Scientific Research Publishing

Journal of Modern Physics, 2023, 14, 1526-1536
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Quantum Unruh Effect on Singularities of Black Holes

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How to cite this paper: Zhang, T.X. (2023) Quantum Unruh Effect on Singularities of Black Holes. *Journal of Modern Physics*, **14**, 1526-1536.
https://doi.org/10.4236/jmp.2023.1411088

Received: August 10, 2023
Accepted: October 28, 2023
Published: October 31, 2023

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Abstract

It is generally believed that matter inside or once entering a black hole will gravitationally fall into the center and form a size-less singularity, where the density goes to infinity and the spacetime breaks down with infinite curvature or gravitation. In accordance to the Unruh effect, one of the most surprizing predictions of quantum field theory, however, it is found from this study that such singularity cannot be actually formed because it violates the law of energy conservation. The total Unruh radiation energy of the size-less singularity is shown to be infinite, much greater than that the collapsing matter can generate. All the energies of the collapsing matter including the gravitational potential energy, deducted, are far below the Unruh radiation energy, increased, for the collapsing matter to form the singularity. The collapsing matter actually formed is shown to be not a size-less singular point but a small sphere with a finite radius, which is found to be dependent of the mass of the singularity sphere, approximately proportional to the square root of the mass. The radius of the singularity sphere cannot be zero, unless the mass also approaches to zero. The result obtained from this study not only provides us a quantum solution to the problem of black hole singularity, but also leads to profound implications to the spacetime and cosmology. The Unruh effect excludes a black hole to form a size-less singularity, which has a finite mass but infinite density, curvature, and Unruh radiation energy. A point-like or size-less singularity can only be massless and naked.

Keywords

Black Hole, Singularity, Gravitation, Quantum Field Theory, Blackbody Radiation

1. Introduction

The Unruh effect is one of the most creative and surprising predictions from quantum field theory [1] [2]. It refers to that an accelerating observer detects a thermal radiation or thermal bath with temperature, which is proportional to the acceleration of the observer, whereas a non-accelerating observer detects none. The Unruh temperature for acceleration is expressed by [3]

$$T = \frac{\hbar a}{2\pi c k_B}, \quad (1)$$

according to quantum field theory. Here, $h = 2\pi\hbar$ is the Planck constant, c is light speed in the free space, k_B is the Boltzmann constant, and a is the acceleration.

As acceleration is indistinguishable from gravitational field in accordance with the Mach principle of equivalence, which was properly introduced by Albert Einstein in the development of his general theory of relativity [4], we have the temperature of Unruh radiation in a gravitational field to be given by

$$T = \frac{\hbar g}{2\pi c k_B} \sim 4.05 \times 10^{-21}\, g, \quad (2)$$

where g is the gravitational acceleration in the gravitational field. Here, we have applied the SI unit system. For a gravitational object with mass M and radius R, the magnitude of the gravitational acceleration at the radial distance r is given by

$$g = \frac{GM}{r^2}, \quad (3)$$

where G is the gravitational constant, and the radiation spectral energy density in the space per unit volume and per unit frequency is determined from the Planck's law of radiation [5], if the radiation is blackbody radiation,

$$u(T,\nu) = \frac{8\pi h \nu^3}{c^3} \frac{1}{\exp\left(\frac{h\nu}{k_B T}\right) - 1}. \quad (4)$$

Then, the Wien displacement law for the Unruh radiation in a gravitational field can be represented based on the gravitational acceleration as

$$\lambda_{\text{peak}} = \frac{b}{T} = \frac{2\pi c k_B b}{\hbar g} \sim \frac{7.16 \times 10^{17}}{g}, \quad (5)$$

where λ_{peak} is the wavelength of the Unruh radiation at the peak of the radiation energy density curve and $b = 2.89 \times 10^{-3}$ m K is a constant of proportionality. The principle of equivalence between a and g with the Unruh effect derives the equivalence between a gravitational field and a thermal radiation background (*i.e.* between g and T). To have a measurable Unruh radiation temperature and wavelength, the gravitational field needs to be extremely strong, such as that of a black hole.

A black hole is an object, from which even light cannot escape due to its strong gravitational field. Applying the Unruh temperature to be the surface temperature of a black hole, Stephen Hawking, in terms of the Stefan-Boltzmann law of blackbody radiation, derived the radiation power of the black hole to be inversely proportional to the square of its mass and the evaporation time of the black hole to be proportional to the cube of its mass [6]. The existence of black holes was theoretically predicted from the Schwarzschild solution of Einstein's general relativity a century ago [7] and has been confirmed recently from the observational detection of gravitational waves by the Laser Interferometer Gravitational Wave Observatory (LIGO) [8]. A star with mass above around twenty solar masses, when it runs out its nuclear fuel, will end as a black hole through a supernova explosion. In addition to the millions to billions of star-like black holes, a galaxy usually has a massive or supermassive black hole at its galactic center. An extremely luminous quasi-stellar object, quasar, is believed to be run or powered by a supermassive black hole with billions of solar masses. The highly energetic events, gamma-ray bursts, are believed to be generated or created when giant stars collapse into black holes or mergers of black holes including neutron stars. Our universe itself may be an extremely supermassive and fully expanded black hole [9]-[14].

The inside of a black hole is still a big mystery, though, conventionally, astrophysicists have believed over a century based on Einstein's general relativity that all mass inside a black hole, due to the strong gravity, will inevitably fall into the center of the black hole and form a size-less or point-like singularity, where the matter has an infinite large density and the spacetime breaks down with an infinite large curvature [15]. The Penrose singularity theorem describes the information of a gravitational singularity inside a black hole [16]. Under the non-negative energy density and strong gravitation conditions, the gravitational singularity forms once a trapped surface occurs. The Hawking singularity theorem interprets the gravitational singularity in the big bang situation [17]. Our universe contained a singularity deep in the past and all matter and energy were emanated in a big bang about 13.8 billion years ago from the singularity. The theory of quantum gravity such as string theory is expected to cure the spacetime singularity problem that plagues the inside of a black hole.

In this paper, we investigate the Unruh effect on the singularities of black holes. First, by integrating the blackbody radiation energy spectral density with respect to the frequency in the entire range (*i.e.* from zero to infinity) and the volume of the entire space outside the object (*i.e.* from the surface to infinite distance), we determine the total Unruh radiation energy that a gravitational object generates around the object and show it to be proportional to the fourth power of the object mass and inversely proportional to the fifth power of the object radius. This result implies that a collapsing matter or object, as it shrinks its radius and hence increases its outside total Unruh radiation energy, emits the Unruh radiation. Then, considering the matter that is inside a black hole and collapsing towards the center, we find that it is impossible to collapse all the matter with finite mass into the center point and forms a size-less or point-like singularity without violation of the law of energy conservation. The reason is because the total Unruh radiation energy emitted for the collapsing matter to form the size-less singularity is infinite large, much greater than the entire energy that the collapsing matter can release or emit, including the gravitational potential energy. Without violating the law of energy conservation, the collapsing matter can only form a singularity sphere with finite radius. Further, we determine the radius of the singularity sphere and shows it is extremely small but proportional to the square root of the mass. A point-like singularity can be formed only when its mass also approaches zero. In this case, a massless singularity is also naked, since its radius is less than the Schwarzschild singular radius. As the universe has finite energy, the big bang singularity might be a massless singularity but with finite Unruh radiation energy. Overall, this study provides us a quantum solution to the problem of black hole singularities, and meantime has profound implications to the spacetime and cosmology.

2. Total Energy of the Unruh Radiation

To find the total Unruh radiation energy produced by a gravitational object with mass M and radius R around it, we first integrate the Planck blackbody radiation spectral energy density, Equation (4), with respect to the frequency ν from 0 to infinity. This obtains the Unruh radiation energy density per unit volume to be proportional to the fourth power of the Unruh temperature as,

$$u_\gamma(T) = \int_0^\infty u(T,\nu)\,\mathrm{d}\nu = \int_0^\infty \frac{8\pi h\nu^3}{c^3}\frac{1}{\exp\left(\frac{h\nu}{k_BT}\right)-1}\mathrm{d}\nu = \beta T^4, \tag{6}$$

where the constant β is given by

$$\beta = \frac{8\pi^5 k_B^4}{15h^3c^3} \sim 7.54\times10^{-16}\ \mathrm{J\cdot m^{-3}\cdot K^{-4}}. \tag{7}$$

Then, we substitute the Unruh temperature (Equation (2)) into Equation (6), multiply the volume element $\mathrm{d}V = 4\pi r^2\mathrm{d}r$, and integrate both sides of Equation (6) with respect to the radial distance r from R to infinity to obtain the total radiation energy in the entire space surrounding the gravitational object as,

$$U_\gamma = \int_R^\infty u_\gamma 4\pi r^2\mathrm{d}r = \int_R^\infty 4\pi\beta\left(\frac{\hbar GM}{2\pi ck_Br^2}\right)^4 r^2\mathrm{d}r = \frac{\hbar G^4M^4}{300\pi c^7R^5} = \alpha\frac{M^4}{R^5}, \tag{8}$$

where the constant α is defined and given by

$$\alpha = \frac{\hbar G^4}{300\pi c^7} \sim 1.01\times10^{-137}\ \mathrm{J\cdot m^5\cdot kg^{-4}}, \tag{9}$$

It is seen that the total Unruh blackbody radiation energy of a gravitational object is proportional to the fourth power of the mass and inversely proportional to the fifth power of the radius. In general, a gravitational object can only have an extremely weak and hence non-detectable Unruh radiation energy surrounded. For a star such as the Sun, we can estimate the total Unruh radiation energy to be $U_\gamma \sim 9.6\times10^{-61}\ \mathrm{J}$. For a neutron star with 1.5 solar masses and radius of 20 km, we can estimate the total Unruh radiation energy to be $U_\gamma \sim 2.56\times10^{-37}\ \mathrm{J}$. For a particle such as a proton, the total Unruh radiation energy is $U_\gamma \sim 1.5\times10^{-169}\ \mathrm{J}$. These estimations indicate that the contributions of the Unruh blackbody radiation from all of the matter to the universe are negligible.

For a black hole, as its mass-radius ratio to be a constant, we have the total Unruh radiation energy and the Unruh radiation temperature on the surface of the black hole to be inversely proportional to the mass of the black hole,

$$U_\gamma = \alpha\left(\frac{c^2}{2G}\right)^5 \frac{1}{M} = \frac{\hbar c^3}{9600\pi GM} \sim \frac{1.36\times10^{-3}}{M} \quad (10)$$

and

$$T = \frac{\hbar g}{2\pi c k_B} = \frac{\hbar c^3}{8\pi k_B GM}. \quad (11)$$

Here, we have used the Schwarzschild mass-radius relation of a black hole,

$$\frac{2GM}{c^2 R} = 1. \quad (12)$$

It is seen that a smaller black hole holds more Unruh radiation energy surrounded. In other words, a black hole, when it loses its mass and becomes smaller, radiates the Unruh radiation. For a star-like black hole with 3 solar masses, we have the total Unruh radiation energy to be $U_\gamma \sim 2.27\times10^{-34}$ J. For a supermassive black hole with one billion solar masses, we have $U_\gamma \sim 6.8\times10^{-43}$ J. A larger black hole has less Unruh radiation energy. These results indicate that the contributions of Unruh blackbody radiation from all of the star-like, massive, and supermassive black holes to the universe are also negligible. The smaller a black hole is, the larger its total Unruh radiation energy is. For a black hole with its total Unruh radiation energy to be equal to its rest energy (or the energy of mass), the mass and radius of the black hole are determined to be, respectively,

$$M = \left(\frac{\hbar c}{9600\pi G}\right)^{1/2} \sim 1.25\times10^{-10}\ \text{kg}, \quad (13)$$

and

$$R = \frac{2GM}{c^2} \sim 1.85\times10^{-37}\ \text{m}. \quad (14)$$

The mass is much less than the Planck mass (~0.57%), and the radius is also much less than the Planck length (~1.2%). A black hole, if it is infinite small, has infinite large Unruh radiation energy. This implies that for a finite size (and hence with a finite energy) black hole to change or evaporate into an infinite small size black hole, an infinite amount of work must be done to the black hole in order to provide or emit the infinite Unruh radiation energy. Therefore, there is no way to complete this change by itself evaporation because such change violates the law of energy conservation. The smaller a black hole is, the harder it forms. This is because a smaller black hole needs more Unruh radiation to be generated around.

As the gravitational field g is inversely proportional to the square of radial distance, the Unruh radiation energy density is inversely proportional to the eighth power of the radial distance. Therefore, the most of the total Unruh radiation energy distribute closely around the gravitational object (e.g. within the two to three object radii). To study how the Unruh radiation distributes its temperature and energy around a black hole, we plot, in **Figure 1**, the Unruh radiation temperature T (red line), the Unruh radiation energy density per unit volume u_γ (green line), and the total Unruh radiation energy difference ΔU_γ (blue line) as functions of the radial distance. Here, we have considered the black hole to have 3 solar masses, and normalized the physical quantities T, u_γ, and ΔU_γ by the values of them at the surface of the object, and defined the total Unruh radiation energy difference by,

$$\Delta U_\gamma = \int_r^\infty u_\gamma 4\pi r^2 \mathrm{d}r = \int_r^\infty \beta \left(\frac{\hbar GM}{2\pi c k_B r^2} \right)^4 4\pi r^2 \mathrm{d}r = \alpha \frac{M^4}{r^5}. \quad (15)$$

It is seen, from **Figure 1**, that over 99.5% of the total Unruh radiation energy are distributed within 3 black hole radii since the Unruh radiation temperature and hence the Unruh radiation energy density rapidly decrease with the radial distance. At 3 black hole radii, the Unruh radiation temperature, radiation energy density, and total radiation energy difference are decreased by factors of 3^{-2}, 3^{-5}, and 3^{-8}, respectively, in comparison with the values at the surface of the black hole.

3. The Unruh Effect on the Singularities of Black Holes

From the general result of the total Unruh radiation energy (Equation (8)), it is seen that a gravitationally collapsing object has an increasing amount of the total Unruh radiation energy. This implies that a gravitationally collapsing object is

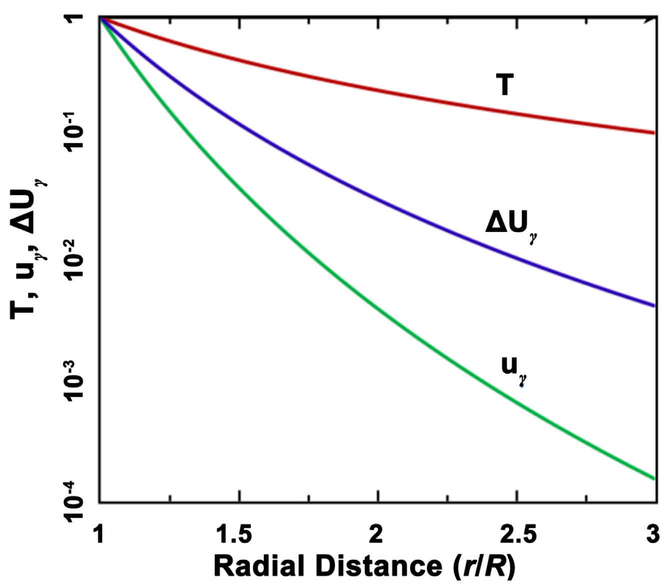

Figure 1. The Unruh radiation temperature, energy density, and total Unruh radiation energy difference are plotted as functions of the radial distance. All quantities rapidly decrease with the radial distance. Over 99.5% of the total Unruh radiation energy is distributed within 3 black hole radii.

emitting the Unruh radiation, which is resulted from (but not equal to) the decrease of the gravitational potential energy as well as any reduction of the mass energy. The more compact the object becomes, the more Unruh radiation it emits. Differentiating Equation (8), we have,

$$\frac{\mathrm{d}U_\gamma}{U_\gamma} = \frac{4\mathrm{d}M}{M} - \frac{5\mathrm{d}R}{R}. \tag{16}$$

Equation (16) shows that a gravitational object emits the Unruh radiation (*i.e.* $\mathrm{d}U_\gamma > 0$), if the rate of changing mass in percentage is greater than 1.25 times the rate of change radius in percentage (*i.e.* if $\mathrm{d}M/M > 1.25\mathrm{d}R/R$). A collapsing matter, as $\mathrm{d}R < 0$, emits the Unruh radiation if it does not loose its mass in percentage at a rate faster than 1.25 times the rate of decreasing its radius in percentage.

For the matter inside a black hole, it may thus emit the Unruh radiation if its gravity is strong and gets stronger as it is collapsing towards the center. Conventionally, it is generally believed that all matter inside or once entering a black hole will be gravitationally collapsed into the singularity point at the center of the black hole. At the singularity point, matter is compressed to a point with infinite large density and the conceptions of space and time completely break down due to infinite curvature or gravitation. Assuming the gravitational field of the collapsing matter inside a black hole is inversely proportional to the radial distance with power ν, *i.e.* $g \propto r^{-\nu}$, we can obtain the total Unruh radiation energy that surrounds the collapsing matter to be $U_\gamma \propto R^{3-4\nu}$. When $\nu > 3/4$, the collapsing matter emits Unruh radiation. For simplicity, the following analysis of the Unruh effect on the singularities of black holes keeps the case of $\nu > 2$.

With the Unruh effect, however, the collapsing matter is converting its gravitational potential energy and mass energy into the Unruh radiation in an increasing rate, which is much greater than the decreasing rate of the gravitational potential energy. The entire mass and its gravitational potential energies will be completely radiated out or consumed before the matter collapses to the center point. Therefore a singularity point with finite mass but infinite small size and infinite big density does not form inside a black hole. The radius of the singularity sphere can be determined based on the conservation of its total energies, which mainly include the Unruh radiation energy, the rest energy, and the gravitational potential energy,

$$\frac{\hbar G^4 m_c^4}{300\pi c^7 r_c^5} + m_c c^2 - \frac{3Gm_c^2}{5r_c} = C, \tag{17}$$

where m_c and r_c are the mass and radius of the collapsing matter; the constant C is finite and can be determined from the total energy of the initially formed black hole. Here we have neglected the thermal and kinetic energies of the collapsing matter on the left-hand side of Equation (17). Since the singularity sphere is small, the first and third terms on the left-hand side of Equation (17) are dominant. Thus, from Equation (17), we can derive approximately the radius of the singularity sphere to be mass dependent as,

$$r_c = \left(\frac{\hbar G^3 m_c^2}{180\pi c^7}\right)^{1/4} \sim 2.22\times 10^{-32}\sqrt{m_c}\ . \tag{18}$$

It is seen that the radius of the singularity sphere cannot be zero, unless the mass approaches to zero. A point-like singularity must be massless and hence naked, because the Schwarzschild radius is proportional to the mass and thus smaller than the radius of the singularity sphere when mass tends to zero. In this case, the radius of the singularity sphere is determined by

$$r_c = \left(\frac{\hbar G^4 m_c^4}{300\pi c^7 C}\right)^{1/5} \propto \left(m_c\right)^{4/5}. \tag{19}$$

A massless singularity limits its mass and size to zero, density to infinite, and the total energy to be a constant, which is mainly in the form of the Unruh radiation. The massless singularity is naked as the radius of the singularity sphere is less than the Schwarzschild radius.

Figure 2 accurately plots, from Equation (17), the radius of the singularity sphere as a function of the mass of the singularity sphere, which is normalized by the mass of the initially formed black hole. As an example, we have considered that the initially formed star-like black hole has mass about 3 solar masses. It is seen that the radius of the singularity sphere is finite about ~5 × 10^{-17} m if the collapsing mass does not convert its mass energy to the radiation and about

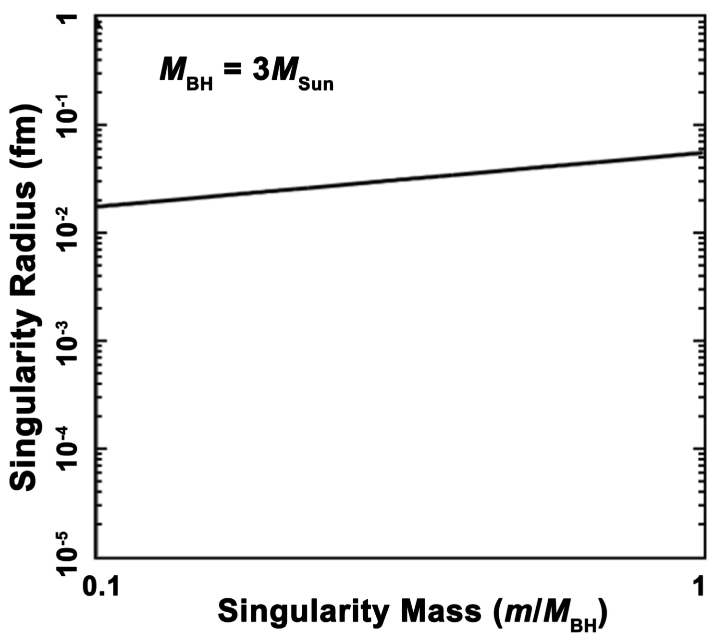

Figure 2. The radius of a singularity sphere versus the mass of the singularity sphere. The radius of the singularity sphere is an implicit function of the mass of the singularity sphere. If the collapsing mass does not completely radiate out its mass inside a black hole, the singularity sphere has a finite radius rather than a singular point. The falling matter should be radiated out due to the Unruh effect before it collapses to form a size-less singularity at the center of a black hole.

~1.74 × 10^{-17} m if the ninety percent of mass are radiated out during the collapsing. Therefore, a singularity point with finite mass and infinite large density does not form from the collapse of matter inside a black hole due to the Unruh effect, a prediction of quantum field theory. This provides a new insight to the matter inside a black hole and a possible solution for the black hole singularity issue.

According to the big bang theory, the universe began as a singularity, which was infinitely small of the size, infinitely dense of the matter, infinitely large but negative of the gravitational potential energy, and infinitely strong and positive of the Unruh radiation energy according to the result of this study. The total energy of the big bang singularity was dominated by the Unruh radiation and hence was infinitely large and positive. A primordial singularity once created, it should be extremely unstable due to having positive infinitely large energy and rapidly expand, which leads to an inflationary and expanding cosmology from the singularity point [18]. Since our universe is finite and hence has a finite energy in total, the primordial or big bang singularity might be massless with a finite Unruh radiation energy. The primordial singularity with a finite mass and thus infinite energy might form multiverses from the big bang and our universe is only the one with a finite energy.

4. Discussions and Conclusions

It should be noted that the total Unruh radiation energy of a gravitational object, calculated in Section 2 and given by Equation (8), is only that outside the gravitational object. To find the Unruh radiation energy inside the gravitational object, we consider that the gravitational object uniformly distributes its matter and hence has a gravitational field inside given by $g = GMr/R^3$. Then, the Unruh radiation energy inside can be obtained as,

$$U_{\gamma,\text{in}} = \frac{\hbar G^4 m_c^4}{420\pi c^7 R^5}. \tag{20}$$

Including the Unruh radiation energies both inside and outside a gravitational object with mass M and radius R, we find the total Unruh radiation energy of the gravitational object in the entire space to be

$$U_\gamma = \frac{\hbar G^4 m_c^4}{420\pi c^7 R^5} + \frac{\hbar G^4 m_c^4}{300\pi c^7 R^5} = \frac{\hbar G^4 m_c^4}{175\pi c^7 R^5}. \tag{21}$$

This only modifies the values of the constant coefficients calculated above such as α given in Equation (9) to be $\alpha = 1.73 \times 10^{-137}$ J·m^5·kg^{-4}. The obtained conclusion on the singularity of black hole does not alter. The radius of the singularity sphere is finite and proportional to the square root of the mass.

According to the principle of maximum force, the tension or force between two bodies including black holes cannot exceed the value of $c^4/(4G) \sim 3.25 \times 10^{43}$ N [19]. This maximum force may be realized by simply multiplying the black hole mass to the gravitational acceleration of the black hole on its surface,

$F_{\max} = Ma = MGM/R^2 = c^4/(4G)$. Here, we have used the black hole mass-radius relation (Equation (12)). To have this maximum self-interaction force, all the black hole mass needs to be distributed on the surface or event horizon of the black hole. In other words, there is no mass within the event horizon; instead all the mass is on the event horizon [20]. Considering that a massive star with 20 or more solar masses, at the end of its life after the fusion power runs out, forms a black hole through a supernova explosion, we usually believe that a large amount of extremely compact matter already exists inside before the event horizon is formed. If there is not a mechanism or physical process to remove all the matter inside against the strong gravitation to the surface or even horizon, conventionally, these inside mass will fall into the center and form a singularity with infinite density and curvatures. With the Unruh effect, as shown by this study, the singularity has a finite radius, which is extremely small, unless its mass also tends to zero as it collapses to the center.

As a conclusion of this study, we have analyzed the quantum Unruh effect on the singularities of black holes. First, we have calculated the total Unruh radiation energy that surrounds a gravitational object and showed that a gravitationally collapsing object emits the Unruh radiation, mainly converted from the reduction of its gravitational potential energy. Then, applying this consequence to the matter that is inside a black hole and collapsing towards the center, we have shown that a size-less singularity with a finite mass cannot be formed if the collapse does not violate the law of energy conservation. As the matter collapses to the center point, its total Unruh radiation energy rapidly tends to infinity. All energies of the matter including the rest and gravitational potential energies will be entirely radiated out or become not enough to generate the required Unruh radiation before the matter collapses into the singularity point. Based on the conservation of energy, we have further determined the minimum size or radius of the singularity sphere and showed that the radius of the singularity sphere is extremely small and proportional to the square root of mass of the singularity sphere. A point-like singularity cannot be formed unless its mass also approaches zero. The results obtained from this study not only provide us a quantum solution to the problem of black hole singularity, but also have profound implications to the spacetime and cosmology. The Unruh effect permits a massless singularity but excludes a finite mass singularity to be formed inside a black hole. The big bang singularity for our finite universe should be massless. A big bang singularity with finite mass and hence infinite Unruh radiation may create multiverses and our finite universe is only one of them.

Acknowledgements

The author acknowledges the support from the IBM-HBCU Quantum Center via the funded project.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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