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Inadequacies in the Current Definition of the Meter and Ramifications Affecting Einstein's Second Postulate of Relativity

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Abstract

The current definition of the meter as based on the time of light transmission and the postulated universal constant light speed is ill-defined and inadequate. The definition fails to identify which second is required, whether to use coordinate or proper time, or which method to construct an exact meter, besides ignoring gravity's effect. In Einstein's 1905 paper that defined special relativity, Einstein stipulated correctly that light traversing the ends of a resting rod takes equal time transmissions in either direction. If that rod is oriented parallel to a constant velocity, a photon from one end of the moving rod takes a longer time span with a universal constant light speed to overtake the receding end and takes a shorter time span to intercept the approaching end of the rod when transmitted in the opposite direction, resulting in a longer roundtrip distance of photons traversing the moving rod versus the resting rod. Length contraction undercompensates this difference. Einstein did not address this issue. However, Einstein claimed the unequal time intervals over the moving rod versus equal intervals over the resting rod are because simultaneous states for the resting observer and resting rod are nonsimultaneous for the constant moving observer. This contradicts his first postulate of relativity: any state of a physical system (*e.g.*, equal timed traverses of photons moving over a rod) is unaffected by a constant translational velocity between inertial reference frames. An in-depth analysis examines Einstein's thought experiment for an adequate redefinition. The analysis reveals one-way photon velocities obey vector velocity addition involving moving photon sources, but it proves by induction that roundtrip photon traverses have an average speed that is identical to the standard light speed c. Thus, Einstein's second postulate of relativity is not general, but is valid for roundtrip traverses of photon transmissions. This may change many physical concepts, since one-way velocities for photons and particles are not limited by the second postulate. A suggested redefinition of the meter is submitted.

Keywords

Light Speed, Time Dilation, Length Contraction, Special Relativity, Gravitation

1. History of the Definition of the Meter

The first definition of the meter was 1E−7 of the distance from the equator to the north pole along a meridian at sea level, which was assumed to go through Paris, as the French metric system was adopted in 1791 using a decimal system [1] [2]. Unfortunately, the Earth is not a perfect oblate spheroid, and the flattening factor of the radii to the equator and the poles was not known then. The meter also varies depending on the meridian of longitude due to the lumpiness of Earth. Over decades of international research, the meter's uncertainty in surveyed length was reduced by improvements in geodesy.

This led to the Meter Convention (Convention du Mètre) of 1875, which mandated the permanent founding of the International Bureau of Weights and Measures (BIPM: Bureau International des Poids et Measures) located in Sèvres, France. This organization was charted to construct and preserve a meter bar as a standard, distribute duplicate bars to other nations, and compare those national standards periodically against the BIPM meter standard. Such bars were distributed in 1889 at the first General Conference on Weights and Measures (CGPM: Conférence Générale des Poids et Mesures) to participating nations to establish the international meter.

James Clerk Maxwell proposed that an emitted light ray could be used to standardize both the meter and the second [3]. This proposal was implemented in 1893 by Albert A. Michelson, who invented the interferometer and measured the meter using light from heated cadmium [4]. BIPM regularly used interferometry with the meter standard until 1960 when the 11th CGPM redefined the meter in the new International System of Units (SI). That definition of the meter was 1650763.73 wavelengths of the orange-red emission of krypton-86 in a vacuum [5].

In 1983, the 17th CGPM redefined the meter in the current terms of the second and the postulated constant speed of light in a vacuum. The current definition of the meter is [6]:

The metre is the length of the path travelled by light in vacuum during a time interval of 1/299,792,458 of a second.

CGPM recommended a laboratory utilize helium-neon lasers to measure the meter. If conducting the test with air, the refractive index n is used to convert the unit of wavelength λ to meters using c, the fixed speed of light in a vacuum in m/s and f as the measured frequency of the source utilizing the equation, $\lambda = c/nf$ [7].

The BIPM issued a clarification of the meter [8] in 2002:

Its definition, therefore, applies only within a spatial extent sufficiently small that the effects of the nonuniformity of the gravitational field can be ignored (note that, at the surface of the Earth, this effect in the vertical direction is about 1 part in 1E16 per meter). In this case, the effects to be taken into account are those of special relativity only.

Initially, this would appear to be possible by calibrating two clocks together by synchronizing to a master time standard to replicate the second and then move one clock about one meter away to construct an exact meter in the laboratory. However, special relativity predicts time dilation with the moved clock so that the two clocks are no longer synchronized to define a meter with a one-way transmission of the emitted photons. So, a roundtrip using photon transmissions is performed so that only one clock is used to time the emission and reception after the reflected photons arrive at the source. The meter's definition lacks this restrictive condition.

2. Unstated Issues Affecting the Definition of the Meter

According to the BIPM clarification, only special relativity effects will affect the determination of the meter if measured at Earth's surface. Special relativity has two types of time—coordinate time and proper time. In Einstein's 1905 paper [9] [10], coordinate time is constructed by having stationary perfect clocks calibrated to a master clock by recording the roundtrip transmission between the master clock and each remote clock and halving that transmission interval to be added to the next time record broadcasted from the master clock to the remote clock. This is Einstein's synchronization method. Any event in the neighborhood of a fixed coordinate location of a calibrated remote clock has the coordinate time at that instant. Effectively, coordinate time is time transmitted by the master clock anywhere in the region of interest provided light speed was infinite instead of finite.

Proper time is the special relativity effect that a perfect clock exhibits due to time dilation caused by the velocity of that clock relative to the resting frame of reference. The problem is a proper second of a moving clock will be slightly longer than the second of the coordinate clock. Unfortunately, the definition of the meter makes no mention of the preferred inertial reference frame, so that the time dilation of the laboratory's proper clock compared to the coordinate clock of the inertial resting frame is nebulous. Many inertial frames of reference are possible: Earth's nonrotational center of mass frame, the nonrotational Earth-Moon barycenter frame, the solar nonrotational barycenter frame, etc., which are needed to determine the velocity of the laboratory relative to that frame of choice to compensate for proper time.

The definition of the meter does not stipulate which second from any timescale should be used. The original second, which is 1/86,400 of a mean solar day, is based on Earth's rotation relative to the Sun, which its elliptical orbit changes the length between contiguous solar days. A tropical year is defined by the mean motion of the Sun's longitude relative to the dynamical equinox set by one complete revolution in Earth's orbit. The calendar year closely follows the tropical year. The original timescale was often called mean solar time or Greenwich Mean Time (GMT) based on meridian transits of stars as the Earth rotated. The second timescale based on the tropical year was Ephemeris Time (ET) and has now been renamed Universal Time (UT1). The International Atomic Time (TAI) is based on the Systèm International (SI) second, which is defined by the calibration between the US Naval Observatory (USNO) and the National Physical Laboratory (NPL) performed between 1955-1958 [11] by comparing ET from the lunar ephemeris to an atomic clock. The UT1 second is slightly longer than the SI (or atomic) second as evidenced by the numerous positive leap seconds inserted into the Coordinated Universal Time (UTC) timescale, which uses the SI second of TAI, so that UTC is retarded to remain within 0.7 seconds of UT1. Atomic clocks are frequency generators which count cycles that are converted into equivalent SI seconds to function as a clock.

Getting a precise SI second via synchronization of a tester's atomic clock with an ultraprecise timing institute is difficult as no master TAI standard exists. BIPM obtains a theoretical EAL (Échelle Atomique Libre or free atomic time scale) from weighting the times of all reported atomic clocks in the participating timing institutes, which each periodically sends its UTC to BIPM and which, in turn, BIPM creates a weighted average timescale. With frequency adjustments by monitoring the past weighted performance and uncertainty of individual clocks, TAI is obtained from EAL. BIPM then reports periodically the time offset and the second's rate of change for each laboratory's atomic clocks compared to the theoretical TAI timescale, which each timing facility is free to steer its clocks accordingly [12] [13].

Gravitational effects on time are missing within the current definition of the meter. In geodesy, the reference surface to measure variations in gravity is the geoid (*e.g.*, mean sea level). The Pound-Rebka experiment [14] measured the change in frequency of photons moving through gravity over a 22.5 m height. Gamma ray photons changed energy, δE, obtaining $\delta E/E = 2.5\text{E}{-15}$, which directly affects frequency f accordingly as photon energy $E = kf$ where k is Boltzmann's constant. For every meter change in increasing altitude, the frequency change is 1.1E−16 that makes the same change in a shorter second of an atomic clock. Some ultraprecise time institutes could detect a 1 m altitude change. When recalibrating the meter by the definition in a laboratory, elevating the test apparatus by a meter can shorten the new meter by 1.1E−7 nm when ignoring the effects of gravity. Elsewhere, it is necessary to choose a reference, such as the geoid where a meter is defined, so that the time interval can be adjusted for very significant gravity changes such as other planets or outer space.

Lastly, when technology has been improved significantly since the adoption of this definition in 1983, some light speed experiments should obtain more precision with more significant figures. Unfortunately, any possible test would be doomed if it obtained a different result than the "exact" value, because critics could claim something was uncompensated in the test results, making the test invalid. Sometimes, even "exact" concepts or values in definitions are wrong. For example, the nautical mile is now defined to be exactly 1852 meters. However, the nautical mile was originally defined as 1 arcminute along the equatorial circle. The Global Positioning System (GPS) in its WGS-84 model and the International Union of Geodesy and Geophysics (IUGG) in its Geodetic Reference System 1980 have the same value of 6,378,137 m for the equatorial radius of Earth [15] [16]. To seven significant figures, the nautical mile is 1855.324 m, which means the exact value in the CODATA standard is off by 3.324 m. Even "exact" definitions need regular scrutiny for accuracy.

3. Timed Measurements Differing from Fixed Distances between Source and Receptor

The current definition of the meter using timed transmissions of photons does not exclude any setup between the emission source and the light receptor. There are many classes of test setups using photon transmissions that can affect the measurements. For example, the gravity field could fluctuate in the laboratory, the accelerated laboratory could gyrate with various motions, the distance between the emitter and receptor could vary during photon transmission, etc. One simple class is a turntable with a constant angular rotation oriented perpendicular to the local gravity. Even within this class, there are infinite possible choices for parameters. This analysis will assume the radius of the turntable is 0.5 meters where both the photon emitter and absorber are fixed on the perimeter of the turntable. Before any measurements are taken in this theoretical experiment, the following definitions are given.

For all points within an acceptably small neighborhood, the coordinate time is given by a perfect clock in that neighborhood, which has been synchronized to a master time standard or timescale where the distance between these clocks remains fixed. Synchronization could be accomplished by Einstein's synchronization method by timing the roundtrip transmission between the clock and the master time standard, divide the transmission interval by two, and add that result to the time tag of any later transmission from the master time standard. Essentially, coordinate time would be identical to the master time standard's broadcast if electromagnetic transmissions had an infinite speed rather than the finite (although very high) speed for the standard speed of light, c, in a vacuum.

Two or more phenomena are simultaneous if they overlap, merge, collide, intersect, etc., or divide, separate, split, etc., at one point at one instant of time. No time dilation exists for an instant of time, and no length contraction is possible for a single point. Nonsimultaneous phenomena occur when they arrive at or emit from the same point at different coordinate times. Simultaneous events affect all phenomena at that location at that instant of time.

Two or more phenomena are synchronized when they occur at separate coordinate locations at the same coordinate time. Synchronization is not to be confused with simultaneity. Often synchronized phenomena are characterized with duplicate actions. For example, synchronized competitions have multiple athletes performing the same actions at separate locations, but any athlete can stop without affecting the other athletes maintaining the synchronization in the competition.

Often, simultaneous is incorrectly stated when events or actions occur at the same coordinate time at different locations. Simultaneity can not be applied on two clocks to maintain the same timescale of coordinate time. Synchronization is used to adjust the two clocks to keep the same coordinate time. It is also obvious such clocks occupy separate locations for synchronization. Two clocks indicating the same time are operating in synchronization. One never states two clocks are simultaneous. Even in common English, simultaneity and simultaneous actions occur at one instant of coordinate time and location. For example, two cars collide in an intersection, which is the result of two cars entering the intersection (same location and same time) simultaneously. Car collisions are not synchronized events. It is incorrect to state two separated events occurring at the same time are simultaneous. When an observer detects two inputs or events arriving at the observer's location at the same time, that constitutes simultaneous detection through sight, sound or touch.

Consider the turntable in Figure 1 to be tested in a laboratory with the photon emitter located at B and the photon absorber at A. The plane of the turntable is perpendicular to the local gravity in the laboratory. In the first test, the turntable is rotating counterclockwise. The emitter broadcasts photons omnidirectionally, but only the photons traveling along the chord BC are intercepted by the absorber, which travels the arc from A to C during the photon transmission. The movement of the emitter during transmission is extraneous and not included in Figure 1. A perpendicular bisector splits the chord AC in two, creating two identical right triangles, CMO and AMO, which make angles ϕ and θ complimentary. The angle AOB is π radians, which makes the supplementary angle, $\Phi = 2\phi$. The arc AC equals $2\theta R = R\omega\Delta t$, where ω is the angular velocity and Δt is the transmission interval from B to C. L is the length of the chord BC, so $\Delta t = L/c$. For simplicity, assume the laboratory houses the platform in a vacuum, making the index of refraction, n, equal to one. Thus, $\theta = \omega L/2c$, and $R\cos(\theta) = L/2 = R\sin(\phi)$. This example shows that the distance between the emitter and absorber is always one meter, but the measurement based on the photon transmission interval multiplied by the photon speed of c is always less than a meter, depending on the value of the angular velocity, ω.

In the second test, the absorber is located at C, and the turntable rotates clockwise in Figure 1. In this case, ω is chosen so that the absorber, initially at C, acquires the photons at A, which is directly across point B, where the photon emitter initially had broadcasted photons to the absorber. Thus, $\omega = 2\theta c/L$ with

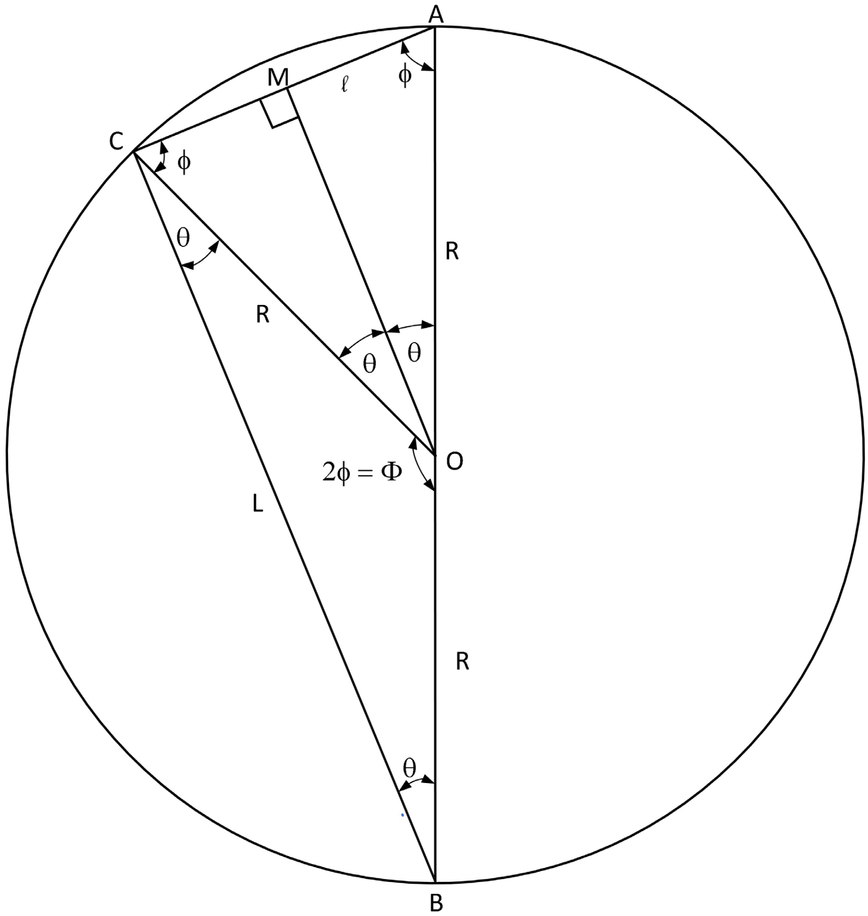

Figure 1. Turntable geometry using photons.

the added requirement that $\cos(\theta) = L/2R$. In this case, the fixed distance between the emitter and absorber is L, but the measurement from photon transmission is the longer distance of $2R$. These two examples illustrate that the current definition of the meter using timed intervals of photon transmission will not replicate the actual fixed distance between the photon emitter and absorber.

This analysis of the turntable only addressed the distances between the two devices. It does allow a comparison between coordinate time and proper time to measure a timed interval. Embellish the experiment with two recorders with antennas serving as observers. One recorder is stationary in the laboratory, and the second recorder is directly above point O. To be technically correct, the second recorder could be rotating to be fixed relative to the turntable, but an omnidirectional antenna rotating on the axis of rotation is equivalent to being fixed in the laboratory. The laboratory has a precise master time standard that broadcasts coordinate time from an antenna located at point O on the turntable. Many designs are possible. Equivalently, the broadcast must consist of a changing signal (*e.g.*, a step function whose vertical slopes denote one ns advances of the master clock) and an accompanying message that denotes the coordinate time tag for each change in the timing signal (*e.g.*, a complete set of messages that denote all the significant figures of the time associated to each ns in the signal). With the foreknowledge that the antenna is 0.5 m away, both the photon emitter and absorber can adjust the time tag at reception by adding a time span of $0.5/c$ to the received coordinate time message so that coordinate time at reception is identical with the master clock.

An atomic clock is integrated with the emitter, and another with the absorber. Each atomic clock is synchronized to the master time standard using Einstein's synchronization method in inertial conditions before the turntable begins rotation. The coordinate time and proper time for each device is identical while the turntable is stationary relative to the laboratory. This ensures the embedded clocks are operationally synchronized.

The turntable can be modified so that the emitter releases photons only once per rotation of the turntable undergoing a constant angular velocity. This can be done, either electronically or mechanically, at some contact point when the emitter reaches position B to release photons omnidirectionally. Simultaneously, when the emitter detects the contact point, it generates a pulse of photons and a message in the coordinate time (as interpolated between the ns pulses with the adjusted coordinate messages from point O) and proper time of the embedded clock. When the absorber receives photons later from the emitter, it generates a message of reception in coordinate time (also interpolated between the ns pulses with adjusted coordinate messages) and its proper time at reception.

When duplicate observers concurrently record data from an event or phenomenon, the results must be identical, especially if all data are transformed to a common reference frame. If theory predicts different results for each observer concurrently witnessing an event, then the theory must be revised, either due to an erroneous assumption, an error in the derivation, or a misinterpretation of the event or phenomenon. In all cases, the theory must be self-consistent.

According to special relativity, the clocks on the moving turntable undergo a constant tangential speed of $\omega R = v$, but both moving clocks remain synchronized as they both move at the identical tangential speed v relative to the laboratory and its master clock. According to special relativity, the proper second of the moving clocks is dilated by the factor $\gamma = (1 - v^2/c^2)^{-1}$ compared to the master standard second. The broadcast coordinate time interval, Δt, between emission and absorption is larger numerically than the broadcasted proper time interval, or $\Delta t/\gamma$ for counted units.

For example, suppose the reader's wristwatch is slower by half than the standard clock. If the actual time elapsed was two seconds, the slow wristwatch counted only one second. If the reader moves at 1 m/s, the calculation predicts a distance of 1 meter traversed using the wristwatch, but actually two standard seconds in coordinate time elapsed for a traveled distance of 2 meters.

The emitter transmits one message at the instant of the photon emission to both recorders in the test setup. The absorber does the same upon photon absorption. The contents of each message do not change, no matter how fast or slow the messages arrived at the recorders, much like receiving letters in one's mail.

Suppose the turntable rotated faster to approach the limit that the arc AC was as long as the chord BC (*i.e.*, the tangential speed approached the limit of c). In the limit, $L = R\omega t = R\omega L/c$, so that $\omega = c/R$. The elapsed coordinate time between emitter and receiver would be the interval, L/c. There would be no elapsed proper time between emitter and absorber as the interval would be infinitely long as $v \to c$ and $\gamma \to \infty$. In this case, the emitter and absorber would have identical proper time tags, implying that the absorber received photons at B, but photons were transmitted from B. The theory is not self-consistent for distances via proper time.

This theoretical test shows time measurements may not obtain the actual fixed distance between an emitting source and absorbing target. In the first example, the distance between the devices is set at one meter (*i.e.*, $2R$ in **Figure 1**), but the measured distance is less than a meter (L in **Figure 1**) as the absorber rotated slightly during photon transmission. In the second case, the distance between the devices is fixed to be less than a meter, but the turntable rotates at the needed angular velocity for the measured time interval to predict exactly one meter distance. It was also discussed that moving clocks on the turntable will have slower proper time tags that will underpredict what the coordinate time tags would predict for distance between the devices. These examples demonstrate that the length between the source and detector was maintained during the measurement, but the velocities of each changed relative to each other, so that the measurement did not replicate the actual distance. The current meter definition allows any setup of precise measurements that produces the wrong meter.

Some may advocate that the current definition of the meter be limited to inertial cases, so that the examples given in this section would be excluded. The problem still remains that the theory does not produce the same results when using coordinate time versus proper time.

4. Einstein's Rod with Traversing Light Beams

In 1873, Maxwell advocated that the transmission of light would serve as a definition of length [3]. Einstein was aware that the electromagnetic equations do not accommodate vector velocity addition, which is a key property in Newtonian mechanics. He discussed this in his 1905 paper "On the Electrodynamics of Moving Bodies" [9] [10], which he established his two postulates for developing his special relativity theory.

To avoid lengthy translations from Einstein's German into English, the author chose the translation of Arthur I. Miller, who was a professor of physics at Harvard and Lowell Universities and who later transitioned as a historian of science due to his keen interest in Einstein's special relativity paper. In his book Albert Einstein's Special Theory of Relativity, he collected and translated many letters from Einstein and his contemporaries and translated Einstein's 1905 paper. The book's appendix contains his translation of that 1905 paper, which differs from previous (and, in places, unacceptable) English translations. Typographical errors in the original *Annalen* version are flagged, which went into the Teubner edition, and additional errors that appear in the Dover reprinted volume The Principle of Relativity. Einstein's and Sommerfeld's footnotes are annotated. This appendix will be the English translation with demarcations for sections, *§*, and line numbers within each section (a new section resets line numbering to 1).

Einstein's 1905 relativity paper does not cite any reference to other papers. Instead, it gives a logical discourse based on experiences, which could mislead. He considered simultaneous events occur with a time associated at a single location. Einstein's example was a train arrived at 7 o'clock, which he meant the small hand of his watch pointed at 7 as a train pulled in ([10], §1, lines 13-17). Einstein footnoted there is an inexactitude of two events occurring at (approximately) the same place. Note he did not define if the watch was a master time standard, or how horologists do synchronize mechanical time pieces to a time standard and monitor how that master time standard defines time for Earth. He admitted that an observer with a clock will not determine time independently of a remote event communicated to the observer by light ([10], §1, lines 24-32).

Einstein considered a clock A at position A and an identical clock B at location B, but there was no way to connect "A time" to "B time" with a common time for locations A and B ([10], §1, lines 33-41). He stated, "The latter time can now be defined by requiring that *by definition* the 'time' necessary for light to travel from A to B be identical to the 'time' necessary to travel from B to A. Let a ray of light start at the 'A time' t_A from A toward B, let it at the 'B time' t_B be reflected at B in the direction of A, and arrive again at A at the 'A time' t'_A." ([10], §1, lines 41-45) He concluded that the two clocks run in synchronization if ([10], §1, Equation (1.1))

$$t_B - t_A = t'_A - t_B$$

Note this involves one-way light transmissions. Einstein assumed this definition of synchronization was free of any contradictions. He further claimed that (1) if a clock at B synchronizes with the clock at A, the clock at A synchronizes with the clock at B, and (2) if the clock at A synchronizes with the clock at B and also the clock at C, then the clocks at B and C synchronize with each other ([10], §1, lines 48-60). He added the requirement ([10], §1, Equation (1.2)) that for length AB

$$\frac{2AB}{t'_A - t_A} = c$$

where c is the universal constant for light speed. Equation (1.2) is Einstein's synchronization procedure between two clocks. Note this time interval is a roundtrip transmission, and one divides the roundtrip interval by 2 and then adds this time span to the next broadcast of a master clock's time to set the remote clock's time tag for synchronization. He stated, "The 'time' of an event is the reading simultaneous with the event of a clock at rest and located at the position of the event, this clock being synchronous, and indeed synchronous for all time determinations, with a specified clock at rest." ([10], §1, lines 57-60) Although he did not state it, other physicists considered this to be coordinate time throughout the reference frame. In any case, the time t is the time recorded by stationary synchronized clocks throughout the resting system.

In Section 2, Einstein defined his two postulates of special relativity ([10], §2, lines 1-12). The translation lists

1) "The laws of which the states of physical systems undergo changes are independent of whether these changes of state are referred to one or the other of two coordinate systems moving relatively to each other in uniform translational motion." and,

2) "Any ray of light moves in the 'resting' coordinate system with the definite velocity c, which is independent of whether the ray was emitted by a resting or by a moving body."

He added, "Consequently,

$$\text{velocity} = \frac{\text{light path}}{\text{time interval}}$$

where time interval is to be understood in the sense of the definition in §1". Technically, postulate (1) is not universal for all equations of force. Newtonian forces are the derivatives of momentum, which a constant velocity $\boldsymbol{v}$ results in zero force for any inertial reference frame (those frames that translate linearly by a constant velocity). The Newtonian force equations remain the same for all inertial frames. This is not true for electromagnetic forces. In particular, the general Lorentz force $\boldsymbol{F}$ is

$$\boldsymbol{F} = q\boldsymbol{E} + q\boldsymbol{v} \times \boldsymbol{B}$$

where $\boldsymbol{E}$ is the electric field intensity, $\boldsymbol{B}$ is the magnetic field induction, q is the charge of the particle, and $\boldsymbol{v}$ is the velocity of the charged particle in the reference frame. For an observer fixed relative to the charges, only the electric term is detected, especially in electrostatic conditions where no magnetic field appears. If that observer is fixed to the reference frame, both electric and magnetic contributions are detected that result in a different observed Lorentz force. If a second observer is moving at a constant velocity $\boldsymbol{V}$ relative to the reference frame, that observer records a dissimilar Lorentz force that is $q\boldsymbol{E} + q(\boldsymbol{v} + \boldsymbol{V}) \times \boldsymbol{B}$. In this case, the observed state of the charged particle does depend on the choice of the reference frame, even if it has a constant translational velocity relative to a "resting" frame. Lorentz published the complete derivation for the Lorentz force in 1895 [17] by adding the electric force to Heaviside's earlier identification of the magnetic force to a charged particle [18]. In theory, one could use the Lorentz force to find the relative velocity between two inertial frames.

Einstein considered that a rigid rod at rest of length L is measured by a measuring rod that is also at rest. The observer would conclude the rigid rod was of length L. He then considered moving this rigid rod at a velocity $\boldsymbol{v}$ and orienting it parallel to its velocity. The observer of the moving rod would ascertain at time t using the stationary, synchronized clocks of the resting system where the ends of the rod would be located and then measure the distance between the predicted two locations. He predicted this distance would not be L. ([10], §2, lines 13-35) He then added two clocks to the two ends, A and B of the rod, that were synchronized with the clocks of the resting system. He further added two moving observers, each fixed to each moving clock. He stated, "Let a ray of light depart from A at the time t_A, let it be reflected at B at the time t_B, and reach A again at the time t'_A. Taking into consideration the principle of the constancy of the velocity of light we find that

$$t_B - t_A = \frac{r_{AB}}{c-v} \text{ and } t'_A - t_B = \frac{r_{AB}}{c+v}$$

where r_{AB} denotes the length of the moving rod—measured in the resting system. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the resting system would declare the clocks to be synchronous." ([10], §2, lines 47-54)

Einstein gave no derivation for the above equations. It is obvious his result does not support his Equation (1.1) that requires equal time intervals for both light transmissions between endpoints A and B. His claim is contradictory that stationary observers agree the clocks are synchronized versus moving observers arguing the clocks A and B are not synchronized. All resting clocks were synchronized initially. The two clocks at the rod's ends at A and B were synchronized to the resting clocks. There is no need for the moving observers to resynchronize the clocks attached to A and B, which Einstein stated the moving observers performed again ([10], §2, lines 45-47). According to Einstein's lemma (2), clock A is synchronized to the resting clocks and clock B is synchronized to the same resting clocks so that clock A is synchronized with clock B. When clocks A and B move, time dilation is the same for both clocks, so these two clocks still remain synchronized to each other even in the moving frame. The time t is from the resting clock time, which is the coordinate time of the resting system, and t is the time listed in the above equations. At time t, endpoint A is at the location of one resting clock, which broadcasts its location with its time. Likewise, endpoint B is at its location of another resting clock, which broadcasts its location with its time t, which is also the same t as the first resting clock where endpoint A happens to be at this instant. The two messages from endpoints A and B are the same for both stationary and moving observers (*i.e.*, a single coordinate point at instantaneous time t), so all observers have the two resting locations at the resting time t, which must be identical between observers. An inconsistent theory reveals some error exists, whether it is an error in a derivation, a false assumption, or misinterpretation of the experimental results. The next section will give an in-depth examination of the photons' path in Einstein's thought experiment.

5. Light Paths between Ends of Stationary and Moving Rods

Einstein's equation that follows his second postulate is equivalent to

$$\text{time interval} = \frac{\text{light path}}{\text{velocity}}.$$

In quantum mechanics, photons emitted from endpoint A combine with the reflective atoms at endpoint B to create excited atoms. New photons are emitted from those excited atoms at B to endpoint A, so that virtually all photons from B follow Snell's law to A. In the roundtrip test, one beam traverses the distance $A \to B$ and the other beam $B \to A$.

As Einstein stated, orient a uniformly moving rod with its length parallel to its velocity, v, relative to the resting frame. When the rod is resting or stationary, the photons traverse a distance of L, the length of the rod. The roundtrip distance for the photons over the resting rod is $2L$, and each time span across is L/c.

However, photons traveling with or opposite to the moving rod experience different distances. For example, runners know that a footrace covers more (or less) ground if the finish line is moved away from (or toward) the runners <u>during</u> the race. This happens also in Zeno's paradox of Achilles and the tortoise. Zeno's paradox relates how the tortoise challenged Achilles to a race with a head start of L, claiming the tortoise would win. Achilles conceded the race without running due to the tortoise's logic that Achilles was always behind the tortoise for an infinite number of time intervals, despite that Achilles was faster than the tortoise. Zeno did not consider that a finite length can be divided into an infinite number of lengths, but the sum of those infinite spans would still be a finite length. Algebra can directly solve the distances that photons traverse to overtake the opposite end of the moving rod. In general, the photon's speed from endpoint A to endpoint B is c_{AB}, and the photon's speed from endpoint B to endpoint A is c_{BA}. In the resting frame, the rod moves with velocity v so that the endpoint B is at B' where the photons from A overtake the receding endpoint at B' in the time span of Δt_{AB} as shown in **Figure 2**. Also, the newly emitted photons from endpoint B intercept the approaching endpoint A at the position A' in the time span of Δt_{BA} as shown in **Figure 3**.

Solve for D in $D/v = (L + D)/c_{AB}$ and replace D in $L_{AB} = L + D$.

Solve for d in $d/v = (L - d)/c_{BA}$ and replace d in $L_{BA} = L - d$. The resulting distances in the resting frame are:

$$L_{AB} = \frac{Lc_{AB}}{c_{AB} - v}, \text{ and} \tag{1}$$

$$L_{BA} = \frac{Lc_{BA}}{c_{BA} + v} \tag{2}$$

It is immediately apparent that $L_{AB} > L$ if $c_{AB} > v$, and $L_{BA} < L$ if $v > 0$. Moreover, if the individual one-way speed of the photon is the standard c, the roundtrip distance is greater than $2L$. Simply add Equations (1) and (2) with the standard c for the roundtrip distance:

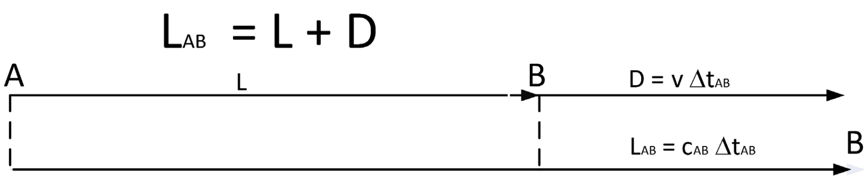

Figure 2. Photons overtaking receding endpoint.

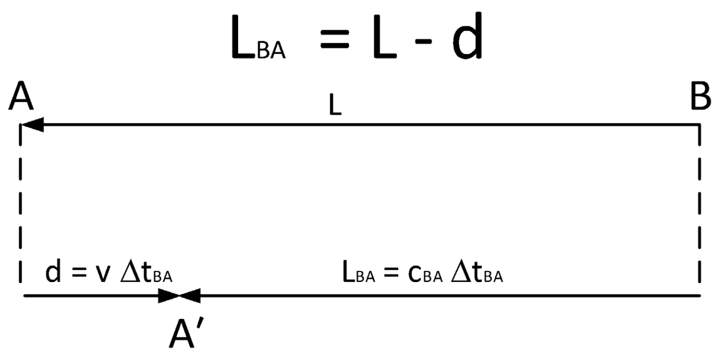

Figure 3. Photons intercepting approaching endpoint.

$$\frac{Lc}{c-v}+\frac{Lc}{c+v}=\frac{2Lc^2}{c^2-v^2}=\frac{2L}{1-\frac{v^2}{c^2}}=2L\gamma^2>2L \tag{3}$$

With a universal c, the photons have a longer roundtrip distance in the resting frame for the uniformly moving rod than if that rod was stationary. Even length contraction from special relativity can not resolve this problem, as length contraction is γL. If one divides the one-way distances L_{AB} and L_{BA} by their respective photon speeds, unequal time spans to intercept the opposite ends of the moving rod occur for the stationary observer while the observer fixed with the moving rod records equal time spans. Einstein correctly required equal time spans, $\Delta t_{AB} = \Delta t_{BA}$, in his Equation (1.1). All observers must concurrently record equal time spans over opposite traverses as mandated by Einstein's first postulate when witnessing the same event. This contradiction implies the theory is not consistent.

Equations (1) and (2) give the actual distances in the resting frame that photons emitted from A or from B travel to the opposite end of the uniformly moving rod. The time to complete a one-way traverse is to divide the respective one-way photon speed into each distance. Since the resting observer and uniformly moving observer must record the same one-way time span, equate their Δt_{AB} time intervals and, next, their Δt_{BA} time spans.

$$\frac{L_{AB}}{c_{AB}}=\frac{L}{c_{AB}-v}=\frac{L}{c}\Rightarrow c_{AB}=c+v\text{, and} \tag{4}$$

$$\frac{L_{BA}}{c_{BA}}=\frac{L}{c_{BA}+v}=\frac{L}{c}\Rightarrow c_{BA}=c-v \tag{5}$$

Equations (4) and (5) show that photon speeds obey vector velocity addition for moving transmitters. The emitted photons from endpoint A obtain the additional $+v$ in the one-way velocity to overtake endpoint B. The second beam of ejected photons from endpoint B after absorption gets the opposite one-way velocity, $-v$, in the antiparallel direction of the uniformly moving rod.

Einstein's equation (1.2) applies to roundtrip transmissions of photons, which agrees with our experiences when the distance between A and B remain fixed during the roundtrip transmissions, yet Equation (1.1) was required for one-way photon transmissions over identical distances. Photon velocities obey vector velocity addition in one-way transmissions. Roundtrip transmissions have an average speed equal to the standard constant speed c as $\left(\left[c+v\right]+\left[c-v\right]\right)/2=c$, as the roundtrip time interval always remains the same. When Einstein replaced any c_{AB} or c_{BA} with c, he essentially was assuming what he was proving before getting the result. He did uncover the mathematical contradiction, but he interpreted that as simultaneity versus nonsimultaneity between observers. However, the theory needs revision.

Einstein's second postulate is an excellent approximation due to the high speed of photons, but it is not exact in general. For example, his last section of his 1905 paper predicted that masses measured longitudinally versus transversely along the electron's velocity will be different ([10], §10, lines 1-81). He mathematically derived the dynamics of the slowly accelerated electron, which resulted in a mathematical difference in longitudinal and transverse mass measurements. No one has published any counter derivation showing any mathematical error in his paper. No national institute of standards has published any finding that a test mass has a diurnal variation, which is expected as Earth's rotation would displace the laboratory's local vertical for a hanging test mass with a component parallel or perpendicular to Earth's orbital velocity. With no known experimental verification of a different mass measurement depending on the direction of a moving mass and no error in the derivation, one must question if one or both of Einstein's postulates are not precisely true.

Arthur I. Miller attempted to show how Einstein could have derived his claim that events simultaneous in one inertial frame are not simultaneous in another inertial frame ([10], p. 264-265). Miller considered a rigid rod at rest in a resting frame that had length L_0 as congruently measured with a "ruler" by an observer in the resting frame. This is a nonsimultaneous activity with end A compared to the "ruler" at one time and another comparison of end B to the stationary "ruler" at another time to get a length of L_0. Einstein defined such a test of the moving rod to have a constant positive velocity v moving in the direction of increasing x and with the rod oriented parallel to the resting frame's x axis. If end A is where the moving rod is at x_1 and end B is at x_2, let $x_1 < x_2$ without loss of generality (if not, swap the subscripts and proceed). Let the resting frame's time t at its origin (*i.e.*, $x = 0$) be transformed to the moving frame's time τ by $\tau = \gamma t$. With $v < c$ in the first frame, length contraction still maintains the moving length $\zeta_2 - \zeta_1 > 0$ as the resting length $x_2 - x_1 > 0$. Miller had two observers fixed in the moving frame get the ζ coordinates of ends A and B of the rod at the same coordinate time of the moving frame. Miller stated the two spatially separated observers mark the position of the ends of the moving rod simultaneously, which is incorrect as the observers are marking synchronously in τ coordinate time.

Miller derived his Equation (C) shown below and inserted it into another transformation equation to get the length contraction equation. He assumed $t_1 = t_2$ to get:

$$\tau_2 - \tau_1 = -\frac{v}{c^2}\left(\zeta_2 - \zeta_1\right) \tag{C}$$

No mathematical fallacy was derived, such as division by zero, but it is a physical fallacy as the time differences and spatial differences are individually positive. Miller's equation has the right side negative compared to the left side that is positive. All transformation equations in Einstein's 1905 paper are derived by Einstein from his second postulate of relativity, which Miller's derivation indicates a contradiction that shows Einstein's postulate is not technically or completely correct. One resolution of the apparent contradiction is that the lengths and times are in the moving frame 2 attached to the moving rod. Then, the velocity of the moving rod in frame 2 is zero, making the case that $\tau_1 = \tau_2$, or that an instant of coordinate time in frame 1 is an instant of coordinate time in frame 2, which now contradicts Einstein's claim of simultaneity versus nonsimultaneity between frames. Either way, Miller's derivation contradicts the second postulate or Einstein's simultaneity claim.

One can test directly if Einstein's second postulate is totally correct in one direction. According to special relativity, length contraction is $\Delta L_{\text{resting}} = \gamma \Delta L_{\text{moving}}$ and time dilation is $\gamma \Delta \tau_{\text{resting}} = \Delta \tau_{\text{moving}}$. Let the integer $k = 299{,}792{,}458$, $\Delta L_{\text{resting}} = 1$ meter, $\Delta \tau_{\text{resting}} = 1$ second. As the speed of light is currently defined:

$$c = k\frac{\text{meters}}{\text{second}} = k\frac{\gamma \Delta L_{\text{moving}}}{\Delta \tau_{\text{moving}}/\gamma} = k\gamma^2 \frac{\Delta L_{\text{moving}}}{\Delta \tau_{\text{moving}}} = c' \tag{6}$$

This means that the speed of light in a constant moving inertial frame has shorter meters and longer second intervals than a resting frame such that one-way light speeds do not have the same universal constant as $k \neq k\gamma^2$. For example, the orbital speed of Earth at apogee and perigee is respectively about 29,300 and 30,300 m/s, which the numerical difference of $k\gamma^2$ would be about 0.199 between these orbital locations. Is Einstein's second postulate always correct and is BIPM's mandate correct that light speed must be exactly 299,792,458 m/s, or is special relativity correct that light speed will have an annual variation on Earth due to its orbital speeds?

Equation (6) demonstrates that special relativity maintains a variable light speed depending on a moving inertial frame's speed relative to a "resting" reference frame. This finding contradicts the second postulate.

A simple test can demonstrate if photons obey vector velocity addition. Set a laser to point horizontally at a partially silvered mirror that is angled at 45 degrees relative to the local plumb line. The reflected beam is aimed vertically to a hemisphere mirror that is a distance, d, of about 10 meters above the partially silvered mirror, so that the hemisphere is centered along the line of transmission. The vertical beam is reflected from the hemisphere of 2 cm radius, r, to the partially silvered mirror below, and some light is transmitted through it to the floor below. Observe if the impact point varies over time as Earth rotates. The hemisphere mirror has a sideway displacement due to the velocity $\boldsymbol{v}$ of Earth around the Sun, forcing the beam to miss the nadir of the hemisphere mirror by an angle of $\theta \approx \sin(\theta) = v\Delta t/r = vd/(cr)$ in radians. The reflected beam from the hemisphere mirror should touch the floor by a maximum of $d\,\theta = 50$ cm from the plumb line assuming a maximum $v \approx 30{,}000$ m/s from Earth's orbit. Even assuming a perpendicular projection of Earth's velocity to the 10 m arm to be a slower speed of 15,000 m/s, this would still cause a 25 cm displacement from the plumb line. If no observed displacement occurs over a day, then the beam is constantly touching the nadir of the hemisphere mirror. This would prove photons obey vector addition of velocity as the beam changes direction and magnitude to remain on the nadir due to the changing horizontal displacement from Earth's orbital velocity (*i.e.*, $\boldsymbol{v}_{\perp}\Delta t$).

The laboratory setup on Earth's surface for this test is in a noninertial frame. All tides show a surface laboratory is noninertial as the walls rotate due to Earth's spin. When high precision is not needed, Newton's equations of motion are satisfactory. For high precision, artificial force terms need to be included in the theory to account for the frame's movement. For example, a Foucault pendulum rotates its plane from Earth's spinning where the rate of change in a vector relative to space (*i.e.*, fixed stars) is equal to the rate of change of that vector in the body frame plus the cross product of Earth's rotation rate with the vector (the last product is related to the Coriolis force).

The Coriolis force is too small compared to photon speed, and the freely falling Earth centered frame does not include Earth's orbital velocity. The worst deviation of the Coriolis effect is on the equator, which has a tangential speed of 465 m/s. Orient a one-meter leg longitudinally, and Earth's rotation would displace the far end 1.55 μm. The hypotenuse that the photon travels is longer than 1 m by 1.20E−12 m. When technology measures light speed with more than 12 significant figures, the Coriolis force will affect a measured speed.

The next freely falling frame is the center of mass for the Earth-Moon system, denoted here as the E-M barycenter. Earth's rotational axis is tilted 23.44˚ from the ecliptic plane, and the E-M barycenter is located approximately 3/4 radially from Earth's center to Earth's surface, but the E-M barycenter plane is 5.14˚ from the ecliptic and the Earth rotates around it in 27.32 sidereal days. Again, the E-M barycenter is not able to provide Earth's orbital velocity, so the next freely falling frame would be the solar nonrotating barycenter (SNB) to be a sufficiently inertial frame. The velocity, V, of the E-M barycenter in the SNB frame can be found at any time and the orientation of the local vertical can be calculated after many transformations and rotations. The sine projection of the orbital velocity to the local vertical will be the perpendicular component needed to predict the directional offset that a beam of photons will miss the nadir. One example is given above resulting in a floor displacement of 25 cm if the perpendicular velocity is 15000 m/s to the local vertical. If no reflected beam is displaced over a day, then the photons must be getting an additional velocity from the mirror added to its photon emission velocity so that the beam has the directional change and magnitude to keep touching the nadir.

A more rigorous demonstration reveals that photons precisely obey vector velocity addition. The Laser Interferometer Gravitational-Wave Observatory (LIGO) is a large-scale physics experiment with three observatories to detect cosmic gravitational waves [19] Twin observatories are near Hanford, Washington, and Livingston, Louisiana, with 4 km long arms within ultrahigh vacuum chambers allowing laser beams to detect gravity waves. VIRGO, located in Italy, joined LIGO in May 2007. **Table 1** lists the locations and azimuth in degrees counterclockwise of each arm from local east at each facility.

Each LIGO facility uses a continuous laser beam that is amplified from 40 watts to 750 watts with power reflecting mirrors. The signals are also magnified with signal recycling mirrors. LIGO has enhanced vibration absorption mechanisms to remove ground vibrations, tremors, solid Earth tides, etc., to isolate the signals. To increase the arm lengths from 4 km, Fabry-Perot cavities are installed at the beam splitting mirror and at the hanging reflection mirror at the end of each arm so that 280 reflections inside the cavities increase the effective arm length to almost 1120 km. An ultrahigh vacuum is maintained so that any gaseous molecule is removed promptly to avoid interference with the photon beams. Also, one of the split signals is inverted so destructive interference is created when recombining the two beams. The original Michelson-Morley interferometer gives constructive interference, but this enhancement allows far easier detection of variations after merging signals. "At its most sensitive state, LIGO will be able to detect a change in distance between its mirrors 1/10,000th the width of a proton." [19]. (*i.e.*, resolution of 1.7E−19 m.)

There were several runs, which LIGO operated continuously: 1) 12 September 2015 to 19 January 2016, 2) 30 November 2016 to 25 August 2017, 3) 1 April 2019 to 30 September 2019, and 4) 1 November 2019 until 27 March 2020, covering over 22 months. In a freely falling, nonrotating frame, which Einstein stated is sufficiently inertial, choose Earth's center of mass as the origin and pick a time during LIGO's operation. Designate this inertial Earth center-of-mass, nonrotating frame as ECN.

As all facilities are in the northern hemisphere, the Earth's rotation will displace the southern end of all arms further eastward than the northern end of the arm. **Figure 4** shows this effect where the left side of every triangle is the arbitrary azimuth of the arm, but Earth's rotation displaces the opposite end of the arm eastward, or to the right in each figure. The 4 km arm is L with the azimuth at emission (going northward) or at reflection (going southward), but the photon's actual displacement due to Earth's rotational effect is D. Note that V_S is greater than V_N depending on the geodetic latitude of each arm's end.

Table 1. LIGO observatories with arm orientations.

LIGO	Location		Local Azimuth	
Facility	Latitude	Longitude	X Arm	Y Arm
Hanford	46˚27'19"N	119˚24'28"W	126˚	216˚
Livingston	30˚33'46"N	90˚46'27"W	198˚	288˚
Virgo	43˚37'53"N	10˚30'16"E	71˚	161˚

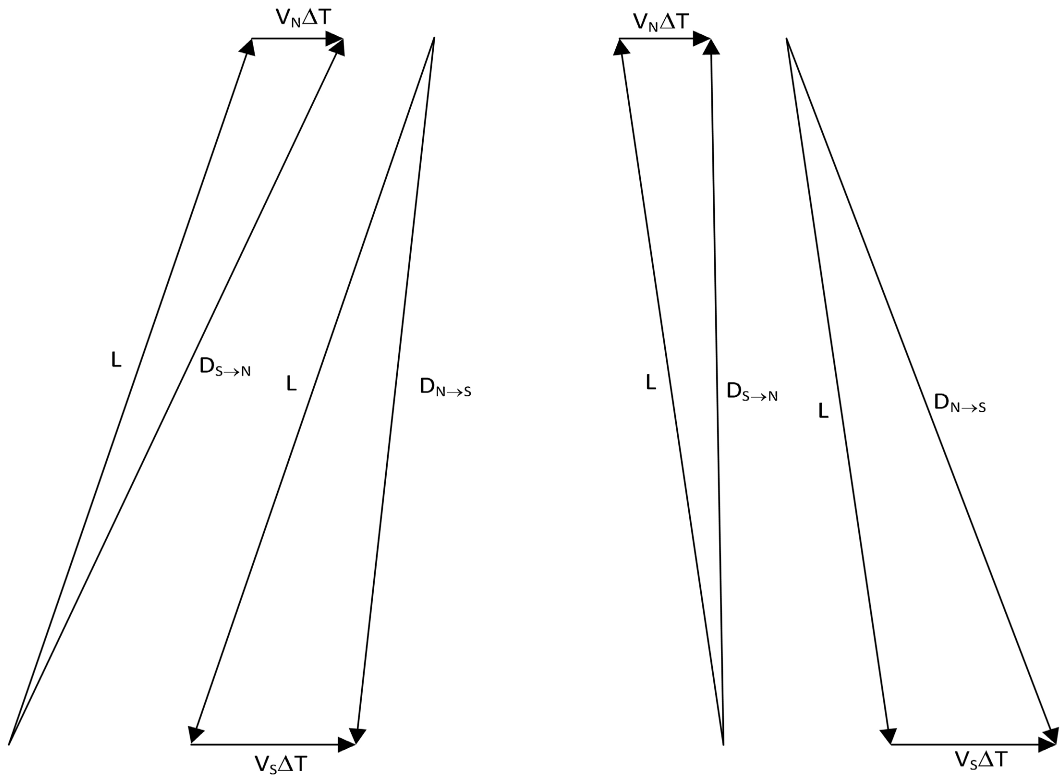

Figure 4. One-way photon displacement from rotation.

The roundtrip distance that photons travel along the x-axis is not equal to the roundtrip distance of the y-axis photons. Both x-axis and y-axis arms are joined at the same point where the beam is split and where the two split beams are recombined, but the opposite ends of both arms are at different latitudes where Earth's rotation from west to east will give different tangential velocities of these opposite ends in the inertial ECN frame. Not only is the outgoing distance unequal to the incoming distance for each arm in the ECN frame, but the difference in the tangential velocities at the opposite ends in the ECN frame prevents the roundtrip distances of the photons in the x-axis arm to be the same as the y-axis arm. All LIGO output over 22 months of continuous operation have resulted in the split beams arriving at the same simultaneous time upon interception (except for the many spontaneous data glitches and the rare detections of gravity waves). The only way for split beams to travel over different distances in the ECN frame and arrive at identical time spans is for photons to have different one-way velocities upon emission.

LIGO observatories are enormous versions of the Michelson-Morley interferometer, which produced a null result when searching for an ether to explain light propagation [20]. One standard answer is length contraction along the axis parallel to the constant velocity will shorten the arm so the roundtrip traverse is identical to the distance traversed in the perpendicular arm. When the split beams are recombined, complete constructive interference occurs. Only length contraction from special relativity is applied to the arm parallel to the velocity of the apparatus. Time dilation should also be applied to the photons traversing the parallel arm, which would make that photon speed different than c. Equation (6) predicts from special relativity that photon speed is a constant of $\gamma^2 c$ in the parallel arm and different than the standard c in the perpendicular arm. If one applied special relativity only to the arm parallel to the Earth's orbital velocity, then destructive interference will be the output. The transverse beam will have a different arrival time at the recombination point due to a standard c speed than the parallel beam with $\gamma^2 c$ speed, even if the roundtrip traverses are identical after length contraction is applied to the parallel arm. The 22 months of continuous output of simultaneous arrival at the recombination point conflicts with this scenario.

The LIGO observatories are noninertial facilities and fixed on Earth's rotating surface. The exact transformation between ECN and the rotating Earth Center, Earth Fixed frame (ECEF) in geographical coordinates is the rate of change of a vector as shown below as an operator equation acting on a vector [21]:

$$\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{ECN} = \left(\frac{\mathrm{d}}{\mathrm{d}t}\right)_{ECEF} + \boldsymbol{\omega}\times \tag{7}$$

where $\boldsymbol{\omega}$ is the angular velocity of Earth's rotation in the cross product. UT1 will be the worldwide time standard for both ECN and ECEF as maintained by BIPM by synchronizing all time standards in all timing institutes. Photons are emitted at the splitting location to traverse either in the x-axis arm or y-axis arm. At the opposite ends of the axes, the hanging mirrors absorb the photons and emit new photons from the excited atoms to return to the recombination point. Let $R_{x,o}$, $R_{y,o}$, $R_{x,r}$, and $R_{y,r}$ denote the displacement vector the photons traverse with footnote x denoting the x-axis, y the y-axis, o for outward direction from the splitting point, and r for the return to the recombination point. The second postulate states "Any ray of light moves in the 'resting' coordinate system with the definite velocity c, which is independent of whether the ray was emitted by a resting or by a moving body." ([10], §2, lines 9-11). When applying the operator Equation (7) to any of the four displacements, the ECN derivative should be the standard c velocity within an inertial frame, but the cross product term differs with direction and with geodetic latitude of the emission point that is the base of each displacement vector. Note that $\boldsymbol{\omega}\times\boldsymbol{R}_{x,o} \neq \boldsymbol{\omega}\times\boldsymbol{R}_{y,o}$ due to differences in the azimuths of the outgoing photons in the arms, and $\boldsymbol{\omega}\times\boldsymbol{R}_{x,r} \neq \boldsymbol{\omega}\times\boldsymbol{R}_{y,r}$ due to different latitudes where the photons were emitted as located at the base of the vectors. With the reputed sensitivity of ΔL = 1.7E−19 m, LIGO is affected by the Coriolis effect, but nothing on the website or in the literature gives any indication that LIGO compensates for it. The Coriolis effect is not ignorable with LIGO's precision that exceeds 13 significant figures. The 22 months of continuous observations of simultaneous arrival of photons through each arm in the ECEF frame require that the one-way photon speeds within the ECEF frame, dR/dt, vary within each arm and direction so that the same time interval within ECEF remains the same in LIGO. That also occurs in the ECN frame as LIGO's photon emitters (*e.g.*, splitters and mirrors) move within ECN. LIGO shows that the theory must be revised. Also, the perpendicular component of Earth's orbital velocity relative to each axis is different and constantly changing, causing each beam to miss the arm's opposite end up to a maximum of 40 cm. LIGO makes no compensation for Earth's orbital velocity affecting each arm's beam. One explanation for equal time transmissions to occur with unequal displacements in inertial frames is photon velocities obey vector velocity addition with moving photon sources. A null result in the laser test using a hemisphere mirror will also reveal that photon velocity is the sum of the standard velocity and the velocity of the emitting source.

Vector addition in three dimensions of two displacements is $\boldsymbol{Z} = \boldsymbol{X} + \boldsymbol{Y}$. As LIGO demonstrated equal time spans for photons to traverse through either arm as $\Delta t_x = \Delta t_y$ (or $\mathrm{d}t_x = \mathrm{d}t_y = \mathrm{d}t$), then the derivative of the addition law of displacements is:

$$\frac{\mathrm{d}\boldsymbol{Z}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{X}}{\mathrm{d}t} + \frac{\mathrm{d}\boldsymbol{Y}}{\mathrm{d}t} \text{ or } \boldsymbol{c}' = \boldsymbol{c} + \boldsymbol{v} \tag{8}$$

So, photons are emitted with a combined one-way speed of the standard light speed plus the velocity of the emitter relative to the resting frame. This verifies by (8) how photons obey vector velocity addition according to (4) and (5) from the ends of the uniformly moving rod.

For photons traveling by a roundtrip over a finite number of traverses, the average speed of the photons is the standard speed of light, c. This is already shown for n = 2 traverses as $\left(c_{AB} + c_{BA}\right)/2 = 2c/2 = c$ in Equations (4) and (5). No n = 1 case exists for a roundtrip, because a reflection requires a minimum of two traverses. Circular tracks are made up of contiguous atoms as a photon is emitted by an atom and absorbed by the nearby atom so that the segments on the order of Avogadro's number approach a circular path. The proof of an average speed of photons in a roundtrip can be done by induction. The initial case for n = 2 is true by (4) and (5). Assume the average speed over a roundtrip excursion for photons is c with n traverses. Let $V_n = v_1 + v_2 + \cdots + v_n$, which is the vector sum of all the velocities from emitters 1 through n that starts from the origin and ends at the end of leg n. The average speed over n + 1 traverses is:

$$\frac{\sum_{i=1}^{n+1} c_i + \sum_{i=1}^{n+1} v_i}{n+1} = \frac{(n)c}{n+1} + \frac{V_n}{n+1} + \frac{c}{n+1} + \frac{v_{n+1}}{n+1} = c \tag{9}$$

The vector sum of V_n and v_{n+1} is the velocity from the origin to the end of leg

n and through the start of leg $n + 1$ back to the origin. The sum of V_n and v_{n+1} equals zero with no net displacement by $n = 2$ case. Reducing the roundtrip from $n + 1$ legs into 2 legs proves the $n + 1$ case by (9). This proof illustrates that the second postulate of relativity is too restrictive and only applies to the case when the photons complete a roundtrip circuit, which the average speed is the standard c.

6. Issues of Varying Photon Speeds

If photon speeds obey vector velocity addition, then a faster speed would elongate the wavelength without changing the frequency so that $c' = f\lambda'$, where c' is any photon speed, f is the associated frequency and λ' is the wavelength. This would mean that photons behave now the same in a vacuum as in a transparent medium. The photon speed in a medium is $c/n = c'$ from experiments (n is the index of refraction and $n \geq 1$ with 1 for a vacuum), and only the speed and wavelength changed in the medium. When photons penetrate different layers of transparent media, the speed and wavelength change with each material, but no waves stack up at the interfaces between materials. This also applies to photons exiting a material into a vacuum or vice versa.

If photon speeds in a vacuum conform to $c' = f\lambda'$, then photons gain speed the farther they move from a gravitated body. In the Pound-Rebka test, photons that traveled up 22.5 m to the absorber had a longer wavelength and a shorter wavelength when traveling down. As gravity weakens with a photon moving away from the mass, there is less deceleration on the photon, which can then move faster. Beams bent by the Sun demonstrate this. Despite the tiny size of a photon, the side of the photon opposite to the Sun has a slightly greater velocity than the side facing the Sun, which makes the beam bend around the Sun. This is much like a tank with its left tread traveling slightly faster than the right tread, so the tank will turn right from a straight heading. This effect impacts all cosmological models and observations.

For example, a redshift in stellar transmissions is an increase in the measured wavelength, which is inferred that the frequency decreased when assuming a universal constant light speed. If photons gain speed the farther away they go from a star, a longer wavelength is created in all stellar emissions. If the net velocity of the star relative to Earth is great enough in the opposite direction, the effect would overcome the lengthening of the wavelength by gravity to produce a shorter wavelength seen on Earth. Under this condition, it would produce the inferred "blueshifts" that are infrequently recorded. If the stars and galaxies have a symmetric normal distribution of velocities toward or away from Earth, then most stellar photons will have lengthened wavelengths (*i.e.*, "redshift") and fewer photons will have shorter wavelengths (*i.e.*, "blueshift") as the needed stellar velocities to overcome a gravitational lengthening would be in one tail of the distribution. This may rectify a controversy over the Big Bang model where some observed galaxies have "redshifts" that place them outside the sphere of observability based on the probable age of the universe according to Hubble's law. In any case, this will require careful analysis by cosmologists as this research indicates "blueshift" observations are due to stellar objects moving from Earth—not towards Earth.

Also, the photon deflection of 1.75" when grazing the Sun can be obtained from Newton's solution of the two-body problem by allowing the Sun and photon to move in hyperbolic orbits against each other. When starting with a photon infinitely far from the Sun, their barycenter is also infinitely far between them. The same gravity the Sun pulls on the photon is also the same pull of the photon exerted on the Sun. This is a two-body problem, which the two bodies execute hyperbolic orbits about the barycenter. As the photon approaches the solar system, the barycenter is moving inside the Sun. The details are obtained with the photon and solar deflection measured against the orbital asymptotes to produce a total deflection of 1.75". [22] Previous classical derivations assumed the Sun was stationary and only the photon is in orbit, which produces half of the observed deflection.

If particles obey vector velocity addition, then a high particle speed could cause misinterpretation of a particle as being different. This may affect at least two particles in the standard model of particle physics. The muon and tauon are identical in all characteristics to the electron, except their masses are different according to special relativity. If a high-energy particle after a collision can exceed the standard light speed, then $E = mc^2$ may misinterpret high speed electrons as muons or tauons for the same electron having a rest mass. A faster photon speed than the standard light speed c would explain all the entanglement experiments where two or more objects at significantly long distances, d, interact with each other within a measured short time interval, Δt (*i.e.*, $d > c\Delta t$).

If high speed or high energy particles can exceed the standard light speed, then a different explanation of high energy γ rays impacting the atmosphere should be considered. High energy cosmic radiation impacts atmospheric molecules creating pions that decay into muons within 26 ns. Electromagnetically, these muons interact with other atmospheric particles before disappearing with a typical lifetime of about 2.194 μs. With a velocity just below c, it would take about 50 μs to reach sea level. The current explanation is that time dilation allows the muons to reach sea level while classical mechanics would predict hardly any would be detected [23]. If electron velocity can exceed c, the cosmic rays may collide with the atmosphere to eject electrons with higher velocities up to say $25c$, which would allow those electrons on average to reach sea level without time dilation.

If photon speeds obey vector velocity addition, then some constants in physics are no longer constants. If photon speeds change c in a vacuum, then permittivity of free space, ε_0, must be a variable as the product $4\pi\varepsilon_0 c^2$ is defined to be exactly 1E7 [24]. The definition of the fine structure constant is $\alpha = e^2/(2\varepsilon_0 hc)$ where e is the elementary charge and h is Plank's constant with ε_0 and c defined. Electric charge e is $1.602176634 \times 10^{-19}$ C, which is a static value for electrons and protons. Then, α equals $2\pi ce^2/(h\ 1E7)$, which implies that either the fine structure constant could vary or Plank's constant h could vary inversely as c varies. Electromagnetic concepts should be revised since a moving photon emitter could change the speed of photons relative to the reference frame and change ε_0. Current interpretation is that any moving photon source has no effect to change the speed of light, which is usually adequate for results of 5 or 6 significant figures and the emitters move with a velocity < 1000 m/s. Maxwell's electromagnetic equations assume c and ε_0 are constants. Curls and divergences are special derivatives that would produce additional terms from the products containing c or ε_0. It is beyond the scope of this paper to delve into the general derivation by expanding Maxwell's equations when c and ε_0 are variables.

If photon speeds obey vector velocity addition, then the meter can not be defined by a one-way photon transmission, as the velocity of the emitter relative to the observer must be compensated in the reference frame. One may have to reestablish the physical meter as the international standard for length.

If photon speeds can exceed c by the emission from a moving satellite, then an alternative explanation of the flyby anomaly is possible. Several deep space satellites have been observed to have a small increase in velocity when nearing the perigee of a flyby of Earth, which often is in the range of 3 to 11 mm/s or about 2E−6 = $\mathrm{d}V/V$. An empirical equation for the anomalous flyby velocity change has been proposed [25], which is

$$\frac{\mathrm{d}V}{V}=\frac{2\omega_E R_E\left(\cos\theta_i-\cos\theta_0\right)}{c} \tag{10}$$

where ω_E is the Earth's angular velocity, R_E is the Earth's equatorial radius, and θ_i and θ_0 are the inbound and outbound equatorial angles of the spacecraft. The tangential velocity at the equator due to Earth's rotation, $\omega_E R_E$, is 465 m/s. Depending on the cosine projection, Equation (10) is 3.1E−6 or slightly less. Equation (10) predicts that the transmitted photon arrived at the ground station slightly sooner than anticipated, which would indicate the photons had a slightly faster velocity (~(1 + 3E−6)c). The effect is tiny, but present, so long as other effects such as atmospheric drag do not cover the effect.

If satellite transmissions obey vector velocity addition, then prograde satellites from the west of a ground station will have a slightly faster transmission velocity and satellites east of the ground station will have a slightly slower transmission velocity. The Global Positioning System (GPS) could improve its GPS satellite orbits by making this compensation in its pseudoranges, which are equivalently the time difference between transmission and reception of the GPS signals. This would also affect ultraprecise timing institutions, which use a GPS satellite in common view to compare between the institutions. Typically, Lab A differences its time with the GPS time from a common view satellite over several minutes for smoothing, and Lab B does the same with the same GPS satellite. Then, each lab exchanges the data. $\mathrm{Lab_A}$ – GPS and $\mathrm{Lab_B}$ – GPS data are calculated, and these sets of data are differenced to get $\mathrm{Lab_A}$ – $\mathrm{Lab_B}$. As shown in the previous paragraph, the effect can be as large as 3E−6. This would affect ultraprecise timing institutions that maintain picosecond precision with their atomic clocks.

Some may argue that prior tests measured the same standard speed of light emitted by a moving particle. For example, γ rays from the decay of π^0 mesons with more than 6 GeV were measured absolutely by timing over a known distance [26]. The test was intended to measure $c + kv$, and the result was $k = (-3 \pm 13) \times 10^{-5}$ for mesons moving near light speed ($\gamma > 45$). The team used two detectors spaced 31.450 m apart to measure the time interval the γ rays traveled. The first detector absorbed the photons from high-speed γ rays and emitted new photons at the usual speed of light. The measured speed recorded by the second detector after light passed through the first detector (*i.e.*, absorbed and reemitted at c) was the standard light speed. This and other tests must be reexamined carefully to ensure that the photon speeds were directly measured without interception to eliminate any misinterpretations of the results.

If photons obey vector velocity addition, then an additional interpretation of the Sagnac effect is needed for some applications. Light is inserted into a ring interferometer and splits in opposite directions. The beams exit the ring at the start/end point and undergo interference. The destructive interference determines how much the ring interferometer rotated after the beams were split. The diagram on the left of **Figure 5** [27] shows a nonrotating ring of radius R would output constructive interference as each beam would travel the same length of $2\pi R$ and exit at the same time. If the ring interferometer rotated as shown on the right side of **Figure 5**, one beam would travel further than the other, so both beams would exit with destructive interference.

It is easy to see as an external observer that one beam takes a longer trip than the oppositely traveling beam when the interferometer rotates, being the standard answer of the Sagnac effect. The rotating interferometer is a frame that is sufficiently inertial when comparing the constant angular velocity ω with the speed of light. The ring is a conduit that bends both beams to traverse in a

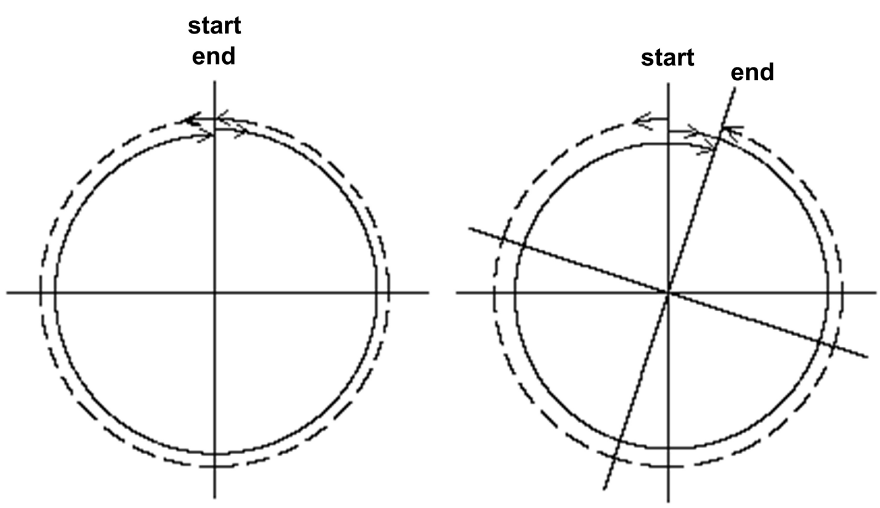

Figure 5. Sagnac effect in circular loops.

circular path. A perpendicular acceleration is required to force the light beams to traverse a circle in the rotating interferometer. Such an acceleration is no different than the perpendicular gravity that exists for the laboratory's light speed test, as any acceleration to the linear paths will be equal to both beams, so the acceleration effects cancel out when differencing the two beams.

Internally, Equations (1) and (2) give the lengths that the beams traverse in the circular interferometer with counterclockwise length for L_{AB} and clockwise length L_{BA}. For a circular traverse, set $L = 2\pi R$ and $v = \omega R$. In internal navigation applications, $c_{AB} \approx c_{BA} \approx c$. The time difference between the combined beams is:

$$\Delta t \approx \frac{2\pi R}{c}\left[\frac{c}{c-\omega R}-\frac{c}{c+\omega R}\right]=\frac{4\pi\omega R^2}{c^2-\omega^2 R^2} \tag{11}$$

If an observer is fixed with the rotating ring, special relativity requires that each beam originates at the same constant light speed c, and each beam travels the circumference of $2\pi R$ inside the inertial rotating frame. This set of assumptions of special relativity for the rotation of the ring undergoing constant angular velocity, ω, would predict both beams exit simultaneously (*i.e.*, constructive interference in the output), but reality contradicts that. Equations (4) and (5) show that the inertial ring interferometer undergoing constant angular velocity ω will impart different one-way velocities to the light beams at the splitter for net velocities of $c + \omega R$ and $c - \omega R$. This explanation will obtain Equation (11) in the rotating ring frame as output for the internal observer. The identical observed interference of output is witnessed for both observers—the external observer fixed to the frame and the internal observer fixed to the circular interferometer. Thus, light obeys vector velocity addition in both linear and rotating reference frames.

As stated before, the second postulate is an excellent approximation, but not exact. The prediction of general relativity for the Mercury perihelion shift is 42.98"/cy, but Meisner, Thorne and Wheeler ([28], p. 1112) determined the residual relativistic perihelion shift was 41.4 ± 0.9"/cy that does not encompass general relativity, which they gave no explanation. A post-Newtonian approximation with coordinate time predicted 40.73"/cy, which is within the error boundary [29]. Shapiro *et al.* [30] chose Icarus for its large orbital eccentricity and periodic close approaches to Earth for observations to compare against general relativity's orbital perihelion shift for a highly eccentric orbit. A dimensionless parameter λ determined how well perihelion predictions compared with observations (unity for general relativity and zero for Newtonian theory). Their first solution for estimating the orbital elements of Icarus and the λ factor was $\lambda = 0.75 \pm 0.08$, which excluded general relativity, but the post-Newtonian approximation of 0.83 fits [29] [30]. Shapiro *et al.* [30] wrote, "we must conclude that either general relativity is incorrect or some other aspect of either our theoretical model or the observations differed from our presumptions." After many more solutions using different models and even varying the angular spread between background stars, that team concluded that the FK4 star catalog had a distortion that caused the anomaly, but subsequent star catalogs such as FK5 show no such regional distortion in comparison to the FK4, which coordinate transformations from FK4 axes are rotations to later star catalogs' axes.

If the one-way speed of photons obeys vector velocity addition due to the velocity of the photon emitter, then the quantity $F = c^2t^2 - x^2 - y^2 - z^2$ is not invariant. The values $F = \pm 1$ define two-sheeted hyperboloids and $F = 0$ yields their asymptotes, which had defined the trajectories of a light ray ([10], p. 268). Minkowski assumed these are invariances to define and describe his four-dimensional space-time model (later used in general relativity), which are based on Poincaré's group properties of the Lorentz transformation, and he used Lorentz's F to seek other invariant quantities under the Lorentz group, as (x, y, z, ict) ([10], p. 238-243). Any derivation based on these quantities is only approximate and limited to only the case when photons complete a roundtrip traverse. So, many derivations using these terms as invariances in quantum mechanics must be scrutinized for valid predictions of particles with one-way velocities.

This is an incomplete list of possible ramifications if the one-way photon speed is not a universal constant in a vacuum. In any case, it will take time to consider these results thoroughly. It would not be surprising if other accepted concepts of physics may need revisions.

7. Conclusions

The word "exact" should be deleted in any updated definition of the meter. The original definition of the meter was 1E−7 of the meridian length from the equator to the north pole at sea level. Due to the irregular shape of the Earth, such a meter would differ depending on the choice of the meridian, even if one did get the quarter meridian length divided exactly into 1E7 units. The International Prototype Meter was the replacement in 1889, which all other national meter prototypes were exactly compared. However, technology improved precision that, by 1927, the definition of the meter was updated to support the prototype with two cylinders to circumvent sagging during measurement and stipulated the temperature to prevent expansion of the prototype. As the BIPM and CGPM acknowledge that gravity can shorten the meter by 1.1E−16 m for every meter the test is elevated in altitude, that alone admits the current meter's definition is never exact. Even an exact definition of the nautical mile of 1852 m is incorrect as it is 1855.324 m due to improvements in geodesic measurements of Earth's equatorial radius. That proves "exact" values must be scrutinized regularly for accuracy and improved precision to redefine any wrong values.

As discovered in the analysis, the one-way speed of photons depends on the velocity of the emitter in the reference frame. This was revealed in both the theoretical analysis of photons traversing the length of Einstein's uniformly moving rod and in LIGO's simultaneous interference on a rotating Earth. Assuming a universal constant speed c, the total distance in the stationary frame that photons move in both directions to traverse a uniformly moving rod exceeds the total distance when the rod is stationary. Length contraction is not enough to rectify the difference in distances, yet the roundtrip time is the same for either the stationary rod or the uniformly moving rod as required by the first postulate of relativity. LIGO shows equal roundtrip times through the x-axis and y-axis arms of participating observatories that are oriented with various azimuths on a rotating Earth. The rotation causes the north ends of each arm to move with slower eastward speeds and less eastward displacements than the south ends, so that the roundtrip distances traversed in an inertial reference frame are different for each arm. Furthermore, the rate of change of a vector (Equation (7)) reveals that the cross product term applied to photon displacements will yield different velocity contributions between the x-axis and y-axis arms due to different orientations. Those contributions are added to the photon velocity in LIGO's frame to produce the photon velocity in the inertial frame. The outgoing velocities will differ with the returning velocities through each arm due to the cross product terms, which makes the photon speed in the inertial frame different than c. Excluding the data glitches and rare gravitational wave detections, LIGO demonstrated the beams arrived simultaneously at the recombination point for 22 months of continuous operation. However, the average speed of roundtrip traverses is the standard photon speed, c, that would allow simultaneous arrival of the split beams at the recombination point, provided the one-way speed of the photons obeys vector velocity addition. For circular interferometers undergoing a constant angular velocity, the external observer easily sees that the output of destructive interference occurs when two beams moving oppositely inside the interferometer cover different distances. For the internal observer fixed with the uniformly rotating interferometer, the output is the same if the interferometer's splitter imparts an additional velocity to the photons moving oppositely to create the same destructive interference recorded by the external observer.

The proposed test using a laser beam, half-silvered flat mirror, and hemisphere mirror will prove if photons do exhibit vector velocity addition. Due to Earth's orbital velocity and Earth rotation, the velocity of the test apparatus will undergo different daily velocities and directions by the changing perpendicular velocity component from Earth's orbital velocity. Photons emitted from the half-silvered mirror will only impact the nadir if their additional velocities contributed by the emitting atoms are the same as the hemisphere mirror is subjected throughout the day. Photons emitted from the hemisphere mirror will hit the same point on the floor if they have the additional velocity of the hemisphere mirror as the floor.

In-depth analysis indicates photon speeds obey vector velocity addition in one-way traverses. In roundtrip tests, the total roundtrip times are always the same. It is proven by induction that the average speed is exactly c in roundtrip tests in either moving or stationary frames. Every precise test measuring photon speeds has been with roundtrip traverses using a reflective surface, which always produced the apparent constant for photon speeds. This was found in both Einstein's thought experiment with the rod and in LIGO's continuously constant reception times. An updated definition of the meter must require a roundtrip test of photons and a fixed distance between the emitter and the reflector during the test to ensure the elapsed transmission time is always the same to construct a meter.

Also, gravity affects the speed of the photons in the vertical direction to be faster the farther away a photon is from the body of mass. For example, the solar deflection of beams near the Sun's surface is bent around the Sun. The side of the photon nearest to the Sun has a slower velocity than the side away from the Sun, which causes the photon trajectory to bend slightly towards the Sun. This is similar to a tank with a left tread moving faster than the right tread so that it turns to the right compared to a straight path. An updated definition of the meter should require the test be conducted in the horizontal plane. If the test is conducted on Earth's surface, the differences due to altitude changes would be in multiples of 1E−7 nm, which is well below the current practical limits of construction. If the meter is attempted in highly different gravitation than Earth's local gravity (*e.g.* another planet or in outer space), then the gravitational effect to construct a meter must be included in the definition.

It is understood that the SI second is implied, but not stated in the current definition of the meter. Other seconds as time units are defined for specific uses. The laboratory time should be coordinate time synchronized to the master time standard. According to special relativity, proper time recorded by moving clocks will produce longer second units than coordinate seconds.

The following suggestion is offered as a possible revision to the current definition of the meter to eliminate the inadequacies discussed in this paper.

The metre is the length between a light source and reflector that is accomplished near Earth's surface in a roundtrip traverse by light in a vacuum during a time interval of 2/299,792,458 of an SI second in coordinate time where the distance between the source and reflector is fixed relative to the Earth during the test and is oriented perpendicular to the local gravity.

If more precise testing of the photon's speed in a vacuum obtains more significant figures, the number in the denominator of the definition's time interval should be updated.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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