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A Model of Modified Newtonian Gravity Alternative to MOND, Consistent with the Properties of Spiral Galaxies and Compatible with Extragalactic Dark Matter

Maciej Rybicki

Independent Researcher, Kraków, Poland
Email: maciej.rybicki@icloud.com

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Abstract

A new model of the modified Newtonian gravity called Compacted & Collapsing Gravity (CCG) is proposed. Similar to the Milgrom's MOND, it allows explaining the flattening of rotation curve in spiral galaxies, thus eliminates the need for dark matter at this level. However, in contrast to MOND, it puts a distinct limit on effective gravity; thereby constraints the sizes of single galaxies in connection to their masses, which complies with observations. In the bigger than single galaxies structures such as galaxy clusters, CCG rather complements than replaces interpretations of the observational data based on dark matter. Besides, the new model provides a plausible explanation to the hierarchical structure of the universe.

Keywords

Spiral Galaxies, Flattening of Rotation Curve, Gravitational Lensing, Dark Matter (DM), Modified Newtonian Dynamics (MOND)

1. Introduction

The numerous and longstanding observations [1]-[8] show that dynamical properties of the spiral galaxies do not comply with the Newtonian law of gravity, also including the (negligible in this case) corrections coming from the Einstein field equations. The orbital velocity of the visible components of galactic matter (mainly stars, gas and dust) should decrease in the direction to the outer disc-edge in accordance with the Newtonian equation:

$$V = \left(GMr^{-1}\right)^{1/2} \tag{1}$$

Hence, orbital velocity should depend on radius as:

$$V \sim r^{-1/2} \quad (2)$$

Instead of that, along the entire radial distance from the galaxy center up to the galaxy outskirts it is observed:

$$V \approx const. \quad (3)$$

The graphs of the functions of orbital velocities vs. galaxy radial distance obtained from the observations of different spiral galaxies each time appear roughly horizontal. The relevant effect called "flattening of rotation curve" is an outstanding problem in the galactic astronomy, known as "galaxy rotation problem". Likewise, the data inferred from the observations of bigger than single galaxy structures such as galaxy groups, clusters and superclusters indicate the properties inconsistent with both Newtonian gravity and general relativity (GR), as long as the sole visible matter is taken into account. Currently, there exist two alternative explanations to this apparent discrepancy. Most of experts, e.g. [9] [10] [11], following and developing the (cited above) hypotheses by Kapteyn, Oort and Zwicky stand for the existence of dark matter (DM), a hypothetic, electromagnetically inactive and therefore invisible form of matter, whose interaction with baryonic matter is restricted to the gravitation. A great impact on consolidation of the DM paradigm came from the large-scale thorough surveys conducted by Vera Rubin and her collaborators [12] [13] [14].

The evidences for dark matter come both from the single spiral galaxies and from the bigger structures such as galaxy clusters. Within the DM framework, the roughly flat function of orbital velocity vs. galaxy radius is the resultant of contribution of two (fictious) functions: a decreasing one—regarding the sole mass of visible matter concentrated in the galaxy bulge and disk, and increasing one—regarding the sole impact of a hypothetic DM galactic halo. Dark matter is thought to explain the anomalies in the gravitational behavior of colliding galaxies. It also provides convincing clue to the observed gravitational lensing of some distant galaxies, otherwise hardly explainable. As it is deduced from the cosmic microwave background, dark matter determines the spatial distribution of visible matter in the largest scale, the so-called "cosmic web" composed of filaments, separated by giant voids.

The various candidates for DM are divided into the "hot", "warm", and "cold" categories, due to different velocities of the relevant particles in the early universe. The other classification, partly intersecting with the one based on temperature, divides DM for the non-baryonic constituents (neutrinos, axions, WIMPs) and the baryonic components of the visible matter (protons, neutrons, electrons), the latter forming however the mostly invisible cosmic objects. The most favored non-baryonic candidates for DM are the weakly interacting massive particles (WIMPs), representing the hypothetic non-baryonic cold dark matter (CDM). WIMPs are too a generic prediction of the various extensions of the Standard Model in the particle physics, such as supersymmetry (SUSY), postulating existence of "supersymmetric particles" being themselves another candidate for DM. Instead, the postulated baryonic forms of DM are the massive compact halo objects (MACHOs) representing the invisible or poorly visible cosmic objects (black holes, neutron stars, brown dwarfs and faint red giants). Finally, DM is also supposed to consist of a mixture of different constituents mentioned above and, possibly, of the other ones yet unpredicted. Dark matter has become an essential constituent of the Lambda-CDM cosmological model. Accordingly, the universe contains 5% ordinary matter and 27% dark matter. All the rest (68%) of total mass-energy content falls on dark energy (DE).

It is paradoxical (bearing in mind the root causes of the DM hypothesis) that most of difficulties connected with DM refer to single galaxies. To name just a few: 1) The DM concept entails the necessity of *ad hoc* adjustments of the DM amounts to fit the Tully-Fisher relation in the low surface brightness galaxies [15]; 2) DM causes the so called "core-cusp problem" [16] [17]—a discrepancy between the distribution of DM inferred from Lambda CDM-model simulations and the one deduced from observations. Let me add my own personal doubt: 3) DM, despite postulated exotic properties, is, after all, supposed to interact with the visible matter and with itself according to the usual rules of gravity; why then the galactic halo does not collapse towards the center of galaxy mass merging with galactic visible matter? If it really forms a roughly spherical nonrotating halo, it should be gravitationally unstable, thus should significantly evolve during billions of years of its hypothetical existence. Meanwhile, if the DM halo really determines the current dynamics of Milky Way (and other spirals in the cosmic neighborhood), it has to be a very up-to-date structure, with no trace of destruction. Certainly, being electromagnetically inactive, DM cannot form stars; however, it should be able to create the black holes in a direct way (*i.e.*, with the omission of star evolution phase). Therefore, although respective conjecture goes against the main thesis of this paper, it is worth considering a possibility that supermassive black holes at the centers of most of large galaxies are in fact of DM origin.

Despite dominating position of the DM paradigm in the current cosmology, a number of researches stand for an alternative solution to the observed discrepancy based on the model called MOND (modified Newtonian dynamics) formulated by Mordehai Milgrom [18] [19] [20]. The core of Milgrom's model is the scale-dependent modification to the Newton's law of gravitation, which either totally excludes DM or at least significantly reduces its need. MOND has been extended to the relativistic form (as a modified GR) in a model called TeVeS, formulated by Jacob Bekenstein [21] and Robert Sanders [22]. Milgrom's model introduces modification to the Newtonian dynamics, interpreted either as correction to the Newton's second law of dynamics, or as a specific adjustment to the Newton's law of universal gravitation (TeVeS does the similar with regard to GR). As far as MOND explains pretty well the rotation curves in spiral galaxies, it is not as much effective in explaining the behavior of galaxy clusters and superclusters. In particular, some of the observed data concerning the discrepancy between the distribution of hot intergalactic gas and gravitational lensing in the Bullet Cluster (formed by colliding subclusters) speak for DM as a factor responsible for the observed deviation [23]. Bullet Cluster is widely thought to provide the best extragalactic evidence for DM, against MOND or TeVeS; however, Milgrom puts in doubt respective assertions [24].

The numerous experiments, to mention the alpha magnetic spectrometer (AMS) mission searching for an excess of positrons or antiprotons in the cosmic ray flux—a supposed effect of colliding dark matter particles according to the supersymmetry theory [25] [26] are expected to settle the question of existence and the real nature of dark matter; yet the results are so far not unambiguous [27]. The searches for "elusive particle" (*i.e.*, the one representing DM) are also conducted in various underground Earth labs [28]. On the other hand, the LISA Pathfinder mission [29], as well as local experiments in Earth labs [30], is aimed at testing Newtonian dynamics in the regime of very small accelerations being the actual domain of MOND. Also, the so-called Pioneer anomaly has been considered in this regard [31] [32] [33]. For now, situation is somewhat confusing. Both DM and MOND (and related TeVeS) have strong arguments on their sides; however, neither of them is able to cover effectively all observations. As far as MOND explains pretty well the galaxy dynamics, DM fits better dynamics and behavior of bigger structures. Concluding, the abundance of hypotheses concerning DM juxtaposed with lacking direct observational evidences means that its nature is still unknown, and even that its existence is still not absolutely certain. Therefore, any reasonable solution aimed at reconciling the contradictory presumptions (such as the one proposed in this paper) should not be excluded in advance. The other noteworthy alternative to DM and, specifically, to the Milgrom's MOND is the recently formulated model called MOUND—Modified Universe Dynamics [34] (see the comment in Acknowledgements).

2. Basics of MOND

The key concept of Milgrom's theory consists in replacing the Newton's second law of motion $F = ma$ by:

$$F = m\mu(x)a \tag{4}$$

where $\mu(x)$ is the interpolating function of the domain $x = a/a_0$, $x > 0$, where a_0 is the postulated natural acceleration constant of the assumed approximate value: $a_0 \approx 10^{-10}\ \mathrm{m \cdot s^{-2}}$, and a is the effective (really experienced) acceleration. In the version of MOND interpreted as an adjustment to the Newton's law of universal gravitation, the force F is identified with the Newton's gravitational force, which specifies Equation (4) to the form:

$$GMmr^{-2} = m\mu(x)a = ma_N \tag{5}$$

(a_N—Newton's gravitational acceleration). The exact form of function $\mu(x)$ is unknown; however, one postulates its specific properties; namely, for all arguments x it is assumed:

$$\mu(x) < 1 \tag{6}$$

Besides, a specific behavior of $\mu(x)$ is postulated for big and small arguments. Namely, for $x \gg 1$, the function converges with Newtonian values:

$$\mu(x) \approx 1 \tag{7}$$

Instead, for $x \ll 1$, the function is expected to give:

$$\mu(x) \approx x \tag{8}$$

Eliminating m from Equation (5) gives the MOND gravitational acceleration in the general case:

$$a = a_N / \mu(x) \tag{9}$$

For the case described by Equation (7) one gets $a \approx a_N$. Instead, the case described by Equation (8) implies:

$$GMr^{-2} = a_N \approx a^2 a_0^{-1} \tag{10}$$

Hence, in that case:

$$a \approx \left(a_0 a_N\right)^{1/2} \tag{11}$$

After rewriting a_N, one gets:

$$a \approx \left(GMa_0\right)^{1/2} r^{-1} \tag{12}$$

Considering orbital velocity: $V = (ar)^{1/2}$, this gives:

$$V = \left(GMa_0\right)^{1/4} \tag{13}$$

This equation implies that in the regime of very small accelerations the orbital speed no longer depends on distance, tending instead to maintain a constant value. There are few MOND functions satisfying the conditions (6), (7) and (8) that fit well the observed rotation curves in galaxies:

$$\mu(x) = x\left(1 + x^2\right)^{-1/2} \tag{14a}$$

$$\mu(x) = x(1 + x)^{-1} \tag{14b}$$

$$\mu(x) = 1 - \mathrm{e}^{-x} \tag{14c}$$

The choice between them is a matter of compatibility with observations.

3. The Reasons for "Modifying" MOND

A general consent among the proponents of Milgrom's theory is that MOND reveals its predicative power in the realm of very low accelerations. According to Bekenstein, "The centripetal accelerations of stars and gas clouds in the outskirts of galaxies tend to be below a_0" [35]. The phrase "tend to" is however somewhat misleading here. Interpreting the obtained data as consistent with a certain nonlinear function tending to zero suggests the possibility of existence of the objects with the effective gravitational acceleration significantly smaller than a_0, *i.e.*, $x = a/a_0 \ll 1$. Detecting such objects at the outskirts of galaxies would indeed speak in favor of MOND. Specifically, Equation (13) suggests that Newtonian acceleration a_N, as calculated from the visible mass and radius, should in some fraction of the observed galaxies significantly drop below the value of the acceleration constant ($a_N \ll a_0$; say, $a_N \sim 10^{-12}\ \mathrm{m \cdot s^{-2}}$ or even smaller)—to give effective accelerations of the value "below a_0", However, the so far observations do not indicate such values. They rather show that centripetal Newtonian accelerations never fall below $10^{-11}\ \mathrm{ms^{-2}}$ (see **Table A1**, **Appendix**). The absence of values smaller than a_0 makes an important argument against MOND; even stronger than against the Newtonian gravity itself—considering the relation between acceleration and radius according to the MOND relation $a \sim r^{-1}$, as compared with Newton's relation $a_N \sim r^{-2}$.

Besides (but in partial connection with the previous argument), the physical consequences of equation $V \approx (GMa_0)^{1/4}$ are hardly acceptable. It namely suggests that the linear size of any galaxy should basically not depend on the galaxy mass, in particular on the mass concentrated in the galaxy bulge. The putative validity of this equation would mean in fact the revival of the pre-Keplerian and pre-Newtonian "celestial mechanics" that treated circular orbits as self-explanatory due to their alleged "ideality". This would mean that orbital motion dispenses with any specific physical cause such as gravitational attraction that, thanks to Newton, gave explanation to the Kepler's three laws. Just a kind of similar (Galilean-like) property follows from MOND's Equation (13), despite its basically gravitational origin.

Let us consider an imaginary example. Assume an errant star (or any other inertial body) freely moving along the trajectory coplanar with the galaxy disc but traced far outside the edge. Assume the centripetal gravitational acceleration significantly smaller than a_0 and the velocity equal to V. The problem consists in the following. According to MOND equation $V = (GMa_0)^{1/4}$, no matter how big the radial distance is, V would refer to the linear orbital velocity. Hence, the mentioned star would always revolve around the center of galaxy. But why it should do so instead of just traveling along the roughly straight path, only slightly affected by the galaxy mass? Equation (13), as deprived of distance factor, does not answer this question. The degree of deviation from the straight trajectory due to gravity, as described either in terms of the Newton's gravitational force or the Einstein's spacetime curvature, depends on the amount of attracting mass and on the distance from the center of mass. Meanwhile, from Equation (13) it follows that linear velocity should always refer to the orbital motion, irrespectively of the distance from the center of mass and from the amount of mass itself. In other words, insofar as orbital motion tends to maintain the constant linear velocity, the ability to revolve does not itself depend on anything. Consequently, the size of given galaxy seems only to depend on the basically random distribution of matter. This looks not as modification of Newtonian gravity, but rather as its demolition!

4. Compacted & Collapsing Gravity (CCG)

Let $\eta(x)$ be the function of the domain $x = a_N / a_0$, where a_0 is the acceleration constant, and a_N is the Newtonian acceleration. Let us notice that this domain is different from the domain of the MOND interpolating function $\mu(x)$ being $x = a/a_0$. Consequently, the CCG function $\eta(x)$ is a "normal" function since a_N is the well-defined argument. Let $\eta(x)$ take the form:

$$\eta(x) = x^{\left(x^k\right)} \tag{15}$$

The coefficient k is a certain constant numerical parameter, such that $\{k \in \mathbb{R} : k \le 0\}$. The following relationship is assumed:

$$\frac{GM}{r^2} = \frac{a}{\eta(x)} = a_N \tag{16}$$

where a denotes the "effective" (*i.e.*, factually experienced) gravitational acceleration. It follows:

$$a = a_N \eta(x) \tag{17}$$

The specific graph of function $\eta(x)$ depends on the assumed value of coefficient *k*. However, it is clear from Equation (15) that, irrespectively of the specific value of *k*, for the argument $x = 1$, the function is $\eta(x) = 1$. This corresponds with the physical prediction according to which, at a distance *r* such that $a_N \approx a_0$, the effective gravitational acceleration falls down more or less violently (depending on the assumed value of *k*) below a_N. The more the function $\eta(x)$ coincides with the Newtonian acceleration together with the increase of absolute value of *k*, *i.e.*, for $k \to -\infty$ (hence, for $\eta(x) \to 1$), the sharper becomes the collapse of $\eta(x)$ starting from $x \approx 1$, that is $a_N \approx a_0$. Likewise, the excess of effective acceleration over a_N within the limit determined by $a_N > a_0$ strongly depends on the specific value of *k*. Until the physical determinants of the function (17) are recognized, setting the exact value of *k* remains a question of fine-tuning with observations. It is pretty obvious that the *k* should be roughly specified as $0 > k > -1$. For the *k* set in this range, the relationship between effective and Newtonian accelerations (*i.e.*, $x = a_N/a_0$) gives the following characteristics:

$$\begin{aligned} &x \gg 1 \to a \approx a_N \\ &x > 1 \to a > a_N \\ &x = 1 \to a = a_0 = a_N \\ &x < 1 \to a \ll a_N \end{aligned} \tag{18}$$

Let us assume the mid-value of the range $0 > k > -1$, *i.e.*, $k = -0.5$, specifying the function $\eta(x)$ as:

$$\eta(x) = x^{\left(x^{-1/2}\right)} \tag{19}$$

The so-defined function seems to be the most appropriate to reconcile the two observed properties: flattening of rotation curve and a distinct limitation of the size of each spiral galaxy (in particular the Milky Way) in connection to its mass. The exemplary relations between $x = a_N/a_0$ and $\eta(x)$ given by Equation (19) are the following:

$x = 10^{10}$ $\eta(x) \approx 1.0002$ (Newton)
$x = 10^{5}$ $\eta(x) \approx 1.037$ ↑
$x = 10^{4}$ $\eta(x) \approx 1.096$ ↑
$x = 10^{3}$ $\eta(x) \approx 1.244$ ↑
$x = 10^{2}$ $\eta(x) \approx 1.584$ ↑
$x = 10$ $\eta(x) \approx 2.071$ ↑
$x = e^{2}$ $\eta(x) \approx 2.087$ (extreme)
$x = 5$ $\eta(x) \approx 2.050$ ↓
$x = 2$ $\eta(x) \approx 1.632$ ↓
$x = 1.5$ $\eta(x) \approx 1.392$ ↓
$x = 1$ $\eta(x) = 1$ (Newton) → (collapse)
$x = 0.5$ $\eta(x) \approx 0.3750$ ↓↓
$x = 0.2$ $\eta(x) \approx 0.0273$ ↓↓
$x = 0.1$ $\eta(x) \approx 0.0007$ ↓↓

The arrows ↑, ↓ stand for the moderate increase and decrease vs. Newton's level respectively; instead, doubled arrows ↓↓ stand for the violent decrease, with "collapse" starting shortly after $x = 1$. "Newton" means effective acceleration nearly or exactly equal to Newtonian centripetal acceleration; "extreme" means the biggest excess of effective acceleration over Newtonian acceleration. Finally, "e" is the base of natural logarithm.

It follows that CCG specified by Equation (19) predicts an excess in the gravitational effective acceleration over Newtonian level within the distance-limit as determined between $a_N \gg a_0$ and $a_N = a_0$ (with the extremum at $x = e^2 \approx 7.39$) and then, for $a_N < a_0$, a violent drop of the effective acceleration below the Newtonian level, with the shortly (also regarding distance from the galaxy center) achieved value $a \approx 0$. In contrast to the predicted by MOND continuous excess in the gravitational acceleration over the Newtonian level tending far from the center of galaxy mass to $a \sim r^{-1}$ (instead of the Newtonian relationship $a_N \sim r^{-2}$), CCG determines a distinct spatial limit applied on that excess and on the effective gravitational interaction in general. This property decidedly constrains the galaxy size, each time in relation to the galaxy mass. From the Newtonian equation $V_N = (a_N r)^{1/2}$, having regard to Equation (17), it follows that the orbital linear velocity predicted by CCG is:

$$V_{CCG} = (ar)^{1/2} = (a_N r\ \eta(x))^{1/2} = \left(GMr^{-1}\eta(x)\right)^{1/2} \tag{20}$$

Hence, the relation between the Newtonian and CCG linear orbital velocities is:

$$V_{CCG} = V_N\left(\eta(x)\right)^{1/2} \tag{21}$$

Consequently, unlike the MOND prediction for very small accelerations, the predicted by CCG linear orbital velocity depends in any case on the radial distance.

5. Origin of the Acceleration Constant

Milgrom suggests a cosmological origin of the acceleration constant, which makes it a present-epoch constant, in general defined as:

$$a_0 = cT_U^{-1} \tag{22}$$

(c—speed of light; T_U—lifetime of the universe). Assuming $c = const.$, this implies a_0 linearly decreasing in cosmic time. In compliance with the Milgrom's suggestion, a_0 is postulated within CCG to decrease in cosmic time too, however not exactly in linear mode, but in connection with the change of some other cosmological parameters. Some recent, perforce highly hypothetical estimates of the linear size of total universe set its value for $R_U \sim 10^{27}$ m [36]—a value slightly exceeding the radius of the observable universe (~10^{26}). Considering estimations of the overall density: ~10^{-27} kgm^{-3}, this gives an approximate value of the total mass: $M_U \sim 10^{55}$ kg—a value exceeding the mass of observable universe by roughly two orders of magnitude. Let us notice that, due to the specific, *i.e.*, either simply or multiply connected topology of the universe (which is a global geometrical property different from curvature), the entire universe can be bigger, equal or even smaller than the observable universe. Admittedly, some researches still maintain that universe can be infinite in size, yet this rather obsolete assumption is nowadays commonly abandoned in favor of the (otherwise equally hypothetic) multiverse concept.

The above estimates confirm the conjecture according to which the total positive energy (according to $E = mc^2$) and the total negative energy (due to gravity) of the entire universe cancel each other out, making the net energy of the universe equal to zero (the concept of "zero-energy universe" [37]). This hypothesis corresponds with another one called the "black hole universe" [38], according to which the universe satisfies the Schwarzschild equation for the radius of static uncharged black hole: $R_S = 2MGc^{-2}$, which translated to the universe's parameters means:

$$R_U \approx GM_U c^{-2} \tag{23}$$

Expressed numerically in SI units this gives:

$$R_U \approx 10^{-11} \times 10^{55} \times 10^{-17} \approx 10^{27} \tag{24}$$

Once the concept of black hole universe is accepted (or assumed), it should be understood as a constant fundamental property of the universe and not as a random coincidence applying for the present epoch only. Consequently, identifying R_U with R_S means that, in accordance with the Hubble's law, also the physical term on the right side of Equation (23) increases at an equal rate. There are several possibilities to meet this requirement; my own research [39] led me to solution: $M_U = const.$, $c = const.$, and $G \propto R_U$. This relationship implies variability of some quantities treated so far as fundamental constants, namely the Newton's gravitational constant G increasing in cosmic time (a conjecture opposite to the decrease of G conjectured by Dirac in his Large Number Hypothesis—LNH). A consequence of G-variability is the variability of other constants containing G, namely, the Planck units (in particular of mass, time and length) and the Chandrasekhar mass-limit. Accordingly, the respective decrease of the latter may account for the observed deficiency of the Type Ia Supernovae progenitors [40].

Rearranging Equation (23) defines the "universal acceleration constant" a_U as:

$$a_U = c^2 R_U^{-1} = GM_U R_U^{-2} \tag{25}$$

Identifying a_U with a_0 gives:

$$a_U \equiv a_0 = c^2 R_U^{-1} \tag{26}$$

In SI units, this gives:

$$a_0 \approx 10^{17} \times 10^{-27} \approx 10^{-10} \tag{27}$$

The so-defined acceleration constant would be equivalent to the MOND acceleration constant $a_0 = c/T_U \approx 6.88 \times 10^{-10}\ \mathrm{m \cdot s^{-2}}$ assuming the equality $R_U = cT$. However, in the current standard cosmology based on Friedman equations, the latter is not precisely satisfied, first of all because the maximal speed of expansion (for extremely distant objects) likely exceeds the speed of light by roughly one order of magnitude. Therefore, the acceleration constant a_0 inferred from $c^2 R_U^{-1}$ is most likely a bit smaller, probably closer to:

$$a_0 \approx 10^{-11}\ \mathrm{m \cdot s^{-2}} \tag{28}$$

Assuming $G \propto R_U$ and $a_0 \propto R_U^{-1}$, one obtains the following relationship (postulated to occur in cosmic time) between increasing Newton's constant and decreasing acceleration constant:

$$c^4 M_U^{-1} = Ga_0 = const. \tag{29}$$

6. Applying CCG to Cosmic Structures

The most specific property of CCG is the prediction of a violent decrease (collapse) of the effective centripetal gravitational acceleration after exceeding the threshold of a_0 by the calculated Newtonian acceleration (*i.e.*, for $a_N < a_0$). This strongly determines an upper limit of the galaxy size in relation to its mass. The Milky Way is a good example. The visible mass of MW is $M \approx 5.8 \times 10^{11} M_\odot \approx 1.2 \times 10^{42}\ \mathrm{kg}$, the radius is $r \approx 4.5 \times 10^{20}\ \mathrm{m}$, the gravitational constant is $G \approx 6.7 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$. This gives Newtonian acceleration at the outskirts:

$$a_N = \frac{GM}{r^2} \approx \frac{6.7 \times 10^{-11} \times 1.2 \times 10^{42}}{2 \times 10^{41}} \approx 4 \times 10^{-10} \left(\mathrm{m \cdot s^{-2}}\right) \tag{30}$$

The obtained value is within the limits of a_0. This would account for the well-known fact, otherwise hardly explainable, that the number of stars in the Milky Way drops violently beyond the distance-limit of roughly 4×10^{20} m from the center. Another noticeable example is the Whirlpool Galaxy M51 (NGC 5194). The visible mass is $M \approx 1.6 \times 10^{11} M_\odot \approx 3.18 \times 10^{41}\ \mathrm{kg}$, the radius is $r \approx 76000\ \mathrm{ly} \approx 3.59 \times 10^{20}\ \mathrm{m}$. Inserting these values to the equation $a_N = GMr^{-2}$ gives $a_N \approx 1.64 \times 10^{-10} \left(\mathrm{m \cdot s^{-2}}\right)$, again matching a_0.

Analogous calculations using data obtained by Sanders and McGauch [41], concerning big sample of the spiral galaxies of different parameters seem to confirm the general prediction of CCG (see **Table A1** in **Appendix**). Bearing in mind the possible uncertainties of estimations, in particular connected with the choice of the mass-to-light relationship, one can conclude that the Newtonian accelerations of cosmic matter observed at the outskirts of galaxies never drop below 10^{-11} m·s^{-2}, usually fluctuating around 10^{-10} m·s^{-2}. This corresponds with the a_0 factor determining the collapse point predicted by CCG, which however does not exclude the possibility of a certain contribution of DM to the total mass in particular galaxies. The main conclusion concerns the relation between mass and radius; namely the radius never seems to be big enough to make Newtonian acceleration smaller than acceleration constant a_0.

In the case of clusters and superclusters of galaxies, the uncertainties as to the total mass become even more distinct. An outstanding example is the Bullet Cluster considered as the best evidence (called therefore the "smoking gun" [42]) for the presence of DM. Its mass is estimated for $\sim 10^{14} M_\odot$, *i.e.*, 10^{44-45} kg, whereas its linear size amounts roughly 300 kpc, *i.e.*, ~10^{22} m. The resultant centripetal acceleration at the outskirts roughly corresponds to a_0. Hence, mass in this case does not provide itself unambiguous evidence for DM—if respective estimates concern the visible mass. In this case, the arguments for DM mainly come from the observed divergence between the visible hot X-ray emitting gas and the observed shape of the gravitational lensing.

Instead, the respective data for Local (Virgo) Supercluster concerning the visible mass and width are: $\sim 10^{15} M_\odot$, *i.e.*, $10^{45\text{-}46}$ kg and ~10^{24} m. The Newtonian acceleration at the outskirts is thus of the magnitude 10^{-12} m·s^{-2}. Hence, in this case, the Newtonian acceleration indeed "tends to be below a_0", which might be considered as an argument in favor of MOND. However, considering the known difficulties of MOND as applied to the bigger than galaxies structures (in particular, connected with the gravitational lensing), this would rather speak for the presence of dark matter.

7. CCG and the Hierarchical Structure of the Universe

As is known from observations, the apparently homogeneous universe in fact consists of different cosmic structures ordered in the multi-level hierarchy starting from single or double stars (partly surrounded by planets) and ending with walls and filaments forming the cosmic web. The spatial limitation on effective gravity predicted by CCG may contribute to this order. A characteristic prediction of CCG is that, in the regime of very small (smaller than a_0) gravitational accelerations cosmic objects should behave in a quite different way than above that limit, compared with Newtonian gravity and, the more, with MOND. Namely, due to the predicted by CCG collapse of effective gravity deep below the Newtonian level for $a_N < a_0$, a given galaxy should not, in principle, "feel" the gravitational attraction coming from any neighboring galaxy taken individually (except for sufficiently close neighborhood). At the same time, it should feel the accumulated attraction coming from the sufficient number of galaxies forming the galaxy cluster to which given galaxy belongs. This may occur when the total visible and invisible mass of all galaxies forming given cluster is big enough to produce gravitational attraction exceeding a_0 within the space it covers. An analogous mechanism would apply both to the smaller and greater cosmic structures. In general, this property of CCG may account for the hierarchical size-order of the universe. This, of course, is an open question demanding careful analysis based on a broad data set.

8. Conclusion

The controversy between the standard DM model and MOND is far from being settled. The arguments for and against each of these hypotheses (or models) give place for other solutions, e.g., the one proposed in this paper. Undoubtedly, CCG follows MOND as to the general idea of modifying the Newton's law of gravity; however, it seems to be free from some difficulties connected with Milgrom's model. Besides, it rather complements than competes the DM concept, the latter being deeply rooted in the standard cosmology. In fact, all we actually have are the mathematical models and observations to test these models. The "simplicity" of given model should not be treated as the decisive criterion. GR, although mathematically incomparably more complicated than Newton's theory, is commonly regarded conceptually "simpler". None of theories, models or rules, including the Newton's law of gravity strictly connecting the force of gravity with the inverse of squared radius, can be regarded correct in advance. This also refers to the Einstein's GR. It is obvious that GR collapses in the presence of extreme gravity due to appearing infinities, and consequently, that singularities predicted by this theory should disappear in the yet unknown quantum gravity theory. On the other hand, it is an open theoretical question (also to be tested in experiments) whether the usual rules of gravity maintain, or change, in the regime of very small accelerations. The latter is just the realm of MOND, as well as of CCG proposed in this paper. For the time being, different possibilities are still on the table.

Acknowledgements

I am grateful to the anonymous Reviewer for turning my attention to the paper by Patrick Tonin [34]. The "Universe gravitational acceleration" proposed therein is in fact identical to my "universal acceleration constant" (Equation (25)), converted from the cosmologically determined Newton's gravitational constant proposed in [39] and [40]. Contrary to the Tonin's claim, Milgrom also suggested cosmological origin of a_0 (Equation (22)). The MOUND velocity formula shares the main property with respective MOND formula; namely for the Newtonian acceleration tending to zero (specifically, of the value significantly smaller than acceleration constant), the MOUND formula for orbital velocity

becomes identical with respective MOND equation (Equation (13)). This entails independence of the orbital velocity from radius in that area, hence disconnection between the mass and size of galaxy. This was exactly the main point of my disagreement with MOND, hence the reason to formulate CCG.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Kapteyn, J.C. (1922) *The Astrophysical Journal*, **55**, 302-327. https://doi.org/10.1086/142670

[2] Oort, J.H. (1932) *Bulletin of the Astronomical Institutes of the Netherlands*, **6**, 249-287.

[3] Zwicky, F. (1933) *Helvetica Physica Acta*, **6**, 110-127. https://doi.org/10.1007/s10714-008-0707-4

[4] Zwicky, F. (1937) *The Astrophysical Journal*, **86**, 217-246. https://doi.org/10.1086/143864

[5] Babcock, H.W. (1939) *Lick Observatory Bulletin*, **19**, 41-51. https://doi.org/10.5479/ADS/bib/1939LicOB.19.41B

[6] Oort, J.H. (1940) *The Astrophysical Journal*, **91**, 273-306. https://doi.org/10.1086/144167

[7] Van de Hulst, H.C., *et al.* (1957) *Bulletin of the Astronomical Institutes of the Netherlands*, **14**, 1.

[8] Volders, L. (1959) *Bulletin of the Astronomical Institutes of the Netherlands*, **14**, 323-335

[9] Erimble, V. (1987) *Annual Review of Astronomy and Astrophysics*, **25**, 425-472. https://doi.org/10.1146/annurev.aa.25.090187.002233

[10] Clowe, D., Gonzales, A. and Markevitch, M. (2004) *The Astrophysical Journal*, **604**, 596-603. https://doi.org/10.1086/381970

[11] Clowe, D., Bradac, M., Gonzales, A., Markevitch, M., Randall, S.W., Jones, C., *et al.* (2006) *The Astrophysical Journal Letters*, **648**, L109-L113. https://doi.org/10.1086/508162

[12] Rubin, V.C. and Ford, W.K. (1970) *The Astrophysical Journal*, **159**, 379. https://doi.org/10.1086/150317

[13] Rubin, V.C., Thonnard, N. and Ford, W.K. (1978) *The Astrophysical Journal Letters*, **225**, L107-L111. https://doi.org/10.1086/182804

[14] Rubin, V.C., Thonnard, N. and Ford, W.K. (1980) *The Astrophysical Journal*, **238**, 471-487. https://doi.org/10.1086/158003

[15] Zwaan, M.A., van der Hulst, J.M., de Blok, W.J.G. and McGaugh, S.S. (1995) *Monthly Notices of the Royal Astronomical Society*, **273**, L35-L38. https://doi.org/10.1093/mnras/273.1.L35

[16] de Blok, W.J.G. (2009) *Advances in Astronomy*, **2010**, Article ID: 789293. https://doi.org/10.1155/2010/789293

[17] McGaugh, S.S., Barker, M.K. and de Blok, W.J.G. (2003) *The Astrophysical Journal*, **584**, 566-576. https://doi.org/10.1086/345806

[18] Milgrom, M. (1983) *The Astrophysical Journal*, **270**, 365-370. https://doi.org/10.1086/161130

[19] Milgrom, M. (1983) *The Astrophysical Journal*, **270**, 371-383. https://doi.org/10.1086/161131

[20] Milgrom, M. (2001) *Acta Physica Polonica B*, **32**, 3612.

[21] Bekenstein, J.D. (2004) *Physical Review D*, **70**, Article ID: 083509. https://doi.org/10.1103/PhysRevD.70.083509

[22] Bekenstein, J.D. and Sanders, R.H. (2012) *Monthly Notices of the Royal Astronomical Society*, **421**, L59-L61. https://doi.org/10.1111/j.1745-3933.2011.01206.x

[23] Clowe, D., Gonzales, A. and Markevitch, M. (2003) *The Astrophysical Journal*, **604**, 596-603. https://doi.org/10.1086/381970

[24] Milgrom, M. (2016) Milgrom's Perspective on the Bullet Cluster. The MOND Pages. http://astroweb.case.edu

[25] O'Brien, T. (2013) AMS Detects Excess of Positrons, Could Suggest Existence of Dark Matter. https://engadget.com

[26] Jeltema, T.E. and Profumo, S. (2008) *The Astrophysical Journal*, **686**, 1045. https://doi.org/10.1086/591495

[27] Persic, M., Salucci, P. and Stel, F. (1996) *Monthly Notices of the Royal Astronomical Society*, **281**, 27-47. https://doi.org/10.1093/mnras/278.1.27

[28] Toomey, E. (2020) New Generation of Dark Matter Experiments Gear Up to Search for Elusive Particle. *Smithsonian Magazine*. https://www.smithsonianmag.com

[29] Trenkel, Ch., Kemble, S., Bevis, N. and Magueijo, J. (2010) Testing MOND/TeVeS with Lisa Pathfinder.

[30] Ignatiev, A. (2014) *Canadian Journal of Physics*, **93**, 166-168. https://doi.org/10.1139/cjp-2014-0164

[31] Anderson, J., Laing, P., Lau, E., Liu, A., Nieto, M. and Turyshev, S. (1998) *Physical Review Letters*, **81**, 2858-2861. https://doi.org/10.1103/PhysRevLett.81.2858

[32] Anderson, J., Laing, P., Lau, E., Liu, A., Nieto, M. and Turyshev, S. (2002) *Physical Review D*, **65**, Article ID: 082004. https://doi.org/10.1103/PhysRevD.65.082004

[33] Hajdukovic, D.S. (2010) *Astrophysics and Space Science*, **330**, 207-209. https://doi.org/10.1007/s10509-010-0467-y

[34] Tonin, P. (2021) *International Journal of Astronomy and Astrophysics*, **11**, 87-94. https://doi.org/10.4236/ijaa.2021.111006

[35] Beckenstein, J. (2006) *Contemporary Physics*, **47**, 387. https://doi.org/10.1080/00107510701244055

[36] Cornish, N.J., Spergel, D.N., Starkman, G.D. and Komatsu, E. (2004) *Physical Review Letters*, **92**, Article ID: 201302. https://doi.org/10.1103/PhysRevLett.92.201302

[37] Berman, M.S. (2009) *International Journal of Theoretical Physics*, **48**, 3278-3286. https://doi.org/10.1007/s10773-009-0125-8

[38] Pathria, R.K. (1972) *Nature*, **240**, 298-299. https://doi.org/10.1038/240298a0

[39] Rybicki, M. (2008) *Apeiron Journal*, **15**, 465-480. http://redshift.vif.com

[40] Rybicki, M. (2016) *Progress in Physics*, **12**, 3-14. http://www.ptep-online.com

[41] Sanders, R.H. and McGaugh, S. (2002) *Annual Review of Astronomy and Astrophysics*, **40**, 263-317. https://doi.org/10.1146/annurev.astro.40.060401.093923

[42] Clowe, D., Bradač, M., Gonzalez, A.H., Markevitch, M., Randall, S.W., Jones, Ch. and Zaritsky, D. (2006) *The Astrophysical Journal Letters*, **648**, L109-L113. https://doi.org/10.1086/508162

Appendix

The table below uses data obtained by Sanders and McGauch listed in the table entitled "Rotation Curve Fits" [40]. Compared with the original, the respective values have been reformulated according to the present need, in the following way. The second column contains visible masses obtained by summation of stellar masses and gas clouds. The third column contains observed linear sizes of galaxies. The fourth column contains respective Newtonian centripetal accelerations obtained from visible mass and radius, according to Newtonian formula $a_N = GMr^{-2}$. All values (expressed originally in $M_\odot$ and kpc) have been converted to SI units. The notation $a(b)$ means $a \cdot 10^b$.

Table A1. Newtonian centripetal accelerations at the outskirts of spiral galaxies, calculated from the total visible mass and the radius, according to $a_N = GM/r^2$.

galaxy#	visible mass (kg)	radius (m)	Newtonian acceleration at outskirts (m/s^2)
UGC 2885	7.12 (41)	4.01 (20)	2.97 (−10)
NGC 2841	6.76 (41)	1.42 (20)	2.23 (−9)
NGC 5533	4.39 (41)	3.52 (20)	2.36 (−10)
NGC 6674	4.36 (41)	2.56 (20)	4.44 (−10)
NGC 3992	3.23 (41)	1.26 (20)	1.35 (−10)
NGC 7331	2.86 (41)	1.39 (20)	9.88 (−10)
NGC 3953	1.62 (41)	1.20 (20)	7.50 (−10)
NGC 5907	2.15 (41)	1.23 (20)	9.50 (−10)
NGC 2998	2.65 (41)	1.67 (20)	6.33 (−10)
NGC 801	2.57 (41)	3.70 (20)	1.25 (−10)
NGC 5371	2.49 (41)	2.44 (20)	2.79 (−10)
NGC 5033	1.94 (41)	1.79 (20)	4.04 (−10)
NGC 3893	9.47 (40)	7.10 (19)	1.25 (−9)
NGC 4157	1.12 (41)	1.54 (20)	3.15 (−10)
NGC 2903	1.15 (41)	6.17 (19)	2.01 (−9)
NGC 4217	8.95 (40)	1.30 (20)	3.53 (−10)
NGC 4013	9.63 (40)	1.08 (20)	5.49 (−10)
NGC 3521	1.42 (41)	7.41 (19)	1.72 (−9)
NGC 4088	8.14 (40)	9.56 (19)	5.94 (−10)
NGC 3877	6.94 (40)	9.26 (19)	5.40 (−10)
NGC 4100	9.19 (40)	8.64 (19)	8.22 (−10)
NGC 3949	3.42 (40)	5.25 (19)	7.83 (−10)
NGC 3726	6.80 (40)	1.70 (20)	1.57 (−10)
NGC 6946	1.07 (41)	1.73 (20)	2.39 (−10)

Continued

NGC 4051	6.55 (40)	1.30 (20)	2.58 (−10)
NGC 3198	5.63 (40)	8.02 (19)	5.84 (−10)
NGC 2683	7.06 (40)	3.70 (19)	3.44 (−9)
NGC 3917	3.14 (40)	8.95 (19)	2.61 (−10)
NGC 4085	2.25 (40)	4.94 (19)	6.15 (−10)
NGC 2403	3.12 (40)	6.48 (19)	4.95 (−10)
NGC 3972	2.23 (40)	6.48 (19)	3.54 (−10)
UGC 128	2.94 (40)	2.84 (20)	2.43 (−11)
NGC 4010	2.25 (40)	8.95 (19)	1.87 (−10)
F 568-V1	1.99 (40)	9.87 (19)	1.36 (−10)
NGC 3769	2.65 (40)	5.25 (19)	6.40 (−10)
NGC 6503	2.13 (40)	5.25 (19)	5.15 (−10)
F 568-3	1.65 (40)	1.23 (20)	7.29 (−11)
NGC 4183	1.85 (40)	1.05 (20)	1.12 (−10)
F 563-V2	1.73 (40)	6.48 (19)	2.75 (−10)
F 563-1	1.57 (40)	1.33 (20)	5.85 (−11)
NGC 1003	2.23 (40)	5.86 (19)	4.34 (−10)
UGC 6917	1.47 (40)	1.05 (20)	8.91 (−11)
UGC 6930	1.45 (40)	9.26 (19)	1.29 (−10)
M 33	1.21 (40)	5.25 (19)	2.92 (−10)
UGC 6983	1.71 (40)	1.11 (20)	9.27 (−11)
NGC 247	1.05 (40)	8.95 (19)	8.74 (−11)
NGC 7793	1.01 (40)	3.40 (19)	5.81 (−10)
NGC 300	7.00 (39)	6.48 (19)	1.11 (−10)
NGC 5585	7.40 (39)	4.32 (19)	2.64 (−10)
NGC 55	4.60 (39)	4.94 (19)	1.26 (−10)
UGC 6667	6.60 (39)	8.64 (19)	5.90 (−11)
UGC 2259	5.40 (39)	4.01 (19)	2.24 (−10)
UGC 6446	8.30 (39)	9.56 (19)	6.06 (−11)
UGC 6818	2.80 (39)	5.86 (19)	5.44 (−11)
NGC 1560	2.60 (39)	4.01 (19)	1.08 (−10)
IC 2574	1.50 (39)	6.79 (19)	2.17 (−11)
DDO 170	1.70 (39)	4.01 (19)	7.04 (−11)
NGC 3109	1.50 (39)	4.94 (19)	4.10 (−11)
DDO 154	1.00 (39)	1.54 (19)	2.81 (−10)
DDO 168	7.00 (38)	2.78 (19)	6.04 (−11)