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Lorentz Transformation Leads to Invariance of the Difference between the Electric and Magnetic Field Intensity

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Abstract

In course of a direct calculation we demonstrate the activity of parameters of the Lorentz transformation entering the original electric and magnetic field vectors $\boldsymbol{E}$ and $\boldsymbol{H}$. The validity of the transformation is shown with the aid of the relation $\boldsymbol{E}^2 - \boldsymbol{H}^2 = \boldsymbol{E}'^2 - \boldsymbol{H}'^2$ which holds for any suitable pair of the vectors $\boldsymbol{E}$, $\boldsymbol{H}$ and $\boldsymbol{E}'$, $\boldsymbol{H}'$. No special geometry of the vector pairs entering $(\boldsymbol{E}, \boldsymbol{H})$ and $(\boldsymbol{E}', \boldsymbol{H}')$ is assumed. The only limit applied in the paper concerns the velocity ratio betweeen v and c which should be smaller than unity.

Keywords

Electric and Magnetic Intensity Pairs, v Denotes the Velocity Ratio between Two Vector Systems

1. Introduction

The aim of the paper is to examine the effect of the Lorentz transformation of the electromagnetic field when the field formulae are general. The Lorentz transformation of the electromagnetic field is a well-known tool applied in numerous motion occasions [1] [2]. But, in spite of its importance, a general kind of the Lorentz transformation concerning three dimensions of the electromagnetic field, seems to be rather seldom discussed. Usually the transformation is limited to a special geometry assumed for a moving particle, or specific values of the applied mechanical parameters.

The main aim of applying the Lorentz transformation seems to be a search for the application of some invariant expressions which remain exactly unchanged.

A well-known example is a tensor built up of the electric and magnetic components [1] [2]:

$$(F_{ik}) = \begin{pmatrix} 0 & H_z & -H_y & -iE_x \\ -H_z & 0 & H_x & -iE_y \\ H_y & -H_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{pmatrix} \tag{1}$$

which represent field intensities belonging to one four-dimensional electromagnetic tensor [2].

2. Requirements Concerning the Lorentz Transformation

In general the Lorentz transformation replaces the original components of the electromagnetic field, viz.

$$\begin{gathered} E_x, E_y, E_z, \\ H_x, H_y, H_z, \end{gathered} \tag{2}$$

by the new components

$$\begin{gathered} E_{x'}, E_{y'}, E_{z'}, \\ H_{x'}, H_{y'}, H_{z'}. \end{gathered} \tag{3}$$

Both kinds of components are coupled according to the fomulae [2]:

$$E_x = E_{x'}, \tag{4}$$

$$E_y = \frac{E_{y'} + \frac{v}{c} H_{z'}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_{y'}}{p(v/c)} + \frac{\frac{v}{c} H_{z'}}{p(v/c)}, \tag{5}$$

$$E_z = \frac{E_{z'} - \frac{v}{c} H_{y'}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{E_{z'}}{p(v/c)} - \frac{\frac{v}{c} H_{y'}}{p(v/c)}, \tag{6}$$

$$H_x = H_{x'}, \tag{7}$$

$$H_y = \frac{H_{y'} - \frac{v}{c} E_{z'}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{H_{y'}}{p(v/c)} - \frac{\frac{v}{c} E_{z'}}{p(v/c)}, \tag{8}$$

$$H_z = \frac{H_{z'} + \frac{v}{c} E_{y'}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{H_{z'}}{p(v/c)} + \frac{\frac{v}{c} E_{y'}}{p(v/c)}. \tag{9}$$

Here

$$p(v/c) = \sqrt{1 - \frac{v^2}{c^2}} \tag{10}$$

where v represents the velocity of the system.

It should be noted that sometimes a simplification is done in which instead of (10) the number

$$p \approx 1 \tag{11}$$

is assumed. Our aim is however, to perform the Lorentz calculation on an accurate basis of (4) - (9), and not with the aid of the Formula (11).

3. Lorentz Transformation and a Search for the Difference $\boldsymbol{E}^2 - \boldsymbol{H}^2$

We find that our application of the Lorentz transformation gives as a result that the invariance property of the difference

$$\boldsymbol{E}^2 - \boldsymbol{H}^2 = \text{const} \tag{12}$$

does hold.

The first term on the left of (12) becomes:

$$\begin{aligned} \boldsymbol{E}^2 &= E_x^2 + E_y^2 + E_z^2 \\ &= E_{x'}^2 + \frac{E_{y'}^2}{p^2(v/c)} + \frac{2H_{z'}\frac{v}{c}E_{y'}}{p^2(v/c)} + \frac{H_{z'}^2\frac{v^2}{c^2}}{p^2(v/c)} \\ &\quad + \frac{E_{z'}^2}{p^2(v/c)} - \frac{2H_{y'}\frac{v}{c}E_{z'}}{p^2(v/c)} + \frac{H_{y'}^2\left(\frac{v}{c}\right)^2}{p^2(v/c)}. \end{aligned} \tag{13}$$

In the next step, the second term on the left of (12) taken without a minus sign gives:

$$\begin{aligned} \boldsymbol{H}^2 &= H_x^2 + H_y^2 + H_z^2 \\ &= H_{x'}^2 + \frac{H_{y'}^2}{p^2(v/c)} - \frac{2\frac{v}{c}H_{y'}E_{z'}}{p^2(v/c)} + \frac{\left(\frac{v}{c}\right)^2 E_{z'}^2}{p^2(v/c)} \\ &\quad + \frac{H_{z'}^2}{p^2(v/c)} + \frac{2\frac{v}{c}H_{z'}E_{y'}}{p^2(v/c)} + \frac{\left(\frac{v}{c}\right)^2 E_{y'}^2}{p^2(v/c)}. \end{aligned} \tag{14}$$

Because of the minus sign which has the expression for $\boldsymbol{H}^2$ in (12) we obtain the following result for the difference in (12):

$$\begin{aligned} \boldsymbol{E}^2 - \boldsymbol{H}^2 &= E_{x'}^2 - H_{x'}^2 + \left(E_{y'}^2 + E_{z'}^2 - H_{y'}^2 - H_{z'}^2\right)\frac{1}{p^2}\left(1 - \frac{v^2}{c^2}\right) \\ &= E_{x'}^2 - H_{x'}^2 + \left(E_{y'}^2 + E_{z'}^2 - H_{y'}^2 - H_{z'}^2\right) \\ &= \boldsymbol{E}^2 - \boldsymbol{H}^2. \end{aligned} \tag{15}$$

Here the full Formula (10) concerning expression $p(v/c)$ is taken into account.

4. Summary

The paper examines the Lorentz transformation extended to the case when the electromagnetic field represented by a general vector formula acting on a system is applied.

We show that also in this situation the difference of the square values of the electric and magnetic field remains equal to a constant term which is uninfluenced by the transformation.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Landau, L.D. and Lifshitz, E.M. (1969) Mechanics. Electrodynamics. Izd. Nauka, Moscow. (in Russian)

[2] Landau, L.D. and Lifshitz, E.M. (1948) Field Theory. 2nd Edition, OGIZ, Moscow. (in Russian)