the LEO was ~3.54 K, and so on. In any case, the LEO was a cold object, not uncommon in physical world.

Power spectra of CMR have been measured with impressive precision and well fitted with the blackbody radiation model [25]. Finer fitting of CMR data with separate models for CMBR and CMFR may be able to differentiate the CMFR as outer LES or image of the LEO, since energy density spectra of image and source should be nearly identical while temperature of outer and inner LESs are less likely to be identical. Further, if the CMFR is verified as image then the source must have been located at the antipode.

## 7. Hubble-Lemaître Correlation

By definition of space, all points of space are and must be identical and equivalent. Therefore, law of physics (LOP) and associated parameters are and must be invariant to location in space. Further, if LOP were RT dependent then it would not have been qualified as LOP. Likewise, if physical constant were RT dependent then it would not have been qualified as physical constant. Therefore, LOP and associated physical constants/parameters must be invariant to RT.

Therefore, if a particle in Rest State (RS) [10] there/then created a photon via quantum transition process according to LOP there/then, energy of the photon created there/then must be identical to that created here/now by identical particle in RS via identical process according to identical LOP. Further, if photon created there/then travels through empty space or void region therein to arrive here/now and is compared to its replica created here/now, the two photons must be found possessing identical amount of energy or otherwise there would have been violation of the law of energy conservation in course of photon traveling. Therefore, there should have been no difference at all in wavelengths of the two photons in such comparison according to the law of Planck on photon energy, since energy of the photons should be identical. Therefore, if photon path in PHS were indeed empty/void then there should have been no such thing as HLC, which, in conjunction with the law of Planck, states that photon travel in PHS shall cost energy of photon.

However, HLC is a fact of observation well established beyond reasonable doubt, regardless of specifics, details, and interpretations of the phenomenon. Further, PHS is finite even if it were FSB, and receding velocity interpretation of HLC does not comply with the observation. Therefore, from the existence of HLC, it is inevitable to construe that photon cannot travel in empty/void region of space but only in media, *i.e.*, nonempty/void region of space, and shall have energy loss in association with such travel in such media. Therefore, by existence of HLC, vacuum is not and cannot be empty/void region of PHS but media of some sort, and photon travel in vacuo is not free but of cost. This is referred to as the law of Hubble-Lemaître (LHL). Accordingly,

$$z_{\mathrm{LHL}} = z_{\mathrm{LHL}}\left[s\right],\ z_{\mathrm{LHL}}\left[0\right] = 0 \ \rightarrow\ z_{\mathrm{LHL}} = a_1 s + a_2 s^2 + \cdots . \tag{25}$$

$z_{\mathrm{LHL}}$ : Redshift of photon due to LHL. *s*: Path length of photon trajectory. $a_k$ : Coefficients of Taylor expansion of continuous differentiable function.

Therefore, for sufficiently small *s*, HLC shall appear as linear function of *s*.

LHL is not the only cause for observed redshift of light from LEO. Other causes include absolute motion induced redshift (MIR) [10], gravitation redshift [26], Doppler Effect, etc. For LEO classified as Galaxy, observed photons therefrom were typically generated by stars and/or particles in far field of center mass. Therefore, gravitation contribution to redshift of Galaxy is generally small and negligible. In contrast, for LEO classified as QSO, observed photons were typically generated in near field of center mass and gravitation contribution to redshift of QSO is generally larger hence not negligible.

LEO and its light emitting particles (LEP) are generally in motion. Therefore, photon emitted from LEO shall have MIR due to absolute motion of the LEP responsible for the photon,

$$z_{\mathrm{MIR}} = 1\Big/\sqrt{1-u_a^2} - 1 . \tag{26}$$

$z_{\mathrm{MIR}}$ : Redshift of photon created by LEP due to absolute motion of LEP in PHS, with respect to reference photon created in RS. $u_a$ : Reduced velocity [10] of LEP in absolute motion in PHS.

Therefore, MIR shall cause emission/absorption lines in spectrum of LEO to shift towards red, pending on mean velocity of absolute motions of the LEPs, and broadening of the lines symmetrically, pending on spreading of the velocities of the motions of the LEPs with respect to their mean velocity. Similarly, MIRs of LEOs shall cause $N_z$ profile to shift towards red and flattening of peaks and troughs therein.

Doppler shift (DS) in finite space [10], *i.e.*, shift of spectral line due to Doppler Effect in finite space, is

$$z_{\mathrm{DS}} = 1\big/\left(1-u_r\right) - 1 . \tag{27}$$

$z_{\mathrm{DS}}$ : DS of LEP caused by relative motion between LEP and observer in RS, with respect to reference photon created in RS. $u_r$ : Reduced velocity of LEP in relative motion with respect to observer in RS, negative if approaching towards observer.

Therefore, DS shall broaden spectral lines of LEO asymmetrically, *i.e.*, sharpening blue edge of a peak and trailing red edge of same, since maxima of blueshift is ½ while that of redshift could approach infinity. Further, $N_z$ profile is most sensitive to relative motion between observer and LEP/LEO that shall cause asymmetric broadening of peaks and filling up of troughs since $z_R$ is only ~0.08, as can be seen in **Figure 1**.

Relative and absolute motions of LEP are generally uncoupled. However, if LEP at local moment of photon emission was moving along geodesic containing observer in RS then the combined DS and MIR is

$$z_{\|}=\frac{1}{\left(1-u_{\|}\right)\sqrt{1-u_{\|}^2}}-1 \rightarrow z_{\|,\min}=\frac{4}{\sqrt{27}}-1,\ u_{\|,\text{at min } z_{\|}}=-\frac{1}{2},\ z_{\|,u_{\|}=-0.839\cdots}=0\,. \quad (28)$$

$z_{\|}$: Combined shift of LEP due to DS and MIR, with respect to reference photon created in RS. $u_{\|}$: Reduced velocity of inline motion of LEP with respect to observer in RS.

That is, maximal blueshift of the inline motion is ~0.23, corresponding to motion of the LEP at ½ SLV towards observer in RS. Further, if velocity of the motion reaches ~84% SLV then the combined shift shall be observed as zero. Beyond this threshold, the shift shall become red even if LEP were moving towards observer in velocity near the speed limit. Such situation is further dramatized when gravitation redshift is factored in. If gravitation redshift of LEP is expressed as [27]

$$z_{\text{GR}}=\left(1-4/\rho\right)^{-1/4}-1 \rightarrow z_{\text{obs}}=\left(\frac{1-4/\rho_{\text{o}}}{1-4/\rho_{\text{s}}}\right)^{1/4}\left(\frac{1-u_{\text{o}}^2}{1-u_{\text{s}}^2}\right)^{1/2}\frac{1+z_{\text{LHL}}}{1-u_r}-1\,. \quad (29)$$

$z_{\text{GR}}$: Gravitation redshift of LEP, with respect to reference photon created in RS free of any field. $\rho$: Reduced distance between LEP and center of gravitation field, in unit of characteristic length of the field (CLF), defined as $\rho\equiv r/r_g$, wherein, $r$ is distance between particle and field center, $r_g$ is CLF, $r_g\equiv G_iM_i/c_i^2$, $G_i$ gravitation constant measured in RS free of any field, $M_i$ restmass in RS causing the field, $c_i$ SLV measured/defined in RS on atomic clock [27]. $\rho_{\text{o}}$: Reduced distance between LEP of reference photon and corresponding field center at moment of photon creation. $\rho_{\text{s}}$: Reduced distance between LEP of LEO and corresponding field center at moment of photon creation. $u_{\text{o}}$: Reduced velocity of absolute motion of LEP of reference photon at moment of photon creation. $u_{\text{s}}$: Reduced velocity of absolute motion of LEP of LEO at moment of photon creation. $z_{\text{LHL}}$: Redshift of observed photon due to LHL.

Therefore, it is rare for LEO to exhibit blueshift if gravitation redshift is not negligible, e.g., typical QSO.

## 8. Associated Light Emitting Objects

Arbitrarily assign a point of PHS as internal origin of the space. Consider a luminescent source in absolute motion along its geodesic near antipode of origin but not towards antipode nor origin, ignoring possible motions of source due to, e.g., clustering with other entities. Suppose some light emitted from the source at a first location is towards antipode then to origin indirectly (first event). Suppose, after a suitable RT duration, some light from the same source at a second location is towards the origin directly (second event). Then, both lights shall arrive at the origin local simultaneously hence be observed by observer at origin as two LEOs in different sky directions of observer. Such LEOs are referred to as associated LEOs since their lights were emitted from one and same source but at different RT. Further, geodesics of the paths of the lights form a two-dimensional sphere of internal diameter $D_i$ containing section of the geodesic of the source during the RT interval. Therefore, path length of first light from first location to antipode, internal distance between second location and antipode, and path length of the source during *the* events form a spherical triangle.

In TDA, assign South Pole as origin of the surface and consider a beacon flying from Albany towards Pittsburgh geodetically. Then, the first event above corresponds to the beacon at Albany and partial of its light towards North Pole, and the second event corresponds to the beacon in Pittsburgh and partial of its light towards South Pole. If both lights arrive at South Pole at same time of observer residing there, then the observer shall see a beacon in direction of Pittsburgh and another one in direction of Maldives. The two beacons found by the observer are due to/caused by the one and same flying beacon, therefore they are associated beacons. Further, geodesics of the paths of the two lights form a two-dimensional sphere. The sphere has the same internal diameter as that of the surface and contains the trajectory of the flying beacon. Therefore, path length of the first light from Albany to North Pole, internal distance between Pittsburgh and North Pole, and path length of the flying beacon between Albany and Pittsburgh form a spherical triangle.

By spherical geometry,

$$\begin{array}{lll} a \equiv \pi l_a / D_i & c_i \tau_a = l_a / u_s & (1+\cos A)\sin b \sin c = \cos a - \cos[b+c] \\ b \equiv \pi l_b / D_i, & c_i \tau_b = D_i + l_b, & (1+\cos B)\sin c \sin a = \cos b - \cos[c+a]. \\ c \equiv \pi l_c / D_i & c_i \tau_c = D_i - l_c & (1+\cos C)\sin a \sin b = \cos c - \cos[a+b] \end{array} \quad (30)$$

$l_a$: Length of path of source between *the* events. $l_b$: Length of path of first light between first location and antipode. $l_c$: Internal distance between second location and antipode. $D_i$: Internal diameter of PHS. *a*: Radian of $l_a$ with respect to center of the sphere. *b*: Radian of $l_b$ with respect to center of the sphere. *c*: Radian of $l_c$ with respect to center of the sphere. $u_s$: Magnitude of reduced velocity of absolute motion of source. $c_i$: SLV in PHS defined on RT. $\tau_a$: RT taken by source travel through $l_a$. $\tau_b$: RT taken by first light travel through antipode to origin. $\tau_c$: RT taken by second light travel directly to origin. *A*: Spherical angle between $l_b$ and $l_c$. *B*: Spherical angle between $l_c$ and $l_a$. *C*: Spherical angle between $l_a$ and $l_b$.

Condition for local simultaneous arrival of the two lights at origin is

$$\tau_b = \tau_a + \tau_c \;\rightarrow\; a = (b+c)u_s. \quad (31)$$

The source is assumed to be in vicinity of the antipode, therefore,

$$\left.\begin{array}{l}0<\dfrac{l_b}{D_i}\ll 1\\[2ex] 0<\dfrac{l_c}{D_i}\ll 1\end{array}\right.,\ \eta\equiv\frac{c}{b}\ \to\ 1-u_s^2\simeq\frac{2\eta\left(1+\cos A\right)}{\left(1+\eta\right)^2},\ \begin{array}{l}u_s\cos B\simeq\dfrac{\eta-\cos A}{1+\eta}\\[2ex] u_s\cos C\simeq\dfrac{1-\eta\cos A}{1+\eta}\end{array}. \tag{32}$$

In addition,

$$A=\pi-\theta,\ \cos\theta=\boldsymbol{p}_1\cdot\boldsymbol{p}_2\ \to\ \cos A=-\boldsymbol{p}_1\cdot\boldsymbol{p}_2. \tag{33}$$

$\boldsymbol{p}_1$: Unit vector at origin pointing to first LEO of associated LEO pair. $\boldsymbol{p}_2$: Unit vector at origin pointing to second LEO of associated LEO pair. $\theta$: Inclusion angle between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$.

Therefore, for associated galaxy pair (AGP) in neighborhood of antipode,

$$\begin{array}{l}1+z_{\text{obs},1}=\sqrt{\dfrac{1-u_o^2}{1-u_s^2}}\dfrac{1+z_{\text{LHL},1}}{1+u_s\cos C+\boldsymbol{u}_o\cdot\boldsymbol{p}_1}\simeq\sqrt{\dfrac{1-u_o^2}{1-u_s^2}}\dfrac{\left(1+\eta\right)\left(1+z_D\right)}{2+\eta+\eta\boldsymbol{p}_1\cdot\boldsymbol{p}_2+\left(1+\eta\right)\boldsymbol{u}_o\cdot\boldsymbol{p}_1}\\[3ex] 1+z_{\text{obs},2}=\sqrt{\dfrac{1-u_o^2}{1-u_s^2}}\dfrac{1+z_{\text{LHL},2}}{1+u_s\cos B+\boldsymbol{u}_o\cdot\boldsymbol{p}_2}\simeq\sqrt{\dfrac{1-u_o^2}{1-u_s^2}}\dfrac{\left(1+\eta\right)\left(1+z_D\right)}{2\eta+1+\boldsymbol{p}_1\cdot\boldsymbol{p}_2+\left(1+\eta\right)\boldsymbol{u}_o\cdot\boldsymbol{p}_2}\end{array}. \tag{34}$$

$z_{\text{obs}}$: Observed redshift of LEO. $z_D$: LHL redshift of photon through light path of length of internal distance $D_i$ in vacuo.

Therefore,

$$\begin{array}{l}1+z_D\simeq\dfrac{y_1y_2}{y_1+y_2}\left(3+\boldsymbol{p}_1\cdot\boldsymbol{p}_2+\boldsymbol{u}_o\cdot\left(\boldsymbol{p}_1+\boldsymbol{p}_2\right)\right)\sqrt{\dfrac{1-u_s^2}{1-u_o^2}},\ \begin{array}{l}y_1\equiv 1+z_{\text{obs},1}\\ y_2\equiv 1+z_{\text{obs},2}\end{array},\\[3ex] 1-u_s^2\simeq\dfrac{1-\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{2}\left(1-\left(\dfrac{\left(y_1-y_2\right)\left(3+\boldsymbol{p}_1\cdot\boldsymbol{p}_2\right)+2\boldsymbol{u}_o\cdot\left(y_1\boldsymbol{p}_1-y_2\boldsymbol{p}_2\right)}{\left(y_1+y_2\right)\left(1-\boldsymbol{p}_1\cdot\boldsymbol{p}_2\right)}\right)^2\right)\end{array}. \tag{35}$$

Reduced velocity of absolute motion of observer at origin, *i.e.*, current location of observer in PHS, is relatively slow, as can be seen in the insert of **Figure 1(a)**. Therefore, define

$$\left.\begin{array}{l}F_z\equiv\dfrac{y_1y_2}{y_1+y_2}\left(3+\boldsymbol{p}_1\cdot\boldsymbol{p}_2\right)\sqrt{1-u_s^2}\\[3ex] 0\le u_s^2\simeq\dfrac{1+\boldsymbol{p}_1\cdot\boldsymbol{p}_2}{2}+\dfrac{1}{2}\dfrac{\left(3+\boldsymbol{p}_1\cdot\boldsymbol{p}_2\right)^2}{1-\boldsymbol{p}_1\cdot\boldsymbol{p}_2}\left(\dfrac{y_1-y_2}{y_1+y_2}\right)^2<1\end{array}\right.\to F_z\simeq 1+z_D. \tag{36}$$

That is, if two galaxies in vicinity of the antipode one $T_D$ ago were indeed an AGP then their $F_z\simeq 1+z_D$. If light from such AGP were not totally attenuated by others then, due to finiteness of PHS,

$$F_z\big|_{\text{AGP near antipode}}\simeq 1+z_{kD},\ k=1,3,5,\cdots. \tag{37}$$

Therefore, plot of number of galaxy pairs versus $F_z$ shall exhibit characteristic peaks, if not swamped by others, at $\sim 1+z_{kD}$ since all pairs meeting the condition shall have near identical $F_z$ if they are indeed AGP.

Consider instead a luminescent source in absolute motion along its geodesic in vicinity of origin. Suppose some light from the source at a first location is towards and passing through origin and to origin again (first event). Suppose, after a suitable RT duration, some light from the same source at a second location is towards antipode then to origin (second event). Then, both lights shall arrive at origin local simultaneously hence be observed by observer at origin as two LEOs in different sky directions of observer. Such are also associated LEOs since they were due to one and same source. Further, geodesics of paths of the lights form a two-dimensional sphere of internal diameter $D_i$ containing section of geodesic of source during the RT interval. Therefore, respective distances of first and second locations to origin and path length of source during *the* events form a spherical triangle. Condition for local simultaneous arrival of the two lights at origin is

$$\tau_b = \tau_a + \tau_c,\ c_i\tau_b = 2D_i + l_b,\ c_i\tau_c = 2D_i - l_c\ \rightarrow\ a = (b+c)u_s\,. \quad (38)$$

$l_b$ : Internal distance between first location and origin. $l_c$ : Internal distance between second location and origin. $\tau_b$ : RT taken by first light to arrive again at origin. $\tau_c$ : RT taken by second light to arrive at origin.

Therefore, for AGP in vicinity of origin, if motion of observer at origin is negligible,

$$F_z\big|_{\text{AGP near origin}} \simeq 1 + z_{kD},\ k = 2,4,6,\cdots. \quad (39)$$

Plot of flux intensities of photons from a LEO as received by observer versus wavelengths of the photons as measured by observer is known as a spectrum of the LEO. Comparing wavelengths of series of characteristic peaks of known species in a spectrum with that of same species in reference state, redshift of the spectrum, hence that of the LEO, is obtainable, referred to as observed redshift of the LEO. Shift of a spectrum can be restored by rescaling its abscissa with $(1+z_{\text{obs}})^{-1}$. Except those effects due to ISE, shift restored spectrum is equivalent to the spectrum taken at location of the LEO at rest with respect to same. Therefore, shift restored spectrum is also known as Rest-frame spectrum (RFS). RFSs of associated LEOs are similar among themselves, if luminescent source of the LEOs in association was regular in morphology, RT stable during *the* events, and optical environments in light paths of the LEOs were similar. RFS of a LEO can be viewed as composed of plurality of spectral bands. For example, in SDSS, flux intensities of RFS are binned to ultraviolet band labeled *u*, green band *g*, red band *r*, 760 nm infrared band *i*, and 910 nm infrared band *z* [18]. With rest-frame flux intensities of corresponding bands as its components, a spectral magnitude vector can be established in association with a LEO,

$$\boldsymbol{f}_m \equiv \left\|\boldsymbol{f}_m\right\|^{-1}\left\{\cdots, f_u, f_g, f_r, f_i, f_z, \cdots\right\}\ \rightarrow\ \lambda_{j,k} \equiv \boldsymbol{f}_{m,j}\cdot\boldsymbol{f}_{m,k}\,. \quad (40)$$

$\boldsymbol{f}_m$ : Normalized rest-frame magnitude vector of RFS of LEO. $f_x$ : Rest-frame flux intensity of spectral band *x* of RFS of LEO. $\lambda_{j,k}$ : Measure of spectral similarity of RFSs of LEO *j* and *k*.

Therefore, if two LEOs are associated then $\lambda$ of the LEOs shall approach one, regardless of difference in optical path lengths of the LEOs. Accordingly, observed LEOs can be paired up by selecting those of maximal $\lambda$ among all possible combinations. Such pairing does not guarantee a chosen pair shall be associated LEOs. However, it is of higher probability that some of such pairs are indeed associated, as can be seen in Figure 4. That is, if none of the $\lambda_{\max}$ pairs are associated then distribution of such pairs in the plot should be statistically identical to that of the random pairs.

From $F_z$ distribution of the $\lambda_{\max}$ pairs in Figure 4, four characteristic peaks are recognizable that centered at 1.154, 1.443, 1.624, and 1.956, respectively. It is thus recognized, per Expression (25), for relatively shorter light path of LEO,

$$\begin{matrix} 0.154 \sim z_{1D} & 0.624 \sim z_{4D} \\ 0.443 \sim z_{3D} & 0.956 \sim z_{6D} \end{matrix}, \rightarrow z_{\text{LHL}} \simeq a_1 s + a_2 s^2, \begin{matrix} a_1 \approx 0.0715 \\ a_2 \approx 0.0007 \end{matrix} \rightarrow \begin{matrix} z_R \approx 0.072 \\ z_D \approx 0.146 \end{matrix}. \quad (41)$$

Accordingly, internal diameter of PHS is refined to $D_i \approx 2.0\,\text{GLY}$.

$\lambda_{\max}$ pairing approach does not ensure LEOs of such pairing are truly associated. Nevertheless, from Figure 4, $\lambda_{\max}$ pairs around centers of the characteristic peaks are more likely being associated ones with respect to those in background continuum. For example, about one third of the $\lambda_{\max}$ pairs around $F_z \sim 1.154$ are likely true AGPs. Therefore, RFSs of $\lambda_{\max}$ pairs around centers of characteristic peaks are worthy of further comparison since, if two LEOs are truly associated then RFSs of the LEOs should be essentially identical down to all details. Some of such examples are listed in Table 1, wherein, best-fit spectra [28] of a LEO pair, upon shift restoration (hence is RFS), are essentially identical in

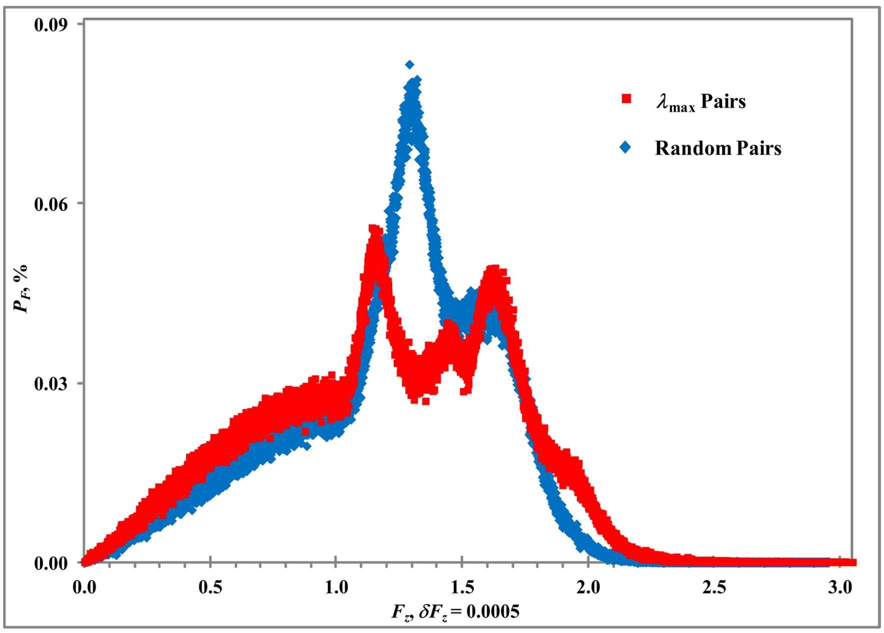



Figure 4. $F_z$ distribution of galaxy pairs. From Gdata, 1 366 802 $\lambda_{\max}$ pairs and 1.353 × $10^6$ random pairs are found satisfying $0 \le u_s < 1$. $P_F \equiv N_{F_z}/N$. $N_{F_z}$: Number of LEO pairs in range of $F_z - \delta F_z/2 \le F_{z,pair} < F_z + \delta F_z/2$. $N$: Total number of LEO pairs.

**Table 1.** Potential associated LEO pairs (example and counterexample).

| No. | Class | Plate | MJD | Fiber ID | $z_{obs}$ | RA (˚) | DEC (˚) | $\theta$ (˚) | $\chi$ | $F_z$ | $u_s$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Galaxy | 449 | 51900 | 521 | 0.089138 (15) | 134.5915890 (10) | 54.9682612 (10) | 93 | 10.2 | 1.158 | 0.69 |
| | | 1821 | 53167 | 39 | 0.080115 (20) | 236.7301552 (10) | 5.2996398 (10) | | | | |
| 2 | Galaxy | 2761 | 54534 | 521 | 0.077140 (16) | 218.8862551 (16) | 18.2527765 (10) | 128 | 10.1 | 1.153 | 0.43 |
| | | 750 | 52235 | 400 | 0.076449 (13) | 358.9248453 (10) | 15.0556286 (10) | | | | |
| 3 | Galaxy | 4535 | 55860 | 202 | 0.337870 (52) | 0.6535550 (26) | 6.7445605 (24) | 146 | 9.2 | 1.434 | 0.29 |
| | | 4743 | 55645 | 895 | 0.431001 (75) | 148.1759879 (84) | 2.8836351 (71) | | | | |
| 4 | Galaxy | 3615 | 56544 | 65 | 0.411465 (60) | 38.0255662 (82) | −0.3776158 (52) | 157 | 9.1 | 1.437 | 0.20 |
| | | 2136 | 53494 | 550 | 0.407973 (89) | 218.8987606 (28) | 23.7369939 (29) | | | | |
| 5 | Galaxy | 4404 | 55513 | 455 | 0.496078 (99) | 25.025871 (13) | 5.217600 (13) | 109 | 9.2 | 1.627 | 0.58 |
| | | 5299 | 55927 | 497 | 0.49332 (13) | 135.494605 (10) | 8.3490581 (96) | | | | |
| 6 | Galaxy | 4554 | 56193 | 416 | 0.51940 (10) | 17.7993941 (99) | 6.9765781 (85) | 127 | 8.9 | 1.625 | 0.44 |
| | | 4971 | 55747 | 455 | 0.50971 (11) | 234.1912541 (98) | 32.990031 (10) | | | | |
| 7 | Galaxy | 9315 | 57713 | 968 | 0.719700 (35) | 43.099316 (25) | −3.675608 (26) | 115 | 8.5 | 1.957 | 0.55 |
| | | 10465 | 58144 | 89 | 0.906741 (56) | 159.598540 (69) | 30.655931 (51) | | | | |
| 8 | Galaxy | 7388 | 56783 | 729 | 0.79255 (32) | 157.657936 (30) | 46.167464 (21) | 113 | 8.4 | 1.956 | 0.55 |
| | | 7573 | 56946 | 159 | 0.80426 (34) | 328.904828 (40) | 20.055175 (38) | | | | |
| 9 | Galaxy | 9442 | 58076 | 132 | 1.19476 (14) | 9.223865926) | 1.269496 (18) | 123 | 7.2 | 2.280 | 0.48 |
| | | 9603 | 58132 | 213 | 1.04457 (16) | 137.179027 (51) | 25.672941 (40) | | | | |
| 10 | Galaxy | 9337 | 57724 | 830 | 1.10674 (18) | 18.247282 (67) | −2.730078 (52) | 131 | 7.0 | 2.289 | 0.42 |
| | | 9626 | 57875 | 373 | 1.18898 (20) | 152.916388 (34) | 25.087244 (41) | | | | |
| 11 | Galaxy | 8853 | 57459 | 334 | 1.214257 (56) | 157.7669164 (73) | 36.2471968 (75) | 125 | 7.2 | 2.405 | 0.46 |
| | | 7584 | 56957 | 50 | 1.25655 (15) | 341.3221471 (91) | 18.7023526 (90) | | | | |
| 12 | Galaxy | 9370 | 58056 | 642 | 1.21366 (12) | 43.90118 (16) | 1.306545 (36) | 115 | 6.6 | 2.407 | 0.54 |
| | | 10469 | 58133 | 949 | 1.21603 (15) | 162.815308 (49) | 26.739980 (60) | | | | |
| 13 | Galaxy | 7894 | 57339 | 586 | 1.276043 (58) | 23.1062869 (73) | −3.6314135 (88) | 126 | 7.4 | 2.459 | 0.45 |
| | | 8525 | 57900 | 666 | 1.300951 (50) | 243.252973 (16) | 44.500398 (16) | | | | |
| 14 | Galaxy | 9425 | 58112 | 776 | 1.25102 (10) | 31.125935 (38) | 4.197383 (33) | 101 | 7.2 | 2.460 | 0.63 |
| | | 10438 | 58142 | 233 | 1.28859 (12) | 137.107902 (35) | 31.401427 (30) | | | | |
| 15 | QSO | 8829 | 57446 | 660 | 1.85486 (51) | 132.5964676 (36) | 39.5001212 (36) | 36 | 9.5 | 1.387 | 0.97 |
| | | 995 | 52731 | 372 | 1.70573 (44) | 149.6369282 (21) | 6.8359664 (19) | | | | |
| 16 | QSO | 8825 | 57451 | 46 | 1.57712 (90) | 152.777661 (10) | 37.5794239 (97) | 33 | 9.4 | 1.383 | 0.96 |
| | | 6755 | 56413 | 272 | 1.62445 (40) | 195.8758710 (18) | 50.2737986 (16) | | | | |
| 17 | QSO | 658 | 52146 | 71 | 1.25190 (39) | 15.4248686 (12) | −10.3746979 (12) | 51 | 9.3 | 1.662 | 0.91 |
| | | 7655 | 57336 | 725 | 1.14894 (35) | 340.1114765 (12) | 26.8826671 (12) | | | | |

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| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 18 | QSO | 11357 | 58522 | 222 | 1.87296 (71) | 168.576748 (13) | 27.798293 (12) | 36 | 8.7 | 1.644 | 0.95 |
| | | 8380 | 57520 | 35 | 1.82241 (69) | 208.793623 (11) | 43.7712393 (95) | | | | |
| 19 | QSO | 596 | 52370 | 340 | 1.77147 (53) | 165.8904117 (18) | 64.9648308 (18) | 47 | 9.0 | 1.964 | 0.92 |
| | | 10744 | 58199 | 834 | 1.68304 (25) | 231.4932974 (37) | 36.8019934 (37) | | | | |
| 20 | QSO | 551 | 51993 | 48 | 1.76501 (41) | 135.2505823 (19) | 50.1176147 (19) | 45 | 8.8 | 1.970 | 0.92 |
| | | 8870 | 57779 | 230 | 1.76643 (48) | 196.6440541 (56) | 36.6435182 (58) | | | | |
| 21 | QSO | 7680 | 58131 | 743 | 1.11471 (44) | 18.6278645 (46) | 29.2413849 (42) | 86 | 9.0 | 2.276 | 0.73 |
| | | 6786 | 56448 | 620 | 1.26468 (14) | 240.13332078 (91) | 57.92251646 (93) | | | | |
| 22 | QSO | 703 | 52209 | 111 | 1.05423 (25) | 34.3600327 (17) | −0.3343066 (15) | 132 | 8.8 | 2.280 | 0.41 |
| | | 10261 | 58462 | 641 | 1.24217 (29) | 176.7597322 (21) | 33.4969193 (23) | | | | |
| 23 | QSO | 4868 | 55895 | 750 | 2.24115 (24) | 135.5217144 (72) | 7.5085638 (68) | 147 | 9.2 | 2.965 | 0.31 |
| | | 4414 | 55882 | 340 | 1.5969 (12) | 346.542250 (17) | 5.080749 (19) | | | | |
| 24 | QSO | 6967 | 56447 | 86 | 2.58015 (46) | 198.0618593 (36) | 60.1424765 (37) | 55 | 9.2 | 2.965 | 0.89 |
| | | 5467 | 55973 | 44 | 2.62203 (45) | 220.0484463 (35) | 7.7458717 (33) | | | | |
| 25 | QSO | 4874 | 55673 | 898 | 2.44835 (71) | 150.4260851 (45) | 6.8423371 (43) | 148 | 8.9 | 3.615 | 0.27 |
| | | 6152 | 56164 | 539 | 2.55060 (49) | 357.7034266 (69) | 9.2241519 (71) | | | | |
| 26 | QSO | 6207 | 56239 | 999 | 2.37018 (47) | 0.7853924 (34) | 17.9778854 (33) | 98 | 8.6 | 3.609 | 0.66 |
| | | 9362 | 57801 | 342 | 2.31871 (82) | 114.6090808 (41) | 34.8680947 (42) | | | | |
| 27 | QSO | 5354 | 55927 | 490 | 3.05248 (71) | 161.5486473 (96) | 8.3511661 (93) | 148 | 7.8 | 4.305 | 0.28 |
| | | 7603 | 56960 | 303 | 3.27885 (98) | 352.7777597 (84) | 22.0103182 (86) | | | | |
| 28 | QSO | 5326 | 56002 | 314 | 3.19809 (23) | 149.5824112 (25) | 16.3230190 (25) | 80 | 7.7 | 4.321 | 0.77 |
| | | 4889 | 55709 | 292 | 3.27797 (11) | 230.8155327 (23) | 5.5131596 (22) | | | | |
| 29 | QSO | 11071 | 58429 | 538 | 3.91470 (94) | 23.4102889 (40) | 7.3528931 (37) | 116 | 7.3 | 5.151 | 0.54 |
| | | 4795 | 55889 | 258 | 3.59479 (32) | 139.8420990 (36) | 3.9518627 (35) | | | | |
| 30 | QSO | 6468 | 56311 | 810 | 3.8086 (11) | 150.953985 (17) | 32.910257 (15) | 86 | 7.0 | 5.151 | 0.73 |
| | | 4998 | 55722 | 838 | 4.05717 (41) | 260.2776744 (23) | 32.9434109 (24) | | | | |
| *a** | Galaxy | 1250 | 52930 | 70 | $3.7 (2.8) \times 10^{-5}$ | 66.04028052 (49) | 26.82959782 (46) | 2 | 7.2 | 0.025 | 1.00 |
| | | 1251 | 52964 | 110 | $9.5 (1.5) \times 10^{-5}$ | 65.36470546 (44) | 28.22729977 (40) | | | | |
| *b** | Galaxy | 2057 | 53816 | 112 | $-0.2 (1.5) \times 10^{-5}$ | 124.5235687 (11) | −1.14925936 (76) | 20 | 7.1 | 0.347 | 0.98 |
| | | 2713 | 54400 | 589 | $4.08 (98) \times 10^{-5}$ | 113.82854071 (67) | 16.16743929 (69) | | | | |
| *c* | Galaxy | 1000 | 52643 | 579 | 0.0685982 (82) | 161.0457000 (30) | 7.5313604 (37) | 8 | 8.9 | 0.023 | 1.00 |
| | | 1222 | 52763 | 254 | 0.073820 (12) | 169.1168975 (83) | 8.455402 (11) | | | | |

**Continued**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *d* | Galaxy | 2764 | 54535 | 363 | 0.043461 (22) | 223.3550093 (15) | 16.6846639 (20) | 11 | 8.7 | 0.023 | 1.00 |
| | | 1817 | 53851 | 106 | 0.033794 (27) | 229.0601828 (27) | 7.1061389 (42) | | | | |
| *e* | Galaxy | 1622 | 53385 | 427 | 0.080614 (17) | 178.0636891 (11) | 8.08752649 (95) | 66 | 9.1 | 1.000 | 0.84 |
| | | 2531 | 54572 | 385 | 0.073568 (17) | 245.1817106 (11) | 11.1220973 (13) | | | | |
| *f* | Galaxy | 993 | 52710 | 494 | 0.088955 (21) | 145.8057466 (11) | 5.9462890 (14) | 64 | 8.8 | 1.000 | 0.85 |
| | | 856 | 52339 | 533 | 0.1161340 (27) | 209.9656330 (14) | 4.6781075 (12) | | | | |
| *g* | QSO | 1771 | 53498 | 621 | 0.63406 (16) | 196.1321372 (32) | 15.3498332 (32) | 19 | 8.1 | 0.052 | 1.00 |
| | | 6010 | 56097 | 706 | 0.68207 (12) | 213.2575478 (62) | 26.5647705 (63) | | | | |
| *h* | QSO | 10914 | 58257 | 670 | 1.07931 (22) | 248.5371148 (21) | 33.6780774 (22) | 5 | 8.1 | 0.043 | 1.00 |
| | | 10902 | 58396 | 498 | 1.08351 (40) | 254.7686030 (21) | 34.4288949 (21) | | | | |
| *i* | QSO | 7617 | 56949 | 498 | 2.03223 (29) | 10.1074341 (21) | 22.2193690 (19) | 28 | 8.6 | 1.000 | 0.98 |
| | | 8733 | 58396 | 286 | 1.9117 (10) | 32.2684782 (34) | 4.7277616 (30) | | | | |
| *j* | QSO | 10444 | 58143 | 110 | 1.60371 (26) | 139.7924895 (11) | 29.7041497 (10) | 33 | 8.3 | 1.001 | 0.98 |
| | | 7401 | 56808 | 150 | 1.77332 (45) | 177.3193299 (34) | 45.7902539 (32) | | | | |

$\chi_{j,k}$: Similarity Index, defined as $\chi_{j,k} = -\log_{10}\left[1-\lambda_{j,k}\right]$. *: Selected from $\lambda_{\max}$ paring of Galaxy class of $-0.0001 \le z_{\text{obs}} \le 0.0001$.

overlapping range of the RFSs except somewhat different flux intensity/background continuum. Nevertheless, such identicalness in the best-fit RFSs still does not guarantee the corresponding pair being associated LEOs, since there are cases, e.g., $a \sim j$ in **Table 1**, of $\lambda_{\max}$ pair of essentially identical RFSs but impossible for LEOs of the corresponding pair to share a common source under normal circumstances, indicating possibility of other causes for LEO association or simply due to the lacking of sufficient spectral details for recognizing uniqueness of each LEO. Therefore, further confirmation/falsification of association of LEOs may need spectrum of a LEO to cover wider spectral range, of finer resolution, with lesser stray light/background interference, etc., such that fingerprints of LEOs may be examined/compared, including line shapes.

It is also possible to verify association of LEOs by observing their angular movements in celestial sphere of observer in motion at origin, pending on internal distance between luminescent source and pole point (origin or antipode). For a LEO in transverse motion with respect to observer, *i.e.*, no relative inline motion between the parties, inclusion angle of the LEO over sufficiently small RT interval is $\tan\theta = \tan a/\sin b$, $a$ being radian of path of the LEO during the RT interval and b radian of internal distance between the LEO and pole point. If the LEO was at equatorial sphere of observer then $b = \pi/2 \to \theta = \pi v_T \delta t / D_i$, $v_T$ is transverse velocity of LEO, $\delta t$ RT interval. Therefore, even if the LEO was in motion at 99% SLV, observed angular movement of such entity over a period of, e.g., three decades would be 0.0096″ (arc second), which is barely above maximal photon deflection (0.0081″) by solar mass near Earth orbit. On the other hand, if the LEO was in transverse motion in vicinity of pole point then $b \ll 1 \to \theta \simeq a/b$. Therefore, if the LEO was in motion in, e.g., 50% SLV and about million light years away from pole point then observed annual movement of such LEO would be ~0.010″, which is detectable, especially with respect to LEO patterns at/near equatorial sphere (fainter than otherwise though), since such LEO pattern is practically stationary and invariant in decades or even centuries regardless of motion of the observer. Therefore, if LEOs of a $\lambda_{\max}$ pair are indeed associated then their angular movements and direction of motion shall be identical hence observed as two LEOs in motion in same speed and direction along same diametrical circle in celestial sphere of observer. With several such AGPs confirmed/verified, velocity and direction of absolute motion of observer can be estimated via Equation (35).

For typical QSOs, gravitation redshift is non-negligible. Therefore, with Expression (29),

$$F_z\big|_{\text{QSO near pole point}} \simeq f_\rho^{-1}\left(1+z_{kD}\right),\ f_\rho \equiv \left(1-4/\rho\right)^{1/4}. \tag{42}$$

$\rho$: Reduced distance between LEP and center of gravitation field, in unit of CLF.

Radius of LES of a spherical QSO is $\rho$, which is private to each and every QSO hence not expected to be identical among QSOs. In other words, for a light emitting particle, e.g., hydrogen atom, orbiting in gravitation field of a quasar, average distance (in unit of CLF) between the orbit and Schwarzschild Sphere of the quasar ($\rho = 4$) should/could have any value positive. Therefore, reduced radius of LES of a quasar should/could have any value $\rho > 4$ hence should not be identical among QSOs. Therefore, $F_z$ distribution of $\lambda_{\max}$ pairs of QSO should not be expected to exhibit any characteristic peak even if all the pairs were associated ones. However, from **Figure 5**, it is evident that exist characteristic peaks of $\lambda_{\max}$ pairs of QSO. Existence of this phenomenon not only indicates that considerable portion of the pairs is indeed associated but also the existence of a common factor of gravitation redshift among the QSOs, *i.e.*, having identical $f_\rho$. In contrast, if none of the $\lambda_{\max}$ pairs were associated then its distribution should be statically identical to that of the random pairs even if all the quasars were of identical $f_\rho$.

From $F_z$ distribution of the $\lambda_{\max}$ pairs of QSO in **Figure 5**, five characteristic peaks are recognizable that centered at 2.284, 2.888, 3.612, 4.280, and 5.096, and recognized as corresponding to $z_{1D}, z_{3D}, z_{5D}, z_{7D}, z_{9D}$, respectively. Thus, with $z_{\text{LHL}}$ approximated as $z_{\text{LHL}} \simeq a_1 s + a_2 s^2$, consistent with that of Expression (41),

$$f_\rho^{-1} \approx 1.99,\ a_1 \approx 0.0722, a_2 \approx 0.008\ \to\ z_R \approx 0.073, z_D \approx 0.148. \tag{43}$$

Therefore, LESs of the QSOs were distant from their respective Schwarzschild Spheres by ~4/15 CLF whether the light was emitted 2, 6, 10, 14, or 18 billion years ago, suggesting the existence of discrete state of LEP in quasar field. There

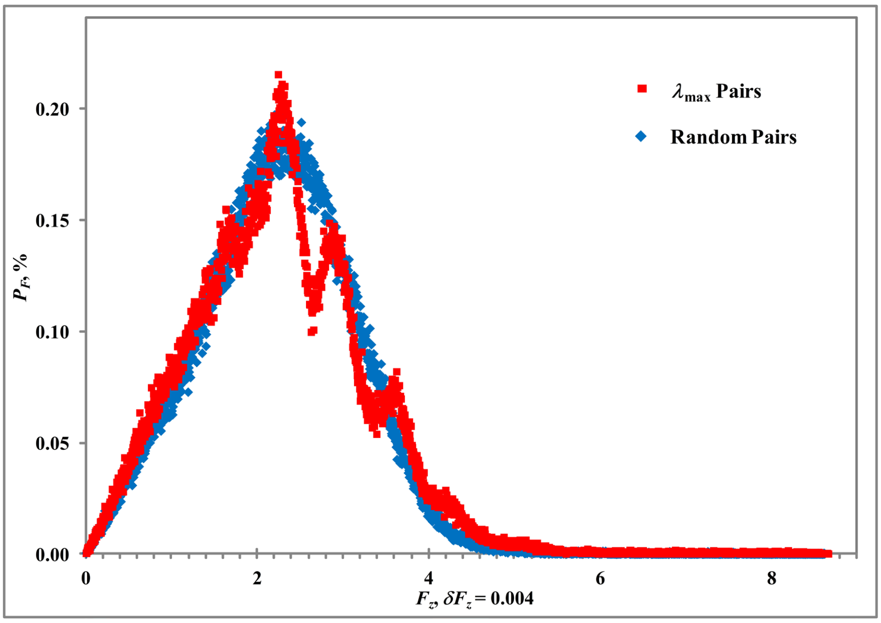


**Figure 5.** $F_z$ distribution of QSO pairs. From Qdata, 416 855 $\lambda_{max}$ pairs and more than $4.128 \times 10^5$ random pairs are found satisfying $0 \le u_s < 1$.

is also a set of smaller peaks recognizable in $F_z$ distribution of Figure 5, centered at 1.33, 1.65, and 1.95, and recognized as corresponding to $z_{2D}, z_{4D}, z_{6D}$, respectively, with $f_\rho^{-1} \approx 1.00$, indicating that minor set of the $\lambda_{max}$ pairs are associated galaxies and/or stars instead of QSOs.

## 9. Recent Findings from JWST Observations

Imagine a replica of Sun at/near equatorial sphere of current observer half $T_D$ ago. Then, apparent magnitude of the star to current observer would be >41.7, which is undetectable even with JWST. However, if the star were at antipode of current observer one $T_D$ ago then it would be observed as if photosphere of Sun were ~2.3 light seconds away with apparent magnitude of −38.3 plus ISE (through internal distance of $D_i$). Further, color-effective temperature of the photosphere would be ~5023 K and observed in all sky directions of current observer hence seen as a cosmic radiation of visible light. If transverse velocity of the star then were 0.01% SLV with respect to observer now, then the super bright cosmic radiation would last for ~13 hours. Similarly, if replica of Milky Way were at/near the equatorial sphere one and one half $T_D$ ago, apparent visual magnitude of the galaxy [29] would be +16 plus ISE (through internal distance of $1.5D_i$), which is beyond detection limit of many ground-based telescopes. However, if the galaxy were centered at current location of observer two $T_D$ ago then it would appear as if observer is viewing from inside of the bulge ~0.65 kpc [30] therefrom but seeing outside of the bulge having an apparent magnitude of about −11.5 plus ISE (through internal distance of $2D_i$). If transverse velocity of the galactic center was 0.2% SLV with respect to observer then the bright cosmic radiation, bisected by a dark lane, would last for ~2 million years. In general, to any observer anywhere/when in finite PHS, if lights were emitted from a LEO half integer $T_D$ ago, then apparent magnitude of the LEO to the observer would be highest in the $T_D$ period ignoring ISE, and if whole integer $T_D$ ago then the LEO would be observed as cosmic radiation (if not hindered by others at preceding pole point). Therefore, cosmic electromagnetic radiation is a common and not extraordinary phenomenon in finite PHS, whether the space was expanding or not.

Massive galaxies (stellar mass > $10^{10}$ solar masses) about 600 million years after Big Bang were found from recent JWST observations [8] but not anticipated at such early stage of the evolution. However, under the paradigm of finite space or nonexpanding FSB, massive galaxies of high $z_{\text{LHC}}$ is but only natural. On the other hand, determination of the masses of these LEOs was based on luminosity measurement involving LISA, that is, effect of finiteness of space/FSB was not considered nor incorporated therein. Therefore, absolute magnitudes, hence masses of these LEOs, may have been overestimated significantly.

The relationship between apparent and absolute magnitude in finite space/FSB is, by definition of the entities,

$$m_\lambda = M_\lambda + 5\log_{10}\left|\sqrt{\frac{1+z_{\text{LHL}}[d]}{1+z_{\text{LHL}}[d_0]}}\frac{\sin[\pi d]}{\sin[\pi d_0]}\right| + f_{\text{ISE}}, \quad \begin{matrix} d \equiv s/D_i, & s \not\subset \text{LES} \\ s_0 \equiv 10\text{ pc}, & s \not\subset \text{FCS} \end{matrix}. \quad (44)$$

$m_\lambda$: Apparent magnitude of LEO, function of photon wavelength. $M_\lambda$: Absolute magnitude of LEO, function of photon wavelength. *s*: Path length from center of LEO to observer. $f_{\text{ISE}}$: Logarithmic function of ISE, variables include photon wavelength and absolute shift thereof, LHL redshift, path length and condition thereof, etc.

Therefore, major factor in determining apparent magnitude of a LEO, which is measurable, is not photon path length *per se* but internal distance of the LEO to nearest pole point of observer, a unique feature of finite space/FSB. Therefore, the observed LEOs could well be due to stars not too far away from pole points.

From recent JWST observations, more than 50% of the observed galaxies having redshift $3 < z_{\text{obs}} < 8$ were found of disk type with Sérsic Index $n \sim 1$ [7], implying that massive and old galaxies have been formed within 2 billion years since the Big Bang, not anticipated at such early stage of the evolution. However, under the paradigm of finite space or nonexpanding FSB, galaxy of any type of high $z_{\text{LHL}}$ is but natural. On the other hand, while observation of the extended features ensured the LEOs observed are not stars, characteristic sizes of these LEOs are on the order of 0.1~1″ (provided pixel resolution of ~40 milliarcseconds) that is rather small for typical disk galaxies. Note that angular size of LEO of 1″ corresponds to ~1 kpc (kilo parsec) at equatorial sphere of observer but only ~5 pc if the LEO were 1 Mpc from pole point. If a disk galaxy were 765 kpc from current observer or antipode thereof, it would be seen similar to Andromeda Galaxy but redshifted significantly and fainter due to ISE. Therefore, apparent morphologic features of the observed LEOs could be due to starburst or core of galaxy under significant distortion due to ISE.

## 10. Discussions

If the universe was begun from a single point or alike, as popularly believed, then size of the universe would have been finite at the beginning and shall be and always be finite thereafter, whether or not observable, since expansion rate of such universe was/is not assumed infinite. If Earthling therein construe from their measurements that the starting event has happened, e.g., 14 GJY ago and are also affirmative that others somewhere else in the universe should also construe the same from their own measurements, then, collection of all such "somewhere", including where of Earthling, forms a set of space-like entities, *i.e.*, where, in the universe, hence size of the set is finite. Therefore, the set resembles a FSB and *the* PHS under the context of expanding universe. In language of relativity theories, it is said that length of any space-like line is finite at any moment of proper time. In addition, space-like line is continuous. Therefore, space-like line of finite length has no ends. Therefore, PHS may be thought of as a collection of all such lines. More appropriately, consider constructing a bundle of *time*-like lines of all *matter* (including radiation and vacuum) in the universe in spacetime fabric of the universe, slicing the bundle orthogonal to *time* at any single moment of the *time* (cosmic or otherwise), and constructing a set by collecting all the points of the bundle at the cut that are simply connected/connectable among the points. Then, under the context of relativity theories, this set is the PHS, and gauge of the set should be finite.

If the PHS was expanding and size of the FSB has become sufficiently large, then observers therein should have observed galaxies, and the farther they look, the more the galaxies would be seen, until edge of observable universe is reached. Therefore, there should have no characteristic peaks of finite space/FSB in plot of galaxy redshift distribution but a cutoff, even if galaxy distribution in the universe was not uniform. If the PHS was expanding and its current size is sufficiently small, then plurality of the characteristic peaks, convoluted by nonuniformity of galaxy distribution in the universe, should show up in the redshift plot, since number of galaxies observed local simultaneously by observer should first increase, then decrease and increase/decrease again, and so on, due to finiteness of the FSB and that of the universe as well. Therefore, those major peaks shown in **Figure 1**, if not mainly caused by instrument limitation and/or survey strategy, provide strong evidence for spacial finiteness of PHS. On the other hand, the ~2.0 GLY size of PHS as measured herein is surprisingly small, even though it is still many orders of magnitude larger than that as estimated by Schwarzschild [31].

If the PHS was expanding, then locations of the characteristic peaks in the redshift plots should be unevenly spaced and the larger the redshift, the more sparse the peaks should be. Therefore, the evenness of the locations of the characteristic peaks, as shown in **Figure 1**, **Figure 4**, and especially **Figure 5**, offered strong evidence that PHS was neither expanding nor contracting during the entire span constrained by the SDSS data.

The most surprising finding though is the identicalness of gravitation redshift among the quasars observed, indicating that the reduced distance is fixed of the LEP of quasar to center of gravitation field of same at local moment of releasing the photon that received eventually by observer. Further, value of the gravitation redshift of $z_{GR} \approx 1$ implies that such LEP was only ~4/15 CLF away from Schwarzschild Sphere of corresponding quasar. Nevertheless, particle-orbiting model shows that mass in circular motion in gravitation field (hence constant gravitation redshift) shall have ground state at six CLFs from Schwarzschild Sphere and in any case is not allowed to be closer to the sphere by one CLF [27]. Wave model, on the other hand, allows mass object at essentially anywhere outside Schwarzschild Sphere. Therefore, the issue remains as a mystery, for now.

In contrast to the above, space lens effect of PHS is rather straightforward even if PHS were FSB and expanding, since, according to GRT, light travelling in PHS shall be along geodesic in same, if no interference of others during the travel. Alternatively, photon created in space can but only travels in that space. Therefore, photon created in PHS shall remain in same until annihilation by others in same, even if HDS exists. Nevertheless, impact of space lens hence finiteness of PHS is not at all small to celestial observation/measurement and interpretation thereof. It also provides new pathways to celestial observation. For instance, it is certain that Milky Way has been in existence for more than four billion years. If Galactic Center has been in motion at, e.g., 1% SLV then the galaxy should be observed with HLC redshift of ~0.3 by Earthling within 40 million light years from the observer, if the galaxy happens to be in accessible sky direction of the observer. If age of the galaxy is older than eight billion years, then it should also be seen somewhere in the sky with HLC redshift of ~0.6 if not hindered by others, and so on. Likewise, if the galaxy has been in motion at 0.1% SLV or less, then the observed CMR may well be due to the clouds surrounding Milky Way.

PHS is finite space or nonexpanding FSB and receding velocity interpretation of HLC is incorrect as evidenced by the observation data. Therefore, energy of photon travelling in PHS has to lose to something/somewhere, hence redshifted thereby/of, but cannot to/in empty/void region of PHS, since nothing is there in such region by definition of empty/void. However, as a matter of fact, photon does travel in vacuum of PHS. Therefore, vacuum is not and cannot be empty/void region of PHS but media of some sort, and HLC has to be caused by energy loss of photon travelling in vacuum. In retrospect, photon is also known as electromagnetic wave. Then, there is nothing really new for energy loss of wave to propagation media, nor constancy of velocity of wave propagation in media under fixed condition, nor alteration of propagation velocity of wave in media by altering condition of media. Vacuum as media for propagation of electromagnetic wave might remind ether in history of physics. However, it can be shown that physical vacuum is merely comprised of ordinary but massless particles.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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## Appendix. Space

Common experience in physical world provides an elementary fact: exists *there* in physical world that object can access to, reside at, and withdraw from. Such entity is referred to herein as *node* and such property of node as *accessibility*. In other words, node is nothing else but entity accessible to/by object and accessibility is the one and only property of the entity.

If there were only one node there in physical world then the phrase "access to, reside at, and withdraw from" would become meaningless. Therefore, the elementary fact also reveals that plurality of *there* exists in physical world. In other words, exists plurality of nodes.

Since exists more than one node there, relationship must exist among nodes, including no relationship. From the elementary fact, there is one and only one type of relationship exists among nodes: *connectivity*. If object exists at node $A$ can access to node $B$ then $A$ and $B$ are said as connected, otherwise they are not. Therefore, *space* is and is defined as set of connected nodes. As to what is construed as object, how it may occupy, reside at, withdraw from node of space and why, etc., are subjects of disciplines of science.

It is plausible logically that plurality of spaces may exist. However, if plurality of spaces does exist then there shall be no relationship among the spaces since, by definition of space, there is no connection between spaces. Therefore, whether or not other space exists shall have no relevance of any kind to any level of details to the space in consideration.

If node $A$ connects to node $B$ and $B$ connects to $C$ then $A$ and $C$ are also connected. If a node is in connection with another node without involvement of any third party then connection of the two is said as *direct*, that is, they are in direct connection. If a node has one and only one direct connection with others then connectivity of the node is defined as one, and so on.

There is no *between* inbetween nodes if connection of the nodes is direct. If there were object inbetween directly connected nodes then there must also have node inbetween since, by definition, object can and can only reside/exist at node. Therefore, if there were entity inbetween directly connected nodes then the entity must be node or none. If the entity is node of space and connection of the nodes has to be bridged by the entity (hence inbetween) then connection of the nodes is not direct; if connection of the nodes does not have to be bridged by the entity then the entity is not inbetween; if the entity is node of other space then if connection of the nodes is bridged by the entity then the other space is in connection with this space hence is not other space; if connection of the nodes is not bridged by the entity then the entity is not inbetween; if the connection is blocked by the entity then connection of the nodes is not direct. If the entity is none, *i.e.*, neither object nor node therefore is nothing, then if the entity is inbetween the nodes but does not block the connection then it is irrelevant to the connection since there is no effect of any kind of existence of such entity to the connection; if the entity does block the connection then the connection is not direct. Therefore, there can be no *between* inbetween directly connected nodes of space. In such context, direct connection of nodes is *continuous*. Since connectivity of nodes of space, *i.e.*, set of connected nodes, is at least one, it is said that space is continuous.

If all nodes of a space are of connectivity two except two that are of connectivity one then such space is referred to as finite space with boundary. In geometric presentation of such space, one-to-one correspondence can be established between nodes of such space and geometric points of geometric line of finite length, hence finiteness of the space. If all nodes of a space are of connectivity two and length of the corresponding geometric line is finite then such space is referred to as finite space with no boundary; otherwise infinite space. All such spaces are said as one-dimensional since connectivity of all nodes of such spaces is two except that of the boundary ones, if any, which is of dimensionality ½. Likewise, if connectivity of all nodes of a space except boundary ones is $2n$ then such space is $n$-dimensional. Further, by definition of connectivity, direct connection of nodes is bidirectional. Therefore, number of directions at any node is twice the connectivity of the node.

Therefore, space is object, probably the most essential object of all. Further, space can be presented as geometric construct in geometry via one-to-one correspondence between nodes of space and points of geometric construct. It is in such context that node of space is also referred to as point or geometric point of geometric entity. Accordingly, if nodes of a space can be exhausted by natural numbers then such space can be presented as web of nodes with the understanding that, other than node of space, there is no *between* inbetween nodes of space therefore space is continuous in any case. Therefore, space, hence PHS, can be infinite, finite with boundaries, finite without boundary, of plurality of directions, plurality of dimensions including rational fraction of such, or of web structure, etc. It is by and only by experiment/observation in physical world, that size, shape, nature, property of PHS may be settled upon.

All points of space are equal/equivalent in terms of accessibility except those at boundary. If some point of space is different from others then there must have cause for causing the difference. In other words, without cause, all points of space are and must be identical in terms of accessibility. On the other hand, cause is synonymous with object and/or effect of object hence still object. In other words, cause means object and object is cause, whatever that may be. Further, it is with respect to the background of no cause that cause is recognized, distinguished, differentiated from. Therefore, by scientific logic/rationale, all points of space are and must be identical in terms of accessibility except boundary ones (because they are boundary ones). More economically, it is said that space is *homogeneous* except at boundary. For same reason, directions in space are and must be equivalent, *i.e.*, there can be no difference of any kind to any details in terms of accessibility along any direction of space (except at boundary), or more economically, space is and must be *isotropic* (except at boundary).

There is fundamental distinction between point of space affected by object residing at the point and boundary point of space. A point of space may be/become difficult to access by object because the existence of other object at the point. However, in principle, such point is still accessible to/by object but may be with effort of/cost to object. On the other hand, boundary point of space cannot be penetrated through nor broken down regardless of effort/cost of object because, by definition of space, there is no *there* outside boundary of space or irrelevant even if there were. More precisely, there is no such thing as *outside* of space, even though the word *boundary* might imply otherwise. The word inside/outside is a relative term pending on perspective and meaningful only if referring to region in space, *i.e.*, subset of space, but meaningless if/when referring to *space* itself. That is, the phrase "outside of space" is self-contradicting to the very meaning of *space*. Nevertheless, as a consequence of narrative inertia, such notion comes natural and may appear meaningful even though it is intrinsically illogical. For same reason, the phrase "boundary of space," as borrowed herein for convenience of communication, has the potential of being misunderstood as implying existence of *there* outside of space or prompting inquisition beyond such. For clarification, "boundary of space" as used herein refers to and only to points of space of lesser connectivity in comparison with that of other points of same and nothing more/less. Likewise, while it may be customary to think of phase such as "entity exists in space," negation of such expression, e.g., "entity not-exists in space," is irrational, not because of error in grammar or logic or anything else but contradiction with the very meaning of the word *space*.

Therefore, by definition, space is but set/collection of connected points accessible to/by object and nothing more/less. Therefore, point of space cannot be created nor annihilated by object exists therein regardless of what the object might be/do since the only property possessed by point of space is accessibility that is not none by definition of node. With the precaution stated above, dimensionality of space can also be understood as boundary of space since it is synonymous with connectivity of node and vice versa. Therefore, dimension of nodes of space hence dimension of space cannot be created nor annihilated by object exists in space.

Similarly, object in one space cannot jump into another space regardless of effort/cost of the object and regardless of how close the spaces may be in their corresponding geometric presentation. If object in one space could cause effect of any kind in other space then, by definition, the object must also be existing in that other space hence that other space must be accessible from this space hence is part of this space by definition of space hence not other space. Conversely, if there were other space out there then, by definition of space, that other space is not accessible from this space hence could not cause any effect of any kind to/from this space hence is of no relevance of any kind/detail to this space hence is or is equivalent to none.