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Gravitational Time Dilation inside the Solid Sphere

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Abstract

Gravitational time dilation directly reflects the difference between gravitational potentials at different altitudes in the gravitational field. At the same time this phenomenon is expected to obey the Einstein's equivalence principle, one of two pillars (apart from general covariance) of general relativity. The experiments aimed at detecting the gravitational time dilation are therefore described as the tests of general relativity or, alternatively, the tests of equivalence principle. When applied to the exterior of a solid sphere, these two interpretations are fully compatible both theoretically and experimentally. However, when applied to the interior of a solid sphere (e.g., to the interior of Earth), they seem to contradict each other. Namely, a strict dependence of the gravitational time dilation on the gravitational potential inside the sphere proves to be at odds with the equivalence principle. This paper reveals this problem and provides solution to it. As a consequence, it is concluded that, contrary to the current belief, the Earth's center is older, not younger, than the Earth's surface. Since all the previous experiments have been performed either on or above the Earth's surface, an experiment performed below the Earth's surface is proposed.

Keywords

Equivalence Principle, General Relativity, G-Force, Gravitational Potential, Gravitational Time Dilation

1. Introduction

Gravitational time dilation is a form of time dilation predicted by general relativity (GR), referring to an actual passage of time—the difference of elapsed time between two events as measured at different altitudes in the gravitational field. It relates to the gravitational potentials (metric tensor) at these altitudes. This phenomenon was originally predicted by Einstein prior to the formulation of GR, as a consequence of applying special relativity to the accelerated frames of reference, hence without direct regard to the gravitational mass (Einstein [1] [2]). It is therefore strictly connected with the equivalence principle, *i.e.*, the Einstein's observation paving the way to GR, according to which there is no experimental difference between the inertial frame of reference and the (local) frame in free fall, as well as between the local reference frame at rest in the uniform in gravitational field (e.g., on the surface of Earth) and the reference frame under uniform acceleration.

Gravitational time dilation has been tested in numerous experiments, to name most important: the Pound-Rebka experiment conducted in 1959 inside the building shaft (tower) of Harvard University (Pound and Rebka [3] [4]); the Hafele-Keating experiment—as a compound effect including both gravitational (due to mass) and kinematic (due to relative velocity) time dilations (Hafele and Keating [5]); Gravity Probe A, performed in 1976 (Vessot *et al.* [6])—the hydrogen maser high precision measurements of the rate of time passage at the altitude of ca. 10,000 km, compared with the measurements of identical maser placed on the Earth's surface; the (Chou *et al.* [7]) experiment with light clocks placed in the Earth's gravitational field, with their altitude differing by only 1 meter. This effect has also a practical relevance: an inclusion of the gravitational time dilation is crucial, apart from the kinematic time dilation, for the correct operation of the GPS (Ashby [8]).

Besides, the experiments aimed at testing the gravitational time dilation in the context of equivalence principle have been conducted using the Mössbauer effect discovered shortly before (Mössbauer [9]). In these experiments, the accelerated system due to rotation of "ultracentrifuge rotor" replaced the gravitational mass (Hay *et al.* [10], Kündig [11]).

Presumably, Richard Feynman was the first who considered this phenomenon in application to the interior of cosmic bodies, specifically to the interior of Earth. According to this great scholar, the inner core (center) of the Earth is, due to the gravitational time dilation, "one or two days" younger than the Earth's crust (surface). Feynman made this illustrative evaluation during his Lectures on Gravitation held at Caltech in 1962/63 (Feynman, Morinigo and Wagner [12]). For a long time taken on trust, the Feynman's estimate has been recently reappraised by Uggerhøj, Mikkelsen and Faye [13], which resulted in its significant correction. Accordingly, the difference between respective ages turned out to be far greater; namely, for the idealized model of the Earth with the assumed uniform density, the center of Earth proved to be 1.58 years younger than the Earth's surface; instead, for the realistic model with factual inhomogeneous mass distribution, this difference increased to 2.49 years. Anyway, no matter if the revealed discrepancy did originate from the Feynman's cursory calculation or from a later misprint in the lecture transcription confusing days with years, the new results differ from the old ones by the magnitude only. Since both estimates share the same theoretical framework, they concordantly state that the Earth's core is younger than the Earth's crust. There is a general agreement that, due to the gravitational time dilation, the time at the center of Earth (and, generally, of any other massive spherical cosmic body, hereinafter referred to as "solid sphere") is passing slower than on the surface. This stems from the established conviction "probated" by the authority of Feynman, according to which, both outside and inside a solid sphere, gravitational time dilation and gravitational potential are linked by the linear relationship.

A hypothetical clock located at the center of Earth occupies the lowest point of the Earth gravity well, which corresponds with the lowest, negative by convention, gravitational potential. Consequently, the respective clock rate is thought to be the slowest one compared to the rate of any other clock located on the radial path both below and above the surface. This conclusion seems also to follow, as the logical extension, from the experiments aimed at detecting the gravitational time dilation performed on, or above, the Earth's surface. The representative (and earliest) example is the Pound-Rebka experiment. The respective rates of time manifesting themselves through the differences in the gamma ray frequency at different altitudes directly reflect the difference between the gravitational potentials. Although both frequencies were measured outside the sphere (at the top and bottom of the building shaft), the obtained result has been extrapolated by Feynman and his successors (including Uggerhøj) on the whole radial path, both outside and inside the Earth. A direct dependence between gravitational time dilation and gravitational potential in both these cases is treated as obvious. Consequently, the time at the center of Earth is thought to lag behind the surface time, in result of which the Earth's inner core is supposed to be younger than the Earth's crust.

2. The Uggerhøj's *et al.* Paper

In introduction to their paper, Uggerhøj, Mikkelsen and Faye (henceforth collectively titled the "Authors") write: "…arguments based on symmetry will convince most skeptics, including those from 'the general public', that there is no gravitational force at the Earth center. Consequently, such an effect [*i.e.*, gravitational time dilation] cannot be due to the force itself, but may instead be due to the 'accumulated action of gravity' (a layman expression for the gravitational potential energy being the radial integral of the force)" [13].

This is a key passage determining the further conclusions of the cited paper. The Authors take for granted that time passes slower at the center of Earth, which unavoidably implies disconnection of the gravitational time dilation from the g-force (interpreted as the proper acceleration). As a consequence, the time dilation must depend directly on the gravitational potential. Hence, in so far as the Authors find justified to reappraise the Feynman's quantitative prediction, they do not intend to question its underlying theoretical framework, determining the general age-relation.

For the sake of transparency (and also with the aim to adopt respective notation), let us quote almost exactly the formal derivation placed in the first part of the cited paper, concerning the relationship between the gravitational time dilation and gravitational potential inside the Earth, for the homogenous distribution of mass. Accordingly, the gravitational potential (Φ) for the exterior of the solid sphere is

$$\Phi_{ext} = -G\frac{M}{r}, \quad r \geq R \tag{1}$$

G—Newton's gravitational constant, *M*—sphere mass, *R*—sphere radius, *r*—distance from the center. Instead, the gravitational potential inside, *i.e.*, in the interior of the sphere is

$$\Phi_{int} = -G\frac{M\left(3R^2 - r^2\right)}{2R^3}, \quad r \leq R \tag{2}$$

These two different expressions share the common result at $r = R$ (at the surface):

$$\Phi(R) = -\frac{GM}{R} \tag{3}$$

Instead, at the center, *i.e.*, for $r = 0$, one has:

$$\Phi(0) = -\frac{3GM}{2R} \tag{4}$$

The respective difference is therefore:

$$\Delta\Phi = \Phi(R) - \Phi(0) = \frac{1}{2}G\frac{M}{R} \tag{5}$$

Consequently, "a difference in gravitational potential implies a time dilation at the point with lower potential" [13], given by the standard gravitational redshift:

$$\omega = \omega_0\left(1 - \frac{\Delta\Phi}{c^2}\right) \tag{6}$$

where ω and ω_0 are the angular frequencies at the center and at the surface, respectively. Combining Equation (6) with the result of Equation (5), and considering $\Delta\omega = \omega - \omega_0$, gives the difference in the frequencies related to the difference between the gravitational potentials:

$$\Delta\omega = -\frac{1}{2}G\omega_0\frac{M}{Rc^2} \tag{7}$$

3. Gravitational Time Dilation and the Equivalence Principle

Let us precisely consider the application of the equivalence principle to our problem. This principle clearly states that, if any effect (hence also the effect of time dilation) takes place in the non-inertial frame due to gravity, it must also take place in the non-inertial frame due to the kinematically determined acceleration. And *vice versa*. Accordingly, an isolated non-inertial observer located, say, in the "windowless box" is basically (*i.e.*, assuming the box small enough to make the tidal forces negligible) unable to detect if the perceived effect is due to "real" gravity or due to "pseudo-gravity" caused by the engine running. The specific examples of pseudo-gravity are the non-inertial systems due to rotation about an axis, e.g., the rotating toroidal spaceship (an idea exploited in the sci-fi movies so far), the hypergravity centrifuge—the lab device also used for pilots and astronauts training, or the ultracentrifuge rotor used in the experiments testing the equivalence principle in the context of gravitational time dilation. The respective centrifugal acceleration, perceived as g-force, is particularly evocative because it eliminates the relative motion between any pair of two radially positioned clocks aimed at comparing the time rates. A formal condition to make this possible consists in applying the rotating frame, so to say comoving with the centrifuge. At the same time, in the stationary lab frame, inertial by assumption, the time dilation takes place due to the linear orbital motion of a given point laid on the centrifuge arm (edge of the rotor), hence it is the SR time dilation. The time dilation measured in the rotating frame must be identical to the time dilation measured in the stationary lab frame. This is because in both frames this effect is absolute; in the rotating frame as it were by definition, and in the lab frame on the same basis as it is predicted by the twin paradox and verified in practice in the Hafele-Keating experiment. Likewise, in both frames, the clock located at the centrifuge pivot can be recognized as the reference clock with the null time dilation.

The particular question is whether and how the pseudo-gravity due to rotation of the centrifuge is similar to the real gravity due to the gravitational mass. According to the equivalence principle, this similarity is both exact and limited. Namely, apart from the demand of locality (in the case of centrifuge, the pseudo-tidal forces are even much more distinct), a striking difference is that proper acceleration (g-force) on the surface of a planet is centripetal, whereas the g-force perceived on the rotating arm is centrifugal. Therefore, of course, the location of the mass-center cannot be identified with the location of the centrifuge axis. Let us consider this more specifically.

Let K be the stationary lab frame, and K' the centrifuge rotating frame. Let O be the central point of the frame K', coincident with the pivot of centrifuge. Let E be the point at the outer end of the centrifuge arm. Let L be the distance between O and E, obviously equal in both frames. The point O represents both the rotating frame K' (as its unique point) and the stationary frame K (as an exemplary point). According to the SR time dilation applied to the frame K, the clock located at E goes slower than the clock located at O. According to GR (on the base of equivalence principle), the numerically identical effect takes place in the rotating frame K'.

Let v be the linear velocity of E in the frame K. The SR time dilation in the K is

$$\gamma_{(K)} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{8}$$

The gravitational time dilation in the frame K', as compared to the non-dilated reference clock at the center (pivot) is defined as

$$\gamma_{(K')} = \left(1 - \frac{|\Phi|}{c^2}\right)^{-1/2} \tag{9}$$

The gravitational potential and gravitational acceleration relate to each other as

$$|\Phi| = g_{(r)} r \tag{10}$$

According to the equivalence principle, the centrifugal acceleration at point *E*, being

$$a_c = v^2 / L \tag{11}$$

can be considered equivalent to the centripetal gravitational acceleration (due to the presence of gravitational mass):

$$g_{(r)} = \left| -GM / r^2 \right| \tag{12}$$

Consequently, also the time dilation factors $\gamma_{(K)}$ and $\gamma_{(K')}$ would be equivalent (equal in value). This can only be achieved if we identify L with r. Then, by multiplying $a_c \times L$ and $g_{(r)} \times r$, we would identify v^2 with $g_{(r)} r$. Can we do that? The short answer is: yes, because time dilations in both K and K' are absolute. And since we deal with the same pair of clocks, both factors have to be identical. However, as far as the purpose to identify L with r is clear, it is not as much clear the reason (possibility) for doing that. For example, the gravitational acceleration (g-force) on the Earth's surface, unitary by convention, is ~9.8 ms^{-2} with the Earth radius being 6.37 × 10^6 m, whereas the same acceleration 1g (and much greater) can be easily obtained using the centrifuge with the arm few meters long only, or the ultracentrifuge with the radius few centimeters only. So, it follows that equal accelerations can be associated with extremely different radii. Hence, how L and r can be identified?

The answer is pretty trivial. Although the equivalence principle implies deep consequences leading to general relativity, we don't need to dig into the GR details. The equality between L and r is taken by assumption, whereas the remaining quantities: either mass or linear velocity, should be considered as variables that have to be adjusted to obtain given preset value of acceleration. There are two options, basically. If we start with the centrifuge arm of definite length, then, to equalize the radius connected with gravity with the arm length, the gravitational mass has to be adjusted to match the preset acceleration. If, in turn, we start with the definite mass and radius due to gravity, then, in order to assume the same length of the centrifuge arm, we have to adjust the linear velocity to match the preset acceleration.

In general (*i.e.*, regardless of the details discussed above), the basis for identifying the "pseudo-gravity" due to kinematic acceleration with the "real" gravity due to gravitational mass is the equivalence principle. This means however that the reason for which the clock located at the outer end of the centrifuge rotating arm lags behind the clock located close to the pivot is that it perceives the centrifugal force indistinguishable from the force of gravity. In both cases, there is one and the same g-force, in formal terms the proper acceleration measurable by accelerometer. The term "perceive" (roughly tantamount to "feel" or "sense") has an unambiguous physical meaning; e.g., exceeding certain critical value of the centrifugal acceleration would result in the damage of clock or, in the case of a trained pilot, in the loss of consciousness. Consequently, if a clock does not actually "perceive" any g-force, one cannot expect it to go slower, hence to undergo the gravitational time dilation. This apparently obvious claim, based on the equivalence principle, has a crucial importance to our problem.

At the center of Earth defined by symmetrical distribution of mass, the lowest gravitational potential coincides with zero g-force. Therefore, to obey the equivalence principle, we shouldn't expect gravitational time dilation to occur there. In fact, it doesn't matter if the clock is located in the center of a planet or in "empty space" far away from any gravity sources. In other words, it is not important whether the g-force is "actually" absent or if it is only "effectively" absent—being neutralized due to the generally conceived free fall (the motion along geodesic), ranging between the rectilinear accelerated motion along the radius and the orbital motion with constant linear velocity, the latter including specific case of a body remaining at rest in any of the five Lagrange points. In both "actual" and "effective" cases, the onboard accelerators (and clocks) do not perceive any g-force, which eventually implies the lack of gravitational time dilation. Otherwise, the equivalence principle would be nothing but a groundless demand. An obvious precondition for the equivalence principle to be valid is the requirement that identical g-forces make two local frames (hence clocks) identical with regard to gravity.

4. Gravitational Time Dilation near the Event Horizon of the Schwarzschild Black Hole

The gravitational properties of the black hole observed from a distance do not basically differ from these of other cosmic bodies. The differences become important only near the event horizon and beyond. The ratio between the time rate near the event horizon of a non-rotating uncharged black hole and the time rate indicated by remote clock is given by equation:

$$\frac{\Delta\tau}{\Delta t}=\left(1-\frac{r_S}{r}\right)^{1/2} \tag{13}$$

$\Delta\tau$—elapsed proper time between two events close to observer located near the event horizon of black hole; Δt—elapsed coordinate time between these same events, measured by distant observer; r_S—Schwarzschild radius; r—radial distance from the center of black hole, provided $r > r_S$. Considering $r_S = 2GM/c^2$, it follows:

$$\frac{\Delta\tau}{\Delta t}=\left(1-\frac{2GM}{c^2 r}\right)^{1/2} \tag{14}$$

The gravitational time dilation factor is therefore:

$$\gamma = \left(1 - \frac{2GM}{rc^2}\right)^{-1/2} \tag{15}$$

This can be alternatively expressed in terms of escape velocity $v_e = (2GM/r)^{1/2}$, written as the fraction of *c*, *i.e.*, $\beta_e = v_e/c$:

$$\gamma = \left(1 - \beta_e^2\right)^{-1/2} \tag{16}$$

5. Gravitational Time Dilation outside and inside the Solid Sphere

Our goal is to reconcile the two seemingly contradictory premises: 1) According to GR, the gravitational time dilation is modeled by the metric tensor, which means that it directly depends on the gravitational potential. This prediction has been confirmed by all previous experiments; 2) Extending this prediction to the interior of a solid sphere violates the equivalence principle. Namely, it contradicts the demand according to which an exemplary windowless box (lab) located at the center of a solid sphere should not differ physically from the identical windowless box located far away from the sources of gravity. This is because, in both cases (in both local inertial frames), the g-force amounts to zero. The previous solution to this problem, represented both by Feynman and Uggerhøj *et al.*, so to say "ignores" the equivalence principle in the application of the gravitational time dilation to the interior of solid sphere. Below, it is proposed an alternative solution consistent with this principle.

Let *R* be the radius of solid sphere, *M*—mass of this sphere, *r*—radial distance from the center of a basically free magnitude, either greater or less than *R*. Let us consider first the gravitational potential Φ as the function of *r*. Due to different (regarding gravity) physical conditions inside and outside the solid sphere, the respective relationship is plotted by two separate functions connected at "inflection point" $r = R$, at which $\Phi_{ext} = \Phi_{int} = -GMR^{-1}$. Hence, the overall dependence of Φ on *r* takes the form of a single graph consisting of two functions on two complementary half-open intervals, according to the conditions specified on the right sides of Equations (1) and (2). At $r = 0$, the function based on Equation (2) reaches the minimum (lowest gravitational potential): $\Phi_{int} = -\frac{3}{2}GMR^{-1}$. The gravitational potential defined according to Equation (2) determines the shape of the gravity well, for $0 \le r \le R$ (**Figure 1**).

In turn, the gravitational accelerations inside and outside the sphere of radius *R* are:

$$g_{(r)_{ext}} = -\frac{GM}{r^2}\hat{\mathbf{r}} = \frac{GM}{r^2} \quad (r \ge R) \tag{17}$$

$$g_{(r)_{int}} = \frac{GM}{R^3}r \quad (r \le R) \tag{18}$$

($\hat{\mathbf{r}}$—the unity vector directed outward). As previously, the respective graph consists of two functions on two complementary half-open intervals "glued together" at $r = R$ (**Figure 2**).

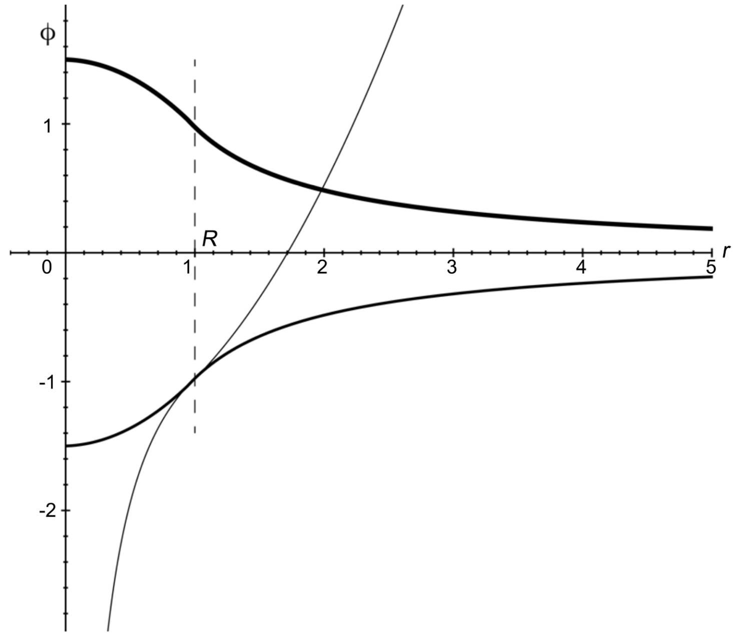

Figure 1. Gravitational potential Φ inside and outside the massive spherical body, as a function of distance (r). The respective graph (lower quadrant, bold line) consists of two functions on two complementary half-open intervals connected at $r = R$. The upper quadrant graph depicts the gravitational potential absolute value, thought to correspond directly to the gravitational time dilation. The constant factors G, M and R are here normalized to unity.

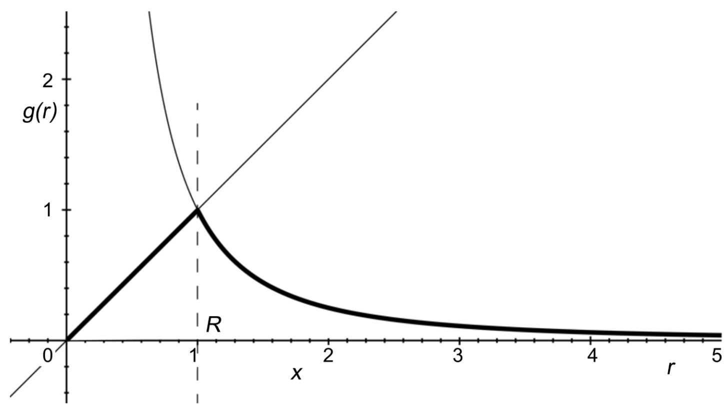

Figure 2. Gravitational proper acceleration $g(r)$ (g-force) inside and outside the solid sphere. The respective graph consists of two functions on the two complementary half-open intervals connected at $r = R$. All constant factors are here normalized to unity.

The gravitational time dilation based on the gravitational acceleration, is, in the general case:

$$\gamma = \left(1 - \frac{|\Phi|}{c^2}\right)^{-1/2} \tag{19}$$

This equation is basically consistent with the gravitational time dilation for the Schwarzschild black hole defined by Equation (13). It is to be noted that gravitational potential Φ is not a quantity directly "perceived". It is rather a mathematical being corresponding to the GR metric tensor. However, the gravitational potential can be factorized into the proper gravitational acceleration (g-force) and the radius. As mentioned before, g-force is a quantity directly "perceivable" by the accelerometer.

Let's rewrite then the gravitational potential as $|\Phi| = g_{(r)}r$ (as we already did in Equation 10). For any case specified as $r \geq R$, this way of defining the gravitational potential is equivalent to the one given by Equation (1), *i.e.*, $\Phi = -G(M/r)$. Hence, outside the sphere, this way of denoting (and defining) Φ is a purely formal operation with no physical consequences. However, for $r \leq R$, things look different. Combining Equation (19) with Equation (10) gives:

$$\gamma = \left(1 - \frac{g_{(r)}r}{c^2}\right)^{-1/2} \tag{20}$$

The next step is the following. We specify $g_{(r)}$ according to Equations (17) and (18) and plug them into the above equation. In result, we obtain the two differing modes to obtain the time dilation factors: one for the exterior and the other one for interior of a solid sphere, both complying with the equivalence principle:

$$\gamma_{ext} = \left(1 - \frac{GM}{c^2 r}\right)^{-1/2} \quad (r \geq R) \tag{21}$$

$$\gamma_{int} = \left(1 - \frac{GMr^2}{c^2 R^3}\right)^{-1/2} \quad (r \leq R) \tag{22}$$

In agreement both with theory and experiments, Equation (21) remains physically identical with Equation (19), expressing linear dependence (proportionality) between the gravitational potential and the gravitational time dilation. Instead, in Equation (22), the factorized gravitational potential makes the actual radius mathematically interacting with the sphere radius, in result of which the gravitational time dilation does not depend any more directly on the gravitational potential. In particular, for $r \to 0$ one has $\gamma(t)_{int} = 1$ (which is also directly obvious considering $g_{(r)} = 0$ at the center). This corresponds with $\gamma(t)_{ext} = 1$ for $r \to \infty$. Hence, the "windowless box" at the center of a solid sphere and the "windowless box" far away in the empty space prove to be identical with each other with regard to the gravitational time dilation, in compliance with the equivalence principle.

6. Numerical Estimation for the Idealized Model of Earth

Let us use the equations obtained in the previous section to estimate the difference in age between the Earth's center and Earth's surface, for the simplified model of Earth with the uniform density. At the center of Earth, *i.e.*, at $r = 0$, Equation (22) reduces to $\gamma_{int} = 1$. Instead, at the surface, *i.e.*, for $r = R$, the

Equations (21) and (22) reduce to a one common equation, namely:

$$\gamma_{\text{surface}} = \left(1 - \frac{GM}{c^2 R}\right)^{-1/2} \tag{23}$$

Let us denote: T_E —the overall reference age of Earth, identified with the age of the Earth's center (T_{center}); T_{surface} —age of the Earth's surface; $\Delta T = T_{\text{center}} - T_{\text{surface}}$. Hence, the difference is given by:

$$\Delta T = T_E - \frac{1}{\gamma_{\text{surface}}} \times T_E \tag{24}$$

It follows:

$$\Delta T = T_E - \left(1 - \frac{GM}{c^2 R_E}\right)^{1/2} \times T_E \tag{25}$$

Substituting: $c^2 \approx 8.99 \times 10^{16}\ \text{m}^2 \cdot \text{s}^{-2}$; $G \approx 6.67 \times 10^{-11}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$; $T_E \approx 4.55 \times 10^9\ \text{yr}$; $M_E \approx 5.97 \times 10^{24}\ \text{kg}$; $R_E \approx 6.37 \times 10^6\ \text{m}$, we obtain after some arithmetic:

$$\Delta T = T_{\text{center}} - T_{\text{surface}} \approx 1.58\ \text{yr} \tag{26}$$

It follows that the center of Earth is approximately 1.58 years older than the Earth's surface—the value equal, but reversely attributed, compared to the figure obtained by Uggerhøj *et al.* The term "approximately" refers here to: 1) assumed homogenous distribution of the Earth mass; 2) assumed constancy of both G and c in cosmic time (precisely, invariability of the factor G/c^2). It is to be noted that, in the light of various "non-standard" theories/hypotheses such as VSL or Dirac's LNH, the latter is not obvious by itself.

7. Conclusions

A principle-based analysis of the problem of gravitational time dilation in the interior of Earth and other cosmic solid spheres reveals fallacy of the current view represented by Feynman, Uggerhøj and other researchers. The quantitative difference between particular estimates eventually appears less important than the incorrectness of the general assumptions commonly shared. Namely, a consistent application of the Einstein's equivalence principle impels us to revise the so far view as to the relationship between the gravitational time dilation and the gravitational potential. It appears that in the case of interior of a solid sphere, the time dilation depends on g-force rather than on the gravitational potential. As applied to the Earth, this means that relation between the ages of Earth's center and Earth's surface is basically different from the one previously formulated; namely, the inner core of the Earth is not younger, but older than the Earth's crust.

As in most cases in physics, an ultimate criterion to settle a given problem is an experiment. To make it happen, we do not need to reach the Earth's core because all that counts here is the general tendency described by Equation (22). The respective test could be basically similar to the Pound-Rebka experiment except being performed not above but below the Earth's surface. This should be neither too difficult nor expensive, considering that devices needed for that purpose are typically on standard lab equipment, and that any of numerous inactive mines endowed with vertical shaft could be used for that purpose.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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