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The Speed of Light Is Not Constant in Basic Big Bang Theory

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Abstract

Starting from the basic assumptions and equations of Big Bang theory, we present a simple mathematical proof that this theory implies a varying (decreasing) speed of light, contrary to what is generally accepted. We consider General Relativity, the first Friedmann equation and the Friedmann-Lemaître-Robertson-Walker (FLRW) metric for a Comoving Observer. It is shown explicitly that the Horizon and Flatness Problems are solved, taking away an important argument for the need of Cosmic Inflation. A decrease of 2.1 cm/s per year of the present-day speed of light is predicted. This is consistent with the observed acceleration of the expansion of the Universe, as determined from high-redshift supernova data. The calculation does not use any quantum processes, and no adjustable parameters or fine tuning are introduced. It is argued that more precise laboratory measurements of the present-day speed of light (and its evolution) should be carried out. Also it is argued that the combination of the FLRW metric and Einstein's field equations of General Relativity is inconsistent, because the FLRW metric implies a variable speed of light, and Einstein's field equations use a constant speed of light. If we accept standard Big Bang theory (and thus the combination of General Relativity and the FLRW metric), a variable speed of light must be allowed in the Friedmann equation, and therefore also, more generally, in Einstein's field equations of General Relativity. The explicit form of this time dependence will then be determined by the specific problem.

Keywords

General Relativity, Friedmann Equation, Big Bang, Cosmic Microwave Background, CMB, Varying Speed of Light, Flatness Problem, Horizon Problem, Cosmic Inflation

1. Introduction

At present, Big Bang theory is about one century old, and it is still our best framework to describe the long term evolution of the Universe. It is considered the dominant theory to describe the evolution of our Universe (however dissident opinions exist). An important observation was the redshift of far-away galaxies, suggesting that we live in an expanding Universe. Furthermore, 3D-space seems to be homogeneous and isotropic. To describe these observations, Big Bang Cosmology (Λ-CDM) was developed, starting from General Relativity [1]. The calculations are based on the Friedmann-Lemaître-Robertson-Walker (FLRW) metric [2]-[8], which says that the scale of space itself increases uniformly, as a function of time. We call Big Bang the total process (continuing today) describing the expansion of space. The physical mechanisms that started this expansion, and the causes of Big Bang in general, are not understood. In Big Bang theory, galaxies are carried away from each other by the expanding space, and the longer the distance, the higher the recession speed. The divergent movement is only observed for very large distances. Nearby galaxies will attract each other gravitationally, and thus they will show motion not related to the universal expansion.

Big Bang Cosmology describes satisfactorily a number of experimental observations, such as the age of the Universe, the observed cosmological redshifts, the existence of a Cosmic Microwave Background (CMB), the homogeneity and isotropy of the Universe and, to a reasonable approximation, the abundance of primordial atomic nuclei. However, there are some important questions, such as the Horizon Problem and the Flatness Problem. In order to solve these difficulties, later theoretical work suggested some new physical mechanisms. At present the most popular of these proposals is Cosmic Inflation [9] [10] [11] [12]. Other proposals are a Bouncing Universe [13], and a Conformal Cyclic Cosmology (CCC) [14]. Also there is the suggestion of Varying Speed of Light (VSL) [15] [16]. Note that at present none of these ideas are generally accepted. They all generate a lot of discussions and new questions. They are additions to basic Big Bang theory that were added later, to fix problems that were not understood from the elementary calculation.

In the present calculation we will start from the standard assumptions and equations of basic Big Bang theory. We will then give a simple mathematical proof that the standard version of this theory implies a variable speed of light, contrary to what is generally accepted. We will show that this solves some of the basic problems of Big Bang Theory. Also we will point out that this has important implications for General Relativity as a whole.

2. General Relativity and the Expanding Universe: The FLRW Metric

The aim of Big Bang theory is to describe the evolution of the Universe as a function of time. To do this the path is studied of light reaching us from a very long time ago. The propagation of this light is influenced by the distribution of matter and other forms of energy in the Universe. Therefore we must start from General Relativity. We will have to solve a differential equation, and in order to obtain the proper solution of this equation we will have to introduce a number of experimentally measured parameters.

In General Relativity the three spatial dimensions and time are taken together to describe one four-dimensional world, space-time. To calculate distances in this space-time we need to introduce a metric. For this purpose, the traditional approach is the Friedmann-Lemaître-Robertson-Walker (FLRW) metric [2]-[8].

The FLRW metric combines time and a homogeneous and isotropic 3D-space with uniform curvature. Light travels through space-time following a radial path along a null geodesic. In the FLRW metric we use the present day speed of light c_0. A time-dependent scale factor $a(t)$ is introduced to allow for uniform expansion (or contraction) of 3D-space. The FLRW metric postulates that the motion of light through space-time is determined by the (zero) line-element:

$$\mathrm{d}s^2 = 0 = c_0^2 \mathrm{d}t^2 - a(t)^2 \frac{\mathrm{d}r^2}{1-kr^2} \tag{1}$$

Here $a(t)$ has the dimension of length and k is a dimensionless constant that describes the overall curvature of space. It can take the values 0, +1 or −1. These values correspond to a flat Euclidian space, a spherical space with constant positive curvature and a hyperbolic space with constant negative curvature respectively. Note that r is a (dimensionless) comoving radial coordinate. It doesn't change when the Universe expands.

From Equation (1) it follows, for non-zero $a(t)$:

$$\frac{a_0 c_0}{a(t)} \mathrm{d}t = \frac{a_0 \mathrm{d}r}{\sqrt{1-kr^2}} \tag{2}$$

Here $a_0 = a(t_0)$, the value of the scale factor at present time.

The distance traveled by a light signal between time t_1 and t_0, and between position r_1 and r_0, can therefore be calculated as

$$\Delta s(t_1, t_0) = \int_{r_1}^{r_0} \frac{a_0 \mathrm{d}r}{\sqrt{1-kr^2}} = \int_{t_1}^{t_0} \frac{a_0 c_0}{a(t)} \mathrm{d}t \tag{3}$$

Here proper distances are calculated. Obviously $\Delta s(t_1, t_0)$ is the distance between the starting and the end point of the light signal, measured by an observer at time $t = t_0$, using fixed (non-expanding) length units.

In order to obtain actual distances from Equation (3), the scale factor $a(t)$ must be determined from General Relativity. This is done by calculating $a(t)$ from the (first) Friedmann equation [2] [3]. This will be illustrated in Section 4.

Note that the FLRW metric is not part of General Relativity. It was added later, by imposing geometric properties on 3D-space (homogeneity and isotropy).

3. Frame of Reference

An important aspect of the General Relativity/FLRW calculation is the status of the observer. The Friedmann equation was developed for an observer on Earth. However every "local" motion of this observer has to be neglected. This means an observer moving only because he/she is carried away from distant galaxies by the expansion of space, *i.e.* a so-called Comoving Observer.

Also note that it is generally accepted that at present our material world is not expanding (and has not expanded) with the Universe, due to the forces keeping material objects together. Therefore we will make all calculations from the point of view of a Comoving Observer, and we will use nonexpanding units along the spatial axes. These non-expanding length units should be determined by the length of (non-expanding) material objects. Note that this is also the frame of reference used by astronomers determining the distance of far-away galaxies, based on the "Cosmic Distance Ladder".

Since 1983 the definition of the meter is based on a (constant) speed of light c_0 [17]. Therefore, in the present calculation we will return to the old definition of the meter, based on the size of material objects, in order to avoid confusion.

4. Calculation of $a(t)$ from General Relativity

The derivation of the FLRW metric is based on a "perfect liquid" model for the Universe, such that a (continuous) density and pressure can be defined for mass and other forms of energy. Furthermore the Universe is supposed to be homogeneous and isotropic. These assumptions are considered acceptable as a lowest order approximation. The variable t appearing in all expressions here is simply the time as measured on the clock of the Comoving Observer.

We will now recall the (first) Friedmann equation. This is a differential equation that was first derived by Alexander Friedmann [2] [3]. The equation has to be solved for the scale factor $a(t)$, describing the expansion of the Universe:

$$\dot{a}^2 - \frac{8\pi G}{3}\rho a^2 = -kc_0^2 \tag{4}$$

Here the dot indicates the derivative with respect to time. G is Newton's gravitational constant and c_0 is the present day speed of light. The curvature parameter k is the same as in the FLRW metric, Equation (1). The time-dependent variable ρ describes the average density of matter and other forms of energy in the Universe. Essentially, the Friedmann equation expresses energy (density) conservation: kinetic energy density plus potential energy density plus energy density related to the overall curvature of space always equals zero.

The Friedmann equation, Equation (4), was obtained by substituting the FLRW metric, Equation (1), in the (0, 0) component of the standard Einstein field equations. Note that the speed of light occurs only in the rhs of the Friedmann equation, in a term proportional to the curvature factor k.

The solution of the Friedmann equation can be found in standard cosmology textbooks (see also [18], where the equation is solved, not only for a constant speed c_0, but also for a time dependent speed of light $c(t)$ in the Friedmann equation). We now illustrate the most elementary solution of the Friedmann equation. This is the basis of standard Big Bang theory. We first introduce some variables and constants that make the calculations more transparent.

The (time dependent) Hubble variable is defined as:

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{5}$$

At present time ($t = t_0$), this reduces to the Hubble constant H_0:

$$H_0 = \frac{\dot{a}_0}{a_0} \tag{6}$$

The (time dependent) critical density ρ_c is defined as:

$$\rho_c = \frac{3H^2}{8\pi G} \tag{7}$$

ρ_c serves to make the distinction between a positively and a negatively curved geometry. When the total density ρ is exactly equal to the critical density ρ_c, the spatial geometry is flat. At present ($t = t_0$), the critical density reduces to

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} \tag{8}$$

In the Friedmann equation, the total density is written as the sum of three terms, describing radiation (ρ_R), matter (ρ_M) (normal and dark) and dark energy (ρ_Λ). The curvature term (ρ_K) is incorporated in the total density:

$$\rho = \rho_R + \rho_M + \rho_\Lambda + \rho_K \tag{9}$$

The variation of ρ_R, ρ_M, ρ_Λ, and ρ_K with $a(t)$ is then described as:

$$\rho_R = \rho_{R,0}\frac{a_0^4}{a^4} \tag{10}$$

$$\rho_M = \rho_{M,0}\frac{a_0^3}{a^3} \tag{11}$$

$$\rho_\Lambda = \rho_{\Lambda,0} \tag{12}$$

$$\rho_K = \rho_{K,0}\frac{a_0^2}{a^2} \tag{13}$$

with

$$\rho_{K,0} = -\rho_{c,0}\frac{kc_0^2}{H_0^2 a_0^2} \tag{14}$$

The Friedmann equation can then be rewritten as

$$H^2(t) = \frac{\dot{a}^2}{a^2} = \frac{H_0^2}{\rho_{c,0}}\left[\rho_{R,0}\frac{a_0^4}{a^4} + \rho_{M,0}\frac{a_0^3}{a^3} + \rho_{\Lambda,0} + \rho_{K,0}\frac{a_0^2}{a^2}\right] \tag{15}$$

Finally, we introduce the relative present-day densities:

$$\Omega_{R,0} = \frac{\rho_{R,0}}{\rho_{c,0}} \tag{16}$$

$$\Omega_{M,0} = \frac{\rho_{M,0}}{\rho_{c,0}} \tag{17}$$

$$\Omega_{\Lambda,0} = \frac{\rho_{\Lambda,0}}{\rho_{c,0}} \tag{18}$$

$$\Omega_{K,0} = \frac{\rho_{K,0}}{\rho_{c,0}} = -\frac{kc_0^2}{H_0^2 a_0^2} \tag{19}$$

The Friedmann equation can then be rewritten as:

$$H^2(t) = \frac{\dot{a}^2}{a^2} = H_0^2 \left[\Omega_{R,0} \frac{a_0^4}{a^4} + \Omega_{M,0} \frac{a_0^3}{a^3} + \Omega_{\Lambda,0} + \Omega_{K,0} \frac{a_0^2}{a^2} \right] \tag{20}$$

The standard method to solve the Friedmann equation from Equation (20) is then to calculate t as a function of a by integration (the normalized integration variable $a' = a/a_0$ has been introduced):

$$t(a) = \frac{1}{H_0} \int_0^{\frac{a}{a_0}} \frac{a' \mathrm{d}a'}{\sqrt{\Omega_{R,0} + \Omega_{M,0} a' + \Omega_{\Lambda,0} a'^4 + \Omega_{K,0} a'^2}} \tag{21}$$

The Friedmann equation, Equation (20), must hold at all times, including $t = t_0$:

$$H_0^2 = H_0^2 \left[\Omega_{R,0} + \Omega_{M,0} + \Omega_{\Lambda,0} + \Omega_{K,0} \right] \tag{22}$$

and thus

$$1 = \Omega_{R,0} + \Omega_{M,0} + \Omega_{\Lambda,0} - \frac{kc_0^2}{H_0^2 a_0^2} \tag{23}$$

This means that, at present, the total energy density, including the curvature of the FLRW Universe, must be equal to the critical density.

Recent experimental values for the parameters used in the present calculation can be taken from the Planck 2015 data [19]:

$$H_0 = 67.3\ \mathrm{km \cdot s^{-1} \cdot Mpc^{-1}} \tag{24}$$

$$\Omega_{R,0} = 9.24 \times 10^{-5} \tag{25}$$

$$\Omega_{M,0} = 0.315 \tag{26}$$

$$\Omega_{\Lambda,0} = 0.685 \tag{27}$$

$$\Omega_{K,0} = 0 \tag{28}$$

Also note that the Hubble time t_{Hubble} is then found to be:

$$t_{Hubble} = \frac{1}{H_0} = 14.53 \times 10^9\ \text{years} \tag{29}$$

From Equations (19) and (28) it is obvious that, with the present-day experimental parameters, space must be almost perfectly flat: $\Omega_{K,0}$ must be equal to zero to a very good approximation, and the boundary condition Equation (23) reduces to

$$1 = \Omega_{R,0} + \Omega_{M,0} + \Omega_{\Lambda,0} \tag{30}$$

which is in good agreement with the experimental results.

We then obtain the solution of the Friedmann equation, for a flat geometry:

$$t(a) = \frac{1}{H_0} \int_0^{\frac{a}{a_0}} \frac{a' \mathrm{d}a'}{\sqrt{\Omega_{R,0} + \Omega_{M,0} a' + \Omega_{\Lambda,0} a'^4}} \tag{31}$$

where the experimental parameters $H_0, \Omega_{R,0}, \Omega_{M,0}$ and $\Omega_{\Lambda,0}$ are given by Equations (24)-(27). Note that $t(a)$ is a function of the ratio a/a_0 only. The absolute value of a is unimportant. Also, since $k=0$, the time function $t(a)$, and thus also the scale factor function $a(t)$, do not depend on c_0.

For the time since the start of Big Bang, t_{BB}, one then finds, by numerical integration:

$$\begin{aligned} t_{BB} = t(a_0) = t_0 &= \frac{1}{H_0}\int_0^1 \frac{a' \mathrm{d}a'}{\sqrt{\Omega_{R,0} + \Omega_{M,0} a' + \Omega_{\Lambda,0} a'^4}} \\ &\cong \frac{0.9506}{H_0} = 0.9506 t_{Hubble} = 13.81\times 10^9 \text{ years} \end{aligned} \tag{32}$$

As a result, we can rewrite Equation (31) as

$$\frac{t}{t_0} = \frac{1}{0.9506}\int_0^{\frac{a}{a_0}} \frac{a' \mathrm{d}a'}{\sqrt{\Omega_{R,0} + \Omega_{M,0} a' + \Omega_{\Lambda,0} a'^4}} \tag{33}$$

In Equation (33), time and length only appear in the ratios $t' = t/t_0$ and $a' = a/a_0$. From the start of Big Bang until today these relative values vary from 0 to 1. Therefore we will use a_0 and t_0 as units of length and time whenever we perform a numerical calculation.

Note that all these results are purely classical. Very close to the origin of Big Bang, distances between material objects become very small, and a more correct calculation should also take into account non-classical (quantum) processes, that are not included in General Relativity.

5. The Calculated Scale Factor Function $a(t)$ Must Be Independent of the Time t_0 When It Is Determined by the Comoving Observer

The calculation of the scale factor $a(t)$, as illustrated here, is based on experimental measurements made at time $t = t_0$, with scale factor a_0. The resulting formulas contain experimental parameters (Hubble constant, densities, …) that vary with time. It is obvious that, in order to be consistent, a theory that calculates the deformation of space-time must always obtain the same scale factor function $a(t)$, independent of the time of the observations. We will illustrate now that this is indeed the case for the standard Big Bang results. We will start from Equation (21), giving the function $t(a)$ as derived by an observer at time $t = t_0$, and we will transform this expression to find the result for $t(a)$, as derived by the same observer at a different time $t = t_1$.

First note that, from Equations (10)-(13):

$$\rho_{R,0} = \rho_{R,1}\frac{a_1^4}{a_0^4} \tag{34}$$

$$\rho_{M,0} = \rho_{M,1}\frac{a_1^3}{a_0^3} \tag{35}$$

$$\rho_{\Lambda,0} = \rho_{\Lambda,1} \tag{36}$$

$$\rho_{K,0} = \rho_{K,1} \frac{a_1^2}{a_0^2} \tag{37}$$

Also, from Equations (7) and (8):

$$\rho_{c,0} = \frac{3H_0^2}{8\pi G} = \frac{H_0^2}{H_1^2} \frac{3H_1^2}{8\pi G} = \frac{H_0^2}{H_1^2} \rho_{c,1} \tag{38}$$

Therefore, Equations (16)-(19) can be rewritten as:

$$\Omega_{R,0} = \frac{\rho_{R,0}}{\rho_{c,0}} = \frac{a_1^4}{a_0^4} \frac{H_1^2}{H_0^2} \frac{\rho_{R,1}}{\rho_{c,1}} = \frac{a_1^4}{a_0^4} \frac{H_1^2}{H_0^2} \Omega_{R,1} \tag{39}$$

$$\Omega_{M,0} = \frac{\rho_{M,0}}{\rho_{c,0}} = \frac{a_1^3}{a_0^3} \frac{H_1^2}{H_0^2} \frac{\rho_{M,1}}{\rho_{c,1}} = \frac{a_1^3}{a_0^3} \frac{H_1^2}{H_0^2} \Omega_{M,1} \tag{40}$$

$$\Omega_{\Lambda,0} = \frac{\rho_{\Lambda,0}}{\rho_{c,0}} = \frac{H_1^2}{H_0^2} \frac{\rho_{\Lambda,1}}{\rho_{c,1}} = \frac{H_1^2}{H_0^2} \Omega_{\Lambda,1} \tag{41}$$

$$\Omega_{K,0} = \frac{\rho_{K,0}}{\rho_{c,0}} = \frac{a_1^2}{a_0^2} \frac{H_1^2}{H_0^2} \frac{\rho_{K,1}}{\rho_{c,1}} = \frac{a_1^2}{a_0^2} \frac{H_1^2}{H_0^2} \Omega_{K,1} \tag{42}$$

From Equation (42) it follows that, if $\Omega_K = 0$ at time t_0, it is zero at all other times.

The results of Equations (39)-(42) can also be formulated as:

$$\Omega_R(t) = \frac{a_0^4}{a^4} \frac{H_0^2}{H^2} \Omega_{R,0} \tag{43}$$

$$\Omega_M(t) = \frac{a_0^3}{a^3} \frac{H_0^2}{H^2} \Omega_{M,0} \tag{44}$$

$$\Omega_\Lambda(t) = \frac{H_0^2}{H^2} \Omega_{\Lambda,0} \tag{45}$$

$$\Omega_K(t) = \frac{a_0^2}{a^2} \frac{H_0^2}{H^2} \Omega_{K,0} \tag{46}$$

Also, from Equation (20) one obtains:

$$H_1^2 = H_0^2 \left[\Omega_{R,0} \frac{a_0^4}{a_1^4} + \Omega_{M,0} \frac{a_0^3}{a_1^3} + \Omega_{\Lambda,0} + \Omega_{K,0} \frac{a_0^2}{a_1^2} \right] \tag{47}$$

We can conclude that, as soon as the scale factor a_1 is known, all other parameters $H_1, \Omega_{R,1}, \Omega_{M,1}, \Omega_{\Lambda,1}$ and $\Omega_{K,1}$ can be deduced from their values at time $t = t_0$.

Substituting the expressions Equations (34)-(42), (47) in the solution of the Friedmann equation, Equation (21), we obtain:

$$t(a) = \frac{1}{H_1} \int_0^{\frac{a}{a_1}} \frac{a'' \mathrm{d}a''}{\sqrt{\Omega_{R,1} + \Omega_{M,1} a'' + \Omega_{\Lambda,1} a''^4 + \Omega_{K,1} a''^2}} \tag{48}$$

Here the new integration variable $a'' = \frac{a_0}{a_1} a'$ was introduced.

It is immediately clear that Equation (48) is the solution of the Friedmann equation, as derived by an observer at time $t = t_1$, with scale factor a_1, Hubble constant H_1 and radiation/mass/dark energy/curvature relative densities $\Omega_{R,1}$, $\Omega_{M,1}$, $\Omega_{\Lambda,1}$ and $\Omega_{K,1}$. We can conclude that the function $t(a)$ as obtained at time t_1 is identical to the function $t(a)$ as obtained at time t_0.

Finally, substituting Equations (39)-(42) in Equation (47) we find immediately that

$$H_1^2 = H_1^2 \left(\Omega_{R,1} + \Omega_{M,1} + \Omega_{\Lambda,1+} + \Omega_{K,1}\right) \tag{49}$$

and thus

$$\Omega_{R,1} + \Omega_{M,1} + \Omega_{\Lambda,1} + \Omega_{K,1} = 1 \tag{50}$$

Therefore, the total energy density, including the energy density related to the overall curvature, remains equal to the critical density at all times.

We can conclude that, in the FLRW Universe, the Comoving Observer always obtains the same time function $t(a)$, independent of the value of the scale factor $(a_0, a_1, a_2, \ldots)$ at which the measurements are made. This is also true for the scale factor $a(t)$ since it is calculated as the inverse function of $t(a)$. Obviously this property is necessary for internal consistency. Otherwise observers at different epochs would obtain different evolution functions for the Universe. This result will be used explicitly in the following sections.

Note that this condition, that the scale factor function $a(t)$ must be independent of the time at which it is calculated by the Comoving Observer, is valid, not only for a flat 3D-space, but also for arbitrary curvature.

6. Time Dependence of the Speed of Light

We will now perform a simple Gedankenexperiment. Consider a Comoving Observer who can carry out measurements of the parameters relevant for cosmology, throughout the history of the Universe. After each measurement session the observer uses General Relativity and the FLRW metric to calculate the scale factor function, and the distance covered by a light signal. Note that an identical evolution function of the Universe must be obtained by the Comoving Observer at all times. At time $t = t_i$ the observer determines the speed of light c_i and the corresponding parameters H_i, $\Omega_{R,i}$, $\Omega_{M,i}$, $\Omega_{\Lambda,i}$ and $\Omega_{K,i}$. From these parameters the observer then calculates the universal scale factor function $a(t)$. Consider the case of a flat 3D-space at $t = t_0$, corresponding to the actual experimentally observed world in which we live. At time t_i the observer will then obtain the distance covered by a light signal since the start of Big Bang, as given by Equation (3):

$$\Delta s\left(0, t_i\right) = \int_0^{t_i} \frac{a_i c_i}{a(t)} \mathrm{d}t = a\left(t_i\right) c\left(t_i\right) \int_0^{t_i} \frac{\mathrm{d}t}{a(t)} \tag{51}$$

The observer can determine $\Delta s\left(0, t_i\right)$ with infinitesimal time intervals $\mathrm{d}t_i$. The speed of light, as seen by the observer at $t = t_i$, can then be found as the derivative of $\Delta s\left(0, t_i\right)$ with respect to t_i. Note that $a(t)$ is independent of t_i, as discussed before: the derivative of $a(t)$ with respect to t_i is zero.

Taking the derivative of both sides of Equation (51) with respect to t_i we then find the speed of light $c(t_i)$ as obtained by the Comoving Observer at t_i:

$$c(t_i) = \frac{\mathrm{d}\Delta s(0,t_i)}{\mathrm{d}t_i} = c(t_i) + \frac{\mathrm{d}\left[a(t_i)c(t_i)\right]}{\mathrm{d}t_i} \int_0^{t_i} \frac{\mathrm{d}t}{a(t)} \tag{52}$$

Therefore, since $\int_0^{t_i} \frac{\mathrm{d}t}{a(t)}$ is a positive quantity, we obtain

$$\frac{\mathrm{d}\left[a(t_i)c(t_i)\right]}{\mathrm{d}t_i} = 0 \tag{53}$$

We can conclude that $a_i c_i$ is a constant quantity as a function of time.

This means that Equation (51) describes light moving at a speed of

$$c(t) = \frac{a_i c_i}{a(t)} = \frac{a_0 c_0}{a(t)} \tag{54}$$

Therefore, for a Comoving Observer, standard Big Bang theory describes light traveling at a variable speed $\frac{a_0 c_0}{a(t)}$, where $a_0 c_0$ is a universal constant.

We will now prove the same fact from internal consistency arguments.

First consider a Comoving Observer at time $t = t_0$. This observer makes all relevant experimental measurements (c_0, H_0, …) and determines the scale factor function $a(t)$. The observer then calculates the distance covered by a light signal from time $t = 0$ to an arbitrary time $t = t_1$. The result is

$$\Delta s(0,t_1) = \int_0^{t_1} \frac{a_0 c_0}{a(t)} \mathrm{d}t \tag{55}$$

However the same observer can also make all relevant measurements (c_1, H_1, …) at time $t = t_1$ and then, using these results, calculate the distance covered by the light signal from time $t = 0$ to this time $t = t_1$. The result is

$$\Delta s(0,t_1) = \int_0^{t_1} \frac{a_1 c_1}{a(t)} \mathrm{d}t \tag{56}$$

In order for the theory to be self-consistent, the results from Equation (55) and (56) must be identical. Therefore, again, we find $a_0 c_0 = a_1 c_1$. $a_i c_i$ is a universal constant. The actual value of $a_i c_i$ is not known, because only the relative value of the scale factor appears in the numerical formulas, not the absolute value.

From Equation (54), the variation of the speed of light for a Comoving Observer can now be calculated by taking the derivative of $c(t)$ with respect to time:

$$\frac{\mathrm{d}c(t)}{\mathrm{d}t} = -\frac{1}{a^2}\dot{a} a_0 c_0 = -\frac{\dot{a}}{a}\frac{a_0 c_0}{a} = -H(t)c(t) \tag{57}$$

Equations (54) and (57) have been suggested before [18] [20] [21] [22]. However, the calculation given above is a simple mathematical proof that, for a Comoving Observer, they follow directly from basic Big Bang theory.

As $a(t)$ increases with time, it is clear from Equation (54) that the speed of light is a decreasing function of time. At present ($t = t_0$), Equation (57) reduces to

$$\frac{\mathrm{d}c}{\mathrm{d}t}(t_0) = -H_0 c_0 \cong -2.1\,\mathrm{cm}\cdot\mathrm{s}^{-1}\cdot\mathrm{year}^{-1} \tag{58}$$

The present-day variation of $c(t)$ is extremely small. Therefore a constant speed of light, as postulated in Special Relativity, is a very good approximation for all "normal" experiments. However, for longer timespans, e.g. in cosmology, the variation of the speed of light becomes important.

For light starting not at $t = 0$, but at a later time t_1, we can write:

$$\Delta s(t_1, t_0) = \int_0^{t_0} \frac{a_0 c_0 \mathrm{d}t}{a(t)} - \int_0^{t_1} \frac{a_0 c_0 \mathrm{d}t}{a(t)} = \int_{t_1}^{t_0} \frac{a_0 c_0 \mathrm{d}t}{a(t)} \tag{59}$$

This means that a light signal starting at a later time must be described using the same function $a(t)$ as before. Therefore, for light generated today, $c(t)$ varies at the same rate as $c(t)$ for light originating from far away and long ago.

7. Analytical Approximations for the Scale Factor Function *a*(*t*), with Error Smaller than 1% over the Complete Range [0, 1]. Speed of Light as a Function of Time

In order to calculate actual distances traveled by a light signal, we will now present sufficiently accurate approximations for $a(t)$. In this section all values of a and t will be expressed in normalized units a/a_0 and t/t_0 respectively.

The function $t(a)$, as given by Equation (33), can be considered in three different regimes, depending on the dominant term under the square root sign. At high values of t the dark energy contribution ($\Omega_{\Lambda,0}$) dominates. At intermediate t matter ($\Omega_{M,0}$) is more important. At early t the radiation contribution ($\Omega_{R,0}$) is dominant. These three regimes also appear in the scale factor function $a(t)$. Using these approximations for $a(t)$ we then calculate the speed of light as a function of time.

7.1. Approximation for *a*(*t*): "Dark Energy and Matter" Regime

A useful expression for $a(t)$, for the regime where matter and dark energy dominate, can be found in the literature [23]. Translated to our units this approximation can be written as

$$a(t) = 0.7947t^{2/3} + 0.0623\left(\mathrm{e}^{t/0.6785} - 1\right) \tag{60}$$

7.2. Approximations for *a*(*t*): "Radiation and Matter" Regime

For low values of t and a, it is obvious from Equation (33) that the term proportional to $\Omega_{\Lambda,0}$ under the square root sign can be neglected. One then obtains

$$t(a) \cong \frac{1}{0.9506} \int_0^a \frac{a'\mathrm{d}a'}{\sqrt{\Omega_{R,0} + \Omega_{M,0} a'}} \tag{61}$$

This integral can be solved analytically. The result is

$$t(a)=\frac{2}{3}\frac{1}{0.9506\Omega_{M,0}^{2}}\left[\left(\Omega_{R,0}+\Omega_{M,0}a\right)^{3/2}-\Omega_{R,0}^{3/2}\right] -\frac{2\Omega_{R,0}}{0.9506\Omega_{M,0}^{2}}\left[\left(\Omega_{R,0}+\Omega_{M,0}a\right)^{1/2}-\Omega_{R,0}^{1/2}\right] \quad (62)$$

This can be rewritten as

$$\left(\Omega_{R,0}+\Omega_{M,0}a\right)^{3/2}-3\Omega_{R,0}\left(\Omega_{R,0}+\Omega_{M,0}a\right)^{1/2}+2\Omega_{R,0}^{3/2}-\frac{3}{2}.9506\Omega_{M,0}^{2}t=0 \quad (63)$$

This is a cubic equation for the unknown $\left(\Omega_{R,0}+\Omega_{M,0}a\right)^{1/2}$. The equation can be solved using standard methods. The discriminant of this equation is

$$D=\frac{3}{2}\times 0.9506\Omega_{M,0}^{2}t\left(\frac{3}{8}\times 0.9506\Omega_{M,0}^{2}t-\Omega_{R,0}^{3/2}\right) \quad (64)$$

Finally $a(t)$ can be calculated from the resulting solutions for $\left(\Omega_{R,0}+\Omega_{M,0}a\right)^{1/2}$.

Depending on the sign of the discriminant different expressions are obtained:

For $D>0$ (*i.e.* for $t>t_D=\frac{8}{3}\frac{\Omega_{R,0}^{3/2}}{0.9506\Omega_{M,0}^{2}}\cong 2.51107\times 10^{-5}$) one finds:

$$a(t)=-\frac{\Omega_{R,0}}{\Omega_{M,0}}+\frac{1}{\Omega_{M,0}}\left[\left(\frac{3}{4}\times 0.9506\Omega_{M,0}^{2}t-\Omega_{R,0}^{3/2}+\left(\frac{9}{16}\times 0.9506^{2}\Omega_{M,0}^{4}t^{2}-\frac{3}{2}\times 0.9506\Omega_{M,0}^{2}\Omega_{R,0}^{3/2}t\right)^{1/2}\right)^{1/3}+\left(\frac{3}{4}\times 0.9506\Omega_{M,0}^{2}t-\Omega_{R,0}^{3/2}-\left(\frac{9}{16}\times 0.9506^{2}\Omega_{M,0}^{4}t^{2}-\frac{3}{2}\times 0.9506\Omega_{M,0}^{2}\Omega_{R,0}^{3/2}t\right)^{1/2}\right)^{1/3}\right]^{2} \quad (65)$$

For $D<0$ (*i.e.* for $t<t_D=\frac{8}{3}\frac{\Omega_{R,0}^{3/2}}{0.9506\Omega_{M,0}^{2}}\cong 2.51106\times 10^{-5}$) the result is:

$$a(t)=-\frac{\Omega_{R,0}}{\Omega_{M,0}}+4\frac{\Omega_{R,0}}{\Omega_{M,0}}\cos^{2}\left[\frac{1}{3}\arccos\left(\frac{3}{4}\frac{0.9506\Omega_{M,0}^{2}t}{\Omega_{R,0}^{3/2}}-1\right)\right] \quad (66)$$

For $t<t_D$ we will use Equation (66). For $t_D<t<0.333$ we will use Equation (65), and for $0.333<t<1$ we will use Equation (64). In this way, it is easy to show that the difference with the exact values of $a(t)$ is smaller than 1% over the complete time interval [0, 1].

Note that time t_D is a purely mathematical quantity. It doesn't have any physical meaning.

7.3. Speed of Light as a Function of Time

Using the approximations for $a(t)$ given above, and using Equation (54), it is now straightforward to calculate the speed of light as a function of time throughout the history of the Universe. The result is shown in **Figure 1**.

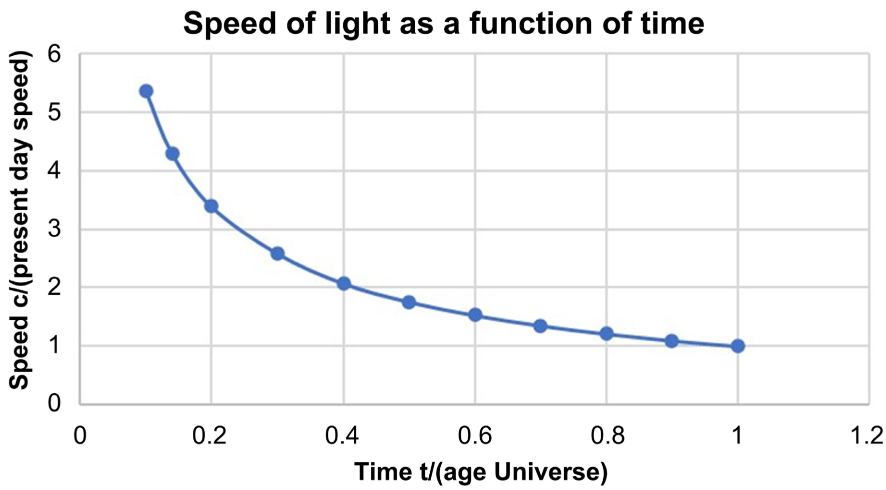

Figure 1. Speed of light (as observed by a Comoving Observer) as a function of time in the FLRW Universe. For $t = t_0$ (present time) the speed of light converges to c_0. For very early times the speed increases strongly, and even diverges for $t = 0$.

8. The Horizon Problem

Our visual horizon is determined by the Cosmic Microwave Background. This is the oldest electromagnetic radiation we can observe. It originated in the first stages of Big Bang, when the temperature of the Universe had cooled down to about 3000 K. At that temperature electrons and nuclei (mainly protons) started to combine to form atoms. After this primordial deionization event, light was free to travel through space, without being scattered, along null geodetics, as described by General Relativity. Some of these photons can still be observed today, and they constitute the CMB. It was found experimentally that the present-day CMB is a nearly perfect example of Black Body radiation, with a temperature of approximately 2.725 K. As the original temperature of the CMB was about 3000 K, it is straightforward to calculate the redshift that produced the present day CMB from the original radiation:

$$z_{CMB} = \frac{3000}{2.725} \cong 1101 \tag{67}$$

Then the corresponding scale factor at CMB time t_{CMB} is (relative to a_0):

$$\frac{a_{CMB}}{a_0} = \frac{1}{1 + z_{CMB}} \cong \frac{1}{1102} \tag{68}$$

From Equation (33) we can calculate the time at which the CMB light started, by numerical integration, using the experimental parameters mentioned before:

$$\begin{aligned} t_{CMB} &= \frac{t_0}{0.9506} \int_0^{\frac{a_{CMB}}{a_0}} \frac{a' \mathrm{d}a'}{\sqrt{\Omega_{R,0} + \Omega_{M,0} a' + \Omega_{\Lambda,0} a'^4}} \\ &\cong 2.644 \times 10^{-5} t_0 \cong 365000 \text{ years} \end{aligned} \tag{69}$$

It is found experimentally that the CMB background radiation is extremely isotropic. Variations in CMB temperature across the sky are smaller than 0.001 Kelvin. This means that all points of our Visible Universe must once have been in thermal equilibrium with each other. If a constant speed of light c_0 is accepted, this is impossible to understand. This is called the Horizon Problem.

To solve this problem we will first calculate the distance covered by a photon from the start of CMB until today, using the approximations introduced in Section 7. From Equation (59) it then follows that this distance can be calculated as

$$\begin{aligned} d_{CMB}(t_0) &= \Delta s(t_{CMB}, t_0) = \int_{t_{CMB}}^{t_0} \frac{a_0 c_0}{a(t)} \mathrm{d}t = c_0 t_0 \int_{t_{CMB}/t_0}^{1} \frac{\mathrm{d}t'}{a'(t')} \\ &= c_0 t_0 \int_{2.644\times10^{-5}}^{1} \frac{\mathrm{d}t'}{a'(t')} \cong 3.28 c_0 t_0 \end{aligned} \tag{70}$$

Therefore, from Equation (70), $3.28c_0t_0$ is, according to standard Big Bang theory, the distance traveled by a CMB photon observed today. Therefore this is the radius of our present-day Visible Universe (corresponding to $3.28\times13.81\times10^9 \cong 45.3$ billion light years, where a light year is defined as the distance covered by a light signal in one year at the "traditional" speed of 299,792,458 km/s).

Taking into account the expansion of space we can now calculate what was the value of the radius of our present-day Visible Universe at $t = t_{CMB}$. According to the definition of the scale factor this is simply

$$\begin{aligned} d_{CMB}(t_{CMB}) &= \frac{a_{CMB}}{a_0} d_{CMB}(t_0) = \frac{1}{1102} \times 3.28 c_0 t_0 \cong 0.00298 c_0 t_0 \\ &= 0.00298 \times 13.81 \times 10^9 \text{ light years} \cong 41.2 \times 10^6 \text{ light years} \end{aligned} \tag{71}$$

Before the time of CMB, at temperatures above 3000 K, photons could not travel freely through space, due to scattering by the primordial plasma. However we can calculate the cumulative distance traveled by a photon that is continually scattered from $t = 0$ to $t = t_{CMB}$. This distance is given by the sum of the individual steps between scattering events (obviously these steps are not in a straight line). The cumulative distance traveled by a photon between time $t = 0$ and $t = t_{CMB}$ can then be calculated numerically as

$$\Delta s(0, t_{CMB}) = \int_0^{t_{CMB}} \frac{a_0 c_0}{a(t)} \mathrm{d}t = c_0 t_0 \int_0^{t_{CMB}/t_0} \frac{\mathrm{d}t'}{a'(t')} \cong 0.0657 c_0 t_0 \tag{72}$$

We can now address the Horizon Problem. According to standard Big Bang theory, the ratio of the cumulative distance covered, since the start of Big Bang, by a photon at $t = t_{CMB}$, to the corresponding radius of our present-day Visible Universe at that time, is

$$\frac{\Delta s(0, t_{CMB})}{3.28 c_0 t_0 \times \dfrac{a_{CMB}}{a_0}} = \frac{0.0657 c_0 t_0}{0.00298 c_0 t_0} \cong 22.0 \tag{73}$$

This large ratio shows that CMB photons were indeed able to make thermal contact between opposite ends of our present-day Visible Universe, evening out temperature fluctuations in the CMB radiation. This is a purely classical process: no quantum gravity considerations are involved.

We will now study the evolution of the ratio (distance traveled by a light signal)/(radius of our Visible Universe) for times before t_{CMB}.

At very early times, the radiation contribution is dominant. Equation (33) reduces to:

$$\frac{t}{t_0} = \frac{1}{0.9506}\int_0^{\frac{a}{a_0}} \frac{a' \mathrm{d}a'}{\sqrt{\Omega_{R,0}}} = \frac{1}{0.9506 \times 2\sqrt{\Omega_{R,0}}}\left(\frac{a}{a_0}\right)^2 \tag{74}$$

and one finds

$$\frac{a}{a_0} = 2^{\frac{1}{2}} 0.9506^{\frac{1}{2}} \Omega_{R,0}^{\frac{1}{4}} \left(\frac{t}{t_0}\right)^{\frac{1}{2}} \tag{75}$$

Therefore, the cumulative distance traveled by a light signal is:

$$\Delta s(0,t) = \int_0^t \frac{a_0 c_0}{a(t)} \mathrm{d}t = c_0 t_0 \int_0^{\frac{t}{t_0}} \frac{\mathrm{d}t'}{a'(t')} = \frac{2^{\frac{1}{2}} c_0 t_0}{0.9506^{\frac{1}{2}} \Omega_{R,0}^{\frac{1}{4}}}\left(\frac{t}{t_0}\right)^{\frac{1}{2}} \tag{76}$$

From Equations (75) and (76) it follows that, in the zero time limit, the ratio (distance traveled by light)/(radius of our Visible Universe) will tend to a constant value:

$$\frac{\Delta s(0,t)}{3.28 c_0 t_0 \times \frac{a}{a_0}} = \frac{1}{3.28 \times 0.9506 \Omega_{R,0}^{\frac{1}{2}}} \cong 33.4 \tag{77}$$

The zero time limit of this ratio is clearly higher than the t_{CMB} value. We will now study the evolution of this ratio in the time interval $[0, t_{CMB}]$. Using the results of Section 7, the cumulative distance traveled by a light signal from time zero to time t can be calculated numerically as

$$\Delta s(0,t) = c_0 t_0 \int_0^{\frac{t}{t_0}} \frac{\mathrm{d}t'}{a'(t')} \tag{78}$$

The results for the ratio of this distance to the corresponding radius of our present-day visible Universe are shown in **Figure 2**. It can be seen that the ratio decreases monotonously, from 33.4 to 22.0, during the time interval from the start of Big Bang to t_{CMB}.

We can conclude that, according to basic Big Bang theory, photons were able to make thermal contact inside our present Visible Universe during the complete time interval before the deionization event at t_{CMB}. A decreasing speed of light gives a simple intuitive explanation of this phenomenon: the light cone "opens up" at early times: photons can cover very large distances due to their very large speeds. The decreasing ratio in **Figure 2** suggests that the quality of the thermal contact in the plasma decreases as a function of time. This may explain the appearance of (small) fluctuations in the CMB temperature.

We have used here the concept of varying speed of light to explain the Horizon Problem. However all formulas and calculations used are standard Big Bang expressions. Therefore there is no Horizon Problem in standard Big Bang theory

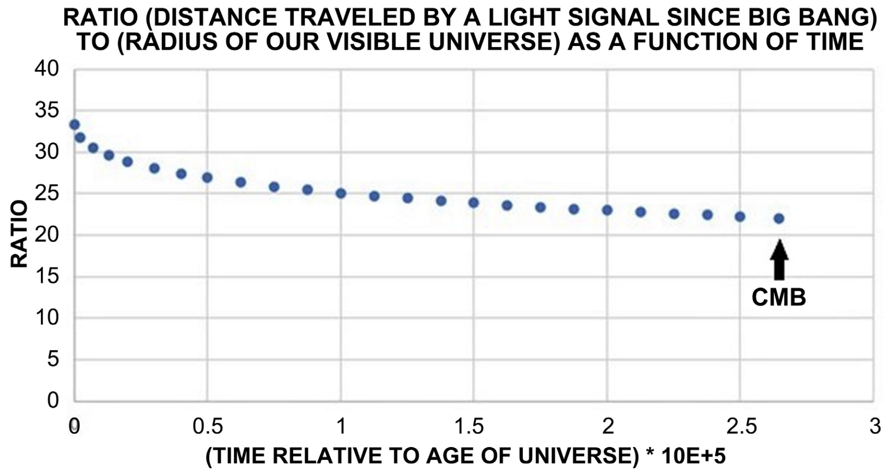

Figure 2. Ratio of the (cumulative) distance traveled by a light signal to the radius of our present-day Visible Universe for the time interval from zero to t_{CMB}. Ratios from 33.4 to 22.0 explain the smoothness of the CMB horizon.

to start with: the formulas show that thermal contact, producing a very smooth CMB, was possible in the very early Universe because of the very high distances covered by light signals at early times. Note that a variable speed of light makes the apparent contradictions intuitively acceptable.

9. For Non-Zero Curvature, the Combination of the FLRW Metric and the Friedmann Equation (and More Generally the Einstein Field Equations) Is Internally Inconsistent

The Friedmann equation, Equation (4), explicitly contains a constant speed of light c_0. However, as shown in the previous sections, the FLRW metric implies a variable speed of light $c(t)=\frac{a_0 c_0}{a(t)}$. Obviously, this situation is inconsistent. Indeed, if the reasoning leading to a variable speed of light is correct, a Comoving Observer at time t_1 should measure a speed of light $c_1=\frac{a_0 c_0}{a_1}$, and thus should start from a Friedmann equation containing a speed of light c_1 instead of c_0. In spite of this inconsistency, the combination of General Relativity and the FLRW metric (*i.e.* standard Big Bang theory) leads to remarkably good agreement with experimental results. To understand this we should note that the inconsistency noted here does not lead to problems for standard Big Bang theory immediately, because, as we know from experiment, we live in a flat 3D-space ($k=0$) and therefore the term proportional to c_0 in the Friedmann equation is zero. Thus, as shown in the previous sections, the scale factor function $a(t)$ does not depend on the speed of light, and using the experimentally measured values (c_1, H_1, $\Omega_{R,1}$, $\Omega_{M,1}$, $\Omega_{\Lambda,1}$, $\Omega_{K,1}$) will automatically lead to the same (correct) solution $a(t)$ of the Friedmann equation as for the $t=t_0$ case. Problems only arise when we ask questions such as "How can the distance covered by a light signal be larger than $c_0 t_0$?" (here the variation of the speed of light is important). The Horizon Problem belongs to this category.

However, for a curved 3D-space, ($k \neq 0$) there is a more direct problem: the standard Friedmann equation, Equation (4), uses the speed of light c_0 at all times, and therefore contradicts the requirements of the FLRW metric. This will lead to problems. A Comoving Observer at time $t_1 \neq t_0$, using the Friedmann equation containing c_0, and not c_1, will obtain an incorrect scale factor function. We can conclude that the evolution of the Universe, *i.e.* the function $a(t)$, as obtained for $k \neq 0$ by an observer at $t_1 \neq t_0$ using a constant speed of light c_0, will be incorrect. The Flatness Problem belongs to this category.

The obvious solution for this problem is to modify the Friedmann equation in such a way that c_0 is replaced by a variable speed $c(t) = \frac{a_0 c_0}{a(t)}$, as required by the FLRW metric. This was done in Ref. [18]. In the following section we will show that this procedure solves the Flatness Problem.

Also note that the Friedmann equation follows from the standard Einstein field equations of General Relativity. Therefore, if we accept a variable speed of light in the Friedmann equation, the speed of light must be allowed to be variable in the Einstein field equations in general as well.

10. The Flatness Problem

Experimentally, the curvature term $\Omega_{K,0}$ today is found to be very close to zero. This is very unexpected in standard Big Bang theory. First, we will now describe the Flatness Problem in more detail, and then we will show how the variable speed of light $c(t) = \frac{a_0 c_0}{a(t)}$ offers a simple and straightforward solution.

10.1. Flatness Problem: Traditional Big Bang Theory

In standard non-zero flatness ($k \neq 0$) Big Bang theory, the evolution of the flatness parameter $\Omega_K(t)$ can be obtained from Equation (46):

$$\Omega_K(t) = \frac{a_0^2}{a(t)^2} \frac{H_0^2}{H(t)^2} \Omega_{K,0} \tag{79}$$

The time derivative of $\Omega_K(t)$ can then be written as:

$$\frac{\mathrm{d}\Omega_K}{\mathrm{d}t} = a_0^2 H_0^2 \Omega_{K,0} \left[\frac{-2\dot{H}}{a^2 H^3} - \frac{2\dot{a}}{a^3 H^2} \right] = -2\Omega_K \left[\frac{\dot{H}}{H} + \frac{\dot{a}}{a} \right] = -2\Omega_K \left[\frac{\dot{H}}{H} + H \right] \tag{80}$$

Now, taking the time derivative of the definition $H = \frac{\dot{a}}{a}$ it is easy to show that

$$\frac{\dot{H}}{H} + H = \frac{\ddot{a}}{aH} \tag{81}$$

Also, from the Friedmann equation, Equation (4), we know that

$$\dot{a}^2 + kc_0^2 = \frac{8\pi G}{3} a^2 \left[\rho_{R,0} \frac{a_0^4}{a^4} + \rho_{M,0} \frac{a_0^3}{a^3} + \rho_{\Lambda,0} \right] \tag{82}$$

Taking the time derivative of both sides of Equation (82) we then find:

$$\frac{\ddot{a}}{aH} = -\frac{4\pi G}{3H}\left[2\rho_R + \rho_M - 2\rho_\Lambda\right] = -\frac{H}{2}\left[2\Omega_R + \Omega_M - 2\Omega_\Lambda\right] \tag{83}$$

Combining Equations (80), (81) and (83) we obtain the time derivative of Ω_K:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Omega_K(t) = \Omega_K H\left[2\Omega_R + \Omega_M - 2\Omega_\Lambda\right] \tag{84}$$

It is immediately clear that $\Omega_K(t) = 0$ is a solution of this differential equation. This is consistent with the experimental value $\Omega_{K,0} = 0$. However, when we focus on early times there is a problem. At early times the radiation and mass contributions Ω_R and Ω_M are dominant. The dark energy term Ω_Λ is negligible. Therefore the numerical factor $(2\Omega_R + \Omega_M - 2\Omega_\Lambda)$ must be positive at these times. Therefore it follows from Equation (84) that even a very small deviation of Ω_K from zero (positive or negative) will grow (exponentially) fast. Small deviations from zero are supposed to occur, e.g. due to quantum processes. Therefore the $\Omega_K(t) = 0$ solution is unstable. It can easily be shown that the value of $\Omega_K(t)$ at early times must have been unrealistically small in order to produce the very low value we observe today. This is known as the Flatness Problem.

10.2. Flatness Problem Solution: Big Bang Theory with a Modified Friedmann Equation Using a Time Dependent Speed of Light

As explained in Section 9, Big Bang theory can be made self-consistent by using $c(t) = \frac{a_0 c_0}{a(t)}$ instead of c_0 in the Friedmann equation. The equation can then be solved using the same methods as before. The modified Friedmann equation is:

$$\dot{a}^2 - \frac{8\pi G}{3}\rho a^2 = -k\frac{a_0^2 c_0^2}{a^2} \tag{85}$$

The flatness parameter $\Omega_K(t)$ can then be written as

$$\Omega_K(t) = \frac{a_0^4}{a(t)^4}\frac{H_0^2}{H(t)^2}\Omega_{K,0} \tag{86}$$

The time derivative of $\Omega_K(t)$ then is:

$$\frac{\mathrm{d}\Omega_K}{\mathrm{d}t} = a_0^2 H_0^2 \Omega_{K,0}\left[\frac{-2\dot{H}}{a^4 H^3} - \frac{4\dot{a}}{a^5 H^2}\right] = -2\Omega_K\left[\frac{\dot{H}}{H} + 2\frac{\dot{a}}{a}\right] = -2\Omega_K\left[\frac{\dot{H}}{H} + 2H\right] \tag{87}$$

From the modified Friedmann equation, Equation (85), it follows that

$$\dot{a}^2 a^2 + k a_0^2 c_0^2 = \frac{8\pi G}{3}a^4\left[\rho_{R,0}\frac{a_0^4}{a^4} + \rho_{M,0}\frac{a_0^3}{a^3} + \rho_{\Lambda,0}\right] \tag{88}$$

Taking the time derivative of both sides of Equation (88) we then find:

$$\frac{\ddot{a}}{aH} + H = \frac{4\pi G}{3H}\left[\rho_M + 4\rho_\Lambda\right] = \frac{1}{2}H\left[\Omega_M + 4\Omega_\Lambda\right] \tag{89}$$

Combining Equations (81), (87) and (89) we find the new differential equation for $\Omega_K(t)$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\Omega_K(t) = -\Omega_K H\left[\Omega_M + 4\Omega_\Lambda\right] \tag{90}$$

This equation is similar to Equation (84), however now the coefficient of Ω_K is always negative. At early times an arbitrary non-zero curvature will decrease exponentially. Space will inevitably evolve to zero curvature. Due to the decreasing speed of light the Flatness Problem has disappeared. Basic Big Bang theory predicts a flat Universe.

11. Discussion and Conclusions. Special Relativity, General Relativity and the Constancy of the Speed of Light

At present, the speed of light is generally considered to be a universal constant. Light rays can change direction, but every observer will always measure the same local speed. This is indeed the basic postulate of Special Relativity [24]. The predictions of Special Relativity have been verified multiple times to a very high degree of accuracy and therefore any claim of a variation of the speed of light can only be accepted if it is confirmed by irrefutable experimental results.

Note that in this paper, to avoid confusion, we define the speed of light from measuring the time it takes for a light signal to cover the length of a material object. This is different from the "constant speed" definition of 1983 [17].

In Special Relativity, in the absence of gravitation, the speed of light is clearly constant. In the presence of gravitation the situation is more unclear. The generally accepted opinion is that the speed of light is constant in that case as well, however dissident ideas exist. Einstein himself suggested a variable speed of light [25], before producing the standard equations of General Relativity. Einstein's variable speed of light calculation for bending of starlight by the Sun contained an error, but an improved version of this calculation, leading to the correct result, was later given by Dicke [26].

As shown in Section 9, the Friedmann equation and the FLRW metric are essentially inconsistent, because the first uses a constant speed of light c_0, and the latter implies a variable speed of light $c(t) = \frac{a_0 c_0}{a(t)}$, for a Comoving Observer. In the case of a flat space (our experimentally observed situation) this only produces problems when we focus explicitly on the variation of the speed of light (the Horizon Problem). In the case of non-zero curvature, the inconsistency leads to problems directly (the Flatness Problem). Introducing the correct variable speed of light in the Friedmann equation solves the Horizon and Flatness Problems immediately, taking away the need for ad hoc additions to Big Bang theory, such as Cosmic Inflation.

The problem is more general, however. The Friedmann equation was derived from Einstein's field equations of General Relativity. Therefore, if we accept the introduction of the FLRW metric as a legitimate procedure, and distances are expressed in units based on non-expanding material objects, we now have an example where it is necessary to introduce a varying speed of light in the Einstein field equations. This suggests a modification of the Einstein field equations in general: they should allow for a variable speed of light. The explicit form of the time dependence of the speed of light (and possibly the spatial dependence as well) will then be determined by the specific problem.

Experimental observations of the speed of light have been made as a function of time, at different locations, and fluctuating results have been obtained [27]. It has been suggested that these fluctuations are related to (tidal) variations of the local gravitational acceleration *g*. If such effects are real indeed, it is obvious that they are small variations compared to the large scale, long term evolution of the speed of light described here. The FLRW metric starts from a homogeneous Universe, and thus ignores perturbations due to local mass concentrations.

We can conclude that, to verify the decrease of the speed of light predicted here, new, more precise, measurements of the speed of light, and its evolution, are needed (the latest experimental measurements, in the 1970s, had an accuracy of approximately 1 m/s). Also it would be interesting to perform such measurements in gravitational environments different from the surface of the Earth. If the speed of light is found to be invariable in deep intergalactic space, we can conclude that the combination of General Relativity and the FLRW metric does not give an acceptable description of our Universe.

The concepts "expanding space" and "variable speed of light" are complementary. This, then, seems to be the physical meaning of the FLRW metric: the expansion of the Universe, and the simultaneous decrease of the speed of light for a Comoving Observer. This explains in a simple way how the distance covered by a light signal since the start of Big Bang can be much larger than $c_0 t_0$, as expected classically from a constant speed c_0.

A natural question is: "Is indeed the speed of light time dependent, or is what we observe just a consequence of gravitational time dilation?" It should be noted that a Comoving Observer doesn't have an objective method to judge his cosmic flow of time. Therefore it seems reasonable to define the speed of light from the time it takes a light signal to cover the length (supposedly constant) of a material object. Future developments of physics will tell us whether this is indeed the best way to describe the reality of our Universe.

Note that, if we write the Planck radiation formula in the frequency form, the speed of light only appears as a multiplicative factor $\frac{1}{c^2}$. Therefore, when the speed of light is altered, the general form of the spectrum remains unchanged, and we still recognize it as blackbody radiation (albeit with a modified temperature, due to the expansion of space (and wavelength)).

The present results can be interpreted as showing that the speed of light, as measured by a Comoving Observer, is caused by the collective mass of the Universe. This is essentially Mach's Principle [28]. Also note that in this view our present-day speed of light has no special meaning whatsoever. It just happens to be, by chance, the speed of light at the time we live. This also means that the speed of light is a variable property of the vacuum, changing as a function of gravitation. Note that this has also consequences for notions such as Planck length and Planck time, since the speed of light appears in their definition.

Observations of high-redshift supernova data have shown that, at present, the expansion of the Universe is accelerating [29] [30]. Interestingly, it was recently noted that this observed accelerating expansion of the Universe can be simply described by a phenomenological decrease of the speed of light by 2.2 cm/s per year [31]. This confirms the present calculation.

Additional claims have been made about experimental observations of a systematic decrease of the speed of light [32]. An interesting case is the Lunar Laser Ranging experiment (LLR). During the Apollo and Lunokhod missions, in the 1970s, mirrors were installed on the Moon, and these are used to determine very precisely the distance between Earth and Moon, by measuring the round trip time of laser pulses. In this way, a systematic increase of the Earth-Moon orbit semimajor axis of 3.8 cm per year has been observed [33]. Also it is straightforward to calculate that a decrease of the speed of light of 2.1 cm/s per year results in an apparent increase of the Earth-Moon distance of 2.7 cm per year. However, this distance is also influenced by tidal effects and other geophysical processes. Another observation is the Pioneer Anomaly, related to radio data probing the position of distant spacecrafts. All these observations, as described in [32] and [33], might suggest a speed of light decrease of the order of 2 cm/s per year, however the results are insufficient to make a final conclusion.

It might be argued that the present results contradict Special Relativity. This is correct indeed. Lorentz invariance is broken because of the transition from a non-expanding to an expanding frame of reference. In the present formalism, $a_i c_i$ is a universal constant. In Special Relativity, where a_i is a constant, this reduces to the standard result that c_i is a universal constant. Also note that, in the calculation presented here, the speed of light remains the maximum speed of communication at any time t. Therefore the speed of light remains the speed of causality, which is an essential aspect of Special Relativity.

Obviously General Relativity is a classical, non-quantum theory, producing divergences for time $t = 0$. These divergences should disappear in a quantum version of General Relativity. However, today we do not have such a quantum theory of gravitation yet.

In "normal" experiments, times are short, and it is perfectly acceptable to use a constant speed of light. For longer times, however, such as in cosmology and astronomy, a varying speed of light will lead to new physics. An obvious example is the solution of the Horizon Problem. This is essentially a consequence of the fact that the light cone is no longer made up of straight lines: it "opens up" at times closer to the start of Big Bang. Similar remarks can be made about the Flatness Problem. As we have shown, basic Big Bang theory with a decreasing speed of light predicts a flat Universe. Note that it is no longer necessary to make claims of extreme fine tuning to explain the observed flatness. On the contrary, basic Big Bang theory makes this flatness mandatory.

Also note that the present results imply that the most important motivations for the development of Cosmic Inflation theory no longer exist. To explain the isotropy of the CMB background and the Flatness of 3D-space we do not need to start from quantum processes, they are natural consequences of classical physical mechanisms.

Recent observations suggest that the speed of light is equal to the speed of gravitational waves to a very good approximation [34]. Therefore gravitational interaction is expected to show the same time dependence as the speed of light. This might be important for our interpretation of the results of gravity, especially at very large distances.

Also the Hubble Tension, *i.e.* the conflicting results for measurements of the present day Hubble constant, as determined from "recent" and "ancient" phenomena, might be related to this variation of $c(t)$. According to the present calculation, the speed of light was extremely high, and rapidly decreasing, at the time when the CMB background radiation was released. It should be investigated how this affects the baryon acoustic oscillations of the primordial plasma, and thus the temperature variations we observe today in the CMB background. If the present calculation is correct, it is clear that standard present day physics cannot be used to describe the situation at time $t = t_{CMB}$.

A very common remark is that in our present situation, on earth, we are not subjected to an FLRW metric, but to a Schwarzschild metric, due to the earth's gravitational field. However note that the earth's gravitational field does not make the gravitational influence of the rest of the Universe disappear. We are subjected to a superposition of local and distant fields. As the variation of the influence of faraway matter (and other forms of energy) is very slow, compared to local effects, these two contributions are decoupled in a natural way. It is reasonable to conclude that our present-day speed of light c_0 is very well approximated by the varying speed of light in intergalactic space (where the FLRW metric is supposed to be valid). Big Bang theory provides a general background, on which local phenomena are superimposed. Also, note that the Earth, the Solar System and the Milky Way are moving through intergalactic space, through regions that are supposed to have been modified by cosmic expansion before.

Finally, a variable speed of light suggests that other physical "constants" might be variable as well. This is certainly the case for ε_0 and/or μ_0, the permittivity and permeability of vacuum, since $c_0 = 1\big/\sqrt{\varepsilon_0 \mu_0}$. Obviously, such variations would seriously complicate the interpretation of experimental results. In this context it is interesting to note that recently it has been argued [35] that physics can be described starting from a Unified Energetic Tension Field based on (ε_0, μ_0) of free space.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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