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Modified White Hole Enthalpy Coupled to Quantum Bose-Einstein Condensate at Extremely Low Entropy

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Abstract

We model the universe as a white hole, and in the process we perform detailed analysis of the enthalpy equation of the modified white hole, and we get a much detailed picture of when and how did; quantum gravity (cosmology) phase, inflationary phase, and the acceleration phase of the universe happened. We determine the field equations of the modified white hole and evolve the scale factor and compare the evolution to the thermodynamic properties of the universe. We also illustrate that the strong energy condition is violated, but both the null energy condition and the strong cosmic censorship are not violated. Lastly, we couple the enthalpy to the Bose-Einstein condensate at extremely low entropy at the quantum gravity (cosmology) regime. Thereafter, we determine the unstable condition of the Bose-Einstein quantum equation which we interpret as the moment when the big bang occurred.

Keywords

Modified White Hole, Quantum Bose-Einstein Condensate, Enthalpy Energy Density, Stability Analysis, Strong Cosmic Censorship

1. Introduction

The concept of duality between thermodynamics and gravity as highlighted by Hubeny [1] and other authors, allow us to approach the problem of the evolution of the universe from purely thermodynamic framework and then make inferences to the gravitational process of the universe including those of quantum cosmology/gravity. Thus, our approach to the problem of the smooth integrated evolution phases of the thermodynamics of the early universe to explain its evolution is along the lines of argument of the Emergent gravity theory discussed in detail by Padmanabhan *et al.* [2], Verlinde [3], and Padmanabhan [4] where enthalpy changes over entropy (time) can be used to describe the evolutionary properties (phases) of the universe from the state of zero entropy. We approach this problem by modeling the evolution of the universe in the setting of a white hole gravity [5] and analyse its thermodynamic properties. Carlip [6] stated that: "a likely successful emergent gravity will require more fundamental principles not yet unknown to allow its emergent properties to be organized into a realistic model of spacetime".

Thus far, the exact nature of the thermodynamic transitions from the vacuum phase, and quantum gravity through to the inflationary phase and into the current evolution phase of our universe is not well understood and captured in a single cosmological model. What we have are assumptions made for each phase of the evolution of the universe with model constraints to be taken care of for the cosmological model concern Gunzigy *et al.* [7], Fukuyama *et al.* [8], Tawfik *et al.* (2013), and Alfaro *et al.* (2018).

Gunzigy *et al.* [7] presented a consistent simple thermodynamic model for the quantum creation of radiation from the decay of vacuum energy. The consistency of their model was with regard to the following natural thermodynamic physics of our universe; the production rate of entropy starts at maximum, the vacuum energy is non-singular and regular, the creation rate is higher than the expansion, and then falls below the expansion rate. In their model, they further observed that the nonadiabatic inflationary era exits smoothly to the radiation era, without a reheating transition. We also observe the same phenomenon in our analysis of the underlying cosmological implication of our enthalpy results, taking into account that quantum gravity theories do not accurately predict the vacuum energy and its thermodynamic properties as we do in this paper and in [5].

Fukuyama *et al.* [8] studied the two stages of the accelerations of the universe; that is the exponential inflationary acceleration, and the mild acceleration using the Bose-Einstein condensate phase of the boson field. They developed a unified model of dark energy and dark matter for the mild acceleration phase of our universe. In the process they observe a phase transition of the Bose-Einstein condensate in which the two phases transform with each other, one is the 1/4 dark energy and the other is the 1/4 normal gas. In our work, we do observe a similar disconnected phase transition, for the enthalpy in the quantum gravity regime for the Bose-Einstein condensate. We interpret this phenomenon as the moment when the big bang occurred. We also observe both acceleration phases for the inflationary phase, and the mild acceleration just after the event horizon of the white hole/or Universe. In addition, we also observe that through enthalpy that the acceleration (or cosmological constant) of the universe just after the inflation phase and outside the event horizon, it is almost constant, very small, and slowly exponentially growing but near zero as observe by [8].

Our results of the modified white hole thermodynamic quantities we derived in [5] and in particular enthalpy, intrinsically display all the cosmological model properties of Gunzigy *et al.* [7], and Fukuyama *et al.* [8]. Most importantly, we further observed through enthalpy, the detailed thermodynamic picture of the pre-big bang, at the moment of the big bang, and post-big bang.

Our paper is structured as follows: In Section 2, we study the modified white hole enthalpy at low entropy and we obtain interesting results regarding the thermodynamics of the hole. In Section 3, we derive the enthalpy energy density of the white hole, and we use it in Section 4, when we couple the white hole equation of state to the quantum Bose-Einstein condensate. In Section 6, we study the stability and the unstable conditions of the Bose-Einstein quantum state of the modified white hole, and then we report on the strong cosmic censorship in Section 7. In Section 8, we conclude the paper.

2. Enthalpy at Low Entropy

From Kubeka *et al.* [5] we have the enthalpy of the modified white hole being given by

$$H(S)=\frac{2}{-\frac{S}{\pi}+b\left(\frac{S}{\pi}\right)^{1/2}}\left[1-2\pi a\ln\left(\left(\frac{S}{\pi}\right)^{1/2}\right)-ab\pi\left(\frac{\pi}{S}\right)^{1/2}\right] \tag{1}$$

where a, and b are the van der Waals fluid constants, and S is the entropy of the modified white hole. At the white hole horizon $S=\pi r^2=12.566$, r is the radius of the white hole. The plot of the enthalpy H against the entropy is given below.

From **Figure 1**, a and b were taken as 1, and 0 respectively. The value of a was chosen to be equal to the energy density of the universe at the big bang, and b was chosen to be equal to the value of the radius of the universe at the singularity $r_0=0$ or zero spacetime volume.

The following fundamental observations from **Figure 1** were noted regarding the behavior of H; that is as S approaches 0.4 from the right, H decreases asymptotically to minus infinitely. Implying an enthalpy singularity at 0.4. This thermal singularity or thermal singularity phase change, is actually the bifurcation critical point reached by the growing quantum vacuum fluctuations where the fluctuated vacuum energy splitted into its constituents particles; 5 percent of plasma of ordinary matter radiation and electromagnetic radiation energy (light), and 95 percent of dark matter and dynamic quantum vacuum energy (cosmological constant and dark energy) [9] [10]. At equilibrium, without the quantum vacuum energy fluctuations, the vacuum energy values of these constituents cancel out, thus in a sense leaving the vacuum empty [11].

We further interpret H at $S<0.4$ as pre-Big bang Anti-de Sitter enthalpy. Dafermos *et al.* [12] formulated the Anti-de sitter instability conjecture and shown that there exist infinitesimally and arbitrarily small perturbations to the

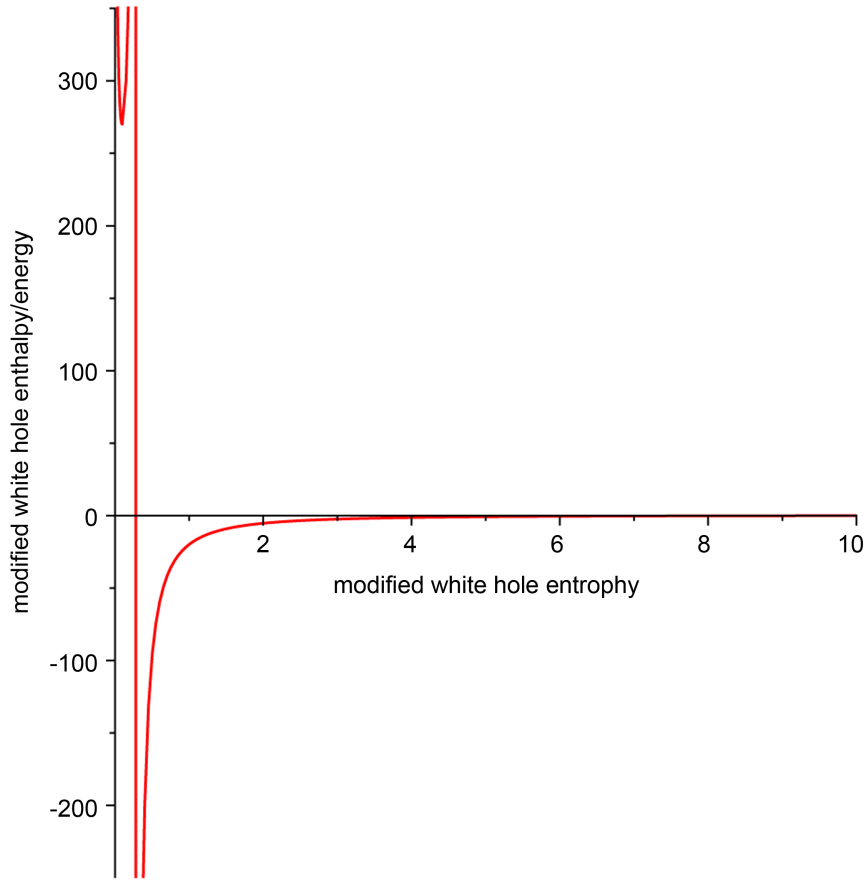

Figure 1. Enthalpy of a modified schwarzschild white hole with values $a=1$ and $b=0$.

anti-de Sitter spacetime initial data that make the spacetime very unstable, and that these perturbations can grow unbounded as noted in the above, and in our case resulting in infinitely uncontrolled thermodynamic explosion, the big bang. It was shown in Kubeka *et al.* [5] that both the vacuum and Anti-de Sitter spacetimes are inherent properties of the modified white hole geometry, our universe. Furthermore, at $S=0.4$, H underwent a thermodynamic phase transition, and at $S>0.4$, H is the enthalpy of the post Big bang. The region where $S<4.3$ and $S>0.4$ is the region when H is negative which indicates the low density of the van der Waals fluid, The region where $S<0.4$ and $S>0$ is the region when H is infinitely positive and fluctuating. Therefore, we explicitly note that the quantum gravity enthalpy was very high and fluctuated from positive infinity near $S=0$ and dropped to just below 300 at about 275 when the $S=0.1$, then it increases asymptotically to infinity again near $S=0.4$. It seems that at $S=0.1$ the infinitesimally quantum gravity self-perturbations started to occur, and the perturbations growth increased so much so that they generated infinite heat resulting in H increasing infinitely near $S=0.1$.

We also observe that the enthalpy phase transition at $S=0.4$ was extremely dramatic and changing from extremely infinite positive value to extremely infinite negative value. We further observe that it is this extremely high value of H

(heat energy) that resulted in the system blowing up at the big bang, with such a shock but at an extremely and infinitesimally small S (in an instant) as it increase resulting in H dropping from very high positive value to very low negative value and thereafter increasing asymptotically approaching 0 between $S = 0.4$ and $S = 4$ because of the inflationary expansion of the universe. It should be noted that this is still the regime of quantum gravity at the singularity when $b = 0$.

Below we plot the graph of the evolution of enthalpy post the Big bang when the universe has just come out of the big bang singularity (when its volume is $b = 0.2$ say). and when $a = 1$.

From **Figure 2** we observe that the exponential increase of enthalpy changed from negative to positive at $S = 4.3$. In particular, between $S = 0$ and $S = 3$, there is an inflationary phase where the enthalpy increase rapidly exponentially and between $S = 0.5$ and $S = 4.3$ the inflationary phase increase starts to slow down because the inflaton field starts to disappear. At $S = 4.3$ the inflationary phase ends. This is still the regime inside the white hole horizon and we interpret the point $S = 4.4$ as the boundary of the quantum gravity regime.

It is important therefore to note that the big bang did not end this quantum state, but in fact, this state continued to exist and generate much more heat exponentially as the white hole enters the inflationary phase between $S = 0.5$ and

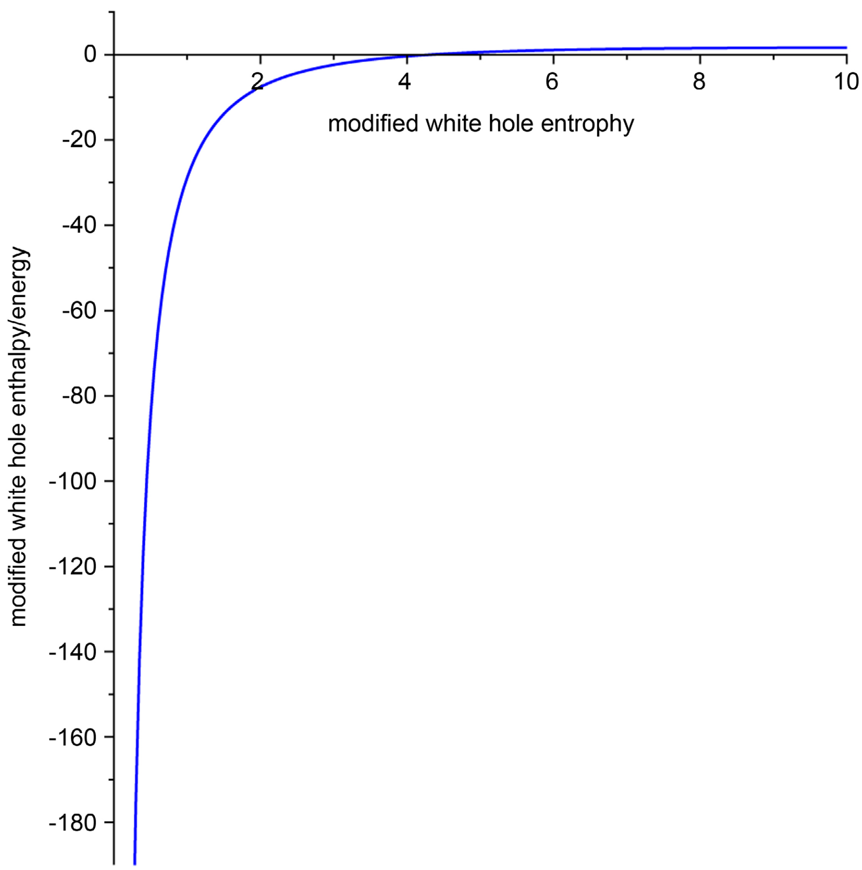

Figure 2. Enthalpy of a modified schwarzschild white hole with values $a = 1$ and $b = 0.2$.

$S = 4.3$. We also observe from **Figure 2** as we go back in time, that negative H confirming the existence of the Hawking radiation [13] past the quantum gravity boundary at $S = 4.3$ to $S = 0$ and the strong energy condition is violated.

We also observed from [5] that modified while hole have the second law of thermodynamics as its intrinsic property, and therefore the null energy condition is not violated and thus causality is not broken [14] [15].

We also observe that the enthalpy of the universe increases asymptotically very small when quantum gravity ends at $S = 4.3$. The asymptotic increase of the enthalpy may be linked to the cosmological fluid particles (galaxies) accelerating due to the cosmological constant as it is observed in the evolution of the universe [16].

From thermodynamic perspective, the above analysis of enthalpy gives us a much more detailed and accurate picture of the physics of quantum cosmology, big bang scenario, inflationary phase, and the acceleration of the matter content (galaxies) of the universe.

3. Enthalpy Energy Density

The enthalpy energy density of the modified white hole is given by

$$\rho_{ent} = \frac{H(S)}{V}$$

where from [5]

$$V = \frac{-5ab\left(\frac{\pi}{S}\right)^{1/2} + 2ab^2\left(\frac{\pi}{S}\right) + 2a - \frac{b}{(\pi S)^{1/2}} + \left(ab\left(\frac{\pi}{S}\right)^{1/2} - 2a\right)\ln\left(\frac{S}{\pi}\right) + \frac{2}{\pi}}{b\left(\frac{S}{\pi}\right)^{1/2} - \frac{S}{\pi}}. \quad (2)$$

The graph of the enthalpy energy density when $a = 1$, and $b = 0$, is

From **Figure 3**, we observe that the enthalpy energy density of our universe seems to be behaving differently inside the event horizon, on the event horizon, and outside the even horizon. In fact, the graph shows thermodynamic phase transition of the enthalpy energy density at $S = 11.9$ just near the while hole horizon $S = 12.566$ inside the white hole. The existence of this thermodynamic phase transition is analogous to the existence of critical phenomena in cosmological gravitational collapse of perfect fluids [17]. This also confirm the thermal energy/gravity duality as highlighted by [1] [2] [3] [4] in the sense that a large black hole corresponds to a hot plasma of the gauge theory degrees of freedom at the Hawking temperature.

The implications of the energy density phase transition inside the white hole near the event horizon in relation to structure formation of the universe after the big bang needs to be investigated further to give some further insight on the horizon problem. Perhaps it is this thermodynamic phase transition that was responsible for the fine turning of the universe, the matter density, and the observed cosmic microwave background radiation.

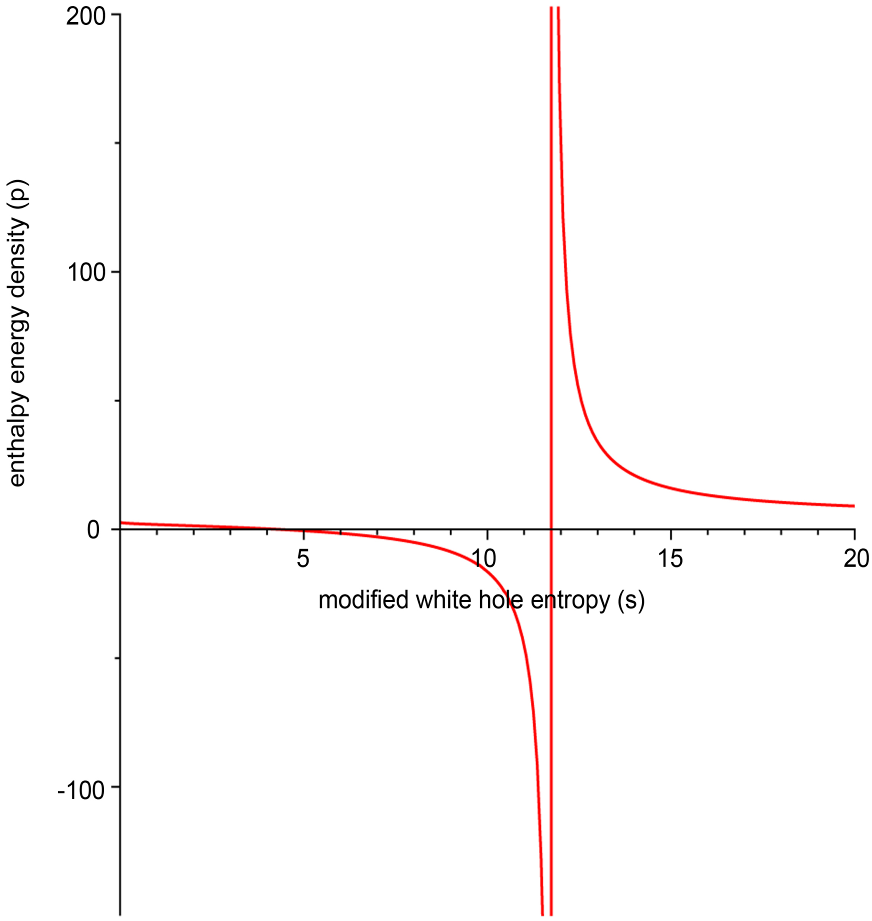

Figure 3. Enthalpy energy density of a modified schwarzschild white hole with values $a = 1$ and $b = 0$.

Furthermore, We also note that from the white hole event horizon, the enthalpy energy density scales down very fast exponentially, and at about $S = 20$ approximately, it starts to be almost constant *i.e.* the enthalpy energy density exhibit a power law scaling near the thermodynamic phase transition *i.e.* critical point and away from the critical point. Also, this thermodynamic phases transition seems to solve the problem of the low-entropy state at the quantum gravity state as demonstrated in **Figure 2** and the subsequent explanations that followed. The negative energy density after inflation ($S = 4.3$) inside the event horizon indicates that the radiation phase is dominating when either Bosons or Fermions have more charge particles than the other.

4. Coupling Modified White Hole Equation of State to Quantum Bose-Einstein Condensate

Since all the thermodynamics properties of the modified white hole inside and outside the horizon do not depend on the total mass of the hole, but on the cosmological constant Λ as illustrated by [5], then we are at liberty to add quantum effects to the van der Waals fluid to enable us to study the stability of the hole at the Planck and at very low entropy.

From [5] we have the modified white hole equation of state given by

$$T(V,P) = -\left(1-\frac{1}{4\pi}\right)\frac{1}{2\pi r^2}\left[\frac{V}{2\pi r^2 - r^2 + br}\left(1-\frac{a\ln(r)}{2}-\frac{ab}{4r}\right)\right.$$
$$\left.+\left(V-\frac{2\pi r^2}{1-\frac{1}{4\pi}}\left(r-\frac{r}{2\pi}+\frac{b}{4\pi}\right)\right)P\right]+\frac{a}{8\pi r}-\frac{ab}{16\pi r}. \quad (3)$$

and from [18] we have

$$P = P(V,T) + ann_a \quad (4)$$

where from Equation (2), $V = \frac{H(S)}{\rho_{ent}}$, and $n = n_a + n_c$, where n_c is the particle density from the Bose-Einstein condensate, and n_a is the active particle density with momentum distribution given by the integral [18]

$$\frac{d}{6\pi^2}\frac{k^2}{e^{\beta[\varepsilon(k)-\bar{\mu}]}+\eta}\frac{1}{\varepsilon k}$$

where

$$\beta = 1/T$$

and

$$\varepsilon(k) = \sqrt{k^2+m^2}$$

is the particle density, and d its the degeneracy factor, m is the particle mass, and

$$\eta = 1$$

for fermions and

$$\eta = -1$$

for bosons,

$$\bar{\mu}$$

is the chemical potential of the quantum Van der Waals fluid and can be calculated if V, T are known. We note that the particles do not touch the inside walls of singularity, that is, they experience the Casimir effect and are in a sense compressed and enclosed by the vacuum energy at extremely low negative enthalpy shown in **Figure 3** with low particle energy density. The cosmological constant in this quantum Planck regime help to facilitate the thermodynamic effects of the particles from their very infinitesimally small but exponentially growing self-perturbing vacuum thermal disturbances. This means that the required potential energy of the particles to overcome the vacuum energy is exponentially high, that is when the enthalpy reaches r_0 ($b = 0$) where $S = 3.1$, and $H = 0$, then the big bang occurred to free up the particles as a soap of plasma radiation of bosons and fermions at extremely high temperatures.

5. The Radial Expansion of the Universe

In this section, with the approach from Ref. [19], we explore that the horizon evolves with time for a dynamical white hole, so that the white hole metric rewrites from Ref. [5] as

$$\mathrm{d}s^2 = f_w(t,r)\mathrm{d}t^2 + f_w(t,r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2, \tag{5}$$

where the evolving horizon is at $r = r_+$ which satisfy the condition $f(t,r_+) = 0$. The Einstein equation, $G_{\mu\nu} = 8\pi T_{\mu\nu}$, in which $T_{\mu\nu}$ is the energy-momentum tensor for the above metric is obtained as follows:

$$G_t^t = G_r^r = -\frac{\partial_{r_+} f_w}{r_+} + \frac{1-f_w}{r_+^2}, \tag{6}$$

$$G_\theta^\theta = G_\phi^\phi = \frac{1}{2}\left(\frac{1}{f_w^2} - 1\right)\partial_{r_+}^2 f_w - \frac{\partial_{r_+} f_w}{r_+} - \frac{(\partial_t f_w)^2}{f_w^3}, \tag{7}$$

where an interesting interpretation can be given from Einstein's equation that clarifies the concept of entropy and energy. The Hawking temperature is written for the evolving horizon as $T = \frac{\partial_{r_+} f_w}{4\pi}$.

To describe thermodynamic identity $\mathrm{d}E = T\mathrm{d}S - P\mathrm{d}V$ in which S is for the horizon entropy and E identifies as the energy of the white hole, we consider the radial Einstein equation G_r^r by assuming minimal coupling with the radial energy-momentum energy tensor T_r^r as $G_r^r = 8\pi T_r^r$. Now to take the corresponding parameters for evolving horizon as follows:

$$P = -T_r^r, \quad V = \frac{4}{3}\pi r_+^3, \quad S = \pi r_+^2, \tag{8}$$

where we will have

$$P = \frac{\partial_{r_+} f_w}{8\pi r_+} - \frac{1}{8\pi r_+^2}, \quad \mathrm{d}V = 4\pi r_+^2 \mathrm{d}r_+, \quad T = \frac{\partial_{r_+} f_w}{4\pi}, \quad \mathrm{d}S = 2\pi r_+ \mathrm{d}r_+. \tag{9}$$

To substitute Equation (9) into the thermodynamic identity becomes

$$\mathrm{d}E = \frac{1}{2}\mathrm{d}r_+, \tag{10}$$

where we have

$$E = \frac{r_+}{2}, \tag{11}$$

in that case, the answer obtained for the energy is expected because the modified white hole of our study with $r_+ = 2M$ has energy $E = r_+/2 = M$. As a result, the structure of Einstein's equation was able to describe the evolving horizon as the above thermodynamic identity for the dynamics of a white hole because the radial change is with respect to the total energy content (*i.e.* the cosmological constant energy included) of the white hole that courses the expansion of r_+. Also, as stated in [5] that the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology is inherently a characteristics of the modified while metric Equation

(5) at later times, then it follows that the Equation (10) is analogous to the scale factor formula of the FLRW universe, and therefore from Equation (10), the Hubble parameter and the Hubble law are respectively written as

$$\mathcal{H}(E)=\frac{\mathrm{d}^2 r_1}{\mathrm{d}r_1} \tag{12}$$

$$\mathrm{d}E=E\int\mathcal{H}(E), \tag{13}$$

in this framework, $\mathrm{d}r_1$ in Equation (10) scales analogous to the FLRW scale factor.

6. Stability Analysis of the Bose-Einstein Quantum State of the Modified White Hole

This section explores the thermodynamic unstable conditions at which the big bang accrued as discussed above when $S=0.4$, and the accompanying underlying thermodynamic stability conditions before the big bang and after the big bang.

For this purpose, we describe the corresponding stability of the modified white hole by the equation of state in terms of three state parameters *P*, *V*, and *T*. This means that when a thermodynamic system is in stable equilibrium, it has the condition $\left.\frac{\partial P}{\partial V}\right|_T\le 0$ and $C_P\ge 0$, and $\left.\frac{\partial P}{\partial V}\right|_T\ge 0$ and $C_P\ge 0$ is the case of unstable equilibrium, and in our case, the moment of the big bang.

For the thermodynamic stability analysis, we take the derivative in the Equations (4) and (2) with respect to the entropy S, and then insert $S=\pi r_+^2$ into it, and thus obtaining the following relation

$$\left.\frac{\partial P}{\partial V}\right|_T=\left.\frac{\partial P/\partial S}{\partial V/\partial S}\right|_T$$

$$=\frac{2\pi r(-r+b)^2\left(Tr^3-\frac{1}{4}ab^2+abr-ar^2\right)}{(-2r+b)^2\left(2a\pi r\left(2b^2-5br+4r^2\right)\ln(r)+4(-2a\pi-1)r^3+b(23a\pi+5)r^2-2(10a\pi+1)b^2r+6a\pi b^3\right)}. \tag{14}$$

Therefore, we find the thermodynamic stability by the condition below

$$T<\frac{\left(b^2-4br_++4r_+^2\right)a}{4r_+^3}. \tag{15}$$

From the Van der Waals temperature relation [5], we obtain the corresponding conditions for P in the following form

$$\begin{cases} P<\dfrac{a(r_+-b)}{4r_+^3}, & r_+>\dfrac{b}{2}, \\ P>\dfrac{a(r_+-b)}{4r_+^3}, & r_+<\dfrac{b}{2}. \end{cases} \tag{16}$$

Exploring another thermodynamic stability condition, *i.e.*, $C_P\ge 0$, we acquire the corresponding stability as

$$\begin{cases} P < \dfrac{-a}{4r_+^2}, & r_+ < \dfrac{b}{2}, \\ P > \dfrac{-a}{4r_+^2}, & r_+ > \dfrac{b}{2}, \\ P < \dfrac{-a(b-r_+)}{4r_+^3}, & r_+ < 0, \\ P > \dfrac{-a(b-r_+)}{4r_+^3}, & r_+ > 0, \end{cases} \quad \text{or} \quad \begin{cases} P < \dfrac{-a}{4r_+^2}, & r_+ > \dfrac{b}{2}, \\ P > \dfrac{-a}{4r_+^2}, & r_+ < \dfrac{b}{2}, \\ P > \dfrac{-a(b-r_+)}{4r_+^3}, & r_+ < 0, \\ P < \dfrac{-a(b-r_+)}{4r_+^3}, & r_+ > 0, \end{cases} \tag{17}$$

where the aforesaid conditions are for case $P \neq 0$. But we can clearly see that C_P always negative for case $P = 0$.

7. Strong Cosmic Censorship

As discussed in the sections above, this condition is not violated by the hole because causality is also not violated because of the second law of thermodynamics. Also, the second law of thermodynamics implies that cosmic censorship will not be violated, and this can indeed be seen from the above **Figure 1** and **Figure 2** that the universe before and after the big bang it is thermodynamically accurately deterministic. These relations were also confirmed by Gwak [20] for a non-charged notating black hole.

8. Conclusion

Carlip [6] pointed out the challenges of the emergent gravity theory advanced by Padmanabhan [2], and stated that a successful emergent gravity cosmological model must satisfy all the observational and the theoretical foundations of general relativity as we know in the literature. Thus, our study as evident from the thermodynamic analysis of our universe in the framework of white hole gravity, does in fact satisfy both quantum gravity (cosmology) and general relativity evolution phenomena in the literature. For instance, in our analysis we found interesting and expected thermodynamic results that shared light on the nature of the thermal transition from pre-big bang to post big bang at the moment of the big bang occurrence. For the first time, it is revealed that the nature of the thermal transition was actually discontinuous for a very infinitesimally short time. Furthermore, we derived the stable and unstable thermodynamic equilibrium conditions of the modified white hole. We then interpreted the unstable thermodynamic equilibrium conditions as those of the moment of the cosmological big bang scenario. The exact time frames of the various evolutionary phases of the universe in the framework of entropy were extracted and a clear picture of the evolution of the universe was obtained. Future work that naturally follow from the results of this paper, is to investigate in depth the dynamics of the evolution of the structure formation, dark energy and dark matter, and also another possible application of the results is in constraining quantum gravity parameters in the theories of quantum gravity and quantum cosmology from a thermodynamics perspective to resolve contemporary issues that arise in uniting quantum gravity phenomenon and General relativity in resolving issues of gravitational singularity predicted by the Big Bang theory particularly the implications of our results on the analysis of the thermodynamic stability of the quantum state of the Bose-Einstein fluid at the big bang epoch.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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