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Ontology of Relativistic Mass

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Abstract

The term "relativistic mass" defined by equation $m = \gamma m_0$ with $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$ has a somewhat controversial history, based on special relativity theory, mathematics, logic, intuition, experiment, and ontology. Key is the ontological framework, specifically whether the framework does or does not include gravity. This paper examines both cases, with detailed analysis of gravitomagnetism and of relativistic mass in collisions.

Keywords

Spacetime Ontology, Comparative Ontology, Local Absolute Space, Relativistic Mass, C-Field Circulation, Hidden Energy Reservoir, Transverse Mass, Longitudinal Mass

1. Introduction

The term "relativistic mass", defined by equation $m = \gamma m_0$ with $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$, has a somewhat controversial history; some physicists believe that mass changes with velocity; others do not.

Trupp [1] claims "*According to the Theory of Special Relativity, the mass of a body has increased when it has gathered speed* (where mass resists acceleration)." Okun [2] states "*The terminology* [relativistic mass] *has no rational justification today*", while Rindler [3] and others retained it as a useful concept. This is a key disagreement about a very major aspect of physics, affecting our understanding of ontology, that is, the nature of physical reality. A physical theory provides a logico-mathematical model of reality that *assumes* an ontology; often, different theories assume different ontologies. Different theories have varied degrees of success, and this has caused many physicists to dismiss ontology as "unknowable". An anonymous reviewer seemed to confirm this: "*Eventually, strange and unintuitive as* $4D$ *is, one quits thinking about ontology.*" However, since the nature of physical reality should be of significance to physicists, this paper presents an ontological analysis of relativistic mass. This paper is not an analysis of Einstein's special relativity of space-time; my recent paper [4] compares *space-time relativity* to *energy-time theory* on a feature-by-feature basis and presents a table of the results.

Analysis proceeds in terms of an ontological framework—specifically whether gravity is included or not. Special relativity excludes gravity, with frames in constant relative motion, unaccelerated with respect to each other, with no privileged frame. An observer in each frame feels himself to be at rest with a *rest mass*, m_0, regardless of his relative velocity with respect to the other frame.

Thyssen [5] claims "*special relativity leaves the debate on the dimensionality of the world underdetermined.*" *i.e.*, it is uncertain whether time has a unique dimension and space has three dimensions D^{3+1} or space-time is a 4D reality, however, one can develop physics for a D^{3+1} universe (*presentism*) or a 4D universe (*eternalism*) in terms of Hestenes multi-vector $X = (ct + \boldsymbol{x})$ based on one's choice of basic assumptions: *local absolute space and time* D^{3+1} or *relative space-time* 4D and corresponding choice of how to apply the Lorentz transformation. The *energy-time* theory D^{3+1} formulation $X = (t + \boldsymbol{x})$ is Lorentz compatible, but, based on metaphysical assumptions of local absolute space and time and inertial mass, yields $m = \gamma m_0$, as opposed to relativistic 4D rotations mixing 3-space and time: $x' = \gamma(v,c)(x - vt)$, $t' = \gamma(v,c)(t - vx/c^2)$.

The non-intuitive mixing of time and space in 4D is problematic in relativity, where primed coordinates (x', t') apply in one frame: unprimed coordinates (x, t) in another. Relativity problems are always posed in terms of two or more inertial reference frames, each with its own universal time dimension, related by the Lorentz transformation. An alternate ontology, D^{3+1}, represents all of space and a universal time dimension covering all of space right now. Two physical frames of interest, one at rest, the other in motion, each have their own spatial map but share a common time. Speed of light is with respect to local absolute space, whereas relativity assumes c = *constant* in all frames. Rindler, a major relativist, noted: "*Each inertial frame now has the properties with which the ether frame had been credited.*" The assumption c = *constant* is necessary for Lorentz to work, but per Rindler: "*Light propagates the same in all inertial frames… It is not for us to ask how*!" If it made sense, we *could* ask how; Rindler admits *it doesn't make sense*.

Beginning with photon relation $x = \pm ct$ we can derive $c^2t^2 - x^2 - y^2 - z^2 = 0$; for another photon $c^2t'^2 - x'^2 - y'^2 - z'^2 = 0$. Constant c allows us to relate these two frames in relative motion $v \neq 0$ via the Lorentz group $(x', y', z', t') = L(v,c)(x, y, z, t)$ and inverse transformation: $L^{-1}(v,c) = L(-v,c)$. This group symmetry, characteristic of geometry, is represented by *rotations*; rotation from $\boldsymbol{x}$ to $\boldsymbol{x}'$ can be reversed by an inverse rotation from $\boldsymbol{x}'$ to $\boldsymbol{x}$. Sobczyk [6] develops a theory of linear algebra based on such null vectors with property $v^2 = 0$. In physics they produce the invariants about which physical theories can be formed.

Einstein concluded, circa 1918, that gravity functioned as the *ether*, but failed to update relativity, which banishes the medium of ether, replacing it with the proclamation that the local space-time coordinate frame accomplishes the required invariance. The gravitational field is assumed present everywhere in space, and, having energy, the field is *material* and is the medium through which electromagnetic waves and gravitomagnetic waves propagate. Light propagation in this local medium is compatible with both Michelson-Morley and Michelson-Gale experiments [7] but violates Einstein's axiom of constant c in all frames and his claim that one cannot detect the speed of the local frame from within the frame.

This paper is organized as follows:

The Introduction discusses ideas about relativistic mass, key to *ontology*—the physical reality of the universe—briefly describing two alternate ontologies. Section 2 traces Voight's decision to vary space and time in analyzing the Doppler effect, instead of varying the dynamic frequency and momentum aspects of the (acoustic) physics of reality. Section 3 treats *relativistic mass* as *Lorentzian mass* in a Galilean framework. Section 4 applies a concept from the quantum theory of fields, super-selection rules, to formulate a Hamiltonian in terms of spin-$\frac{1}{2}$ particles and spin-1 helical C-field circulation. In Section 5, the Lenz Law nature of the gravitomagnetic field is discussed. Section 6 asks "*How weak is the C-field*?" Local gravity holds the moon to the earth and promises pain when anyone jumps from a high place. Gravity is *not* weak; what about the C-field? Section 7 defines C-field energy and asks how this energy is to be accounted for. Section 8 treats the issue of relativistic mass in collisions. Section 9 summarizes the paper.

2. Voight's Transformation

The source of the idea that time and space change with local velocity appears to be Voight's 1887 analysis of the Doppler effect, based on the generalized wave function: $\psi = \psi_0 \sin(\boldsymbol{k}\cdot\boldsymbol{r} - \omega t)$, where the phase angle is a D^{3+1} product of $\{\boldsymbol{k},\omega\}$ with $\{\boldsymbol{r},t\}$. Voight chose parameters of space and time to vary for two observers, rather than the momentum and frequency aspects of the wave carrying the Doppler shift. His coordinate-based analysis underlies Einstein's relativistic interpretation of space and time, with associated concepts: "time-dilation" and "length contraction". The corresponding space-time ontology derived from Lorentz is based on 4D-geometry; the ability to transfer from one 4D frame (x,y,z,t) to another frame (x',y',z',t'), with basic motion fixed by uniform velocity $\boldsymbol{v}$ between the frames. Einstein's lack of acceleration removes *force* from the picture; the transformation from an event in one frame to its corresponding event in the other frame is independent of mass, so mass does not appear in the Lorentz transformation.

In energy-time theory [8] the *clock slowing* mechanism is explained using Galilean transformation; arguments exist against length contraction, which has never been measured and, per Rindler, probably never will be. In space-time theory t' is the *time dimension* in the primed frame, different from the t dimension in the unprimed frame; incompatible with physicists' intuition, while the energy-time definition of t' is that of time *measurement*, not time dimension.

Time-dilation, the key "proof" of relativity, can be derived in the ontology of local absolute space and time, by assuming that mass is a function of velocity $m = m(v)$. When one frame is accelerated with respect to another, clocks in the accelerated frame effectively gain mass and hence resist acceleration. All clocks are based on some form of simple harmonic oscillator, in which a restoring force returns a displaced mass to its equilibrium position, where it overshoots and is displaced in the opposite direction; the increase in inertia causes mass to accelerate more slowly, so clocks do run slower when moving. That mass will be minimum when $v = 0$ implies a preferred frame in which mass is minimized. The *space-time symmetry principle* forbids preferred frames, so rest mass is not associated with *any* frame, but with *every* frame. Any observer in a moving frame sees rest mass m_0; in D^{3+1}-ontology only the rest frame S has $m \equiv m_0$ while 4D-ontology assigns velocity zero to *every* object at the origin of any S' inertial reference frame: $m \equiv m_0$; when relativists transform (x, y, z, t) and $(x, y, z, t)'$ they reset rest mass:

$$\begin{matrix} \text{Rest} & & \text{Moving} \\ \begin{bmatrix} x = 0 \\ \dot{x} = 0 \\ m = m_0 \end{bmatrix} & \Rightarrow & \begin{bmatrix} x' = 0 \\ \dot{x}' = 0 \\ m' = m_0 \end{bmatrix} \end{matrix} \tag{1}$$

Einstein essentially invented "slices" of physical reality in which the objects of interest move with uniform velocity with respect to each other. He *excluded from his theory* periods of physical acceleration necessary to provide the relative velocity to objects initially at rest in a local frame and he mapped 4D-ontology into "slices" of D^{3+1}-ontology as seen in **Figure 1**. The velocity curve shows constant relative velocity of relativity as shaded regions, while the acceleration portions of the curve exist only in D^{3+1}-ontology, between the slices.

Recognition of the *relativistic reset* $m(v) \to m_0$ of mass as the basis of the inertial reference frame, *automatically excludes all inter-frame kinetic energy*; allowing the observer to switch from one frame to the other and to retain the relevant *rest mass* in accordance with *space-time symmetry*, the key postulate that *there is no preferred frame*. This enables geometric transformation from one frame to the other and back, but makes it is impossible to tell which inertial frame is stationary and which is moving. In other words, the relativistic approach effectively *resets* the rest mass in each frame, while causing parameters of space and time to vary from observer to observer according to Lorentz transformation. So, relativity, based on ontologically questionable assumptions, always contains paradoxes, places where logic breaks down. Per Susskind [9]:

"*Special relativity…is counter-intuitive…full of paradoxical phenomena*."

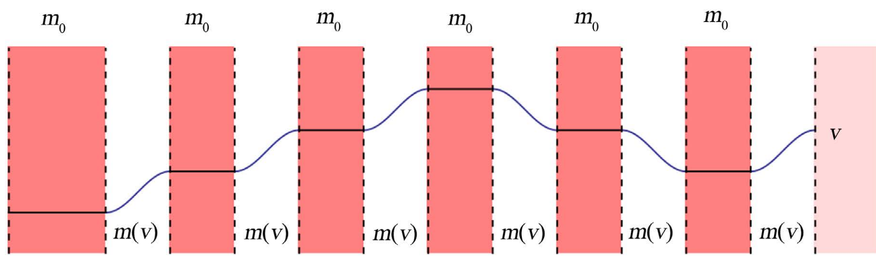


Figure 1. Map 4D-ontology onto "slices" of D^{3+1}-ontology.

3. Relativistic Mass = Lorentzian Mass

4D *time-space*-rotation is thus established at the expense of kinetic energy according to Lorentz transformation, while according to *energy-time theory* inertial mass is transformed by inertial factor γ to Lorentzian mass, $m = \gamma m_0$; *time and space are Galilean in nature*. Lucas and Hodgson [10] say of inertial mass: "*If we insist on retaining Newtonian dynamics, and Newtonian definitions of velocity and acceleration, then we can still obtain relativistically correct results if we pay the price of allowing the mass to depend on the velocity.*"

Space-time theory Energy-time theory

$$\begin{Bmatrix} m' = m_0 \\ x' = \gamma(v)(x - vt) \\ t' = \gamma(v)(t - vx) \end{Bmatrix} \quad \begin{Bmatrix} m' = \gamma(v) m_0 \\ x' = x - vt \\ t' = t \end{Bmatrix} \tag{2}$$

Einstein's relativity states the equivalence of inertial frames of reference. Weinberg [11] distinguishes this from the Galilean principle of relativity, obeyed by Newtonian mechanics, by the transformation connecting coordinate systems in different inertial frames. Of course, physics is fundamentally independent of coordinate systems—they can have no effect on physical reality. Per Weinberg: "*A symmetry transformation is a change in our point of view that does not change the results of possible experiments.*" Although the mathematics of the Poincare group is simpler than that of the Galilean group, Weinberg notes:

"*…there is nothing to prevent us from formally enlarging the Galilean group, by adding one more generator to its Lie algebra, which commutes with all the other generators, and whose eigenvalues are the masses of the various states.*"

In Equations (2), *energy-time theory* is seen to consist of the Galilean transformation and a generator whose eigenvalues are the masses of the various states in relative motion. This is the formal explanation of *relativistic mass*.

Weinberg [12] also points out that since $d\tau = dt/\gamma$ we obtain:

$$m \frac{d}{dt}[\gamma v] = q\left[\boldsymbol{E} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}\right]. \tag{3}$$

"*It is a special feature of electromagnetic force that the only changes in the equation of motion introduced by special relativity is the replacement off mass m in the momentum with* $m = \gamma m_0$, [*and thus*] *treat* γm_0 *as a relativistic mass.*"

In other words, the concept of relativistic mass is conceptually useful. Weinberg also points out that, based on Maxwell's equations, "*we have no a priori knowledge of the Lorentz transformation properties of the electric and magnetic fields*."

The use of relativistic mass, $m = \gamma m_0$ in *energy-time theory* obviates the need for Lorentz transformation on space and time. This is ontologically correct at the level of special relativity, which does *not* incorporate gravitation. *Energy-time theory*, like special relativity, does *not* include a theory of gravity. When, however, one adds the primordial field theory to energy-time theory then kinetic energy of the moving mass is shown to represent storage of energy in the C-field circulation, and mass is invariant, as claimed.

4. Super-Selection Rules

Weinberg, in *The Quantum Theory of Fields*, observes that it may not be possible to prepare a system in a state represented by $\Psi_A + \Psi_B$. "It is widely believed to be impossible to prepare a system in a superposition of two states whose total angular momenta are integers and half-integers, respectively." In such cases, there is a "superselection rule" between the different classes of states. In this section we invoke a *superselection rule* explanation of relativistic mass. Elsewhere, I derive a fermion from the primordial field with spin-$\frac{1}{2}$, while the field circulation induced by particle motion has $U(1)$ symmetry, and hence integer spin. According to the superselection rule, these states are not super-imposable into one state, and must be developed separately. In other words, the mass/energy of the moving particle is a function of two classes of energy:

$$E = f\left(mc^2, mv^2\right) \tag{4}$$

where mc^2 has been shown to have half-integral spin, and where mv^2 will be seen to have integral spin representing momentum-induced circulation of the C-field. In *energy-time theory* the Hamiltonian is derived for *relativistic mass* $m = \gamma m_0$, with $E = mc^2$ and $\boldsymbol{p} = m\boldsymbol{v}$:

$$\begin{aligned} &E = \left(m_0^2 c^4 + c^2 p^2\right)^{1/2} \Rightarrow E^2 = m_0^2 c^4 + c^2 p^2 = m_0^2 c^4 + c^2 \gamma^2 m_0^2 v^2 \\ &\Rightarrow E^2 = \gamma^2 m_0^2 c^4 \end{aligned} \tag{5}$$

Dividing both sides by $m_0^2 c^4$ we obtain:

$$\frac{E^2}{m_0^2 c^4} = 1 + \gamma^2 \frac{v^2}{c^2} \Rightarrow \gamma^2 = 1 + \gamma^2 \frac{v^2}{c^2} \Rightarrow \gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \tag{6}$$

In other words, separating the energies according to spin thus yields:

$$E = \sqrt{\left(m_0 c^2\right)^2 + \left(cp\right)^2} = m_0 c^2 \sqrt{1 + v^2/c^2} \sim \underbrace{m_0 c^2}_{\text{spin-}\frac{1}{2}} + \underbrace{\frac{m_0}{2} v^2}_{\text{spin-1}}, \tag{7}$$

which is the way orthogonal entities must be added. Energy-time theory, supporting the concept of relativistic mass, retains the Lorentz γ-factor, but applies it to inertial mass, rather than to abstract space and time. In D^{3+1}-ontology the velocity $\boldsymbol{v}$ of an object is with respect to rest frame *S*, *local absolute space*, and any change of $\boldsymbol{v}$ is via accelerating force: $m\boldsymbol{a} = \mathrm{d}\boldsymbol{p}/\mathrm{d}t = \mathrm{d}(\gamma m_0 \boldsymbol{v})/\mathrm{d}t$ while in 4D-ontology the momentum relation $\boldsymbol{p} = \gamma m_0 \boldsymbol{u}$ is applied where $\boldsymbol{u}$ is the velocity of the object in a reference frame, *not* the velocity of the reference frame relative to another.

5. The Lenz Law Nature of the Gravitomagnetic Field

Energies involved in relativistic mass include rest mass mc^2 and momentum $\boldsymbol{p}$. Circa 1893, Heaviside [13] extended Newtonian gravity, based on analogy with Maxwell's equations, with a key equation describing the circulation of the gravitomagnetic field, which we call the C-field:

$$\nabla \times \boldsymbol{C} = -\boldsymbol{p} + \frac{\partial \boldsymbol{G}}{\partial t} \tag{8}$$

In (8) we let physical constants $c = g = \hbar = 1$ where c is the speed of light and g is Newton's gravitational constant, with $\boldsymbol{G}$ being the gravitational field. Instead of momentum, $\boldsymbol{p}$ is momentum density:

$$\boldsymbol{p} = \frac{\boldsymbol{P}}{\int \mathrm{d}^3 x} = \frac{m}{\int \mathrm{d}^3 x}\boldsymbol{v} = \rho \boldsymbol{v} \tag{9}$$

For momentum used in the Hamiltonian, we have $\boldsymbol{P} = m_0 \boldsymbol{v} = \int \mathrm{d}^3 x \nabla \times \boldsymbol{C}$. That is, momentum is the volume integral of the C-field circulation induced by the momentum density $\boldsymbol{p}$. Temporarily ignore change in gravitational field, $\frac{\partial \boldsymbol{G}}{\partial t}$, and consider only $\nabla \times \boldsymbol{C} = -\boldsymbol{p}$. The force $\boldsymbol{F}$ that accelerates rest mass $\boldsymbol{F} = m_0 a = \mathrm{d}\boldsymbol{p}/\mathrm{d}t$ gives rise to a change in circulation of the C-field:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\nabla \times \boldsymbol{C}) = \frac{-\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t}}{\int \mathrm{d}^3 x} \tag{10}$$

The negative sign in Equation (8) is associated with the direction of circulation, that is, momentum density $\boldsymbol{p}$ induces a left-handed circulation about the momentum. However, in the force formula, any negative sign associated with change in momentum density $\mathrm{d}\boldsymbol{p}/\mathrm{d}t$ will have the same meaning as current flow in Lenz's Law of electromagnetic theory. As a reminder of the physics of Lenz's Law, consider the classic distributor found on gasoline engines. The collapsing magnetic field induced by flowing electric current is interrupted by mechanically breaking the connection. Lenz's Law states that the direction of the electric current induced in a conductor by a changing magnetic field is such that the magnetic field created by the induced current opposes changes in the initial magnetic field. When the current conductor is broken, the change in current is immediate, and this large derivative (rate of change) induces a strong force to keep current flowing in the inductor. Since electric "force" is the "emf" electric field or induced *electro-motive force*, a sufficiently strong emf will ionize atoms and produce a "spark". This spark is timed to ignite the fuel in the cylinder near top-dead-center, thus driving the piston down for the power stroke.

In other words, the current flowing in the coil (inductor) induces a magnetic field that is sustained by the continued current flow. When the flow is interrupted, the collapsing magnetic field causes an electric force that attempts to keep the current flowing. The dual of electric current density flow in electromagnetism is mass density flow in gravitomagnetism: $\boldsymbol{J} \Leftrightarrow \boldsymbol{P}$ therefore:

$$\nabla \times \boldsymbol{B} \sim \boldsymbol{j} \quad \text{and} \quad \nabla \times \boldsymbol{C} \sim -\boldsymbol{p} \tag{11}$$

implies

$$\frac{\mathrm{d}}{\mathrm{d}t}(\nabla \times \boldsymbol{B}) \sim \frac{\mathrm{d}\boldsymbol{j}}{\mathrm{d}t} \quad \text{and} \quad \frac{\mathrm{d}}{\mathrm{d}t}(\nabla \times \boldsymbol{C}) \sim -\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}. \tag{12}$$

Thus, the gravitomagnetic dual of Lenz's Law is such that change in momentum (force $\mathrm{d}\boldsymbol{P}/\mathrm{d}t$) induces a C-field circulation. For a charged particle we might use electric field $\boldsymbol{E}$ to accelerate the particle, inducing both B-field and C-field circulations. If charges are balanced, $\sum q = 0$, then we can mechanically accelerate the mass, producing only C-field circulation. When all forces are removed, the mass will essentially "coast" forever, that is, *momentum is conserved*. Yet, according to Feynman [14]: *the reason why things coast for ever has never been found out*: "*The law of inertia has no known origin*." But, from the above analogy, a decrease in momentum will generate a corresponding decrease in C-field circulation, and this will, in turn, generate a force that compensates for the initial decrease, thereby maintaining momentum. Thus, acceleration increases the C-field circulation, while deceleration is opposed by the existing circulation. The same physical reasoning applies to a particle "tunneling" through a finite potential barrier. The change in momentum as the particle begins to penetrate the barrier is opposed by the corresponding force associated with the change in circulation. The particle is effectively accelerated by the collapsing C-field circulation until the circulation disappears.

6. How Weak Is the C-Field?

A potentially major impediment to understanding gravitomagnetic circulation as kinetic energy, and hence as "relativistic mass", is the label "weak field approximation", based on derivation of Heaviside's equations from Einstein's nonlinear field equations in curved space-time. For weak fields the higher order terms in the approximation can be ignored and higher order terms dropped, leaving the "linear" Heaviside equations. For those physicists, probably the majority, who believe that curved space-time is the true nature of gravity, this is a convincing argument. Yet, physicists such as Clifford Will, working with *gravitomagnetic post-Newtonian* physics, have remarked upon the unexpected effectiveness of the equations in strong-field problems [15]. And Einstein's gauge field tensor can be derived from Heaviside's equations [16].

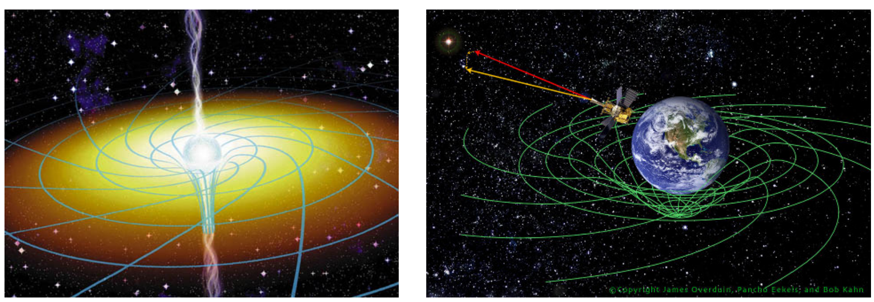
The ontology of gravitational field versus curved space-time is treated in [17] and [18]. The ontology of the gravitomagnetic field can explain century old paradoxes associated with curved space-time ontology. The gravitational field underlies the "ontology of relativistic mass".

The existence of the C-field was proved in 2011 via the *Gravity Probe B* experiment [19] where the results confirmed general relativity to within a few percent. But this too may give the impression of "weak field", largely because the experiment was performed in orbit and the density of the Earth as seen from orbit is relatively low. Contrast this with the density of atomic nuclei seen from nuclear distances. The key parameter in $\nabla \times \boldsymbol{C} \sim \rho \boldsymbol{v}$ is mass density ρ. For a macro-object in motion, a mass, the relevant C-field circulation energy is the sum of the energy for each nucleus times the number of nuclei of which the mass is constituted.

Finally, as analyzed in "*Quasi-Local Mass*", energy-density is *not defined* in general relativity; this too probably accounts for the fact that many physicists are oblivious to the ontology of the C-field circulation, conceiving of it instead as "frame dragging" in curved space time.

For the moment, take both ontologies seriously. Assume, per general relativity, a twisting of the local spacetime fabric with respect to the undisturbed background of reality. Contrast this with the circulating flow of the gravitomagnetic C-field. A 2D rubber sheet is often used to illustrate curved space time, but this perspective is lacking. Consider the images in **Figure 2** [20] and [21]; in 2a a superluminal jet source is postulated to be a black hole spinning near the extreme theoretical limit. In 2b the spin of Earth is shown "dragging" curved space around it. Since the Earth has been spinning daily for billions of years; this "rubber band" concept of space-time would imply that the rubber sheet must be wrapped up infinitely tight, a paradoxical idea. On the other hand, *the C-field conception is based on circulation of the energy in the field*, and simply contributes to the angular momentum of a spinning object; the circulating field surrounds the object. We next look at the energy of the field.

What has yet to be measured is the strength of the C-field at the atomic and nuclear level. Since Heaviside's equations are density based, one expects the greatest C-field strength at the nuclear level. Erroneous belief in the "weak field approximation" has prevented consideration of this realm of physics.



(a) (b)

Figure 2. Images of "frame dragging" in general relativity. (a) NASA image of frame dragging around a black hole, and (b) Frame dragging as measured by Gravity Probe B.

7. The Energy of the C-Field

The energy density of any physical field is proportional to the square of the field strength, for example, the energy density of the electromagnetic field is $\sim\left(E^2+B^2\right)$. The formula for C-field energy density is:

$$\left(\frac{c^2}{g}\right)(\boldsymbol{C}\cdot\boldsymbol{C}) = \text{C-field energy density} \tag{13}$$

If we multiply energy density by local volume, we obtain the dimensional relation:

$$\int \mathrm{d}^3x\left(\frac{c^2}{g}\right)(\boldsymbol{C}\cdot\boldsymbol{C}) \Rightarrow \left(l^3\right)\cdot\left(\frac{m}{l}\frac{1}{t^2}\right)=\frac{ml^2}{t^2}\approx mv^2 . \tag{14}$$

The C-field has been shown to be real, and therefore to have finite energy density, but this local energy density has not been treated in any standard treatment of kinetic energy (of motion). Thus, we seem to be faced with two choices:

If C-field energy is not kinetic energy, then it must be added to kinetic energy in any real physical situation, else C-field circulation energy is kinetic energy; no new energies need be accounted for.

An alternative gravitomagnetic approach is dual to geometric algebra-based treatments of Maxwell's equations; Arthur [22] develops D^{3+1} and 4D models in detail. We follow his treatment of D^{3+1}-Maxwell equation $\left(\nabla+\partial_t\right)F=J$ with F the field tensor and J the source, $J=\rho(1+v)$. Multiply both sides by $\left(\nabla-\partial_t\right)$ to obtain:

$$\left(\nabla^2-\partial_t^2\right)F=\left(\nabla-\partial_t\right)J . \tag{15}$$

In source-free space $J=0$ and Equation (15) becomes the wave equation, which, in terms of a plane wave, reduces to $\left(-k^2+\omega^2\right)F=0$. Making use of natural units $g=c=\hbar=1$ and the quantum equivalents: momentum $p=\hbar k$ and $E=\hbar\omega=\hbar C$, we obtain:

$$\left(-k^2+\omega^2\right)\Rightarrow -p^2+C^2=0 \tag{16}$$

For unit mass this implies

$$p^2=C^2 \tag{17}$$

/ \ <energy densities>

kinetic C-field

In other words, kinetic energy, is again seen to be physically represented by energy of gravitomagnetic circulation induced by momentum $\boldsymbol{p}$. Almost every energy in physics is associated with a potential or energy field—kinetic energy may be unique in having no field correlate. Heaviside theory of gravitomagnetism implies that the essentially undefined mechanism of storage of *energy of motion* is actually C-field circulation energy, bringing our most basic energy into agreement with all other field energies.

8. Relativistic Mass in Collisions

The chiral nature of C-field circulation (left-handed) implies that particles moving in opposite directions generate C-fields circulating in opposite directions. If these particles collide, counter-rotating C-fields should effectively cancel each other, that is, the relativistic mass of each cancels out. Does this happen? A recent paper by Trupp, dealing with "*The Interaction between an Accelerated Mass in Straight Motion and a Hidden Energy Reservoir as a Strict Mathematical Consequence of Special Relativity*", states:

"*The total energy of two objects that undergo a symmetrical, elastic, head-on collision is therefore not conserved, thus requiring the involvement of a hidden reservoir of energy.*"

He claims that "*an apparent disappearance of energy has been noticed in particle physics already, but its consequences have been ignored*". This recalls Cannoni's statement [23] that the *law of velocity addition* is known to be violated in relativistic colliders. It appears as if violations of Einstein's special relativity are essentially ignored in the literature, rather than the usual approach in which any experiment that contradicts a theory is typically considered to invalidate the theory. Additionally, paradoxical elements of relativity are downplayed. Einstein, Lorentz, and others distinguish between "transverse mass" and "longitudinal mass". R. C. Tolman [24] observed that:

"*If, however, mass is a quantity to which a conservation law applies, the mass of a body cannot well be different in different directions…*"

For a particle with momentum $\boldsymbol{p}_z$ in the *z*-direction a longitudinal force $\boldsymbol{F}_L = \frac{\mathrm{d}\boldsymbol{p}_z}{\mathrm{d}t}$ is applied in the *z*-direction, acting in the same direction as the original force that resulted in $\boldsymbol{p}_z = \nabla \times \boldsymbol{C}$. From the above discussion of Lenz's Law, we observe that any further acceleration or deceleration in this direction will be opposed by existing C-field circulation, and this resistance to further acceleration is interpreted as increased (relativistic) mass.

On the other hand, a transverse force $\boldsymbol{F}_T = \frac{\mathrm{d}\boldsymbol{p}_y}{\mathrm{d}t}$ is orthogonal to momentum $\boldsymbol{p}_z$ and therefore orthogonal to $\nabla \times \boldsymbol{C}$. In this case local C-field circulation will <u>not</u> resist acceleration by $\boldsymbol{F}_T$ and thus will not be interpreted as other than rest mass, *i.e.*, relativistic mass is directional. This does not lead us, as it led Einstein and others, to distinguish between *transverse mass* and *longitudinal mass*.

This demonstrates the power of ontology to resolve issues. Equations of motion that are ontologically unexplained or otherwise inappropriate can lead to confusion: "*the mass of a body cannot well be different in different directions.*" In energy-time ontology and Heaviside-based gravity the mass of the body is always the same (rest) mass in all directions, but a body in motion has direction-dependent momentum, and correspondingly direction-dependent circulating field-based resistance to acceleration.

In his analysis, Trupp concludes that Epstein [25] was correct when he postulated that the only way to avoid the inconsistency of two different masses is the following:

"*…when a body is accelerated by means of an external, technical force, an additional, hidden force turns up.*"

This "hidden" force is the Lenz-Law-like force associated with $\boldsymbol{p}_z = \nabla \times \boldsymbol{C}$. This direction dependent force, $\frac{\mathrm{d}}{\mathrm{d}t}(\nabla \times \boldsymbol{C}) \sim -\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}$ is hidden from previous special relativity experiments and hidden theoretically by the erroneous *weak field approximation* of general relativity.

Trupp states initially that he is by no means attempting to modify or refute the basic assumptions of special relativity; he chooses to analyze a "fixed acceleration", a very artificial construct, which, nevertheless leads him to conclude that:

"*The energy of the hidden, work-performing force is fed from a hidden reservoir in space.*"

He states the relativistic mass or energy generated by Epstein's hidden force is not converted into elastic energy in a head-on collision, but

"*flows off into the unknown where it had come from.*"

We are not committed to the proposition that special relativity should not be refuted, and do not argue Trupp's details. It is obvious from the rest of our paper that the "*hidden reservoir in space*" is C-field circulation, which stores the energy as angular momentum (equal to linear momentum, in agreement with partitioning of energy).

In other papers I have shown that the C-field resolves other century-old paradoxes built into special and general relativity. Therefore, it is interesting that Trupp and others deduce that the nature of the resolution is *hidden*, a somewhat ontological deduction of the nature of reality.

9. Summary

It seems strange that an actual storage mechanism for kinetic energy is generally ignored. Yet, kinetic energy, *the energy of motion*, is typically the first introduction to energy in high school or even earlier. Young physicists simply learn the definition of such and absorb it before going on to learn of many other forms of energy, all of which are associated with some energy storage mechanism. If and when they encounter gravitomagnetic field energy, they are biased, both by the original acceptance of kinetic energy and the later biases of "weak field approximation" and of "frame dragging". It is hardly surprising that when physicists encounter *relativistic mass* and use the relativistic Hamiltonian to relate it to kinetic energy, it is with some degree of confusion. As noted above, *C-field energy is real*, and must either be *equated to* kinetic energy or *added to* kinetic energy. From thorough consideration of the density-based nature of C-field circulation, and the density of atomic and nuclear constituents of matter, it appears that equating momentum-induced C-field circulation energy to momentum associated kinetic energy is the most natural interpretation and is completely compatible with the ontology of relativistic mass.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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