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Quantum Mechanics: Internal Motion in Theory and Experiment

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Abstract

Dispersion dynamics applies wave-particle duality, together with Maxwell's electromagnetism, and with quantization $E = h\nu = \hbar\omega$ (symbol definitions in footnote) and $p = h/\lambda = \hbar k$, to special relativity $E^2 = p^2c^2 + m^2c^4$. Calculations on a wave-packet, that is symmetric about the normal distribution, are partly conservative and partly responsive. The complex electron wave function is chiefly modelled on the real wave function of an electromagnetic photon; while the former concept of a "point particle" is downgraded to mathematical abstraction. The computations yield conclusions for phase and group velocities, $v_p \cdot v_g = c^2$ with $v_p \geq c$ because $v_g \leq c$, as in relativity. The condition on the phase velocity is most noticeable when $p \ll mc$. Further consequences in dispersion dynamics are: derivations for ν and λ that are consistently established by one hundred years of experience in electron microscopy and particle accelerators. Values for $v_p = \nu\lambda = \omega/k$ are therefore systematically verified by the products of known multiplicands or divisions by known divisors, even if v_p is not independently measured. These consequences are significant in reduction of the wave-packet by resonant response during interactions between photons and electrons, for example, or between particles and particles. Thus the logic of mathematical quantum mechanics is distinguished from experiential physics that is continuous in time, and consistent with uncertainty principles. [Footnote: symbol E = energy; h = Planck's constant; ν = frequency; ω = angular momentum; p = momentum; λ = wavelength; k = wave vector; c = speed of light; m = particle rest mass; v_p = phase velocity; v_g = group velocity].

Keywords

Wave Packet, Reduction, Phase Velocity, Group Velocity, Resonant Response, Dispersion Dynamics, Quantum Physics

1. Introduction

In *The principles of quantum mechanics* [1], Dirac mentioned "internal motion" in the electron, but he neither explained nor described it. He also claimed to have calculated the speed of the electron to be equal to the speed of light *c*, and this was confusing because special relativity shows that the speed of a massive particle cannot exceed *c*, and is infinitely massive at this speed. Relativity further implies that the quantum wave packet in massive particles contains two velocities, namely the group and phase velocities, that have the geometric mean $(v_g \cdot v_p)^{1/2} = c$ [2] [3]. The group velocity corresponds to the velocity of a quantized object relative to a given reference frame, and may have any value $0 < v_g < c$ that is used to calculate time dilation, space contraction, relativistic mass *etc*. Meanwhile, within the context of 19th century wave-particle duality after modification for 20th century quantization, the phase describes wavelength, momentum, frequency, energy etc. The phase velocity in a free, massive particle is typically faster than *c*, and is, for that reason, not independently measured, but it is usually known from the product of frequency and wavelength that, typically, *are* known or measured. *In this paper we demonstrate consequences for the widespread wavelength, frequency and velocity measurements that are routine in experimental physics, especially in transmission electron microscopy, even when* v_p *is ignored or denied in theory.*

The wave group is generally understood in frequency modulated and amplitude modulated communication theory. The group can be used to explain many properties in quantum physics including diffraction in general; but also intrinsic spin [4], dual diffraction in logarithmic structures [5], reduction of the wave packet by resonance [6], etc. Further possible explanations may well be available for structures of elementary particles.

2. Dispersion Dynamics

The dynamics combine 19th Century wave-particle duality with special relativity and 20th Century quantum mechanics, by means of a symmetric wave group with finite spatial and temporal coherence. We find firstly the equation for rest mass m [2]:

$$m^2c^4 = \hbar^2\omega^2 - \hbar^2k^2c^2 \tag{1}$$

where $\hbar$ is the reduced Planck constant; c the speed of light; ω the wave angular momentum; and k its wave vector. Write an equation in a form that quantizes the infinite wave e^X where:

$$\varphi = A \cdot \exp\left(\frac{X^2}{2\sigma^2} + X\right)$$

with imaginary:

$$X = \mathrm{i}\left(\bar{\omega}t - \bar{k}x\right) \tag{2}$$

and where $\bar{\omega}$ is the mean angular frequency, $\bar{k}$ the mean wave vector, and A a normalizing amplitude. The wave group envelope in **Figure 1** has the normal, Gaussian distribution in time t and space, simplified here for the direction of

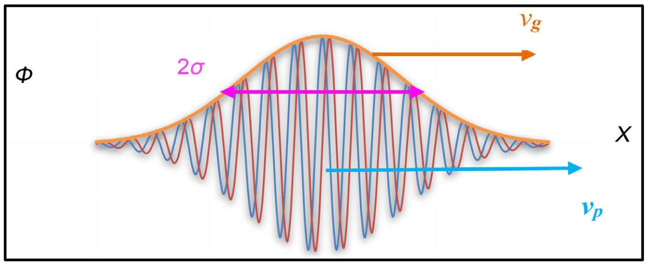

Figure 1. Normal wave packet including conservative function (orange) enveloping infinite, responsive, elastic, complex wave (red and blue), with uncertainty 2σ (pink double arrow) [2]. The label X may represent any of the four variables x, k_x, ω, or t. In massive particles, the group velocity v_g < c (orange); the phase velocity $v_p > c$ (blue).

propagation x.

2.1. Massless Bosons

When rest mass $m = 0$, the energy of this wave follows Planck's law for energy E and follows de Broglie's hypothesis for momentum p:

$$E = h\nu = \hbar\omega$$

and

$$p = \frac{h}{\lambda} = \hbar k \tag{3}$$

where h is Planck's constant, and where ν and λ are frequency and wavelength respectively. In wave mechanics, as in relativity, the products:

$$\nu\lambda = \frac{\omega}{k} = c \tag{4}$$

and, since, in the present case, $E = pc$, then the derivative of $h\omega = \hbar kc$ gives:

$$\frac{\mathrm{d}\omega}{\mathrm{d}k} = c \tag{5}$$

which is the beat velocity of an unbalanced tuning fork and, by integration, the group velocity of the symmetric wave function [2]. Massless particles, in free space, have phase and group velocities that are both equal to c. They progress in phase, at their respective centers.

The wave function $\phi = e^X$ has unit amplitude for all x and t. However, by definition, in a mono-atomic crystal having conditions where all atoms scatter an electron beam *in phase*, the structure factor is the sum of all the atomic scattering factors in the unit cell. However, when half of the atoms scatter in anti-phase, the structure factor is zero, and the corresponding diffraction line is forbidden. The wave function ϕ is therefore genuine with typical properties. The energy of the quantum is proportional to the area under then envelope.

2.2. Massive Fermions

When $m > 0$, it is well known that the relativistic mass m' of a particle increases with its velocity [7]:

$$E = m'c^2 \tag{6}$$

Since

$$E^2 = p^2c^2 + m^2c^4 \tag{7}$$

and, by definition of the Lorentz factor γ:

$$m' = \gamma m = \frac{m}{\sqrt{1-\beta^2}} \tag{8}$$

where $\beta = v_g/c$. Consider the transmission electron microscope. We need to work with the most convenient reference frame which might include one of the following: near the field emission tip maintained at accelerating voltage V; or at the laboratory specimen chamber; or within the specimen, etc. The last option is needed in fine measurements, but it is specific to specimen composition, precise orientation, and to the probe energy. The appropriate adjustment is comparatively small [8], a few electron volts in energy, so we will calculate parameters in the laboratory frame as a general approximation. This becomes more valid for probes of energy $pc > 100$ eV.

Where does the accelerating energy Ve (voltage x electronic charge) of the probe enter into Equation (1) for mass? Though a magnetic field would be velocity dependent, that is not the case with electrostatic interaction, so it is included as a scalar like the coulomb potential in Schrödinger's equation. The accelerating energy is added to the rest mass energy, *i.e.* independent of momentum. The frequency of the probe is therefore given by:

$$\nu = \frac{E}{h} = \frac{mc^2 + Ve}{h} \tag{9}$$

The wavelength of the probe depends on momentum through de Broglie's hypothesis, so that when applying Equation (7) (**Figure 2**) and the Pythagoras theorem: $p^2c^2 = E^2 - m^2c^4$; $p = \pm\sqrt{E^2 - m^2c^4}/c$ and:

$$\lambda = \frac{h}{p} = \frac{hc}{\left(E^2 - m^2c^4\right)^{1/2}} = \frac{hc}{\left((Ve)^2 + 2(Ve)mc^2\right)^{1/2}} \tag{10}$$

From which the phase velocity $v_p = \nu\lambda$ is calculated (**Table 1**). Meanwhile, following Equation (8), $\gamma = E/mc^2 = 1/\sqrt{1 - v_g^2/c^2}$ and the group velocity:

$$v_g = c\sqrt{1-\gamma^{-2}} \tag{11}$$

excepting low energies when $E \to mc^2$, the product of the group and phase velocities tends increasingly to c^2 as the probe becomes more relativistic (**Figure 3**), so that when $pc \gg mc^2$, $v_p \approx v_g \approx c$. More generally, the Pythagorean relationships illustrated in **Figure 2** underpin the complex nature of ϕ: since mass displacement varies with time while momentum displacement varies in space, complex ϕ combines them. Notice that in the *free* electron probe the potential energy Ve is positive; whereas in Schrodinger's solutions for electrons in the bound hydrogen atom, the potential energy eV and eigenvalues are negative.

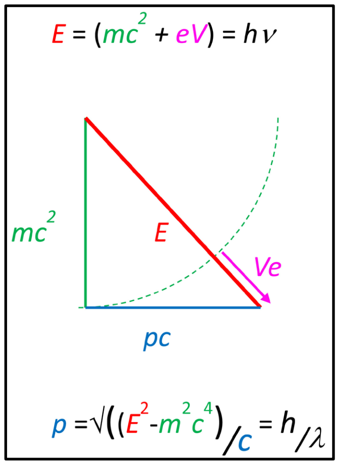

Figure 2. The frequency ν and wavelength λ of the probe (Equations (6)-(10)) are related by Pythagoras' theorem as in relativity Equation (7).

Table 1. Calculated values for an electron probe, following Equations (6)-(11) and **Figure 2**. In the 6th column, the product $v_p \cdot v_g \simeq 1$ (at low energies there is an arithmetic rounding error on the square roots); and in the 7th column, the ratio $v_p/v_g > 4000$ in a probe of energy 10 eV (**Figure 3**).

$\log_{10}(V)$	$\log_{10}(\nu)$	$\log_{10}(\lambda)$	$\log_{10}(v_p)$	$\log_{10}(v_g)$	$v_p \cdot v_g/c^2$	$\log_{10}(v_p/v_g)$
1	20.0919	−9.4114	10.6805	6.2743	**1.0028**	**4.41E+00**
2	20.092	−9.9114	10.1806	6.7732	**1.0003**	**3.41E+00**
3	20.0927	−10.412	9.68115	7.2725	**1**	**2.41E+00**
4	20.1003	−10.913	9.18682	7.7668	**1**	**1.42E+00**
5	20.1695	−11.432	8.73787	8.2158	**1**	**5.22E−01**
5.69897	20.3882	−11.847	8.54088	8.4128	**1**	**1.28E−01**
6	20.5627	−12.06	8.50319	8.4504	**1**	**5.27E−02**
7	21.4051	−12.928	8.47733	8.4763	**1**	**1.03E−03**
8	22.3857	−13.909	8.47683	8.4768	**1**	**1.12E−05**

This occurs because the positive kinetic binding energy of a *bound* state is less than the Coulomb potential energy. $\left|\left\langle pc^2/(2m)\right\rangle\right| < \left|\left\langle Ve\right\rangle\right|$ (For a *free* anti-particle, such as the positron, the relationships between m, p and E would need adjustment [3]).

To understand **Figure 3**, return to Equation (1). Differentiating with respect to k gives the important result:

$$\frac{\omega}{k} \cdot \frac{\mathrm{d}\omega}{\mathrm{d}k} = c^2 = v_p \cdot v_g \tag{12}$$

where the group and phase velocities are inversely related. The more familiar group velocity is used in the Lorentz factor (the inverted bracket in Equation (8)) in special relativity. At an acceleration of 10 eV, the free electron has a phase velocity over 4 orders of magnitude greater than its group velocity.

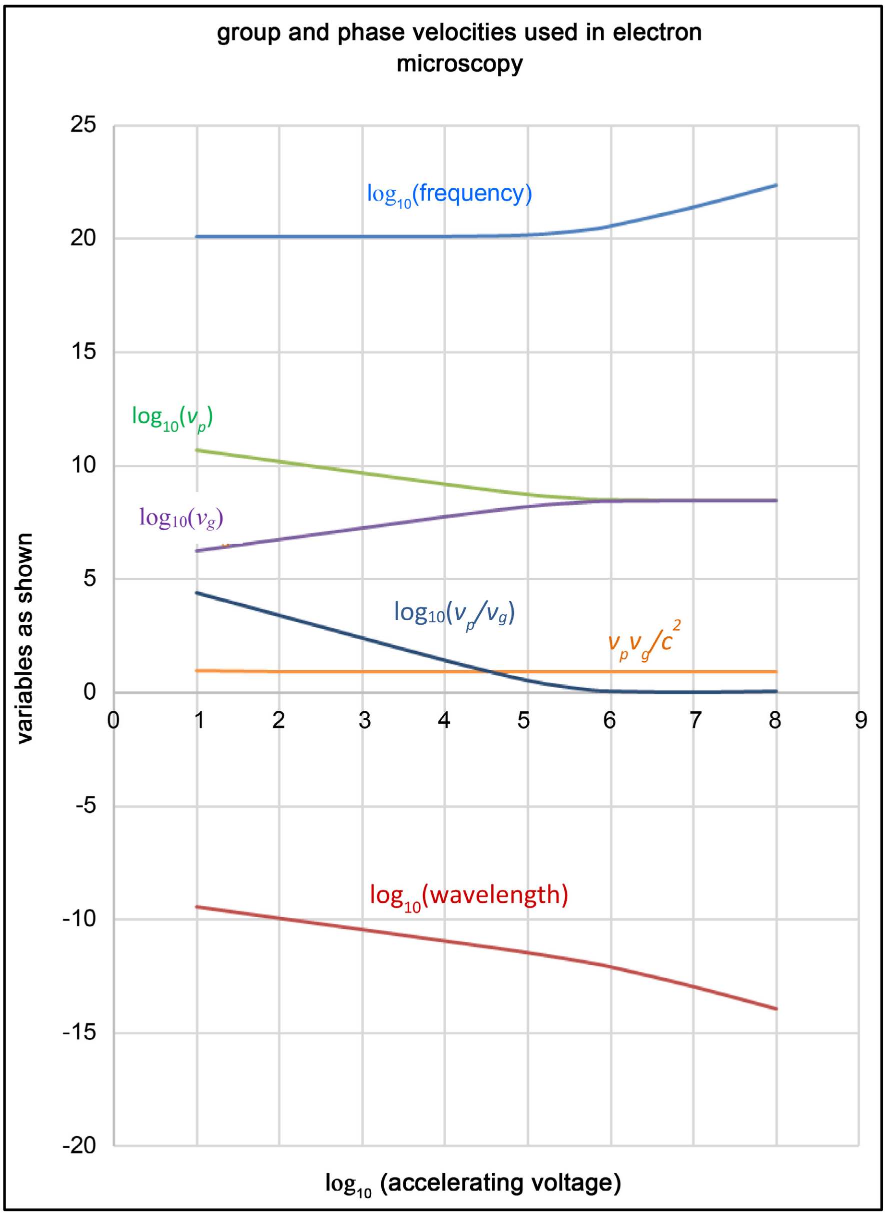

Figure 3. Plots of values calculated for parameters in various electron probes of voltage V in **Table 1**, including from top down: the frequency (blue) in SI units; the phase velocity v_p (green), the group velocity v_g (purple); the ratio of phase/group velocities (navy blue); the product of phase with group velocities (yellow); and the wavelength (red). Notice the systematic relativistic changes when $Ve \simeq mc^2 \simeq 0.5$ MeV, *excepting* the constant product $v_p \cdot v_g$!

Anti-particles behave a little differently (**Appendix 1**).

3. Uncertainty Expectation and Limits

Dispersion dynamics have been employed in various applications, already mentioned. One of those is "expected uncertainty" that we now extend to uncertainty limits' in wave mechanics. Equation (2) describes a wave function with expected uncertainty in time $\Delta t = 2\sigma$ when $x = 0$, The Fourier transform of a Gaussian is

Gaussian, so the uncertainty in angular momentum may be written:

$$\int \exp\left(-t^2/2\sigma^2\right)\cdot\exp\left(\mathrm{i}\omega t\right)\mathrm{d}t = \left(\sigma/\sqrt{2}\right)\exp\left(-\omega^2\sigma^2/4\right) \quad (13)$$

Then normalizing the final exponent between the variables $\omega\sigma/2 = \pm 1$, yields, $\Delta\omega = 4/\sigma$. In the dual uncertainty, σ cancels, so the expected uncertainty is:

$$\Delta\omega\cdot\Delta t = 8 \quad (14)$$

i.e. an order of magnitude larger than Heisenberg's limit. Likewise, $\Delta k_x\cdot\Delta x = 8$ *etc*. The limit occurs when a wave outreaches its group (**Figure 4**). This occurs when $\Delta x = 2\sigma \le \lambda_{limit}/2 = \pi/k_{limit}$; *i.e.* Lt. $\Delta x\Delta k_x \ge \pi$.

4. Discussion

4.1. Faster than Light

The evidence in electron microscopy and in creation and annihilation experiments is so copious and longstanding that the claim that "Nothing travels faster than light" should now be considered unscientific, because phase velocity is not "nothing". Obviously, Einstein meant only that $v_g < c$ [7]; the range of v_p is limited to space within the corresponding wavefunction. Nevertheless, his views were realistic [9]: he accepted hidden variables with their explanatory power [10].

However, we should wonder why, for the free electron, since λ and ν were certainly calculated many decades ago, it passed unnoticed that their product is faster than light. The fact is not found in common textbooks, and it may never have been discussed in journals. Other characteristic features of dispersion dynamics are also neglected. If we are to understand what, in general, we do as scientists, we are obliged to speculate on the multiple reasons that have produced this outcome.

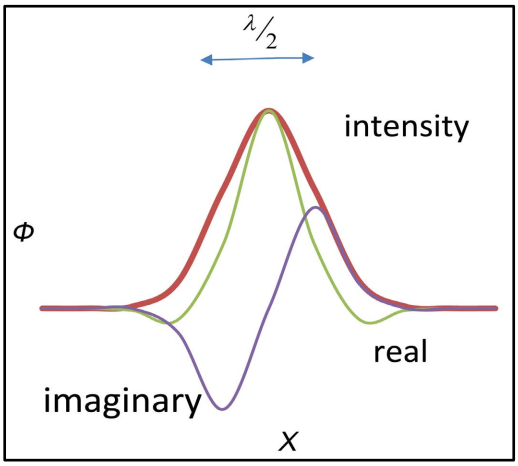

Figure 4. The uncertainty limit occurs when the wavelength of the semicarrier-wave equals the 2σ in the group (**Figure 1**). The uncertainty σ cancels in the dual limit, so that $\Delta x\cdot\Delta k_x \ge \pi$. This cut-off limit avoids singularities at longer wavelengths.

4.2. Logic

Obviously therefore, the method described here cannot be called mainstream. However, our logic [2] does adhere to falsifiability [11] [12]; but weakly to Bohr's phenomenology [13] and yet less to Dirac's preference for beauty and simplicity [14] over tested truth. Our method is realistic: the quantum is a property; not an entity that has been multiplied without necessity. The property is not a substantial mathematical axiom: but is a consequence of harmonious interaction of matter with regularities in spatial harmonics (as in Schrödinger's solutions), or with crystalline material (as with momentum quanta in diffraction patterns), including logarithmically periodic quasicrystals [15], etc.

4.3. Schrödinger's Limitation

Our dispersion dynamic is relativistic, unlike Schrödinger's equation that is in principle classical. Approximate relativistic corrections can however be applied [16]; the greatest difference is that our states are for free electrons in an electron microscope, whereas Schrödinger's states are bound atomic electrons where the total energy $E < mc^2$. This comparison is illustrated in **Appendix 2**.

A further difference between the two methods is our separation of Equation (1) into a conservative part (green) and a responsive part (pink) [2]. The former provides the condition for particulate properties, energy, momentum, charge, mass, spin etc.; the response part provides conditions for interference, annihilation, creation, entanglement etc.:

$$m_0^2 = \omega^2 - k^2 = (\omega + k)(\omega - k) \tag{15}$$

These two operations remain separable in the corresponding wave function for the free particle:

$$\phi(x,t) = A \cdot \exp\left(\frac{X^2}{2\sigma^2} + X\right)$$

with

$$X = \mathrm{i}(\bar{\omega}t - \bar{k}x) \tag{16}$$

and elastic uniform response intensity: $(\mathrm{e}^X)^* \cdot \mathrm{e}^X = 1$ for all x and t inside the normal envelope. These formulae are a direct expression of wave-particle duality.

4.4. Dirac's Influence

A significant reason for the general ignorance of the phase properties in dispersion dynamics is Dirac's opinion that the wavepacket is unstable [1]. This claim is falsified by two facts: one is that many photons are currently observed by cosmologists and others, even though the microwaves are many billions of years old; the second fact is that unstable photons would break Newton's first law of motion. These facts suggest that the high mobility of the phase is constituent in its stability.

His influence is widespread: his relativistic theory by rank 8 matrices is precise and widely used, particularly with Heisenberg's representation of quantum mechanics by non-commuting canonical variables. Dirac showed that the latter representation provides similar results to Schrödinger's alternative, which however is more commonly used for practical reasons, especially in chemical applications.

5. Conclusions

In the extensive field of electron microscopy, the wavelength of the electron is extremely well established. So, moreover, is the wave frequency in annihilation and creation measurements of photons, of electrons and of positrons. The phase velocity, as the ratio of two known quantities, is therefore well-known even though it is difficult to measure directly. This difficulty is obviously due to the small net effect of the push and pull of a wave operating at speed greater than the group velocity of a massive particle that is typically imagined to be a "point particle". These velocities are also simple consequences of the most well-established of 20th century physical theories, namely special relativity. It is reasonable to expect that direct resonance measurements of v_p will in future become possible, *i.e.* interactions of wave on wave over finite intervals and finite distances—like interactions calculated between two exchange electrons in atomic physics.

Dispersion dynamics resolves a major anomaly in de Broglie's hypothesis in quantum mechanics: He makes the wavelength λ of the electron *inversely* proportional to momentum $p = mv = h/\lambda$; whereas in wave mechanics the momentum is proportional, since velocity: $v_p = \nu\lambda$. In the former case, the group velocity v_g is a property of the conserved envelope as in de Broglie's hypothesis; the phase velocity v_p is its inverse.

Following Equations (6)-(11), the phase velocity of an electron, accelerated to an energy of 10 eV, where $\nu\lambda > c$, is over 10^4 times its group velocity: The geometric mean of the two velocities is c, the speed of light. Obviously, wave phases travel faster than the speed of light, contrary to common assumptions that are based on group velocity, and that ignore internal motion. The facts of internal motion apply to various aspects of physics, including reduction of the wave-packet during measurement [2] and, furthermore, to the logic of quantum theory [2]. It is the business of physics to know what it does: theories that are spooky, discontinuous and instantaneous, not only offend rules of temporal uncertainty, but flag simplified short cuts. A wave function that does not describe internal motion can hardly be as complete as was claimed by Bohr [9].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix 1

Dirac taught that the mass of the positron m_p is negative and that its momentum is positive [1]. The combination of these properties raises unphysical singularities in v_p and v_g when $|m_p| \simeq |pc|$ [3]. To avoid the singularities, it is consistent in dispersion dynamics, to treat each of the mass, and momentum, and energy of the anti-particle as negative. Then the minimum energy required to create an electron-positron pair, for example, is the *difference in energy* between the two particles' energies; rather than the sum of individual mass energies.

Appendix 2

Compare Equation (7) for free electrons (**Figure 2**) with the bound state for which the eigenvalue ε is negative (**Figure A1**). The virial theorem, where $pc \simeq \langle eV \rangle / 2 \simeq \varepsilon$, together with the de Broglie relation, yield (while avoiding singularities at $|mc^2| = |pc|$):

$$c\sqrt{\langle p^2 \rangle} \simeq \frac{eV}{2} = \frac{hc}{\lambda} \quad \text{(A2.1)}$$

or

$$\lambda = \frac{h}{\langle |p| \rangle} \quad \text{(A2.2)}$$

and from Pythagoras' theorem:

$$E^2 = \left(mc^2 - \varepsilon\right)^2 + p^2c^2 \quad \text{(A2.3)}$$

since $\varepsilon \simeq pc$, *where* $E = h\nu$. The wave functions described by Pauling and Wilson [17] are more detailed.

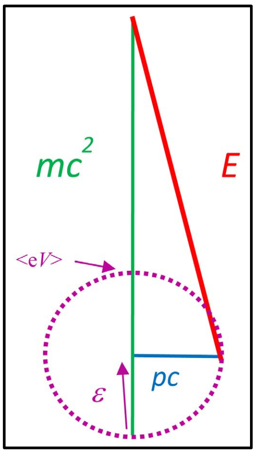

Figure A1. Compare the free electron configuration in **Figure 2**, with an atomic electron in a bound, ground state, having approximate energy, mass, momentum distributions (*i.e.* where $\varepsilon \simeq pc \simeq eV/2$, as in the virial theorem). Then, after appropriate adjustment for scalar and vector values, $E \simeq h\nu$ and $p \simeq h/\lambda$.