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On Galaxies Rotation Curves: Gravitomagnetism Rather than MOND and Missing Mass

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Abstract

The contribution of gravitomagnetism to ordinary Newton law is presented. This seems to account for the rotation curves of galaxies. The equivalence of the missing mass conjecture and of the MOND fit is proved.

Keywords

Galaxies Rotation Curves, MOND, Dark Matter, Gravitomagnetism

1. Introduction

Ever since Zwicky's work [1] coming to the recent H 21 cm lines for the M33 galaxy, the mismatch between luminous and dynamical data for orbiting systems has prompted a considerable amount of speculations.

The simple application of Newton's law

$$\frac{v^2}{r} = \frac{GM}{r^2}$$

yields the well known relation for the Keplerian velocity of orbiting objects

$$v_K = \sqrt{\frac{GM}{r}}$$

where the fall off in r is, as mentioned, totally contradicted by experimental data (as shown in **Figure 1**). However orbital velocities are much bigger than escape velocities which imply that these systems are really bound.

The situation can be roughly summarized as: the falloff vs. the distance from the core of orbiting particle does not follow Newton's law but seems to flatten approximately to a constant, or even increase as if Newton's force would show at

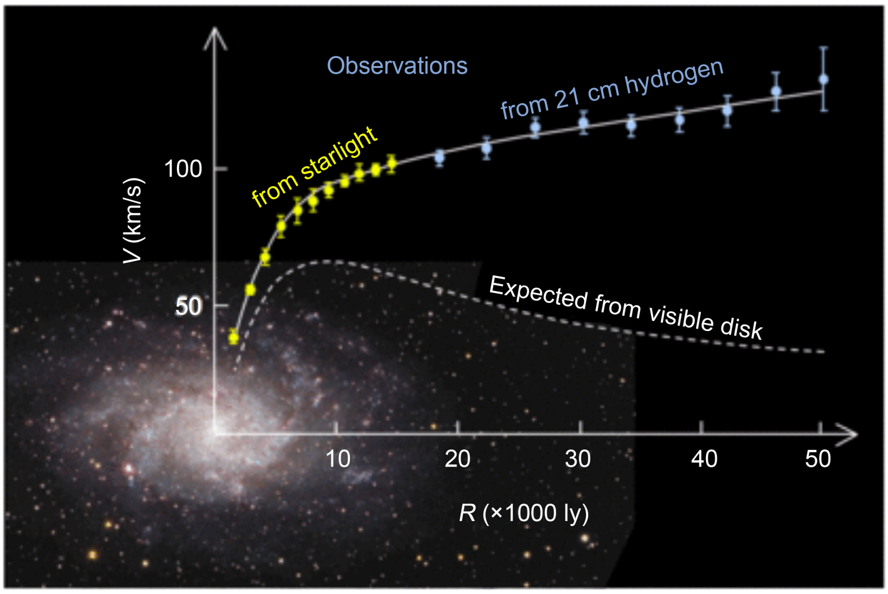

Figure 1. M33 rotation curve.

these scales an $1/r$ dependence instead of the usual (tested at our scales) $1/r^2$ thus justifying the proposal of undetectable mass beyond the (observed) core.

This has led to invoke the existence of the notorious dark (or missing) mass, entity with remarkably metaphysical properties: invisible but nevertheless gravitationally interacting and of just the right amount to account for data, thus increasing with increasing scale of the systems. In plain words: very reminiscent of **ether** and even more complicated!

However the extension to these scales of Newton's law cannot be taken for granted and Milgrom [2] has alternatively suggested that two parameters be essentially involved in the description of gravitation: the velocity and the acceleration of the orbiting particles. Whereas for the first one in going from the solar system to these scales no great differences occur, for the second one goes from $v^2/r \simeq 10^{-2}\ \text{m/s}^2$ to $10^{-10}\ \text{m/s}^2$. This is almost a truism as can be immediately got by inspection of **Figure 1**. Actually corrections to Newton's law have already been found, although expressed in a different form (as a correction to the Keplerian period [3], not to be confused with frame dragging and geodetic precession which depend on the spin of the giroscope) in the case of the rotation of the earth as the small gravitomagnetic clock effect.

Thus it seems rather funny that people who have underlined the relevance of General Relativity (GR) for the fundamental tests (which can however be got just from Special Relativity plus the inertia principle [4]) have clinched to ordinary Newtonian formulation for situations where small effects (gravitomagnetic of relativistic origin) may be expected.

The consideration of GR effects however has been recently investigated with contradictory conclusions [5] [6] [7] who however end up by confirming that effects are of the order of ($v^2/c^2 \simeq 10^{-6}$) and therefore negligible, essentially countering the phenomenological arguments by Milgrom. We are going to prove here with very simple arguments that this is not the case. Whether this implies the presence of missing mass in GR will not be addressed here.

Actually in the galaxy formation it is tacitly assumed that the core does not rotate and that therefore these effects are observed in an inertial reference frame. However according to our understanding of the galaxies formation mechanism this is not so. Indeed in the collapse process conservation of angular momentum requires objects to increase their orbital velocity more and more so that the core should be spinning even faster than the orbiting objects.

2. Beyond Newton. Gravitomagnetism and Coriolis Force

Therefore the basic equation has to be regarded

$$\frac{v^2}{r} = \frac{GM}{r^2} + 2hv \tag{1}$$

where the last term of the r.h.s. represents the gravitomagnetic field h of the core rotating with angular velocity ω_G and the first one the Newtonian contribution which yields the Keplerian orbiting velocities.

Notice the similarity of the second term with the Coriolis force. Indeed in an approximate vector treatment of gravitation it has been show the identity of the Coriolis force and of gravitomagnetism [3]. In that sense what is generally regarded just as a kinematical effect is in reality due to a dynamical interaction, the rotation of the source which generates a "new" force.

This Coriolis like force, which appears in non inertial frames, increases the tangential velocity in the rotating frame and results in a total velocity in the external inertial frame (we on earth in a first approximation) given by this velocity plus the velocity of rotation (as measured from the Doppler shift of H21 lines in the orbital plane).

Results are given by the positive root of Equation (1).

$$\bar{v} = hr + \sqrt{h^2 r^2 + \frac{GM}{r}}$$

However for our purposes it is enough to consider separately the two terms of the r.h.s. of this equation.

Now the gravitomagnetic force has been shown to be proportional to the angular velocity as

$$h = \varepsilon_G \omega_G \tag{2}$$

where the coefficient ε

$$\varepsilon_G \simeq \frac{GM}{c^2 r}$$

shows because of the factor $1/c^2$ its relativistic origin and is a measure of the (relativistic) strength of gravitation. Its form can be even understood on elementary grounds: indeed $\frac{GM}{r}$ represents the velocity squared so that ε_G is just the advertised

$$\frac{GM}{c^2 r} \simeq \frac{v^2}{c^2}$$

The first relevant feature of gravitomagnetic forces is their different (*with respect to the Newtonian term*) *r behaviour. They are indeed* (*as gravitational waves* [8]) *proportional to* $1/r$ *instead of* $1/r^2$.

A very simple evaluation of the gravitomagnetic term alone would yield

$$\bar{v} \simeq \frac{GM}{c^2}\omega_G$$

where the term of the r.h.s. in fraction, with the given numerical values of M33, yields 10^{13} to be compared to the term on the l.h.s. of 10^5. Thus an ω_G of the order of 10^{-8} which means that a rotation period of the core of some years would do the job. So the core would indeed rotate, as conjectured with a greater velocity than the outskirts. Of course this hypothised core angular velocity has nothing to do with the angular velocity of orbiting objects near the origin which only can be read off from the figure.

Consider then the relative importance of the second term of Equation (1) to the Newtonian one

$$\frac{\bar{v}}{v_K} \simeq 2hr\sqrt{\frac{r}{GM}} = 2h\sqrt{\frac{r^3}{GM}} = 2\frac{h}{\omega_K} \tag{3}$$

i.e. the ratio of the galaxy centre gravitomagnetic field to the Keplerian frequency.

A plot of v for the values $v = 10^5\ \text{m/s}$, $r = 10^{20}\ \text{m}$ and $M_G \simeq 10^{11} M_S \simeq 10^{40}\ \text{kg}$ is reported in **Figure 2**.

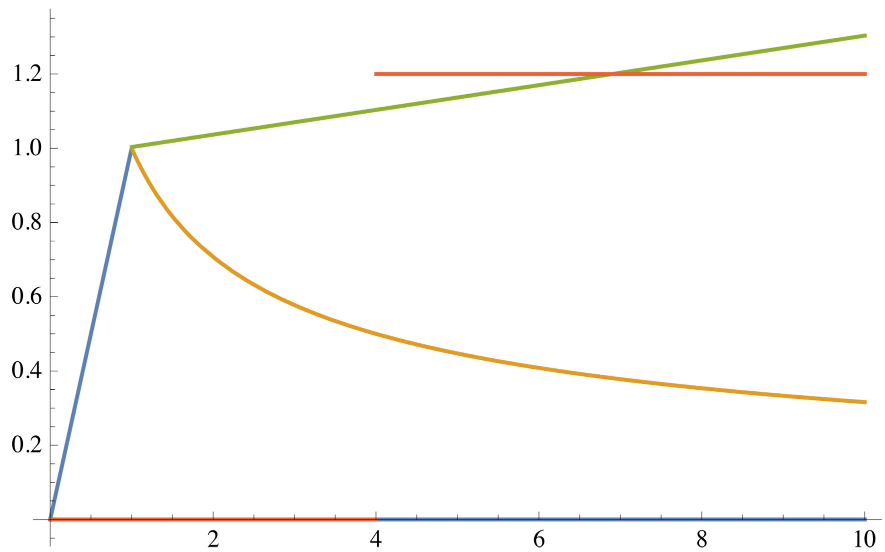

Figure 2. Rotation curve of the galaxy M33. The inner part is obtained by assuming that the core has a uniform mass density with $M_G \simeq 10^{11} M_S$. The yellow curve represents the Newtonian $v \simeq \sqrt{1/r}$ and the green curve with the additional effect of gravitomagnetism. In the vertical axis units of v in 10^5 m/s and in the horizontal axis of r in 10^{20} m. The horizontal line is the MOND fit with $v^4 \simeq M$.

Of course this treatment is admittedly and necessarily approximate in the sense that the core does not rotate all at the same angular velocity. This is anyhow the most reasonable and economic position we can take.

It is also immediate to get for the centripetal gravitomagnetic acceleration

$$2hv \simeq 2h^2 r \simeq 10^{-10}\ \mathrm{m/s^2} \tag{4}$$

i.e. obviously the same value demanded by MOND.

Let finally consider the reason for the absence of this gravitomagnetic effect on the earth motion.

Indeed it is immediate to see that for the sun, whose rotation period is of the order of

$$\omega_S \simeq 10^{-6}\ \mathrm{s^{-1}}$$

and

$$\varepsilon_S \simeq 10^{-8}$$

resulting in a gravitomagnetic field

$$h_S \simeq 10^{-14} \tag{5}$$

and therefore in a totally negligible velocity and acceleration.

3. MOND Vs. Missing Mass? Conclusions

Final remarks about MOND and missing mass. The first one proposes to modify the second law of Newton force as

$$F = \frac{GMm}{r^2} = ma \to m\left(1/\left(1 + a_0/a\right)\right)a$$

where *a* stands for the Newtonian acceleration and the factor $\left(1/\left(1+a_0/a\right)\right)$ the simplest interpolating function which for values of a_0 much bigger than a yields a factor a/a_0 so that the second term turns out to be a^2/a_0 with the desired result $v^4 = GMa_0$.

One can however alternatively manipulate the preceding equation as

$$\frac{GM\left(1 + a_0/a\right)}{r^2} = a \tag{6}$$

which can immediately interpreted as an ordinary Newton equation with standard $a = v^2/r$ but with an increased mass $M = M\left(1 + a_0/a\right)$, which represents an interesting bookkeeping help, confirming at the same time the usefulness of the virial theorem. This renders totally idle all considerations about alternatives in the modification of the equation of motion or the law of gravitation.

4. Conclusions

1) The physically relevant parameter is not the acceleration (although it is as a consequence) but the radius r. It is this quantity which makes the interaction of non relativistic particles weak.

2) Therefore large distances make the possibility of highlighting the $1/r$ beha-

viour of small gravitomagnetism comparable to the ordinary dominating $1/r^2$ Newtonian term.

3) Thus the gravitomagnetic force, adding to the usual Newtonian one in non inertial frames, provides the extra attraction which increases the velocities and in a Newtonian context had to be attributed to a bigger mass than that responsible for the luminosity.

4) As a final comment in addition to missing energy which had been disposed of as due to matter creation in the black hole model [9] also the missing mass paradigm has been strongly questioned.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Zwicky, F. (1933) *Helvetica Physica Acta*, **6**, 110-127.

[2] Milgrom, M. (1983) *Astrophysical Journal*, **270**, 371-383. https://doi.org/10.1086/161131

[3] Christillin, P. and Barattini, L. (2013) The Machian Contribution of the Universe to Geodetic Precession, Frame Dragging and Gravitational Clock Effect. arXiv: 1206.4593v2

[4] Christillin, P. and Morchio, G. (2019) Relativistic Newtonian Gravitation. arXiv:1707.05187v2

[5] Ludwig, G.O. (2021) *The European Physical Journal C*, **81**, Article No. 186. https://doi.org/10.1140/epjc/s10052-021-08967-3

[6] Lasenby, A.N. Hobson, M.P. and Barker, W.E.V. (2023) Gravitomagnetism and Galaxy Rotation Curves: A Cautionary Tale. arxiv:2303.06115v1

[7] Astesiano, D. and Ruggiero, M.L. (2023) Can General Relativity Play a Role in Galactic Dynamics? arxiv:22111.11815v2

[8] Christillin, P. (2022) Gravitation for the Simple Mind. Aracne, Rome.

[9] Christillin, P. (2023) *Journal of Modern Physics*, **14**, 18-30.