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Surface Waves in a Relativistic Plasma Stream Propagating in a Duct: Kinetic Theory

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Abstract

Dispersion relation of surface waves generated by a relativistic plasma stream in an infinite duct surrounded by vacuum is derived by means of relativistic Vlasov equation. The kinematic boundary condition imposed on the distribution function, the specular reflection conditions on the four sides of duct, can be satisfied by placing infinite number of fictitious surface charge sheets spaced by the duct widths. By placing appropriate fictitious surface charge sheets one can effectively deal with the extended electric field introduced in the Vlasov equation and treat kinetically the surface waves in semi-infinite, slab, and duct plasmas on equal ground. The relativistic duct dispersion relation is compared with the earlier non-relativistic surface wave dispersion relation.

Keywords

Surface Wave, Relativistic Plasma, Duct, Slab

1. Introduction

We investigate surface waves generated by a relativistic plasma beam travelling in a duct interfaced with vacuum by using relativistic Vlasov equation. Surface waves propagate along the interface between two different media while being attenuated in the perpendicular direction. Surface waves are the normal modes that are given rise to in bounded plasmas by satisfying the kinetic and the electrodynamic boundary conditions on the interface between two media. The electrodynamic boundary conditions are the connection formulas matching the fields of the two media, which can be mathematically worked out from the governing equations themselves if the density gradient across the plasma and the other side is very steep. "A sharp interface" is synonymous to a theoretically in-

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finite density gradient across the boundary. In this case, the connection formula can be obtained by "infinitesimal integration" across the interface, which is the operation performed on a certain relevant equation in the manner $\int_{-\epsilon}^{\epsilon}(\cdots)\mathrm{d}x$, where ϵ is a positive infinitesimal. If the quantity $(\cdots)$ is a perfect differential, this operation yields a non-vanishing surface term that contributes to the connection formula. Usually the surface term is surface charge or surface current, In this way, the well-known electromagnetic and dynamic boundary conditions on the boundary can be derived [1]. We might say that the surface wave in the plasma and the vacuum side wave are two different manifestations of "the same wave" created in an extreme inhomogeneous plasma. The electromagnetic boundary conditions are the same in both non-relativistic and relativistic plasmas.

Integrating the Maxwell equation

$$\nabla\times\boldsymbol{B}=\frac{4\pi}{c}\boldsymbol{J}+\frac{1}{c}\frac{\partial\boldsymbol{E}}{\partial t}$$

over an infinitesimal segment $(-\epsilon,\epsilon)$ across the interface $x=0$, we obtain

$$\left[B_y\right]=\frac{4\pi}{c}J_z^*$$

where $[\cdots]$ signifies the jump across the interface, $\boldsymbol{J}^*=\int_{-\varepsilon}^{\varepsilon}\boldsymbol{J}\mathrm{d}x$ is the surface current. In a cold plasma, the normal component of the electric field is discontinuous by the amount of surface charge σ: $\left[E_x\right]=4\pi\sigma$. On the other hand, we have the relation $J_z^*=u\sigma$ due to the charge conservation law, where u is the drift velocity of the plasma in the z direction, and c is the speed of light. Thus we have the characteristic boundary condition in a cold streaming plasma [1] [2]:

$$\left[B_y\right]=\frac{u}{c}\left[E_x\right] \tag{1}$$

or equivalently,

$$\left[D_x\right]=\frac{ck_z}{\omega}\left[B_y\right] \tag{2}$$

where D_x is the normal to the interface component of $\boldsymbol{D}$, the electric displacement, k_z is the z component of the wave vector, and ω is the wave frequency. The casual use of $\left[D_x\right]=0$ or $\left[B_y\right]=0$ in a drifting plasma leads to erroneous results as discussed in earlier works [1] [3]. The physical origin of the boundary relation in Equation (1) is due to the surface current formed in a cold streaming plasma as is evident in the above derivation. Equation (1) is valid in non-relativistic as well as in relativistic plasmas [2].

In bounded Vlasov plasmas, the kinematic boundary condition that is usually referred to as the specular reflection condition is assumed to be satisfied on a sharp boundary [4], regardless of whether the plasma is described by non-relativistic distribution function $f(\boldsymbol{r},\boldsymbol{v},t)$ or relativistic distribution function $f(\boldsymbol{r},\boldsymbol{p},t)$ (see Equation (9) below). Most of the works on bounded kinetic plasmas were dealt with non-relativistically. Non-relativistic kinetic theory of surface waves in semi-infinite plasmas is well-known [5] [6]. Non-relativistic kinetic dispersion relation of surface wave in a slab plasma was worked out earlier [7].

In this work, we investigate surface waves of a moving relativistic Vlasov plasma in a duct. We consider an infinite duct formed by intersections of four planes: $x = 0, a$ and $y = 0, b$, with $-\infty < z < \infty$. We shall discuss the specular reflection boundary condition in terms of $f(\boldsymbol{r}, \boldsymbol{v}, t)$, rather than in terms of $f(\boldsymbol{r}, \boldsymbol{p}, t)$, since the reflection of $\boldsymbol{v}$ is entirely equivalent to the reflection of $\boldsymbol{p}$ in the kinetic equation (see Equation (12) below). Thus the specular reflection conditions require for the distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ to satisfy

$$f(0, y, z, v_x, v_y, v_z, t) = f(0, y, z, -v_x, v_y, v_z, t) \quad \text{on} \quad x = 0 \quad \text{plane}$$

$$f(a, y, z, v_x, v_y, v_z, t) = f(a, y, z, -v_x, v_y, v_z, t) \quad \text{on} \quad x = a \quad \text{plane}$$

$$f(x, 0, z, v_x, v_y, v_z, t) = f(x, 0, z, v_x, -v_y, v_z, t) \quad \text{on} \quad y = 0 \quad \text{plane}$$

$$f(x, b, z, v_x, v_y, v_z, t) = f(x, b, z, v_x, -v_y, v_z, t) \quad \text{on} \quad y = b \quad \text{plane.}$$

In our duct-bounded plasma, the kinematic conditions on the four planes are satisfied by introducing extended electric field in the fashion:

$$E_x(-x, y, z) = -E_x(x, y, z), \quad E_x(2a - x, y, z) = -E_x(x, y, z) \tag{3}$$

$$E_y(x, -y, z) = -E_y(x, y, z), \quad E_y(x, 2b - y, z) = -E_y(x, y, z) \tag{4}$$

This scheme is workable if $f_0(\boldsymbol{p})$, the zero order distribution function, is invariant with respect to the reflections $p_x \to -p_x$ and $p_y \to -p_y$, and clearly this reflectional property is satisfied by the moving Maxwellian to be introduced later. Equations (3) and (4) are the important conclusion of the above discussion that is valid in relativistic as well as in non-relativistic kinetic equations.

The function $E_x(x)$ as defined in Equation (3) is a periodic function of a piecewise continuous function of period "a" extending over the range $-\infty < x < \infty$ with discontinuity at $x = \pm 2na$ with a jump of A_1 (say) and with discontinuity at $x = \pm(2n-1)a$ with a jump of A_2 (say), where n is integer. The profile of the piecewise function $E_x(x)$ is plotted in the book by Lee [8]. The algebra involved in carrying out the Fourier transform of the piecewise discontinuous functions with aforementioned discontinuous jumps is quite laborious [7]. However, it turns out that, after all the algebraic hard work, the discontinuities that are present in the extended field components $E_x(x, y)$ and $E_y(x, y)$ in Equations (3) and (4) at the locations $x = \pm 2na$ and $x = \pm(2n-1)a$ and $y = \pm 2nb$ and $y = \pm(2n-1)b$ are mathematically (as well as physically) tantamount to placing fictitious surface charges at the corresponding jump locations in the form

$$\begin{aligned} S(x, y, z, t) = {} & A_1 \sum_{n=0,1,2,\cdots} \delta(x \pm 2na) + A_2 \sum_{n=1,2,\cdots} \delta(x \pm (2n-1)a) \\ & + B_1 \sum_{n=0,1,2,\cdots} \delta(y \pm 2nb) + B_2 \sum_{n=1,2,\cdots} \delta(y \pm (2n-1)b) \end{aligned} \tag{5}$$

This is the crucial part of improvement in this work as compared with the earlier work [7]. The surface charges are associated with the surface currents by satisfying the charge conservation equation

$$\frac{\partial S}{\partial t} + \nabla \cdot \hat{\boldsymbol{z}} \boldsymbol{J}_s = 0 \tag{6}$$

Therefore, we can assume the presence of the fictitious surface currents

$$\boldsymbol{J}_s(\boldsymbol{k}, \omega) = \hat{\boldsymbol{z}} \frac{k_z}{\omega} S(\boldsymbol{k}, \omega) \tag{7}$$

The surface charges in Equation (5) and the surface currents in Equation (7) should be included in the Maxwell equations for our duct plasma wave analysis.

In this work, we consider a streaming plasma which moves along the axial direction (z-direction) with a relativistic speed u. In Section 2, we introduce the relativistic Vlasov equation. In Section 3, the boundary value problem satisfying the kinetic and electromagnetic boundary conditions are solved to find the dispersion relation of the duct surface wave. Section 4 furnishes discussions, and compares the dispersion relation with the recent work of Lee and Cho on non-relativistic duct flow [9].

2. Basic Equations

We begin with the relativistic equation of motion for electrons

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -e\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \tag{8}$$

where $\boldsymbol{p}$, the relativistic momentum, is

$$\boldsymbol{p} = m\gamma\boldsymbol{v} \tag{9}$$

with m being the rest mass and $\boldsymbol{v}$ being the particle velocity, and

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{10}$$

An easy way to write down the relativistic Vlasov equation is to note that the particle orbit as given by Equation (8) is the characteristics of the Vlasov equation. Indeed, the characteristic equation of the following equation is Equation (8):

$$\frac{\partial}{\partial t} f(\boldsymbol{r}, \boldsymbol{p}, t) + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - e\left(\boldsymbol{E}(\boldsymbol{r}, t) + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \frac{\partial f}{\partial \boldsymbol{p}} = 0 \tag{11}$$

Linearizing (11) gives

$$\frac{\partial}{\partial t} f(\boldsymbol{r}, \boldsymbol{p}, t) + \boldsymbol{v} \cdot \frac{\partial f}{\partial \boldsymbol{r}} - e\left(\boldsymbol{E}(\boldsymbol{r}, t) + \frac{1}{c}\boldsymbol{v} \times \boldsymbol{B}\right) \cdot \frac{\partial f_0(\boldsymbol{p})}{\partial \boldsymbol{p}} = 0 \tag{12}$$

where $f_0(\boldsymbol{p})$ is the zero order equilibrium distribution function which will be specified later. We also have the Maxwell equations for electrons. Ions are assumed to be stationary and only form the neutralizing background.

$$\nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \tag{13}$$

$$\nabla \times \boldsymbol{B} = \frac{4\pi}{c}\boldsymbol{J} + \frac{1}{c}\frac{\partial \boldsymbol{E}}{\partial t} + \hat{\boldsymbol{z}} J_s \tag{14}$$

where the plasma current $\boldsymbol{J}$ is

$$\boldsymbol{J}(\boldsymbol{r},t) = -\frac{eN}{m}\int \mathrm{d}^3 p\, f(\boldsymbol{r},\boldsymbol{p},t)\frac{\boldsymbol{p}}{\gamma} \tag{15}$$

with N being the zero order equilibrium number density. The fictitious surface current J_s corresponding to the surface charge S in Equation (5) takes the form

$$\begin{aligned} J_s = {} & A_1' \sum_{n=0,1,2,\cdots} \delta(x \pm 2na) + A_2' \sum_{n=1,2,\cdots} \delta(x \pm (2n-1)a) \\ & + B_1' \sum_{n=0,1,2,\cdots} \delta(y \pm 2nb) + B_2' \sum_{n=1,2,\cdots} \delta(y \pm (2n-1)b) \end{aligned} \tag{16}$$

$$\nabla\cdot\boldsymbol{E} = -4\pi e\int f\mathrm{d}^3 p + S(x,y,z) \tag{17}$$

$$\nabla\cdot\boldsymbol{B} = 0 \tag{18}$$

We Fourier transform all the dependent variables in the basic equations, including f, $\boldsymbol{E}$, and $\boldsymbol{B}$, and assume the form $\sim \exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t)$. Then Equation (12) gives

$$f(\omega,\boldsymbol{k},\boldsymbol{p}) = i\frac{e}{m}\frac{1}{\omega - \boldsymbol{k}\cdot\boldsymbol{v}}\left(\boldsymbol{E} + \frac{1}{c}\boldsymbol{v}\times\boldsymbol{B}\right)\cdot\frac{\partial f_0}{\partial \boldsymbol{p}} \tag{19}$$

where $\boldsymbol{v}$ reads as the function of $\boldsymbol{p}$ as given in Equation (9). Using Equation (19) in the Fourier transform of Equation (15) yields

$$J_l = -\frac{ie^2 N}{m}\int \mathrm{d}^3 p\, p_l \left[\frac{E_i + \frac{1}{c} e_{ijk} v_j B_k}{\gamma(\omega - \boldsymbol{k}\cdot\boldsymbol{v})}\right]\frac{\partial f_0}{\partial p_i} \tag{20}$$

where l is the Cartesian index and e_{ijk} is the Cartesian tensor called Levi-Civita symbol, and repeated indexes are summed over. We evaluate the current in Equation (20) for a cold streaming plasma whose zero order distribution function is

$$f_0(\boldsymbol{p}) = \delta(p_x)\delta(p_y)\delta(p_z - p_0) \tag{21}$$

where

$$p_0 = \gamma_0 m u,\ \gamma_0 = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}} \tag{22}$$

which corresponds to the streaming velocity $\boldsymbol{v}_0 = \hat{\boldsymbol{z}}u$. Integrating by parts Equation (20) gives

$$J_l = \frac{ie^2 N}{m}\int \mathrm{d}^3 p\, f_0 \left(\delta_{il} + p_l \frac{\partial}{\partial p_i}\right)\left[\frac{E_i + \frac{1}{c} e_{ijk} v_j B_k}{\gamma(\omega - \boldsymbol{k}\cdot\boldsymbol{v})}\right] \tag{23}$$

The following relations are useful for evaluating the above integral:

$$\gamma(p) = \sqrt{1 + \frac{p^2}{m^2 c^2}},\ \frac{\partial \gamma}{\partial p_j} = \frac{p_j}{\gamma m^2 c^2} \tag{24}$$

$$v_i = \frac{cp_i}{\sqrt{m^2c^2+p^2}},\ \frac{\partial v_i}{\partial p_j} = \frac{1}{m\gamma}\left(\delta_{ij} - \frac{v_i v_j}{c^2}\right) \tag{25}$$

Using Equation (21) in Equation (23), we obtain after a long algebra,

$$\boldsymbol{J}(\boldsymbol{k},\omega) = \frac{ie^2N}{m\gamma_0\omega'}\left[\boldsymbol{E} + \frac{u}{c}\hat{\boldsymbol{z}}\times\boldsymbol{B} + \hat{\boldsymbol{z}}\left\{-\frac{u^2}{c^2}E_z + \frac{u}{\omega'}\left(\boldsymbol{k}\cdot\left(\boldsymbol{E}+\frac{u}{c}\hat{\boldsymbol{z}}\times\boldsymbol{B}\right) - \frac{u^2}{c^2}k_z E_z\right)\right\}\right] \tag{26}$$

where $\omega' = \omega - k_z u$ is Doppler-shifted frequency.

Using $\boldsymbol{B} = \frac{c}{\omega}\boldsymbol{k}\times\boldsymbol{E}$ and writing in components, we have

$$J_x = \frac{ie^2N}{m\omega\gamma_0}\left(E_x + \frac{uk_x}{\omega'}E_z\right) \tag{27}$$

$$J_y = \frac{ie^2N}{m\omega\gamma_0}\left(E_y + \frac{uk_y}{\omega'}E_z\right) \tag{28}$$

$$J_z = \frac{ie^2N}{m\omega\gamma_0}\left[\frac{u}{\omega'}\left(k_xE_x + k_yE_y\right) + \frac{E_z}{\omega'^2}\left(u^2\left(k_x^2+k_y^2\right) + \frac{\omega^2}{\gamma_0^2}\right)\right] \tag{29}$$

The array formed by the coefficients of the electric field components is the conductivity tensor, as defined by $J_i = \sigma_{ij}E_j$.

Combining Equations (13) and (14) with the aid of Equations (27)-(29) yields a vector equation for $\boldsymbol{E}(\boldsymbol{k},\omega)$ in the form

$$\boldsymbol{k}\times(\boldsymbol{k}\times\boldsymbol{E}) + \frac{\omega^2}{c^2}\varepsilon_{ij}E_j = -\hat{\boldsymbol{z}}\frac{i\omega}{c}J_s \tag{30}$$

where

$$\varepsilon_{ij} = \delta_{ij} + \frac{4\pi i}{\omega}\sigma_{ij} = \left(1 - \frac{\omega_p^2}{\gamma_0\omega^2}\right)\delta_{ij} - \frac{\omega_p^2}{\gamma_0\omega^2}U_{ij} \tag{31}$$

is the dielectric tensor, and ω_p is the plasma frequency, and

$$U_{ij} = \begin{pmatrix} 0 & 0 & \frac{uk_x}{\omega'} \\ 0 & 0 & \frac{uk_y}{\omega'} \\ \frac{uk_x}{\omega'} & \frac{uk_y}{\omega'} & U_{zz} \end{pmatrix}$$

$$U_{zz} = \frac{u^2}{\omega'^2}\left(k^2 - \frac{\omega^2}{c^2}\right) + \frac{2k_z u}{\omega'}$$

Putting Equation (31) into Equation (30) gives a 3 by 3 matrix equation for E_j:

$$\left[\left(1 - n^2 - \frac{\omega_p^2}{\gamma_0\omega^2}\right)\delta_{ij} + \frac{k_ik_j}{k^2}n^2 - \frac{\omega_p^2}{\gamma_0\omega^2}U_{ij}\right]E_j = -\frac{ic}{\omega}\hat{\boldsymbol{z}}J_s \tag{32}$$

where $n^2 = c^2k^2/\omega^2$ is the refractive index, and J_s is the Fourier transform of the fictitious surface currents in Equation (16):

$$J_s(\boldsymbol{k},\omega)=\delta(k_y)\left[A_1\Sigma_0 e^{\pm i2nak_x}+A_2\Sigma_1 e^{\pm i(2n-1)ak_x}\right] +\delta(k_x)\left[B_1\Sigma_0 e^{\pm i2nbk_y}+B_2\Sigma_1 e^{\pm i(2n-1)bk_y}\right] \tag{33}$$

where A's and B's may be functions of k_z and ω, and the double signs are summed over, and the notations Σ_0 and Σ_1 are the summations in Equation (16).

Equation (32) can be inverted after spending a considerable amount of time to solve for $\boldsymbol{E}(\boldsymbol{k},\omega)$:

$$E_x=-\frac{J_s}{\Delta}\frac{c^2k_x}{\omega^2}\left(k_z-\frac{u}{c^2}\frac{\omega_p^2}{\gamma_0\omega'}\right) \tag{34}$$

$$E_y=-\frac{J_s}{\Delta}\frac{c^2k_y}{\omega^2}\left(k_z-\frac{u}{c^2}\frac{\omega_p^2}{\gamma_0\omega'}\right) \tag{35}$$

$$E_z=\frac{J_s}{\Delta}\left(1-\frac{\omega_p^2}{\gamma_0\omega^2}-\frac{c^2k_z^2}{\omega^2}\right) \tag{36}$$

where

$$\Delta=\left(1-\frac{\omega_p^2}{\gamma_0\omega^2}-\frac{c^2k^2}{\omega^2}\right)\left[1-\frac{\omega_p^2}{\gamma_0\omega^2}-\frac{\omega_p^2}{\gamma_0\omega^2}\left(U_{zz}-\frac{u^2}{\omega'^2}\left(k_x^2+k_y^2\right)\right)\right] \tag{37}$$

After further algebra, Δ becomes

$$\Delta=\left(1-\frac{\omega_p^2}{\gamma_0\omega^2}-\frac{c^2k^2}{\omega^2}\right)\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right) \tag{38}$$

Also we obtain

$$B_x=\frac{c}{\omega}\frac{J_s}{\Delta}k_y\left(1-\frac{\omega_p^2}{\gamma_0\omega\omega'}\right) \tag{39}$$

$$B_y=-\frac{c}{\omega}\frac{J_s}{\Delta}k_x\left(1-\frac{\omega_p^2}{\gamma_0\omega\omega'}\right) \tag{40}$$

In addition, we put $B_z=0$ since we investigate the transverse magnetic mode.

3. Boundary Equations

In order to apply the boundary conditions, the electric and magnetic field components in the Fourier $\boldsymbol{k}$ space should be inverted to the fields in the ordinary $\boldsymbol{r}$ space by performing $\int_{-\infty}^{\infty}\mathrm{d}k_x\int_{-\infty}^{\infty}\mathrm{d}k_y\,e^{ik_xx+ik_yy}(\cdots)$. The integrals involve infinite series through the surface charge J_s, but the infinite series are nicely summed at the particular positions corresponding to $x=0,a$ and $y=0,b$. Thus, we apply the boundary conditions along the two infinite lines: $(x,y,z)=(0,0,z)$ and (a,b,z) with $-\infty<z<\infty$. The two lines correspond to the two seams of the duct which are diagonally opposite.

When the inversion integrals are performed, the following formulas are useful, which can be verified by a simple change of variable, as is shown in earlier work

[7]. We have integrals of the type in the inversion integrals

$$Q(x)=\int_{-\infty}^{\infty}\mathrm{d}k_x k_x \Phi(k)\mathrm{e}^{ik_x x}\left[A_1\Sigma_0\mathrm{e}^{\pm i2nak_x}+A_2\Sigma_1\mathrm{e}^{\pm i(2n-1)ak_x}\right] \tag{41}$$

where $\Phi(k)$ is an even function of k_x. Then, we have

$$Q(0)=A_1\int_{-\infty}^{\infty}\mathrm{d}k_x k_x \Phi(k_x) \tag{42}$$

$$Q(a)=-A_2\int_{-\infty}^{\infty}\mathrm{d}k_x k_x \Phi(k_x) \tag{43}$$

We also have integrals of the type

$$R(x)=\int_{-\infty}^{\infty}\mathrm{d}k_x \Phi(k)\mathrm{e}^{ik_x x}\left[A_1\Sigma_0\mathrm{e}^{\pm i2nak_x}+A_2\Sigma_1\mathrm{e}^{\pm i(2n-1)ak_x}\right] \tag{44}$$

Then, we have

$$R(0)=2\int_{-\infty}^{\infty}\mathrm{d}k_x \Phi(k_x)\left(A_1S_1+A_2S_2\right) \tag{45}$$

$$R(a)=2\int_{-\infty}^{\infty}\mathrm{d}k_x \Phi(k_x)\left(A_1S_2+A_2S_1\right) \tag{46}$$

where

$$S_1=\frac{1}{2}+\mathrm{e}^{2iak_x}+\mathrm{e}^{4iak_x}+\cdots \tag{47}$$

$$S_2=\mathrm{e}^{iak_x}+\mathrm{e}^{3iak_x}+\cdots \tag{48}$$

Formulas in Equations (42), (43), (45), and (46) are useful for evaluating the integrals. Let us evaluate:

$$\begin{aligned}
&E_x(0,0,z)\\
&=-\frac{c^2}{\omega^2}\left(k_z-\frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)\int_{-\infty}^{\infty}\mathrm{d}k_x\int_{-\infty}^{\infty}\mathrm{d}k_y\frac{k_x}{\Delta}\Big[\delta(k_y)\Big(A_1\Sigma_0\mathrm{e}^{\pm i2nak_x}\\
&\quad+A_2\Sigma_1\mathrm{e}^{\pm i(2n-1)ak_x}\Big)+\delta(k_x)\Big(B_1\Sigma_0\mathrm{e}^{\pm i2nbk_y}+B_2\Sigma_1\mathrm{e}^{\pm i(2n-1)bk_y}\Big)\Big]\\
&=-\frac{c^2}{\omega^2}\left(k_z-\frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)\int_{-\infty}^{\infty}\mathrm{d}k_x\frac{k_x}{\Delta}\left(A_1\Sigma_0\mathrm{e}^{\pm i2nak_x}+A_2\Sigma_1\mathrm{e}^{\pm i(2n-1)ak_x}\right)\\
&=-\frac{c^2}{\omega^2}\left(k_z-\frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)A_1\int_{-\infty}^{\infty}\mathrm{d}k_x\frac{k_x}{\Delta}
\end{aligned} \tag{49}$$

where we used Equation (42). In the last (also in the later) integral, k^2 hidden in Δ is $k^2=k_z^2+k_x^2$. We have

$$\begin{aligned}
&E_x(a,b,z)\\
&=-\frac{c^2}{\omega^2}\left(k_z-\frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)\int_{-\infty}^{\infty}\mathrm{d}k_x\mathrm{e}^{ik_x a}\int_{-\infty}^{\infty}\mathrm{d}k_y\mathrm{e}^{ik_y b}\frac{k_x}{\Delta}\Big[\delta(k_y)\Big(A_1\Sigma_0\mathrm{e}^{\pm i2nak_x}\\
&\quad+A_2\Sigma_1\mathrm{e}^{\pm i(2n-1)ak_x}\Big)+\delta(k_x)\Big(B_1\Sigma_0\mathrm{e}^{\pm i2nbk_y}+B_2\Sigma_1\mathrm{e}^{\pm i(2n-1)bk_y}\Big)\Big]\\
&=-\frac{c^2}{\omega^2}\left(k_z-\frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)\int_{-\infty}^{\infty}\mathrm{d}k_x\mathrm{e}^{ik_x a}\frac{k_x}{\Delta}\left(A_1\Sigma_0\mathrm{e}^{\pm i2nak_x}+A_2\Sigma_1\mathrm{e}^{\pm i(2n-1)ak_x}\right)\\
&=-\frac{c^2}{\omega^2}\left(k_z-\frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)(-A_2)\int_{-\infty}^{\infty}\mathrm{d}k_x\frac{k_x}{\Delta}
\end{aligned} \tag{50}$$

where we used Equation (43). Analogous integrations yield

$$E_y(0,0,z) = -\frac{c^2}{\omega^2}\left(k_z - \frac{u\omega_p^2}{c^2\omega'\gamma_0}\right) B_1 \int_{-\infty}^{\infty} \mathrm{d}k_y \frac{k_y}{\Delta} \tag{51}$$

$$E_y(a,b,z) = -\frac{c^2}{\omega^2}\left(k_z - \frac{u\omega_p^2}{c^2\omega'\gamma_0}\right)(-B_2) \int_{-\infty}^{\infty} \mathrm{d}k_y \frac{k_y}{\Delta} \tag{52}$$

In the above (also in the later) $\int \mathrm{d}k_y$ integral, k^2 hidden in Δ is $k^2 = k_z^2 + k_y^2$.

$$B_x(0,0,z) = \frac{c}{\omega}\left(1 - \frac{\omega_p^2}{\omega\omega'\gamma_0}\right) B_1 \int_{-\infty}^{\infty} \mathrm{d}k_y \frac{k_y}{\Delta} \tag{53}$$

$$B_x(a,b,z) = \frac{c}{\omega}\left(1 - \frac{\omega_p^2}{\omega\omega'\gamma_0}\right)(-B_2) \int_{-\infty}^{\infty} \mathrm{d}k_y \frac{k_y}{\Delta} \tag{54}$$

$$B_y(0,0,z) = -\frac{c}{\omega}\left(1 - \frac{\omega_p^2}{\omega\omega'\gamma_0}\right) A_1 \int_{-\infty}^{\infty} \mathrm{d}k_x \frac{k_x}{\Delta} \tag{55}$$

$$B_y(a,b,z) = -\frac{c}{\omega}\left(1 - \frac{\omega_p^2}{\omega\omega'\gamma_0}\right)(-A_2) \int_{-\infty}^{\infty} \mathrm{d}k_x \frac{k_x}{\Delta} \tag{56}$$

We encounter with different type of integral in

$$\begin{aligned} E_z(0,0,z) = {} & \left(1 - \frac{\omega_p^2}{\gamma_0\omega^2} - \frac{c^2k_z^2}{\omega^2}\right) \int_{-\infty}^{\infty} \mathrm{d}k_x \int_{-\infty}^{\infty} \mathrm{d}k_y \frac{1}{\Delta}\Big[\delta(k_y)\Big(A_1\Sigma_0 \mathrm{e}^{\pm i2nak_x} \\ & + A_2\Sigma_1 \mathrm{e}^{\pm i(2n-1)ak_x}\Big) + \delta(k_x)\Big(B_1\Sigma_0 \mathrm{e}^{\pm i2nbk_y} + B_2\Sigma_1 \mathrm{e}^{\pm i(2n-1)bk_y}\Big)\Big] \end{aligned}$$

which becomes

$$\begin{aligned} E_z(0,0,z) = {} & \left(1 - \frac{\omega_p^2}{\gamma_0\omega^2} - \frac{c^2k_z^2}{\omega^2}\right)\Bigg[\int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{\Delta}\Big(A_1\Sigma_0 \mathrm{e}^{\pm i2nak_x} + A_2\Sigma_1 \mathrm{e}^{\pm i(2n-1)ak_x}\Big) \\ & + \int_{-\infty}^{\infty} \frac{\mathrm{d}k_y}{\Delta}\Big(B_1\Sigma_0 \mathrm{e}^{\pm i2nbk_y} + B_2\Sigma_1 \mathrm{e}^{\pm i(2n-1)bk_y}\Big)\Bigg] \end{aligned}$$

which we write in the form

$$\begin{aligned} E_z(0,0,z) = {} & 2\left(1 - \frac{\omega_p^2}{\gamma_0\omega^2} - \frac{c^2k_z^2}{\omega^2}\right)\Bigg[\int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{\Delta}\big(A_1S_1(ak_x) + A_2S_2(ak_x)\big) \\ & + \int_{-\infty}^{\infty} \frac{\mathrm{d}k_y}{\Delta}\big(B_1S_1(bk_y) + B_2S_2(bk_y)\big)\Bigg] \end{aligned} \tag{57}$$

where we used Equation (45), and

$$S_1(\xi) = \frac{1}{2} + \mathrm{e}^{2i\xi} + \mathrm{e}^{4i\xi} + \cdots, \quad S_2(\xi) = \mathrm{e}^{i\xi} + \mathrm{e}^{3i\xi} + \cdots \tag{58}$$

Analogously, we obtain

$$\begin{aligned} E_z(a,b,z) = {} & 2\left(1 - \frac{\omega_p^2}{\gamma_0\omega^2} - \frac{c^2k_z^2}{\omega^2}\right)\Bigg[\int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{\Delta}\big(A_1S_2(ak_x) + A_2S_1(ak_x)\big) \\ & + \int_{-\infty}^{\infty} \frac{\mathrm{d}k_y}{\Delta}\big(B_1S_2(bk_y) + B_2S_1(bk_y)\big)\Bigg] \end{aligned} \tag{59}$$

where we used Equation (46).

- **Vacuum solution**.

Vacuum solutions should solve

$$\left(\nabla^2 + \frac{\omega^2}{c^2}\right)\boldsymbol{B} = 0 \tag{60}$$

and

$$\boldsymbol{E} = \frac{ic}{\omega}\nabla\times\boldsymbol{B} \tag{61}$$

Equation (60) is solved by

$$\boldsymbol{B} \sim \mathrm{e}^{ik_z z}\mathrm{e}^{\pm k_x x}\mathrm{e}^{\pm k_y y} \tag{62}$$

with constraint $k_x^2 + k_y^2 = k_z^2 - \frac{\omega^2}{c^2} \equiv \lambda^2$ and $\nabla\cdot\boldsymbol{B} = 0$. Furthermore, we assume $B_z = 0$ since we consider the TM (transverse magnetic) mode.

The vacuum regions corresponding to (or exterior to) the lines $(0,0,z)$ and (a,b,z), which we designate as (*i*) and (*ii*), respectively, are:

Vacuum region (*i*) $x<0,\ y<0$, where we have

$$B_x^v(i) = H_x\mathrm{e}^{ik_z z}\mathrm{e}^{k_x x}\mathrm{e}^{k_y y} \tag{63}$$

$$B_y^v(i) = H_y\mathrm{e}^{ik_z z}\mathrm{e}^{k_x x}\mathrm{e}^{k_y y} \tag{64}$$

$$k_x H_x + k_y H_y = 0 \tag{65}$$

$$E_z^v(i) = \frac{ic}{\omega}\left(H_y k_x - H_x k_y\right)\mathrm{e}^{ik_z z}\mathrm{e}^{k_x x}\mathrm{e}^{k_y y} \tag{66}$$

$$E_x^v(i) = \frac{c}{\omega}k_z H_y\mathrm{e}^{ik_z z}\mathrm{e}^{k_x x}\mathrm{e}^{k_y y} \tag{67}$$

$$E_y^v(i) = -\frac{c}{\omega}k_z H_x\mathrm{e}^{ik_z z}\mathrm{e}^{k_x x}\mathrm{e}^{k_y y} \tag{68}$$

Vacuum region (*ii*) $x>a,\ y>b$, where

$$B_x^v(ii) = G_x\mathrm{e}^{ik_z z}\mathrm{e}^{-k_x x}\mathrm{e}^{-k_y y} \tag{69}$$

$$B_y^v(ii) = G_y\mathrm{e}^{ik_z z}\mathrm{e}^{-k_x x}\mathrm{e}^{-k_y y} \tag{70}$$

$$k_x G_x + k_y G_y = 0 \tag{71}$$

$$E_z^v(ii) = \frac{ic}{\omega}\left(-G_y k_x + G_x k_y\right)\mathrm{e}^{ik_z z}\mathrm{e}^{-k_x x}\mathrm{e}^{-k_y y} \tag{72}$$

$$E_x^v(ii) = \frac{c}{\omega}k_z G_y\mathrm{e}^{ik_z z}\mathrm{e}^{-k_x x}\mathrm{e}^{-k_y y} \tag{73}$$

$$E_y^v(ii) = -\frac{c}{\omega}k_z G_x\mathrm{e}^{ik_z z}\mathrm{e}^{-k_x x}\mathrm{e}^{-k_y y} \tag{74}$$

Putting $(x,y) = (0,0)$ or (a,b) in the above equations gives the vacuum side values of the relevant quantities.

4. Dispersion Relation

We enforce the following boundary conditions to connect the plasma and the vacuum fields: $[E_z]=0$, $[B_y]=\frac{u}{c}[E_x]$, $[B_x]=-\frac{u}{c}[E_y]$.

Along line $(0,0,z)$

$[E_z]=0$ gives, per Equations (57) and (66),

$$\left(1-\frac{\omega_p^2}{\gamma_0\omega^2}-\frac{c^2k_z^2}{\omega^2}\right)\left(A_1I_1+A_2I_2+B_1J_1+B_2J_2\right)=\frac{ic}{\omega}\left(H_yk_x-H_xk_y\right) \tag{75}$$

where

$$I_i=2\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x}{\Delta}S_i\left(ak_x\right),\ J_i=2\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x}{\Delta}S_i\left(bk_y\right),\ \left(i=1,2\right) \tag{76}$$

$[B_y]=\frac{u}{c}[E_x]$ gives

$$A_1Q\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right)+\frac{\omega}{c}H_y=0 \tag{77}$$

where

$$Q=\int_{-\infty}^{\infty}\mathrm{d}k_x\frac{k_x}{\Delta}=\int_{-\infty}^{\infty}\mathrm{d}k_y\frac{k_y}{\Delta} \tag{78}$$

$[B_x]=-\frac{u}{c}[E_y]$ gives

$$B_1Q\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right)-\frac{\omega}{c}H_x=0 \tag{79}$$

Along line (a,b,z)

$[E_z]=0$ gives

$$\begin{aligned}&\left(1-\frac{\omega_p^2}{\gamma_0\omega^2}-\frac{c^2k_z^2}{\omega^2}\right)\left(A_1I_2+A_2I_1+B_1J_2+B_2J_1\right)\\&=\frac{ic}{\omega}\left(-G_yk_x+G_xk_y\right)\mathrm{e}^{-k_xa}\mathrm{e}^{-k_yb}\end{aligned} \tag{80}$$

$[B_y]=\frac{u}{c}[E_x]$ gives

$$A_2Q\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right)-\frac{\omega}{c}G_y\mathrm{e}^{-k_xa}\mathrm{e}^{-k_yb}=0 \tag{81}$$

$[B_y]=-\frac{u}{c}[E_x]$ gives

$$B_2Q\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right)+\frac{\omega}{c}G_x\mathrm{e}^{-k_xa}\mathrm{e}^{-k_yb}=0 \tag{82}$$

In addition, we have, per $\nabla\cdot\boldsymbol{B}=0$ and $B_z=0$,

$$k_xH_x+k_yH_y=0 \tag{83}$$

$$k_x G_x + k_y G_y = 0 \tag{84}$$

Thus, we have 8 equations for 8 unknowns; $A_1, A_2, B_1, B_2, H_x, H_y, G_x, G_y$. Eliminating H_x, H_y, G_x, G_y gives

$$A_1\left(I_1 + i\frac{c^2}{\omega^2}\frac{\eta}{\xi}k_x Q\right) + A_2 I_2 + B_1\left(J_1 + i\frac{c^2}{\omega^2}\frac{\eta}{\xi}k_y Q\right) + B_2 J_2 = 0 \tag{85}$$

$$A_1 I_2 + A_2\left(I_1 + i\frac{c^2}{\omega^2}\frac{\eta}{\xi}k_x Q\right) + B_1 J_2 + B_2\left(J_1 + i\frac{c^2}{\omega^2}\frac{\eta}{\xi}k_y Q\right) = 0 \tag{86}$$

$$k_y A_1 = k_x B_1 \tag{87}$$

$$k_y A_2 = k_x B_2 \tag{88}$$

where

$$\xi = 1 - \frac{\omega_p^2}{\gamma_0 \omega^2} - \frac{c^2 k_z^2}{\omega^2}, \quad \eta = 1 - \frac{\omega_p^2}{\gamma_0^3 \omega'^2} \tag{89}$$

Eliminating B_1 and B_2 gives

$$A_1\left[k_x I_1 + k_y J_1 + \frac{ic^2}{\omega^2}\frac{\eta}{\xi}\left(k_x^2 + k_y^2\right)Q\right] + A_2\left(k_x I_2 + k_y J_2\right) = 0 \tag{90}$$

$$A_1\left(k_x I_2 + k_y J_2\right) + A_2\left[k_x I_1 + k_y J_1 + \frac{ic^2}{\omega^2}\frac{\eta}{\xi}\left(k_x^2 + k_y^2\right)Q\right] = 0 \tag{91}$$

The above two equations for A_1 and A_2 yield the dispersion relation in the form

$$k_x \int_{-\infty}^{\infty} \frac{dk_x}{\Delta}\frac{1 \pm e^{iak_x}}{1 \mp e^{iak_x}} + k_y \int_{-\infty}^{\infty} \frac{dk_y}{\Delta}\frac{1 \pm e^{ibk_y}}{1 \mp e^{ibk_y}} + i\lambda^2 \frac{c^2}{\omega^2}\frac{\eta}{\xi}\int_{-\infty}^{\infty} k_x \frac{dk_x}{\Delta} = 0 \tag{92}$$

where we used

$$\int_{-\infty}^{\infty} \frac{dk_x}{\Delta}\left[S_1\left(ak_x\right) \pm S_2\left(ak_x\right)\right] = \frac{1}{2}\int_{-\infty}^{\infty} \frac{dk_x}{\Delta}\frac{1 \pm e^{iak_x}}{1 \mp e^{iak_x}} \tag{93}$$

In regard to the Fourier variables k_x and k_y outside the integrals, we imposed the constraint $k_x^2 + k_y^2 = k_z^2 - \omega^2/c^2 \equiv \lambda^2$. Therefore it is convenient to transform

$$k_x = \frac{b\lambda}{\sqrt{a^2 + b^2}}, \quad k_y = \frac{a\lambda}{\sqrt{a^2 + b^2}} \tag{94}$$

[k_x, k_y inside the integrals are dummy and let them be there as they are.] The transform in Equation (94) satisfies the constraint and the relation $ak_x = bk_y$. In fact it can be derived from the latter and the constraint. Then the dispersion relation takes the form

$$\frac{b}{\sqrt{a^2 + b^2}}\int_{-\infty}^{\infty} \frac{dk_x}{\Delta}\frac{1 \pm e^{iak_x}}{1 \mp e^{iak_x}} + \frac{a}{\sqrt{a^2 + b^2}}\int_{-\infty}^{\infty} \frac{dk_y}{\Delta}\frac{1 \pm e^{ibk_y}}{1 \mp e^{ibk_y}} + i\lambda \frac{c^2}{\omega^2}\frac{\eta}{\xi}\int_{-\infty}^{\infty} k_x \frac{dk_x}{\Delta} = 0 \tag{95}$$

If either a or $b \to \infty$, we recover the slab dispersion relation in the non-relativistic limit [7].

$$\int_{-\infty}^{\infty}\frac{\mathrm{d}k_x}{\Delta}\frac{1\pm \mathrm{e}^{iak_x}}{1\mp \mathrm{e}^{iak_x}}+i\lambda\frac{c^2}{\omega^2}\frac{\eta}{\xi}\int_{-\infty}^{\infty}k_x\frac{\mathrm{d}k_x}{\Delta}=0 \tag{96}$$

where η and ξ are given by Equation (89) with $\gamma_0=1$.

It is recalled that k^2 hidden in Δ is: $k^2=k_z^2+k_x^2$ in $\int \mathrm{d}k_x$-integral and $k^2=k_z^2+k_y^2$ in $\int \mathrm{d}k_y$-integral. Thus, let us change the integration variables, both k_x and k_y, in Equation (95) to κ:

$$\begin{aligned}&\frac{b}{\sqrt{a^2+b^2}}\int_{-\infty}^{\infty}\frac{\mathrm{d}\kappa}{\Delta}\frac{1\pm \mathrm{e}^{ia\kappa}}{1\mp \mathrm{e}^{ia\kappa}}+\frac{a}{\sqrt{a^2+b^2}}\int_{-\infty}^{\infty}\frac{\mathrm{d}\kappa}{\Delta}\frac{1\pm \mathrm{e}^{ib\kappa}}{1\mp \mathrm{e}^{ib\kappa}}\\&+i\lambda\frac{c^2}{\omega^2}\frac{\eta}{\xi}\int_{-\infty}^{\infty}\kappa\frac{\mathrm{d}\kappa}{\Delta}=0\end{aligned} \tag{97}$$

where $k^2=k_z^2+\kappa^2$. In regard to the double signs in Equation (97), the upper (lower) signs correspond to the symmetric (anti-symmetric) mode which also occurs in a slab plasma. For a square duct ($a=b$), Equation (97) reduces to the form identical with the slab dispersion equation Equation (96), except for factor $\sqrt{2}$. This reduction is due to the x-y symmetry. To recover slab dispersion relation from Equation (92), we take $k_y\to 0$, $b\to\infty$, and put $k_x=\lambda$. We can take $k_y\to 0$ since the y-direction has a translational invariance in a slab.

The duct dispersion relation in Equation (97) can be contour-integrated for a cold plasma, giving

$$\frac{b\Gamma}{\sqrt{a^2+b^2}}\tanh\frac{a\Gamma}{2}+\frac{a\Gamma}{\sqrt{a^2+b^2}}\tanh\frac{b\Gamma}{2}+\sqrt{k_z^2-\frac{\omega^2}{c^2}}\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right)=0 \tag{98}$$

where $\Gamma=\sqrt{k_z^2-\frac{\omega^2-\omega_p^2/\gamma_0}{c^2}}$. For the anti-symmetric mode, tanh-function above is replaced by coth-function.

5. Discussion

In a bounded plasma, one way of solving Vlasov equation by satisfying the specular reflection condition is to extend the plasma electric field in the manner of Equation (3). The job of Fourier transforming such as piecewise continuous periodic function, extending to infinity, is laborious. In this work, we present an alternative way of avoiding the hard algebra by placing sheets of fictitious surface charges at the location of discontinuities of the electric field. The magnitudes of the surface charges are undetermined constants, but they can be determined through the connection formula with the vacuum side field—resulting in the dispersion relation of the surface wave. This method enables one to deal with semi-infinite, slab, and duct plasmas in a common work-frame. Taking $b\to\infty$ in Equation (98) gives

$$\Gamma\tanh\frac{a\Gamma}{2}+\sqrt{k_z^2-\frac{\omega^2}{c^2}}\left(1-\frac{\omega_p^2}{\gamma_0^3\omega'^2}\right)=0 \tag{99}$$

which is the slab ($0 < x < a$) dispersion relation. Taking $a \to \infty$ in Equation (99) gives the semi-infinite plasma dispersion relation

$$\Gamma + \sqrt{k_z^2 - \frac{\omega^2}{c^2}} \left(1 - \frac{\omega_p^2}{\gamma_0^3 \omega'^2} \right) = 0 \tag{100}$$

Equation (100) agrees with the semi-infinite dispersion relation obtained by Lee [3]. If $u = 0$, Equation (99) agrees with the slab dispersion relation obtained from the fluid theory worked out by Gradov and Stenflo [10].

For a square duct, putting $a = b$ in Equation (98) yields

$$\sqrt{2}\Gamma \tanh \frac{a\Gamma}{2} + \sqrt{k_z^2 - \frac{\omega^2}{c^2}} \left(1 - \frac{\omega_p^2}{\gamma_0^3 \omega'^2} \right) = 0 \tag{101}$$

which is similar to the slab dispersion relation. This is because the complete symmetry between x and y coordinates makes the three-dimensional problem a two-dimensional problem practically.

The Doppler-shifted frequency ω' appearing in Equation (98) represents the streaming effect in the lowest order of $\frac{u}{c}$. The higher order effect enters through the relativistic factor γ_0 and U_{zz} in Equation (31). The relativistic effect manifests itself through the attenuation constant Γ per $\frac{\omega_p^2}{\gamma_0}$ and in the dispersion relation per $\frac{\omega_p^2}{\gamma_0^3}$. Recently Lee and Cho [9] investigated surface waves in a non-relativistically streaming Vlasov plasma in a duct. Their result is identical with Equation (98) upon putting $\gamma_0 = 1$.

A visual understanding of the extended electric field may be grasped by plotting the extended function [8]. A useful reference for relativistic Vlasov equation is, among others, Momtgomery and Tidman [11], in which the velocity-version of Vlasov equation (in contrast to the momentum-version employed in this work) is presented in detail. This work may find applications in laboratory or astrophysical situation where electromagnetic waves propagate through certain channels. This work might be useful for analysis of a proton beam travelling in a duct.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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