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Real Quanta and Continuous Reduction

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Abstract

Previous theories of quasicrystal diffraction have called it "Bragg diffraction in Fibonacci sequence and 6 dimensions". This is a misnomer, because quasicrystal diffraction is not in integral linear order n where $n\lambda = 2d\sin(\theta)$ as in all crystal diffraction; but in irrational, geometric series τ^m, that are now properly indexed, simulated and verified in 3 dimensions. The diffraction is due not to mathematical axiom, but to the physical property of dual harmony of the probe, scattering on the hierarchic structure in the scattering solid. By applying this property to the postulates of quantum theory, it emerges that the 3rd postulate (continuous and definite) contradicts the 4th (instantaneous and indefinite). The latter also contradicts Heisenberg's "limit". In fact, the implied postulates of probability amplitude describe hidden variables that are universally recognized, in all sensitive measurement, by records of error bars. The hidden variables include momentum quanta, in quasicrystal diffraction, that are continuous and definite. A revision of the 4th postulate is proposed.

Keywords

Quasicrystal, Icosahedra, Hierarchic, Periodic, Harmonic, Irrational, Geometric Series, Metric, Resonant Response, Dispersion Dynamics

1. Introduction

Einstein claimed Bohr's theory is incomplete: "the wave function does not provide a complete description of the physical reality" [1]. Their views represented two physics in schism [2]. Quanta are fundamental. Our theory of diffraction in quasicrystals is falsifiable and verified [3].

The quanta are not only harmonic; but harmonic in dual series: geometric and linear. Many have believed the quantum is real, rather than conceptual and axiomatic. The quasicrystal proves its reality. The formula for the free electron or photon probe, that consistently and realistically describes interactions by the dual wave-particles, can be further used to describe the reduction of the wave packet in space and time. The real probe bridges a short-cut that is taken by the method of mathematical probability amplitudes. The quantum finds new expression in the peculiar diffraction that we observe in quasicrystals [3].

Consider the quantum wave-packet in momentum space: in a scattering crystal, both the probe wave-packet and Bragg diffraction are periodic, while their interaction is *harmonic* in space and time, by *linea*r, *integral* orders. By contrast, diffraction from quasicrystals occurs *in geometric series* of *irrational* orders [4]. This scattering corresponds to hierarchic structure in the quasicrystal. It turns out that the quasi-Bloch waves—that are generated by the hierarchic scatterer and that mediate the quasi-Bragg diffraction—are dual harmonic in both geometric and linear series: the periodic probe scatters into geometric space.

How, we ask, does this realistic, dual harmony in the quasicrystal compare with the following 4 postulates of orthodox quantum mechanics [5]:

1) *Representational completeness of* ϕ. The rays of Hilbert space correspond one-to-one with the physical states of the system.

2) *Measurement.* If the Hermitian operator A with spectral projectors $\{P_k\}$ is measured, the probability of outcome k is $\langle\phi|P_k|\phi\rangle$. These probabilities are objective, *i.e.* indeterminate.

3) *Unitary Evolution of isolated systems:*

$$|\phi\rangle \to U|\phi\rangle = \exp\left(-\hbar^{-1}Ht|\phi\rangle\right)$$

and therefore deterministic and continuous.

4) *Evolution of systems undergoing measurement.*

If Hermitian operator A with spectral projectors $\{P_k\}$ is measured and outcome k is obtained, the physical state of the system changes discontinuously:

$$|\phi\rangle \to |\phi_k\rangle = P_k(\phi)\Big/\sqrt{\langle\phi|P_k|\phi\rangle}$$

Notice the opposites in the 3rd and 4th postulates: the unitary evolution is continuous and deterministic; the measurement is discontinuous and indeterminate. Heisenberg's uncertainty "limit" seems to have been arbitrarily discarded. The examination of dual harmonies in the wave-packet, preserves his limits.

In this paper, we consider first the probe, with group velocity and phase velocity variables; then illustrate dual quanta in quasicrystals; and finally describe the evolutionary reduction of the wave-packet using the known variables. The treatment is not so much probabilistic as classically crystallographical. Wherever measurement predictions are calculated to be the same in realistic theory as in probabilism, the theories are, in logic, equally "true"; however, the dual harmonics in quasicrystals demand redefinition of the quantum, and they are consistent with continuous change during measurement.

2. Wave-Packet

Consider the interaction, in time and space, between an X-ray or electron wave-packet and a quasicrystal. This stable wave-packet [6] is deduced from the combination of wave-particle duality, Maxwell's electromagnetism, special relativity, Planck's law, and the de Broglie hypothesis, all expressed in simplified units with unified reduced Planck constant $\hbar = 1 = c$, the speed of light. The rest mass (zero for the photon) of the probe is given by:

$$m_o^2 = \omega^2 - k^2 = (\omega + k)(\omega - k) \tag{1}$$

where ω is its angular frequency; k its wavevector; and m_o its rest mass (zero for the photon). The equation is separable into conservative and responsive parts. For a normal free particle, the wave function may be expressed[1]:

$$\varphi(t,x) = A\exp\left(\frac{X^2}{2\sigma^2} + X\right) \text{ with imaginary: } X = i\left(\bar{\omega}t - \bar{k}x\right) \tag{2}$$

where uncertainty σ depends on initial conditions that determine coherence of a packet in space and time (in manifold rank $\Re^4$) and where A^2 is a normalizing constant[2]. The variables $\bar{\omega}$ and $\bar{k}$ are mean values. The Gaussian envelope function is conservative and contains energy, momentum, mass density, intrinsic spin etc. The imaginary factor in exp(X) is responsive as an infinite wave with uniform density for all x and t: $(e^X)^*(e^X) = 1$. It describes interference, superposition, entanglement, creation, annihilation, harmony, resonance, etc. Equation (2) is not only *stable* mathematically with energy and momentum conservation; but it represents stable photons from the microwave background that have travelled 13 billion light years (cf. [7]).

The packet has many special properties. Differentiation of Equation (1) provides the equations for dispersion dynamics [6] in simplified units, including:

$$\frac{\omega}{k} \cdot \frac{\mathrm{d}\omega}{\mathrm{d}k} = 1 \tag{3}$$

where the normalized phase velocity of the wave $v_p/c = \omega/k$. Notice this phase velocity, *in vacuo*, is—for particles with mass $m_o > 0$—faster than the speed of light c. It does not conflict with relativity because the phase does not carry energy and is not measurable directly, but it is a real part of physics and we apply it below. The beat velocity is the normalized group velocity, $v_g/c = \mathrm{d}\omega/\mathrm{d}k$ [3] [6]. Notice that the group, velocity $v_g/c = (\mathrm{d}\omega/\mathrm{d}k)^{-1}$ = energy/momentum, and is the velocity of the reference frame in relativity, *i.e.* less than the speed of light. For a free particle, $v_g/c = k/m'$, *i.e.* momentum/relativistic mass.

The phase velocity v_p is the ratio of the two most measured variables in atomic physics and is very easily derived from the free particle wave equation e^X, while it is totally ignored in texts about quantum theory and even denied [7]. Its inverse v_g by comparison, that is theoretically derived with comparative difficulty as beat velocity, is the dominant variable in special relativity and even in quantum mechanics. As we illustrate below, the neglect of v_p causes major confusion, *e.g.* in collapse.

The probe is uncertain in space and time [8]:

[1]We let ϕ stand for a density function of either photon or electron.

[2]In the simplest case with k linear, $A^*A = 2\Big/\left(\int \exp\left(X^2/\sigma^2\right)\mathrm{d}\tau\right)$.

$$\Delta\omega \cdot \Delta t = 8;\ \Delta k_i \cdot \Delta i = 8,\ i = x, y, z \tag{4}$$

since σ (in Equation (2)) cancels after Fourier transformation. We shall consider uncertainty in collapse after reporting on the reality of dual harmonics in quasicrystals.

Notice that this physical result is sharply distinguished from signal processing of electromagnetic waves: in *the latter case*, c depends on physical laws that are invariant in all inertial reference frames, so that, wherever $m_o = 0$, $v_p = c = v_g$. The speeds may be measured by reflections of signals into space, or from interferometry. However, in *the former case*, as applied to electron microscopy, $v_g \ll c$ so that $v_p \gg c$. We will apply this internal motion of the wave packet in double slit interference.

Moreover, the wave-packet has remarkable properties that have been overlooked in standard quantum mechanics. The packet enables spatial entanglement in the propagation direction as in the transverse directions, and also enables action at distances, with speeds faster than c, in waves representing massive particles. We shall return to this property in the context of Young's double slit experiment, but are uncommitted regarding and Bell's inequalities and observations from crossed polarizers.

3. Hierarchic Structure

The probe just described diffracts off a hierarchical quasicrystal (QC) with icosahedral symmetry in its diffraction pattern. Not only is the pattern forbidden in Bragg diffraction from crystals, but so also are the diffraction orders which are in geometric series that is emphatically inconsistent with the integral orders n in Bragg's law: $n\lambda = 2d\sin(\theta)$, where $\lambda = 2\pi/k$ is the probe wavelength; d is the interplanar spacing for a particular diffraction beam; and θ is half the scattering angle. The law is very well understood in terms of harmonic reflections of the periodic probe from atomic sites that are periodic and crystalline. By contrast, quasicrystal diffraction is often misnamed "Bragg diffraction in Fibonacci series", which is a contradiction in terms for reasons already given. *A priori*, relationships between n, λ, d and θ are undefined. We had to work out both the law of quasicrystal diffraction and to understand the harmonies that are required between the periodic probe and the geometric series diffraction. It is obvious that the quasicrystal is structured from *hierarchically* arranged icosahedra because—especially after the unit cell is identified and measured—this introduces the geometric series with the point group symmetry of the pattern. Moreover, the unit cell is icosahedral and is extremely dense owing to the precise diameters not only of *Mn* and *Al* atoms in the first quasicrystal observed [4] but of all of the diatomic 1:6 quasicrystals subsequently reported. The structure is therefore *uniquely* icosahedral.

Furthermore, complete indexation in three dimensions was developed from a stereogram of the icosahedral structure, both for the principal axes as for the diffraction planes that are normal to them. The indexation was three dimensional and geometric, which excludes the prior usage of six dimensions. Dimensions should not be multiplied without necessity. The indexation of the diffraction pattern is complete [4].

From the hierarchic structure, quasi-structure factors could be calculated by formulae [4] that are modified from classical structure factors for crystals. The modifications included, firstly a scaling factor c_s to compensate for the (linear) aperiodicity of the structure that causes surprisingly sharp diffraction; and secondly an iterative procedure that summed quasi-structure factors over a large quasicrystal of selected order so as to account for the aperiodicity of the unit cell. By numerically scanning values for c_s, it was found to maximize at a unique value for all quasi-Bragg reflections. Applying this value, the calculations provided a range of intensities that matched very well the wide range of experimental values observed in the diffraction patterns. Not surprisingly, the structure factor intensities were all close to zero at Bragg scattering angles, but consistent with experimental line intensities at scattering angles that were larger than defined by Bragg's law[3]. The divergence from the Bragg condition depends on the coherence factor, $c_s = \theta/\theta'$: the Quasi-Bragg law was therefore deduced:

$$\tau^m \lambda = 2d' \sin(\theta') \tag{5}$$

for the simplest index (h, 0, 0) with $\tau = \left(1+\sqrt{5}\right)/2$, *i.e.* the golden section, and with interplanar spacing:

$$d' = a'\left(h^2 + k^2 + l^2\right)^{-1/2} \tag{6}$$

having generally irrational values h, k, l, for each a member of the series 0, τ^{-1}, 1, τ, …, τ^m, …, and having quasi-lattice parameter a', which was consistently measured in the conventional way [9]. The primes indicate modified versions of Bragg variables as they apply to QC diffraction (Equation (5)).

4. Summary of the Analytic Derivation

Whether the QC diffraction series is Fibonacci or geometric is nominal, since the following identities can easily be demonstrated by mathematical induction: [9]:

$$\tau^m = F_m(1,\tau) = \delta_{m,1} + F_m(0,1) + F_m(0,1)\tau \tag{7}$$

where the brackets define the bases for the m^{th} term of the Fibonacci series F_m and $\delta_{m,1}$ is the Kronecker delta. The geometric series is irrational, but Equation (7) shows how it can be separated into a natural part, $\delta_{m,1} + F_m(0, 1)$, and irrational part, $F_m(0, 1)\tau$. By substituting the last τ by 3/2, an approximate natural value for τ^m is obtained, and subtraction of this approximate natural value from the original irrational geometric number, yields an extraordinary and exact value

[3]In crystals, structure factors are calculated from atoms in a unit cell, and the number of values is therefore restricted. In pure *Al*, for example, there are only two values: 0 (when half the atoms scatter in antiphase) or 4. In quasicrystals, the quasi-structure factors are calculated over all the scattering atoms in selected structural orders of clusters, superclusters *etc.* This calculation is performed iteratively.

$1/c_s$. which we will call also the metric function (Equation (8)). The match is emphatically extraordinary because the comparison is between numeric and analytic answers, that are the same for all m, *i.e.* $\tau - 0.5$.

The exact match clarifies the nature of c_s. The irrational part of the index provides a scaling factor for the scattering of probe by specimen, that results in dual harmonics in the diffraction (Section 4). The scaling factor describes a ratio between a Bragg angle from a cubic crystal having lattice parameter a and integral indexation, from a corresponding quasi-Bragg angle in quasicrystals with quasi-lattice parameter a' and geometric indexation. The ratio results from path differences between neighboring rays in the quasi-Bragg scattering [10]. The irrational part, phase shifts the ray paths that are longer than in Bragg diffraction. The result is that corresponding quasi-Bragg diffraction angles are not only sharply defined; but are fractionally larger by ~11.18… % after allowing for different indexations in electron microscopy where $\sin(\theta) \sim \theta$. The fraction is the peculiar consequence of the ideal icosahedral, hierarchic structure. The diffraction pattern is a map of quantized momentum transfers. The dual harmonics determine the momentum quanta that define quasicrystal diffraction patterns. The following illustration is by quasi-Bloch waves that are stimulated by the interaction of the probe that scatters inside the quasi-lattice.

5. Dual Harmonies That Occur in Irrational, Geometric Order

The diffraction mechanism by quasi-Bloch waves in QCs at the quasi-Bragg condition, illustrates consequences of the irrational QC diffraction. In a crystal oriented to a first order Bragg condition, an advancing high-energy electron-beam interacts with the reflecting lattice to form two momentum dispersed Bloch wave bands [11]. Relative intensities of the zeroth order and first order beams depend on specimen thickness and on specimen orientation, and they form regular fringes in wedge foils; and lattice images in high resolution imaging [12]. The images are commensurate with the unit cell and with all cells periodically repeating. This is represented in the blue wave of **Figure 1**. However, these periodic Bloch waves are incommensurate with the hierarchic quasi-lattice that is geometric and irrational. However if their scale is multiplied by the metric function (Equation (8)) [3] [9];

$$\frac{1}{c_s} = 1 + \frac{\tau^m - F_{m+4}/2}{F_{m+1}} = \frac{1}{0.894} \tag{8}$$

the (red) wave becomes commensurate with the geometric quasi-lattice both long-range, and simultaneously at linear short-range on each geometric intercept, *i.e.* for all m. In Equation (8), F_m represents the Fibonacci sequence, base (0, 1). The quasi-Bloch wave is translationally invariant about all geometric intercepts $a'\tau^m$. Notice that the spacings between intercepts are in Fibonacci series that are represented by the denominator in Equation (8), F_{m+1}.

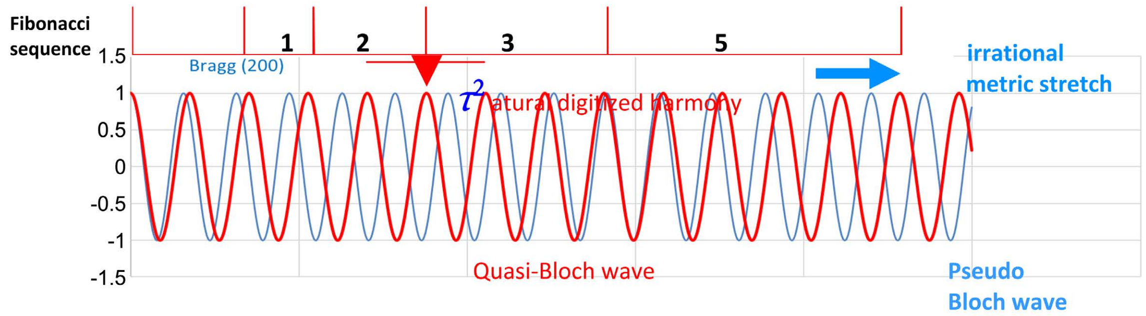

Figure 1. Crystalline Bloch waves (blue) are commensurate with their unit cell and corresponding periodic crystal lattice at the Bragg condition. When this wave is stretched horizontally by the inverse coherence factor $1/c_s$, the quasi-Bloch-wave (QBW in red) commensurates with the irrational, geometric and hierarchic, quasi-lattice. Its geometric order is represented by the intercepts on the horizontal line above it. The digitized number of periodic cycles between successive intercepts is in Fibonacci sequence (denominator in equation 8), and the diffraction is logarithmically periodic. The natural and irrational parts of the indices are separable: the irrational part is expressed by the metric stretch; the natural part scatters with sharp, coherent diffraction [3].

Most Important is the fact that the quasi-Bloch wave is dual harmonic. The irrational part of any index is represented by the metric function (Equation (8)) and this digitizes the periodic probe, which commensurates with the hierarchic lattice. The fractional increase in ray paths causes an 11.18… per cent increase in scattering angle. The dual harmony enables the periodic probe to scatter coherently from the hierarchic lattice onto a geometric reciprocal lattice with a peculiar and precise quasi-lattice constant a' [3].

It is obvious that the dual harmony forces the quantization of the momentum that is evident in the diffraction pattern. *It is reasonable to make the hypothesis that all quantization is the result of—not the cause of—harmonic dynamic variables.* Further confirmation may, in future, be found from multi-slice calculations of quasi-Bloch wave intensities as probe interacts with specimen. This becomes more feasible now that c_s is known, understood and applied with geometric band-gaps in momentum space [11].

Notice that this discovery of dual harmony in QC diffraction is realistic rather than mathematical: the solution is three dimensional, geometric, and classical with harmonies in space and time. By contrast mathematicians have digressed with six dimensions for "Fibonacci sequences" in abstruse tiling and unexplained diffraction [13].[4]

6. Hidden Variables

> "It was [Einstein's] almost solitary conviction that quantum mechanics is logically consistent but that it is an incomplete manifestation of an underlying theory in which an objectively real description is possible—a position he maintained until his death" ([14] p. 433).

Einstein's EPR thought experiment has not resolved his differences with

[4]Incidentally, another example of dual harmony is the 12 point chromatic scale in Western music, which is, conversely, irrational short range ($\times\sqrt[12]{2}$) and linear long range (×2).

Bohr[5]: the former realistic, the latter probabilistic. The difference is partly nominal because a realist variable that is truly hidden can be represented, in epiphenomenal mathematics, by a probability amplitude. That is why mathematicians have been content to "choose" their lattice parameter [3] [4], instead of measuring it, and thereafter to "apply" Bragg's law in six invented dimensions. By contrast, the metric function is derived, observed and verified in three dimensions. The derivation occurs by applying observed, irrational indices to a modification of the classical theory, and the derivation is therefore realistic.

This is not to deny that the probability amplitude, that is used for example in elementary particle interactions, has been extraordinarily successful. However, the standard theory is indistinguishable from a realist theory when the probability amplitude expresses hidden variables. This is consistent with "mad-dog Everettianism" [5]: we have the Schrödinger equation and a wave function and that is all, with no metaphysics and no phenomenalism.

Sometimes a "hidden variable", such as phase velocity, becomes useful to explain a long-known property such as intrinsic spin, and to discover new dependent properties such as its magnetic radius [15]. As another example, the hidden variable that determines the direction of spontaneous emission is, by momentum conservation, atomic recoil, the same recoil as is acknowledged in the Schrodinger eigenvalues by reduced mass on the electron. In this paper, we use phase velocity that derives directly from special relativity and can be used to describe the otherwise instantaneous and problematic collapse of the wave packet when a measurement is made (Section 6). The phase velocity is hidden because it is faster than the speed of light, and because it does not carry energy; this is carried by conservation in the group velocity. However, when the angular frequency and wavelength of an interacting particle is known, the phase velocity is sometimes informative. It is simpler in concept (ω/k) than the more easily measured group velocity $\mathrm{d}\omega/\mathrm{d}k$. For many such reasons, the wavefunction cannot be complete. The realistic quantum that we have described implies continuity in the final realistic collapse upon measurement (*cf.* postulate 4).

7. Operators and Reduction of the Wave-Packet

Equations (1) and (2) describe a *stable* wave-packet because $\bar{\omega}$ and $\bar{\boldsymbol{k}}$ are conserved. They represent photons that are more than 13 billion light years old, when measured in the microwave cosmic background, to an accuracy of $1{:}10^5$. They are mathematically conserved by mean energy and momentum; so can hardly be unstable (cf. [7]). The equations also apply to free electrons in high energy electron diffraction.

To show that the photon is consistent with mainstream quantum mechanics, we need to show that it responds consistently with known operators. Consider firstly, the energy operator in Schrödinger's equation:

[5]Though it is obvious that if reference frames are not held stable, Einstein could not measure, by conservation laws, the momentum on Bohr's electron.

$$\hat{\varepsilon} = -i\hbar \cdot \partial\phi/\partial t \tag{9}$$

Then by applying to Equations (2) with the chain rule:

$$\left\langle \phi^* \middle| -i\hbar\frac{\partial}{\partial t} \middle| \phi \right\rangle = \left\langle \phi^* \middle| \omega(2X+1) \middle| \phi \right\rangle \tag{10}$$

the integral over X, in the antisymmetric first term of the bracket operating on symmetric ϕ is zero. The second term provides the expectation $\langle \varepsilon \rangle = \bar{\omega}$, in absence of Schrödinger's central potential etc.

Similarly,

$$\langle \boldsymbol{k} \rangle = -\bar{\boldsymbol{k}}_x \tag{11}$$

With this consistency, we proceed to consider the reduction of the wave-packet in space and time. Particularize with observations on a Young's slit experiment in strong beam and weak beam (**Figure 2**) ([16] p. 262). After taking account of different λ and m_o, electrons produce corresponding interference patterns to Young's. Suppose an *electron is a point particle* that may be incident on slit B as a single time-resolved event. In weak beam, individual scintillations are observed in the image plane (green pattern), but the pattern is different if slit A is open (upper red pattern) or closed (lower red pattern). Bohr claimed that the calculated wave function is a probability amplitude. He held no way of predicting precisely where an individual event would be recorded on the image plane. Einstein objected that his interpretation of probability amplitudes implies "spooky action at a distance", which is unsatisfactory as an explanation because information about the state of A would be needed at B by a speed faster than light.

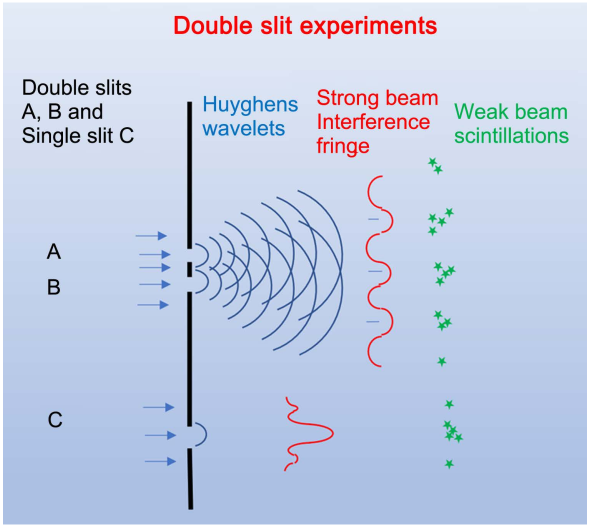

Figure 2. A bright incident beam, transmitted by Young's double slit, forms a regular interference pattern in the image plane (upper red). When the beam intensity is weak, scintillations may be counted on the plane (green) while, after a long time, the pattern approximates to the strong beam pattern. A single electron passing through slit B would require "spooky action at a distance" to respond to either slit A open (upper red) or A closed (lower red).

However, when we consider the wave function to be a probability amplitude that is due to hidden variables, including v_p, then the information at A (whether open or closed) may be carried to B through those variables in the following way.

The wave-packet described by Equation (2) is extended by σ_x in both time and x-space so that transverse waves have time Δt to interact after passing through the slit(s), and the interference is as Young observed it (**Figure 2**) with the transverse uncertainty σ_y that can be estimated. There is no spooky action at a distance, and no instantaneous collapse: Notice that across the wavefronts in **Figure 2**, time is constant and locally Newtonian. The waves, as they advance, interact long range with scintillator atoms in their general path: some will resonate in phase causing the transverse density function to accelerate across the front in response, further exciting a scintillating molecule. Any resonant molecule will compete with other molecules to absorb energy from the electron, so that energy will eventually be captured when the wave front becomes localized. Subsequently decay occurs by photo-emission.

Electron-scintillator resonance corresponds to photon resonance, which is simple since in vacuo, the components $v_p^i = v_g = c$. Absorption depends on the oscillator strength $|\langle|e\mathbf{r}/4\pi\epsilon_0|\rangle|^2$ around the excited molecule. Typically, since the scintillation energy of de-excitation is similar to the probe photon energy, its absorption is an all-or-nothing event.

Consider the "collapse" of Bohr's wave-packet that is supposed to occur when an event is recorded by scintillation or chemical reaction on photographic emulsion. In the standard theory, by definition of the wavefunction, the event is only probable and never predictable for particular quanta. However, as with the interference pattern, a realistic wavefunction undergoes a different sequence. The real interference occurs throughout the space between slits and image plane, as a superposition of excitations from the double slit. Moreover, this pattern extends in both time and space (Equations (2)). As the superposition approaches the image plane it interacts, in time and space, with the chemicals on the image plane, some of which will—depending on hidden variables—resonate, typically through electric permittivity. Resonances will appear and there may be a mutual forward-backward response through the extended wave-packet, leading to concentration of the wave-packet near a molecule and localization as the excitation grows. Final absorption will occur in time Δt~$8/\Delta\omega$, within typical scattering angle of <45˚ from the axial line through the slits. The absorption event is, within this time scale, all-or-nothing, and contrasts with the gradual decay of energy when the probe is a high energy electron beam. The resonance occurs continuously.

High-energy electron diffraction may scintillate more than one atom sequentially with small energy losses, so that the absorptive beam spread is small across a thin detector. Furthermore, over a short decay path, multiple excitations, if they occur, will result typically in a single recorded pulse owing to the uncertainty in time σ (in Equation (2)) controlling the resonant interaction.

Because the quantum has finite uncertainty, and because the electron has intrinsic magnetic moment with dimensions $L^3T^{-1}Q$ in length, time and charge respectively [15], we can drop the supposition that the quantum is a point particle. Instead, Huyghens' wavelet is real and is described mathematically in the $\Re^4$ complex space. Consider further, the phase velocities that are described in the wave-packet (Section 2) in all three spatial dimensions. On the photon they are all equal to c: $v_p = v_g$. However in the electron with finite m_o, $v_p > c$, and the transverse velocities are greater than the velocity in the propagation direction, $v_p^y, v_p^z \gg v_p^x$, since transverse momenta are small: $1/v_p^i = v_g^i = p^i/m'^i$; $i = x, y$ or, z (momentum/relativistic mass m', in simplified units). Applications of these principles to Bell's inequalities and to crossed polarizers will be described in future work [17][6].

8. Conclusions

Easy it is for a mathematician to invent axioms that describe an infinite wave that is attached to a quantum as if the wave were a probability amplitude for the positions and momenta of atoms in an ideal gas. The invention was discontinuous and indefinite during measurement, and therefore not subject to the laws of physics. It is lucky that such an invention should have been useful in developing the standard model for elementary particles.

However, it is difficult for a physicist to discover the continuous and definite laws in physics that can be used to predict future measurements on such complicated systems. This is done here by employing the reality of the wave function, including its physical properties of phase velocity that is measured by real components ω, k_i, ν, λ etc.

We propose a change in postulate 4 of quantum theory (in the Introductory section above) to account for physical variables that can always be described, even if not actually measured on individual atoms:

4. Evolution of systems undergoing measurement.

If Hermitian operator A with spectral projectors $\{P_k\}$ is measured and outcome k is obtained, the physical state of the system changes continuously: $|\phi\rangle \to |\phi_k\rangle = P_k(\phi)\Big/\sqrt{\langle\phi|P_k|\phi\rangle}$ within time $\Delta t \sim 8/\Delta\omega$.

The notions, that the probability amplitude is extended in time and space, while that the quantum is a point particle, are multiplication of entities. By contrast, we have shown how the real wave-packet describes the effects of Young's slits completely, as it does indeed for other diffraction effects: Quasicrystal diffraction has proved the quantum to be dual harmonic and real in this instance. We understand that phase velocity $v_p = \omega/k$ is hidden in the sense that it is measured through its inverse, $v_g = \mathrm{d}\omega/\mathrm{d}k$. Independently, the constituent va-

[6] The following identities, that are used in this paper, are consistent. Relativity: $E^2 = p^2c^2 + m_o^2c^4$; $E = \hbar\omega = m'^2c^4 = m_o^2c^4\Big/\sqrt{1-\beta^2}$; $\beta = v_g/c$; $p = \hbar k = m'v_g = m_o v_g\Big/\sqrt{1-\beta^2}$. Corrollary: Frequency ν, angular frequency ω, wavelength λ, and wavector k are all relativistic; m_oc^2 is normally constant. On the analysis used, after massive annihilation, energy conservation would require the rate of increase in c is half the rate of decrease in rest mass m_o: $\mathrm{d}c/\mathrm{d}m_o = -c/2m_o$.

riables are also measurable.

In physical quantum mechanics, reduction of the packet is continuous in time, having typical uncertainty, $\Delta t = 8/\mathrm{d}\omega$. This contradicts postulate 4 in mathematical quantum theory, where the evolution is instantaneous. On their hypotheses, arbitrary change in reference frame is a mathematical option for Bohr in the EPR experiment [1]; but is confusing in the wider scope of physics.

In other words, the fact of real quanta in quasicrystals implies that extreme probabilism is an analytic theory of math. If Probabilism expresses the effects of hidden variables in measurement, then it is indistinguishable from real physical theory since they describe the same experimental results.

9. Postscript on Intuition

Many, from freshman undergraduates to seasoned mechanists, have been mystified by physical quanta. Einstein famously objected, "God does not play dice with nature".

> "[Einstein] was said to reject the idea of a personal God, but I am fairly sure he meant by that the anthropomorphic figure of the Blake pictures—God with a great beard. He accepted the idea of a spirit of righteousness and for one who had not fed on the Gospels that is surely a just paraphrase of what the true idea of God might mean." [18]
>
> Dirac on Einstein: "He wasn't merely trying to construct a theory to agree with observation. So many people do that. Einstein wrote quite differently. He tried to imagine, 'If I were God, would I have made the world like this.' And according to the answer to that question, he would decide whether he liked a particular theory or not." [19]
>
> Pauli quipped, "Dirac has a religion, 'There is no God and Dirac is his prophet'" [20].

All science begins with intuition. One intuition need not depend on another. Einstein's calculation of the perihelion of Mercury and concern for light bending during the 1917 total eclipse show Dirac mistaken on the necessity for evidence, at least for general relativity. Intuition belongs not to the core logic of science that has been described by Popper [21]; but to the psychology of scientists before they formulate a law and begin to systematize and verify it. Einstein's profound belief in objective reality for the wave function [1] ([22] ch.25c) has been debated over a long time e.g. ([14] p. 433).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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