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Particle Creation from Yang-Mills Gravity

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Abstract

The genesis of physical particles, a foundational aspect of physics, is still a mystery. Quantum field theory creation operators provide an abstract mechanism to bring particles into existence. The assumption of a primordial field underlies the Standard Model (SM), yet the forces have failed to converge to such a field. Current treatments of a superfluid-based universe [Huang, Volovik, and Svistunov, Babaev, Prokof'ev] focus heavily on vortices and Yang-Mills theory, so we analyze self-interaction of the primordial field in the context of Yang-Mills. We show that a self-stabilizing higher-order self-interaction interpretation of the Yang-Mills non-Abelian term yields a stable quantum gravity explanation of the mass-gap. In future we will address the spin-$\frac{1}{2}$ and conserved charge aspects in terms of this fundamental theory of particle creation.

Keywords

Self-Stabilized Field, Self-Organizing Structure, N^{th}-Order Dynamics, Heaviside Equations, Solitons

"*The invention of Yang and Mills was not the first non-Abelian gauge field known to physicists*; *the gravitational field has that honor.*" Bryce DeWitt.

1. Introduction

Particle creation is still a mystery. Since 1954 the Yang-Mills gauge field theory of self-interaction has been believed to be the appropriate framework in which to formulate the problem, but it has so far been impossible to explain the "mass-gap" issue. The *mass-gap* is the finite value of the lowest particle mass above the vacuum energy state. The insurmountability of this problem has inspired a million-dollar *Millennium Prize*, but the prize has been unclaimed for two decades. In this paper I analyze the Yang-Mills formalism and propose a reinterpretation of the non-Abelian self-interaction term that is dynamic in nature. I show that this interpretation leads to a stable state for particle mass with finite energy above the vacuum state. In the next section I summarize the Quantum Field Theory approach and contrast this with the approach taken herein.

2. Quantum Field Theory Approach to Particle Creation

Quantum field theory (QFT) provides a bookkeeping system with symbolic *creation operators* bringing particles into existence while *annihilation operators* subtract particles from the ledger. The operators operate on particle-specific quantum fields. In QFT the quantum fields are more fundamental than the particles, which are viewed as excited states of the fields. For quanta such as photons, the oscillations in the field were viewed as arising from oscillators, which exist at every potential minimum. Zee [1] presents QFT as a mattress, idealized as a 2D lattice of mass points connected to each other by springs, a series of harmonic oscillators. He remarks that, even after a century has passed, the whole subject of QFT remains rooted in this harmonic paradigm; unable to break from the basic notions of oscillations and wave packets. He hopes to get beyond this conception, yet the math formalism fit the oscillator ladder so beautifully with "raising" and "lowering" operators promoted to "creation" and "annihilation" operators. The idea then extended to particles as excited states of quantum fields, with each particle arising from a specific field—the *electron field*, the *muon field*, etc. such that, when Feynman [2] developed a quantum field theory of gravity he treated gravity as the "31st field". Instead of a "*field per particle*"; we assume a "*field per universe*", a primordial field existing at the Big Bang, and ask how the field produces a known particle spectrum based on a "mass gap", or finite energy above the vacuum state. The *Standard Model* assumes all forces converge to such a primordial field, but such has not yet been shown.

Physics is largely based on formulating interactions as changes induced by sources, represented as $\nabla \boldsymbol{\psi} = \boldsymbol{j}$, where ∇ is a change operator that generates changes in the field $\boldsymbol{\psi}$ induced by source $\boldsymbol{j}$, separate from field $\boldsymbol{\psi}$. For primordial field $\boldsymbol{\psi}$ nothing is separate from $\boldsymbol{\psi}$; only field $\boldsymbol{\psi}$ exists. Thus, any change operator operating on field $\boldsymbol{\psi}$ must be equivalent to $\boldsymbol{\psi}$ interacting with itself. This *Self-Interaction Principle* [3] is represented by self-interaction eqn:

$$\nabla \psi = \psi \psi \tag{1}$$

To be meaningful, field ψ and operator ∇ must depend on some variable parameter ξ, so we extend our formalism via $\psi \to \psi(\xi)$ and $\nabla \to \partial_\xi$ with two formal solutions—for scalar ξ and for vector $\boldsymbol{\xi}$.

$$\psi(\xi) = -\xi^{-1}, \quad \psi(\boldsymbol{\xi}) = \boldsymbol{\xi}^{-1} \tag{2}$$

We assign physical meaning to these terms; if scalar ξ = time, then ξ^{-1} is frequency; if vector $\boldsymbol{\xi}$ = location in space, then $\boldsymbol{\xi}^{-1}$ is inverse distance. Corresponding operators are $\nabla_t = \partial/\partial t$ and $\nabla_{\boldsymbol{r}} = \partial/\partial \boldsymbol{r}$ so we attempt to solve self-interaction Equation (1). Almeida [4] noted: "*choice of a particular algebra is irrelevant from the point of view of the mathematical validity of the equation,* *but it may make a significant difference to the perception and comprehension of the physics behind the equation*." If so, the question arises as to the optimal algebra for solution of the self-interaction equation. Einstein and Wheeler viewed physics as geometry, with differential geometry the optimal algebra. Quantum physicists evolved Hilbert-space algebra and group theory symmetry represented by matrix algebra. In 1965 Hestenes evolved Clifford algebra to Geometric Algebra; the only mathematical framework in which every term has both an algebraic and a geometric interpretation [5]. For 3-spatial-dimensions-plus-time the terms include scalars, vectors, bivectors, trivectors, and pseudoscalars, interpreted as duality operators represented by *i*, that transform an entity into its dual. The new relation is geometric product $\boldsymbol{uv} = \boldsymbol{u}\cdot\boldsymbol{v} + \boldsymbol{u}\wedge\boldsymbol{v}$. Bivector $\boldsymbol{u}\wedge\boldsymbol{v}$ is a directed area representing rotation of $\boldsymbol{u}$ into $\boldsymbol{v}$. Duality operator i transforms this bivector into an axial vector: $\boldsymbol{u}\wedge\boldsymbol{v} = i\boldsymbol{u}\times\boldsymbol{v}$. Substituting the vector derivative for $\boldsymbol{u}$ the geometric product is:

$$\begin{array}{ccccc} \nabla\boldsymbol{v} & = & \nabla\cdot\boldsymbol{v} & + & \nabla\wedge\boldsymbol{v} \\ / & & | & & \backslash \\ \text{gradient} & = & \text{div} & + & \text{curl} \end{array} \tag{3}$$

No other math formalism has this relation. When $\psi = \boldsymbol{G}(\boldsymbol{r},t) + i\boldsymbol{C}(\boldsymbol{r},t)$ and $\nabla = \nabla + \partial_t$, then Equation (1) takes the form

$$(\nabla + \partial_t)(\boldsymbol{G} + i\boldsymbol{C}) = (\boldsymbol{G} + i\boldsymbol{C})(\boldsymbol{G} + i\boldsymbol{C}) \tag{4}$$

Expansion of (4) in terms of geometric products and grouping of like terms yields:

Self-Interaction equations	**Heaviside equations**	
$\nabla\cdot\boldsymbol{G} = \boldsymbol{G}\cdot\boldsymbol{G} - \boldsymbol{C}\cdot\boldsymbol{C}$	$\nabla\cdot\boldsymbol{G} = -\rho$	(5a)
$i\nabla\cdot\boldsymbol{C} = i2\boldsymbol{G}\cdot\boldsymbol{C}$	$\nabla\cdot\boldsymbol{C} = 0$	(5b)
$\partial_t\boldsymbol{G} - \nabla\times\boldsymbol{C} = \boldsymbol{G}\times\boldsymbol{C} \pm \boldsymbol{C}\times\boldsymbol{G}$	$\nabla\times\boldsymbol{C} = -\rho\boldsymbol{v} + \partial_t\boldsymbol{G}$	(5c)
$i\nabla\times\boldsymbol{G} + i\partial_t\boldsymbol{C} = 0$	$\nabla\times\boldsymbol{G} = -\partial_t\boldsymbol{C}$	(5d)

The equations on the left-hand side of (5) derive from (4) in straightforward fashion. With physical meaning assigned to field ψ, one obtains the equations on the right side, derived in 1893 by Heaviside [6], wherein $\boldsymbol{G}$ is gravity and $\boldsymbol{C}$ is the gravitomagnetic field. Decades later the eqns were erroneously labeled the *weak field approximation* to Einstein's non-linear field equations.

Self-interaction Equation (6a) yields the Heaviside-Newton equation. The Poynting-like $\boldsymbol{G}\times\boldsymbol{C}$ terms are momentum density and can be transported in opposite directions, based on initial and boundary conditions imposed locally; hence the ± in (5c); they are represented as $\rho\boldsymbol{v}$ in Heaviside (5c), while field energy density terms, $\boldsymbol{C}\cdot\boldsymbol{C}$ and $\boldsymbol{G}\cdot\boldsymbol{G}$, are represented by ρ in (5a). The time independent gravitational field in (5d) is irrotational, shown by Michaelson-Gale in 1925.

If local field density accelerates, then local gravitomagnetic circulation alters appropriately; the moving density drives the local field. If local density decelerates, change in circulation induces a *gmf*, a *gravito-motive force* $\boldsymbol{F} = -\mathrm{d}\boldsymbol{p}/\mathrm{d}t$

to drive the particle forward. In the vacuum state (the local ether) this Lenz-law-like behavior explains conservation of momentum, which Feynman claimed was inexplicable. In (5b), $\nabla \cdot \boldsymbol{C} = 0$, we use of vector identity $\nabla \cdot \nabla \times \boldsymbol{A} = 0$ to replace $\boldsymbol{C}$ with a potential vector $\nabla \times \boldsymbol{A}$. Compatible with Equation (5) are the gauge field equations:

$$\boldsymbol{C} = \nabla \times \boldsymbol{A}, \quad \boldsymbol{G} = -\nabla \phi - \partial_t \boldsymbol{A}, \quad \partial_t \phi + \nabla \cdot \boldsymbol{A} = 0 \tag{6}$$

The first two Equations in (6) define the fields in terms of the four-potential A, while the last equation specifies the Lorenz gauge condition, $\partial_\mu A^\mu = 0$. The scalar potential $\phi = -m/r$, and vector potential $\boldsymbol{A} = \boldsymbol{v}$. In analogy with Maxwell's equations, we formulate gauge field four-potential $A = \{\phi, \boldsymbol{A}\}$. Since $\boldsymbol{G} = -\nabla \phi + \partial_t \boldsymbol{A}$ if ϕ is constant then $\boldsymbol{G} = \partial_t \boldsymbol{A}$, but since $\boldsymbol{G}$ is the acceleration of gravity, then $\boldsymbol{G} = \mathrm{d}\boldsymbol{v}/\mathrm{d}t \Rightarrow \boldsymbol{A} = \boldsymbol{v}$. Since $\boldsymbol{C} = \nabla \times \boldsymbol{A}$ then $\boldsymbol{C} = \nabla \times \boldsymbol{v}$ is dimensionally correct; $|\boldsymbol{C}| \sim t^{-1}$. With gravitational potential $\phi = -M/r$ the $\boldsymbol{G}$-field has spatial dependence $|\boldsymbol{G}| \sim r^{-2}$; correct for Newtonian mass. For the primordial field, as shown in several of the references, $|\boldsymbol{G}| \sim r^{-1}$. Physically, all Newtonian mass is treated as entirely within the sphere of radius *r*, whereas the mass of the primordial gravitational field is based only on the portion of the field within the sphere. In all cases, with local mass density ρ the interaction energy density of the field is $\boldsymbol{j} \cdot \boldsymbol{A}$ where $\boldsymbol{j} = \rho \boldsymbol{v}$. Heaviside current density $\boldsymbol{j}$ is momentum density $\boldsymbol{p} = \rho \boldsymbol{v}$; the interaction density of the field is $\boldsymbol{p} \cdot \boldsymbol{A} = \boldsymbol{p} \cdot \boldsymbol{v} = \rho v^2$. The field strength matrix constructed from the above [7] is shown:

$$F_{\mu\nu} = \begin{bmatrix} 0 & G_x & G_y & G_z \\ G_x & 0 & -C_z & C_y \\ G_y & C_z & 0 & -C_x \\ G_z & -C_y & C_x & 0 \end{bmatrix} \tag{7}$$

A full unification of gravitation, electromagnetism, the strong and weak nuclear forces, has not yet been derived. Nevertheless, the four fundamental interactions are generated by a single principle, the gauge principle [8]. Weyl, in 1929, derived the conservation laws and expressed the Riemann tensor in the tetrad form: $R^a_{\mu\nu b} = \left[D_\mu, D_\nu\right]^a_b = \partial_\mu A^a_{\nu b} - \partial_\nu A^a_{\mu b} + A^a_{\mu c} A^c_{\nu b} - A^a_{\nu c} A^c_{\mu b}$. For Yang-Mills, expression of field strength $F_{\mu\nu} = \left[D_\mu, D_\nu\right]$ as commutation was not common at the time; direct expression as a curl was so simple: Weyl's equation is expressed $R = \partial \wedge A + [A, A]$. Yang stated that, when they presented their theory, they had no idea it might be related to gravitation:

"…*when Mills and I worked on non-Abelian gauge fields, our motivation was completely divorced from general relativity, and we did not appreciate that gauge fields and general relativity are somehow related.*"

Little surprise that, in search of a generalization of isotopic spin for application to the nuclear physics of the "50's, Yang and Mills, as particle physicists, did not have tetradic formulations of general relativity in mind, nor the fiber bundle approach developed through differential forms. Today our preferred framework is Hestenes" Geometric Calculus.

3. Aspects of Isospin

Initially Pauli added spin to the Hamiltonian based on energy $\boldsymbol{\mu}\cdot\boldsymbol{B}$ in magnetic field $\boldsymbol{B}$ where magnetic moment $\boldsymbol{\mu}$ is proportional to spin $\boldsymbol{s}$ of the charge, conceived classically. The equation of motion $\dot{\boldsymbol{s}}=\boldsymbol{B}\times\boldsymbol{s}$ results [9] in spin precessing about the $\boldsymbol{B}$-field lines of force in two stable configurations, $\pm\boldsymbol{\mu}\cdot\boldsymbol{B}$. Pauli invented 2 × 2 matrix operator $\hat{\sigma}$ to satisfy $\hat{\sigma}|s\rangle=\pm|s\rangle$ for state $|\boldsymbol{s}\rangle=\begin{pmatrix} up \\ dn \end{pmatrix}$, with $\sigma_x=\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y=\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, $\sigma_z=\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

Heisenberg conceived of the known nucleons, proton and neutron, as a single particle with two states, ignoring electric charge. Instead of up or down *spin* state he formulated the nucleon state with internal "*isospin*" symmetry to allow Pauli's $\boldsymbol{\sigma}$ matrix to switch between internal symmetry states, $\psi=\begin{pmatrix} \text{proton} \\ \text{neutron} \end{pmatrix}$. Matrices are representations of group symmetry, yet isospin is *not* an exact symmetry; it is only *approximate* since the masses of the proton and neutron are not equal. The matrices $\{\sigma_x,\sigma_y,\sigma_z\}$ represent the 2 × 2 Pauli spin matrices of quantum mechanics. Hestenes constructs an equivalent orthonormal basis of three bivectors $\{\beta_x,\beta_y,\beta_z\}$ satisfying $\beta_x\beta_y=-i\beta_z$. The algebras (with Kronecker delta δ_{jk} and Levi-Civita alternating symbol ϵ_{jkl}) are written:

Pauli matrix algebra   Hestenes bivector algebra

$$\sigma_j\sigma_k=-\delta_{jk}-i\varepsilon_{jkl}\sigma_l, \qquad \beta_j\beta_k=-\delta_{jk}-i\varepsilon_{jkl}\beta_l \tag{8}$$

Bivector algebra is identical to spin matrix algebra, by inspection. Since the algebras are identical, their physical implications should be the same; our expressed preference is for the geometric algebra formulation with geometric elements providing visible structures. Attempts to make gauge fields visible in differential geometry center around fiber bundles, with cartoon-like representations of the type shown in Huang's *Fundamental Forces of Nature* [10]. As Penrose has remarked [11] Yang-Mills isospin fields don't exist in the physical world as far as we know. They are *non-physical* abstractions.

Systems coupled to the electromagnetic field possess global gauge invariance *before the coupling is turned on*, so Schrödinger's equation is invariant under a constant phase change $\psi\to e^{i\alpha}\psi$ where α is constant, since $\partial\left(e^{i\alpha}\psi\right)=e^{i\alpha}\partial(\psi)$. Global phase has no physical consequence. Based on Noether, global gauge invariance guarantees existence of a conserved current that yields charge conservation, and Yang and Mills hoped to find such gauge conservation principles in their treatment of isospin. But the system is <u>not</u> invariant under local transformation $\psi\to e^{i\beta(x)}\psi$ since Schrödinger's equation is not invariant: $\partial\left(e^{i\beta(x)}\psi\right)\neq e^{i\beta(x)}\partial(\psi)$. Global gauge invariance is extended to local gauge invariance by replacing derivative ∂ with covariant derivative *D*:

$$\partial\to D,\quad D\to\partial+\frac{iqA}{\hbar}. \tag{9}$$

In quantum mechanics qA is combined with momentum p therefore $qA/\hbar$ has dimension 1/*length*, appropriate to the derivative term. Such derivatives in physics typically represent translations or rotations in local space, parallel transport along a path. A $U(1)$ rotation through angle θ can be represented by $e^{i\theta}$ phase factor, which, for infinitesimally small angles, reduces to $1+i\theta$. An arbitrary rotation about a fixed axis can be constructed from successive infinitesimal rotations about that axis. For three axes there are three possible infinitesimal rotations: $1+i\theta_1 L_1$, $1+i\theta_2 L_2$, $1+i\theta_3 L_3$. While the $U(1)$ group of transformations about one axis is Abelian (commuting), continuous transformations about 3 axes form a non-Abelian Lie group satisfying $\left[L_a, L_b\right]=i\varepsilon_{abc}L_c$, (see bivector algebra of Equation (8)). The L_i cannot be numbers since they do not commute. Since any 2 × 2 matrix is a linear combination of the Pauli spin matrices; a generator of rotations about 3 axes can thus be represented $L_a=\sigma_a/2$, with general transformation $U=\exp\left(\frac{i}{2}\omega_a\sigma_a\right)$ where ω_a are real numbers. The 2 × 2 unitary matrix U has symmetry group $SU(2)$ so isospin is an "internal" symmetry with $SU(2)$ symmetry by construction; operation on any two-component wave function $\psi=\left[\psi_1,\psi_2\right]^{\mathrm{T}}$ with rotation U satisfies $\psi\to U\psi$. In this way the geometry of classical physics is applied to abstract internal symmetry such as isospin.

Yang and Mills, in terms of the infinitesimal charge generator L_a of $SU(2)$, replaced derivative ∂ by covariant derivative $D=\partial+\frac{ig}{\hbar}L_a A_a$ in equation of motion $\left(\partial-igA\right)\psi=0$ where A_a is a 4-vector gauge field with three internal components corresponding to the generation of the gauge group of isospin rotations. D generates a coupling between the particle and the gauge field with interaction energy density $j_a A_a$ where j_a is conserved isotopic spin current density. "*But in the real world, isotopic spin is not conserved; the gauge symmetry is not exact.*" Yang and Mills next guessed that adding quadratic terms to the field strength would represent self-interaction of the gauge field:

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu+ig\left[A^\mu,A^\nu\right] \tag{10}$$

Yang-Mills gauge theory is based on an abstract, non-physical, idea of *approximate* isospin symmetry. Yang-Mills theory does <u>not</u> explain the mass gap that is the key to particle physics, so we switch to the *exact* symmetry derived from the fundamental principle of self-interaction:

$\nabla\psi=\psi\psi \;\Rightarrow\;$ Heaviside equations $\Rightarrow$ Einstein field equations.

Whereas general relativity is derived from an approximate principle, the *Equivalence Principle*, the Heaviside equations are derived from an exact principle, the *Self-Interaction Principle*. There are several consequences of these facts, treated in [12] [13] [14] [15]. Two key facts: 1) Heaviside theory is equivalent to curved space theory, and 2) Heaviside's equations hold at all scales, from Planck scale to Cosmic Microwave Background.

4. Details of Yang-Mills Theory

Yang & Mills [16] formulate B_μ with 12 independent components: 4 × 4 less diagonal elements. For a two-component wave function, ψ, describing a field with isospin $\frac{1}{2}$, the isotopic gauge transformation $\psi = S\psi'$ where S is a 2 × 2 matrix with determinant unity, and all drivatives of ψ appear in combination $\left(\partial_\mu - i\epsilon B_\mu\right)\psi$ where B_μ are 2 × 2 matrices for $\mu = 1,2,3$. Invariance requires

$$S\left(\partial_\mu - i\epsilon B'_\mu\right)\psi' = \left(\partial_\mu - i\epsilon B_\mu\right)\psi .$$

The Yang-Mills isotopic gauge transformation on B_μ, corresponding to $A'_\mu = A_\mu + \frac{1}{e}\frac{\partial \alpha}{\partial x_\mu}$ is

$$B'_\mu = S^{-1} B_\mu S + \frac{i}{\epsilon} S^{-1} \frac{\partial S}{\partial x_\mu} \tag{11}$$

with the last term like the gradient term in the gauge transformation of electromagnetic potentials. To obtain gauge invariant field strengths they define the analog of the electromagnetic case

$$F_{\mu\nu} = \frac{\partial B_\mu}{\partial x_\nu} - \frac{\partial B_\nu}{\partial x_\mu} + i\epsilon\left(B_\mu B_\nu - B_\nu B_\mu\right) \text{ with } F'_{\mu\nu} = S^{-1} F_{\mu\nu} S \tag{12}$$

Yang and Mills next introduce isotopic spin "angular momentum" matrices $\tau^i \left(i = 1,2,3\right)$ which correspond to the isotopic spin of the field ψ under consideration. The B field is then defined as $B_\mu = 2b_\mu \cdot \tau$ where both b_μ and τ are 3-component vectors in isotopic space. Interaction with any field ψ of arbitrary isospin requires replacing ordinary derivative of ψ by $\left(\partial_\mu - i\epsilon b_\mu \cdot \tau\right)\psi$ with τ representing isotopic spin "angular momentum" as above. The isotopic-gauge covariant field strengths $F_{\mu\nu}$ are expressible $F_{\mu\nu} = f_{\mu\nu} \cdot \tau$ where

$$f_{\mu\nu} = \frac{\partial b_\mu}{\partial x_\nu} - \frac{\partial b_\nu}{\partial x_\mu} - 2\epsilon b_\mu \times b_\nu \tag{13}$$

and $f_{\mu\nu}$ transforms like a vector under an isotopic gauge transformation. The field equations derive from the total Lagrangian density

$$\mathcal{L} = \frac{1}{4} f_{\mu\nu} \cdot f_{\mu\nu} . \tag{14}$$

Finally, they define

$$\Im_\mu = J_\mu + 2\epsilon b_\nu \times f_{\mu\nu} \tag{15}$$

with equation of continuity $\partial \Im_\mu / \partial x_\mu = 0$ and the supplementary condition (corresponding to the Lorenz gauge) $\partial b_\mu / \partial x_\mu = 0$ which eliminates the scalar part of the field in b_μ. Equation (15) shows that isotopic spin arises from both spin $\frac{1}{2}$ field (J_μ) and from the b_μ field itself, thus making the field equations for the b_μ field nonlinear. This is as far as we will carry Yang and Mills theory in its original form. Writing for the Clay Mathematics Institute, Jaffe and Witten:

"*There is no known way of deriving the mass gap from the original theory.*"

5. Angular Momentum and Yang-Mills

Linking to our primordial field $\psi = \boldsymbol{G} + i\boldsymbol{C}$ we identify gravitomagnetic gauge field $\boldsymbol{v}$ with the Yang-Mills b_μ field. The problem is to create a mass-gap that has evaded physicists since the introduction of the theory. The gravitomagnetic field has also evaded physicist's standard model of particle physics, suggesting a need to reinterpret non-linear fields.

Yang and Mills introduce and discuss isotopic spin "angular momentum" in quotes and are unsure what it means physically. They adapt Pauli's *SU*(2) spin matrices to Heisenberg's *isospin*; a mathematical formalism applied to an abstract *internal* symmetry. The nature of spin, at least classically, is rotation, and rotation in 3D space entails angular momentum. Exactly what is entailed in the space of internal symmetry, represented by gauge field b_μ, is unknown. However, the nature of this gauge field is captured by the curl operation, so it must somehow entail an analog of angular momentum, as Einstein and deHaas [17] showed to be possessed by the magnetic field. Yang and Mills "*define isotopic gauge as an arbitrary way of choosing the orientation of the isotopic spin axis at all space-time points*."

The matrix $S^{-1}\dfrac{\partial S}{\partial x_\mu}$ appearing in Equation (11) is a linear combination of isotopic spin "angular momentum" matrices τ^i ($i = 1,2,3$) corresponding to isotopic spin of the field we are considering. The B_μ matrices contain a linear combination of matrices $B_\mu(x) = \sum_{a=1}^{n} b_\mu^a(x)\tau_a$ or $B_\mu = 2b_\mu \cdot \tau$ where b_μ and τ are 3-component vectors in isotopic space. In Heaviside isotopic space, the b_μ vector is the v_μ velocity vector determining the linear combination of the bivector angular momenta.

Although there is no well-defined idea of isotopic spin "angular momentum", gravitomagnetic C-field possesses angular momentum; and is proportional to angular momentum: $\boldsymbol{C} = \left(g/c^2\right)\boldsymbol{r}\times\boldsymbol{p}$ with dimension t^{-1}/l^3. For $F_{\mu\nu}$ depicted in **Figure 1** we pair C_y with $-C_y$, and cyclical iterations, where the index represents the axis about which these components of the field rotate. In other words, the formalism contains the angular momentum aspect of the components. The C-field components are compatible with the three bivectors shown in the 3-space representation at the right, defined by the *x*, *y*, and *z* axes. *The nature of C-field circulation*, *from every perspective*, *is angular momentum*.

Consider Yang-Mills term $\epsilon\left[A_\mu, A_\nu\right]$. The ϵ corresponds to the *isospin charge* analogous to electric charge *q* that interacts with electromagnetic gauge A_μ in the Hamiltonian, appearing as qA_μ, the momentum term. For the C-field, ϵ corresponds to mass, hence $\epsilon\boldsymbol{A} \to m\boldsymbol{v}$, the field momentum (actually $\rho\boldsymbol{v}$ the momentum density). In the original Yang-Mills the $\mu = 1$ term interaction with the $\nu = 2$ term concerns the mA_xA_y term. The geometric algebra product $A_xA_y = A_x \cdot A_y + i\left(A_x \times A_y\right)$. The scalar product vanishes while the curl is proportional to A_z. The curl is antisymmetric, so we have

$$A_xA_y - A_yA_x = 2A_z. \tag{16}$$

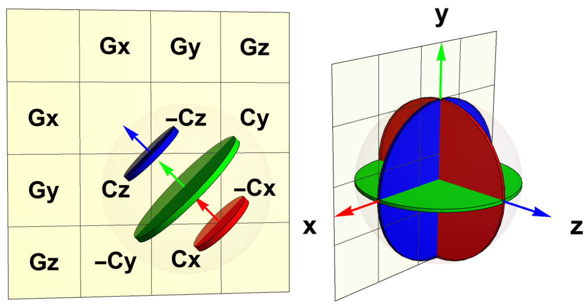

Figure 1. The circulating field, the C-field, can be labeled by the (row, col) component or by the orthogonal axis about which the (row, col) component circulates. For example, the (*x*,*z*) element is labeled C_y and the (*z*, *x*) element is labeled $-C_y$ since both of these terms rotate about the y-axis; similarly for the other components. These rotations are shown abstractly in the representation of the field strength $F_{\mu\nu}$ matrix on the left. The right-hand illustration maps the three bivector diagrams into 3-space. Colors are used for visual convenience and for suggested correlation with $SU(3) \times SU(2) \times U(1)$ symmetry.

Isotopic gauge covariant field equations $f_{\mu\nu}$ are expressible in terms of Yang-Mills gauge field $\boldsymbol{b}_{\mu\nu}$

$$\boldsymbol{f}_{\mu\nu} = \underbrace{\frac{\partial \boldsymbol{b}_\mu}{\partial x_\nu} - \frac{\partial \boldsymbol{b}_\nu}{\partial x_\mu}}_{C_{\mu\nu}} - \underbrace{2m\boldsymbol{b}_\mu \times \boldsymbol{b}_\nu}_{mv_{\mu\nu}} \tag{17}$$

In this case the kinetic term of the Lagrangian, $\mathcal{L} = \frac{1}{4}\boldsymbol{f}_{\mu\nu} \cdot \boldsymbol{f}_{\mu\nu}$, will contain a product term proportional to $(C_{\mu\nu})(mv_{\mu\nu})$ and a quadratic term $(mv_{\mu\nu})^2$. The scalar multiplier (g/c^2) has dimension $l/m = \text{length/mass}$ hence $(g/c^2)(mv_{\mu\nu})^2 \Rightarrow (mv^2)\cdot\text{length}$. The product term corresponds to $\boldsymbol{f}_{\mu\nu} \times 2m\boldsymbol{v}_{\mu\nu}$ which, in Equation (15), shows up as a new source term. In other words, our treatment of the gravitomagnetic gauge field matches Yang-Mills' original treatment.

If it were obvious how to achieve mass gap at this point, it would have been solved in 1954.

6. Higher-Order Self-Interaction

The Yang-Mills $\epsilon[A_\mu, A_\nu]$ term covers all gauge field component interactions, discussed above in terms of the original Yang-Mills paper. Yet neither mass gap nor quark confinement can be formulated successfully in this approach, so we examine a different self-interaction framework. The gravitomagnetic C-field has energy density, hence mass density, and circulates or rotates about an axis in space. The motion of the field, at any local point, results in momentum density at that point. But momentum density is the source current generating C-field circulation to begin with. Thus, the field itself induces more field and these fields interact; exactly what the Yang-Mills non-Abelian term is supposed to represent. *Therefore, we should investigate the real physical field interacting with itself instead of an abstract "internal" symmetry.* The mass density of the second order induced circulation field is not equal to the mass density that induced the first circulation. The self-induced circulation is iterative; the first induced field induces a second order field circulation, and this, at any local point, induces a third order circulation, etc.

Physical spin is associated with circulation of the C-field; $\nabla \times \boldsymbol{C}$ represents bivector circulation, a spinning region of field such as a cross-section through a vortex, possessing angular momentum. Figure 2 illustrates first and second order induced fields caused by source momentum density, p_0. Higher order inductions of C-field circulations can be illustrated successively.

The first conclusion is that successive orders do *not* interact to any degree; the force $\boldsymbol{p} \times \boldsymbol{C}$ is always orthogonal to the velocity, hence the work done is zero: $\text{Work} = \int \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{x} = 0$. Alternate orders, on the other hand, *do* interact, as they are parallel or anti-parallel. To schematically illustrate this, we take the tangent vectors to the circulation loops at the nearest and farthest points and "square the circle", using the straight lines as heuristic devices to facilitate the expression of forces involved via analogy with electromagnetic forces between parallel currents (Figure 3).

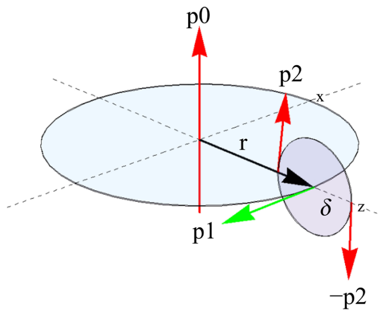

Figure 2. Momentum density $\boldsymbol{p}_0$ (red) induces C-field circulation at position $\boldsymbol{r}$. The C-field circulation at $\boldsymbol{r}$ yields momentum density $\boldsymbol{p}_1$ (green) orthogonal to $\boldsymbol{p}_0$. Momentum $\boldsymbol{p}_1$ induces the C-field at distance δ from $\boldsymbol{p}_1$. This induced C-field yields momentum density $\boldsymbol{p}_2$ (red) with components parallel and anti-parallel to $\boldsymbol{p}_0$.

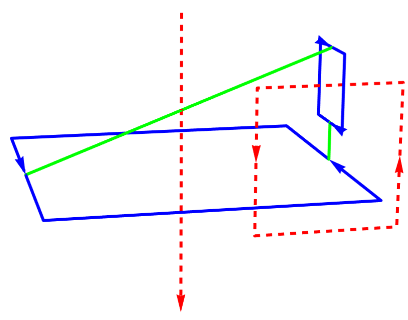

Figure 3. Focusing on (blue) loop1 and loop3 of the structure; source current and second order induction, loop2, are shown as red dashed lines. Since the loop3 bottom current is parallel to the rightmost current of loop1, the currents exert attractive forces upon each other, while top of loop3 is parallel to the current at the left of loop1 so the currents attract each other. The attractive force lines are shown in green. Similar same arguments apply to anti-parallel currents which exert repulsive forces (not shown).

The self-linking field formalism of Figure 2 shows that second-order induction reinforces the primary inducing agent, *i.e.*, local momentum density $\rho \boldsymbol{v}$. The electromagnetic force $\boldsymbol{F}_{ij}$ between two current elements $\mathrm{d}\boldsymbol{j}_i$ and $\mathrm{d}\boldsymbol{j}_j$ a distance $\boldsymbol{r}_{ij}$ apart guides us to write the gravitomagnetic equivalent.

$$\mathrm{d}\boldsymbol{F}_{ij} = \frac{\left[\mathrm{d}\boldsymbol{p}_j\right] \times \left[\mathrm{d}\boldsymbol{p}_i \times \boldsymbol{r}_{ij}\right]}{r_{ij}^3}. \tag{18}$$

Since $\mathrm{d}\boldsymbol{C}_i = \mathrm{d}\boldsymbol{p}_i \times \frac{\boldsymbol{r}_{ij}}{r_{ij}^3}$ where $\mathrm{d}\boldsymbol{p}_i$ is the mass current element inducing the field then $\mathrm{d}\boldsymbol{p}_j \times \mathrm{d}\boldsymbol{C}_i$ and Equation (18) is seen to be compatible with the Lorentz force law $\boldsymbol{F} = \boldsymbol{p} \times \boldsymbol{C}$ for the force on momentum $\boldsymbol{p}$ in gravitomagnetic field $\boldsymbol{C}$. In Figure 2, first-order C-field induction from momentum source density $\boldsymbol{p}_0$, is used to derive second order C-field induction from the momentum of the first-order field, $\boldsymbol{p}_1 \sim \boldsymbol{C}_1 \cdot \boldsymbol{C}_1$. Figure 3 focuses attention on loop1 and loop3 of the structure, showing the source current, and second order induction, loop2, as dashed lines. The bottom current in loop3 is parallel to the rightmost current of loop1, and therefore the currents exert attractive forces upon each other. Similarly, the current at the top of loop3 is parallel to the current at the left of loop1 and the two currents attract each other. The same arguments apply to the anti-parallel currents which exert repulsive forces. The above follows from

$$\mathrm{d}\boldsymbol{F}_{01} = \mathrm{d}\boldsymbol{p}_1 \times \mathrm{d}\boldsymbol{C}_0 = 0 \quad \text{since} \quad \boldsymbol{C}_0 \parallel \boldsymbol{p}_1 \tag{19}$$

$$\mathrm{d}F_{02} = \mathrm{d}\boldsymbol{p}_2 \times \mathrm{d}\boldsymbol{C}_0 \neq 0 \quad \text{since} \quad \boldsymbol{C}_0 \perp \boldsymbol{p}_2 \tag{20}$$

The force between $\boldsymbol{p}_0$ and $\boldsymbol{p}_1$ is zero since these mass density current flows are orthogonal to each other. On the other hand, the force acting between $\boldsymbol{p}_0$ and $\boldsymbol{p}_2$ is maximal or minimal according to whether these flows are parallel or anti-parallel.

This schematic organization guides calculation of the forces involved in the self-interaction of a turbulent primordial field. We seek first a qualitative understanding of dynamic behavior. All squares in the diagrams represent extensions of the tangent vectors depicted in Figure 3 and restore the dashed red loop2 to its true circular form. With this revision current loop3 should rotate about loop2, under the influence of the forces, eventually rotating into the *xy*-plane as depicted in Figure 4.

Loop3, shown in blue above loop1, is simply a slice through a torus surrounding loop2. It has no independent existence such that it can be pulled down into the plane. Nevertheless, if a "slice" is pulled into the plane, the field that *replaces* that slice will experience the same forces; the net result is a dynamic tension that tends to shrink the system of circulations into a lower energy configurational state. The final state of an arbitrary slice is depicted in Figure 5.

Despite having higher order constructions, the behavior is almost certainly governed by interactions between 1st and 3rd order induced circulations, as shown in Figure 4, consisting of the loop1 currents into and out of the page and the two loop3 circulations, each with parallel currents into and out of the page. To formalize these interactions, we define the interaction between momentum density currents p_i and p_j as $f\left[p[i], p[j]\right]$ divided by the absolute distance between the currents and construct the interaction matrix over all six relevant currents, shown numbered in Figure 5.

7. Path Integrals over the Lattice

In Figure 3 a (blue) loop3 is vertically aligned over one leg of loop1. Figure 4 is a snapshot of loop3 rotating about loop2 from the initial vertical state ($\theta = 0$) to the horizontal state in the loop1 plane ($\theta = -\pi/2$). The loop is symmetric and supports an inverse image behavior from another loop3 on the left side of the diagram. To proceed from the initial state to the final state, we step through a sequence of rotations. The paths through the local space surrounding loop1 are traced out by rays originating on loop1 and rotating by $\mathrm{d}\theta_i$ rotations from $\theta = 0$ to $\theta = -\pi/2$. Two paths are traced—the lower leg of loop3, and the upper leg of loop3 as loop3 rotates from vertical to horizontal. This lattice of points defines the points at which we want to calculate forces between loops. Four snapshots of such lattice-based dynamics are shown in Figure 6.

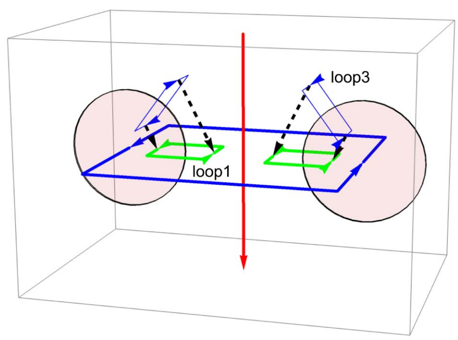

Figure 4. Cartoon snapshot depicting third-order loop (blue) dynamics interacting with first order loop (blue) of C-field circulation induced by (red) source momentum $\boldsymbol{p}_0$.

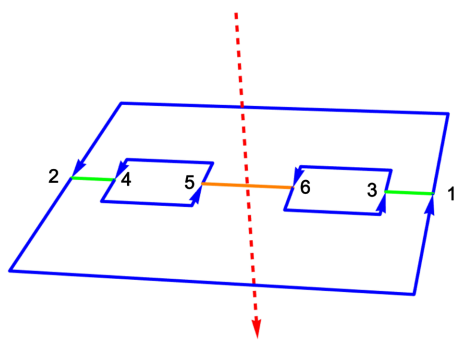

Figure 5. The result of dynamic forces acting on slices of higher order loops whose currents are numbered as shown. In this progression the configuration shown exerts attractive forces (green) between higher order loops and lower order loops, and repulsive forces (orange) between displaced higher order loops. This behavior follows at all orders.

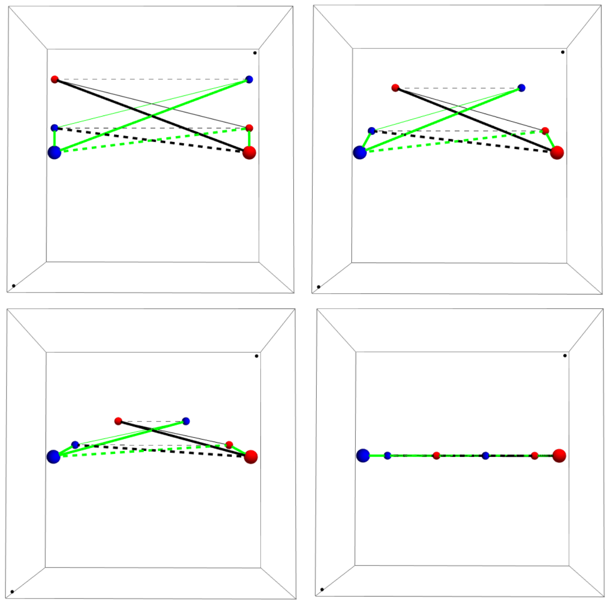

Figure 6. Shows positions and directions from initial vertical state represented by parameter $\theta = 0$, successively transforming to parameter $\theta = -\pi/2$. Attractive forces are shown as solid lines, with repulsive forces represented by dashed lines. Black lines represent forces to be calculated, while green lines represent symmetric forces; identical to forces corresponding to black lines.

A time-sliced adaptation of **Figure 4** shows the relevant portions of the current loops directed into the page (red) and out of the page (blue). Attractive forces are shown as solid lines, repulsive forces by dashed lines. Black lines represent forces to be calculated, while green lines represent symmetric forces, identical to forces corresponding to black lines. Thick lines correspond to one power factor α while thin lines are interactions with factor α^2. **Figure 6** shows positions and directions from initial vertical state (parameter $\theta = 0$) successively transforming to $\theta = -\pi/2$.

Calculations of work done by the forces have the form $W_{ij}(\theta) = \boldsymbol{F}_{ij}(\theta) \cdot \mathrm{d}\boldsymbol{x}_i(\theta)$ where indices *i* and *j* vary from one to six as shown in **Figure 5** and θ varies from 0 to $-\pi/2$ as currents 3 and 6 move from initial vertical position into the *xy*-plane. The displacement $\mathrm{d}\boldsymbol{x}_i(\theta) = \boldsymbol{x}_i(\theta) - \boldsymbol{x}_i(\theta + \mathrm{d}\theta)$. Examination shows that $|\mathrm{d}\boldsymbol{x}_5|$ and $|\mathrm{d}\boldsymbol{x}_6|$ are greater than $|\mathrm{d}\boldsymbol{x}_4|$ and $|\mathrm{d}\boldsymbol{x}_3|$ for the same $\mathrm{d}\theta$.

In **Figure 6**, for example, when currents 5 and 6 come together in the plane from initial vertical position, they oppose each other and the field between them increases, hence the energy density of the field increases, representing positive work shown by W_{56}. Currents 3 and 5, on the other hand, are parallel and attract each other, minimizing their joint field between them and reducing the energy, thus representing negative work, shown by W_{35} in **Figure 7**.

The forces and displacements are calculated for every step of travel along the lattice path. The current force $\boldsymbol{F}_{ij}(\theta)$ applied over $\mathrm{d}\boldsymbol{x}_i(\theta)$ describes the work done for that step. The inner product $\boldsymbol{F}_{ij}(\theta) \cdot \mathrm{d}\boldsymbol{x}_i(\theta)$ is maximum when $\boldsymbol{F}_{ij}(\theta)$ and $\mathrm{d}\boldsymbol{x}_i(\theta)$ are parallel. Since we began calculations at $\theta = 0$, the initial $\mathrm{d}\boldsymbol{x}_1(0)$ is $(-\mathrm{d}x,0,0)$ while the initial force $\boldsymbol{F}_{ij}(0)=\alpha(-x,-y,0)$. The two vectors are not parallel. They become parallel when $\boldsymbol{F}_{ij}(\theta_k)\parallel \mathrm{d}\boldsymbol{x}_i(\theta_k)$. Thus, in Figure 8 the energy is seen to peak at $\theta_k \sim -\pi/6$. From that point onward, each successive step will lead to a lower energy state, and mass-energy density of the field structure becomes more "locked-in". In this way particles emerge with mass-energy greater than the vacuum state.

8. Brief Summary of Physics

The above physics is based on vorticity as the ubiquitous aspect of turbulent superfluid. Energy flows from large vortices to smaller vortices, which are circulating regions in the ultra-dense gravitomagnetic gauge field, with positive energy over a small region. The motion of the local field circulation induces further circulation and this in turn induces even higher order circulation. The topology is such that orders differing by one do not interact, whereas orders that differ by

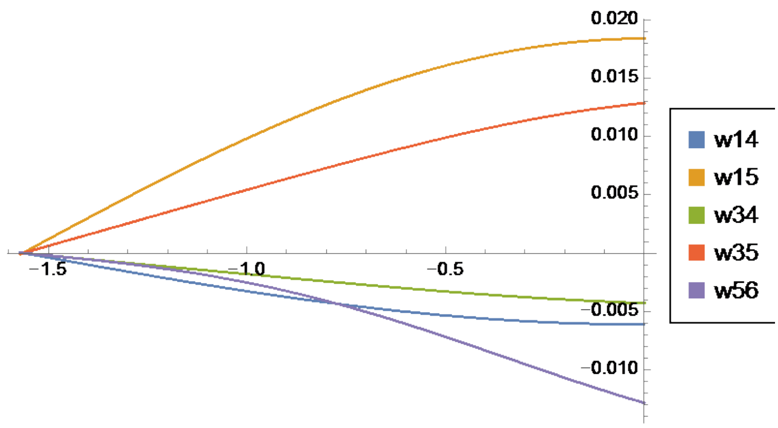

Figure 7. The work $W_{ij}(\theta)$ representing the interactive force $\boldsymbol{F}_{ij}(\theta)$ between momentum currents i and j at angles θ. The horizontal axis runs from $\theta=0$ to $\theta=-\pi/2$ while the vertical axis represents work $W_{ij}(\theta)=\boldsymbol{F}_{ij}(\theta)\cdot \mathrm{d}\boldsymbol{x}_i(\theta)$. At $\theta=-\pi/2$ all displacements are down while all forces are horizontal, so no work is done at the final angle.

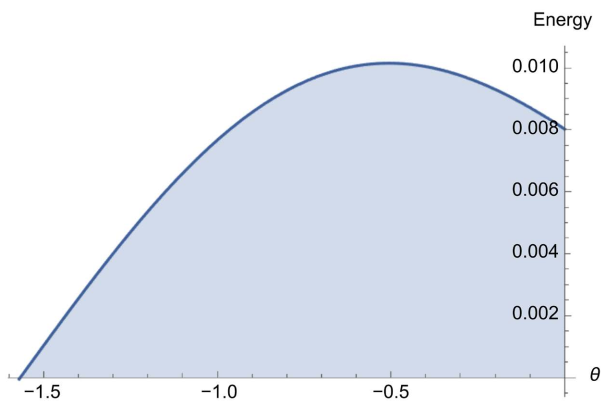

Figure 8. Summing all the (arbitrarily scaled) works involved, we find that the net energy decreases, thus the self-interaction of the field leads to a more stable configuration.

an even number *do* interact. We have employed a fractal structure and defined a path based on the relevant self-interactions. The forces act to move the induced flows into the primary circulation plane while shrinking the boundary of the field. This movement drives the system to a lower, but still positive, energy which is denser than the initial vortex energy. This ultra-dense stabilized spin energy is the "mass-gap" that yields particle mass greater than the vacuum state. We have thus shown that the primordial field "condenses" to a positive energy structure that we identify as a fundamental particle. We know that this particle will have quantized spin and quantized charge, and these aspects must be included to calculate the actual mass of the particle. This is in progress.

9. Discussion of Results

The years 1954 and 1964 witnessed revolutionary mathematics introduced into physics—Yang-Mills non-Abelian gauge theory and Hestenes' geometric calculus, supporting physical intuition relevant to Yang-Mills theory wherein the non-Abelian term $\left[A_\mu, A_\nu\right] \sim A_\mu \times A_\nu$ represents the interaction of the gauge field with itself. This term $\epsilon\left[A_\mu, A_\nu\right]$ is supposed to cover all the gauge field self-interactions, yet neither mass-gap nor confinement has been formulated successfully in this approach. "Free" gauge field propagation through space can conceivably self-interact, but nothing stable arises from such interactions. It is intuitively obvious that a "mass gap" can arise only locally, which seems to imply either a boundary or a local potential well. Alternatively, a local circulation may be invoked, almost certainly the reason for the focus on "angular momentum" in isospin space, which <u>is</u> abstract, based on approximate symmetry. As Weinberg states [18]: "*Many symmetries…were approximate because they weren't fundamental symmetries at all; they were just accidents.*" Despite their approximate nature, there has been much focus on symmetry aspects in Yang-Mills. In "*Yang-Mills Origin of Gravitational Symmetries*" [19] many gravitational symmetries are derived linearly, including general covariance, two-form gauge invariance, local supersymmetry, and local chiral symmetry, following flat-space Yang-Mills theory. They remark that an important improvement would address the issue of dynamics as well as symmetry. Finally, "*we might speculate that the supergravity* ϕ_μ*, the left Yang-Mills* $V^i(L)$*, the right Yang-Mills* $A_\mu^{i'}(R)$ *and the spectator* $\Phi_{ii'}$ <u>*live in different worlds*</u> *with their own Lagrangians.*" By contrast, our approach begins with <u>one</u> primordial world and nothing else, with an implicit self-interaction equation, the solution of which leads straightforwardly to higher order self-interactions. The stability of these interactions yields a mass gap that failed to appear from prior symmetry analysis.

In our derivation of Yang-Mills from the *Self-Interaction Principle*, the Heaviside C-field circulation is proportional to classical angular momentum. Equation (16) shows bivector-based $\epsilon\left[A_\mu, A_\nu\right] \Rightarrow m\left(A_\mu, A_\nu\right) = 2mv_{\mu\nu}$. The Yang-Mills non-Abelian term is proportional to angular momentum of the gravitational gauge field, rather than some isotopic "angular momentum" in abstract space;

our approach is based on a real physical field, with real angular momentum, formulated in terms of bivector rotations in real 3-space. $\epsilon\left[A_\mu, A_\nu\right]$ represents different interacting flows of the same field, locally distributed over space. The key Heaviside equation involving mass current density $\rho\boldsymbol{v}$ induces local circulation $\nabla\times\boldsymbol{C}=-\rho\boldsymbol{v}$ of the C-field with local energy density $\sim\boldsymbol{C}\cdot\boldsymbol{C}$ energy circulating with velocity v' in the medium. This momentum density $\rho\boldsymbol{v}=\left(\boldsymbol{C}\cdot\boldsymbol{C}\right)\boldsymbol{v}'$ will, in turn, induce second-order C-field circulation.

This is *Self-Interaction*; if stable, it will lead to a mass gap. Our approach demonstrates stability via iterative self-induction and shows that, while alternating inductions do not interact, even-order self-induced structures *do* interact, and do so in a manner that *increases the stability of the locally circulating field structure*. In this calculation the scale is unknown, and the coupling parameters are not rigorously specified, but the dynamical behavior of the model is correct. The forces between points on the lattice represent gauge field flows in and out of the plane of the paper which are time-linked. The dynamical energy exchange is a function of flow topology and distances. Global scale parameters will not change the direction of the energy evolution, only the magnitude of the effect. The resulting self-interactions lead to a lower energy and a greater density state. Like a skater pulling in her arms, a decreasing radius leads to increased angular velocity. For simplicity we have suppressed this "shrinkage" of the structure, but in reality we expect the radius of the circulating field to decrease and the local velocity v' to increase, such that $m'v'r'$ is the conserved angular momentum, where v' is the increased rotational velocity, m' is the relativistic mass $m'=\gamma\left(v'\right)m_0\sim\gamma\left(v'\right)\left(\boldsymbol{C}\cdot\boldsymbol{C}\right)$ and r' is the reduced radius. Quantized angular momentum ($\sim\hbar$) should prevent this circulation from shrinking to an infinitely dense "point" particle, and therefore should evolve to a finite sized toroidal field structure whose mass spectrum is based on parameters to be specified, but whose existence as a stable field structure has been demonstrated. Following papers will address half-integer spin and electric charge aspects of the particle, however they will inevitably trace back to this re-interpretation of the Yang-Mills term representing non-Abelian self-interaction: $\epsilon\left[A_\mu^{(i)}, A_\nu^{(i+2)}\right]$ where (*i*) refers to induction order.

10. Summary

Our goal has been to formulate higher-order self-interaction of the gauge field and re-interpret the non-Abelian term based on this, to derive fermions from the gravitational gauge field. Previous papers have shown the derivation of Heaviside's equations from an exact principle, the *Self-Interaction Principle*, is equivalent to Einstein's nonlinear field equations derived from the *Equivalence Principle*, and have treated general relativity-based problems such as *Quasi-Local Mass*. Key is that the Heaviside derivation is field-strength independent, whereas Einstein's derivation erroneously implies that Heaviside is a "weak field approximation".

A *mass-based* understanding of gravity, as well as the *weak field approximation* misunderstanding, causes physicists to generally ignore gravity in particle physics. A *mass-density-based* understanding of gravity leads to a gravitational basis for particle physics. Burinskii [20] has suggested that particles arise from gravity with the structure of the Kerr black holes, while Christian and Diether [21] suggest particle radii on the order of the Planck length. In other words, mass densities associated with the big bang are effectively limitless. Huang [22], Volovik [23], and others view the primordial field as a *superfluid*. Circa 2006 physicists at the LHC were expecting a *quark gas* from heavy-ion collisions but instead [24]

"*It is well known that the properties of the Yang-Mills plasma turned out to be unexpected…the plasma is similar rather to an ideal liquid than to a gluon gas interacting perturbatively.*"

They conclude with an analogy between phenomena in Yang-Mills theory with physics of *superfluidity*. Our underlying premise has been the *superfluid nature of the primordial field*, with ultra-dense fields, in which we identify higher-order self-induction modes.

Einstein (1919) asked "*Do gravitational fields play an essential part in the structure of the elementary particles of matter*?", suggesting the possibility of a theoretical construction of matter out of gravitational field and electromagnetic field alone. From't Hooft's perspective: "*Einstein's theory of general relativity has a mathematical structure very similar to Yang-Mills theory.*" And Zee remarks: "*there is increasing evidence that the Einstein theory of gravity is just Yang-Mills squared.*" Yet the *Millennium prize* declares that:

"*Yang-Mills theory is now the foundation of most of elementary particle theory, but the mathematical foundation is still unclear.*"

Physical ideas may have been the source of Yang-Mills failure on key issues, not *mathematical* ideas. *Density*-based gravity may open realms of physics to gravitational phenomena that have been overlooked since Newton. This paper has presented a density-based re-interpretation of Yang-Mills gauge field *self-interaction* leading to stable gravitational gauge field structures to explain the mass gap. Future papers will explore half-integer spin and genesis of electric charge. These two issues should allow derivation of mass of fermions whose mass-gap was derived herein.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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