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Lorentz Transformation Derived from Relativity of Time

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Abstract

The principles of special relativity and Einstein's simple derivation of the Lorentz transformation are reviewed. A new simple derivation of the Lorentz transformation is developed in this paper, by a new approach of light-pulse observation or time-dilation observation. Therefore, under the two principles of special relativity, there exist two equivalent simple derivations of or two equivalent approaches to the Lorentz transformation. Einstein's approach emphasizes or highlights relativity of space while our approach emphasizes or highlights relativity of time. This research reveals, in a particular way, the equivalence of relativity of space and relativity of time in special relativity. Combination of Einstein's approach and the approach developed in this paper makes the methodology of simple derivation of the Lorentz transformation complete and perfect.

Keywords

Special Relativity, Lorentz Transformation, Relativity of Space, Relativity of Time

1. Introduction

Lorentz published a space-time transformation between inertia systems in a research paper "Electromagnetic phenomena in a system moving with any velocity less than that of light" in 1904 [1]. Larmor (1900) and Poincaré (1906) deduced, respectively, the Lorentz transformation [2] [3]. In 1905, Albert Einstein published his famous paper "On the electrodynamics of moving bodies", in which the two principles of special relativity were suggested and the Lorentz transformation was derived [4] [5]. Authors derived the Lorentz transformation with variant postulates or assumptions [6]-[12].

Einstein published a simple derivation of the Lorentz transformation [13]

[14]. Other simple derivations of the Lorentz transformation were given by authors [15]-[23]. Jozsef derived new forms of the Lorentz transformation and Minkowski's equation [24]. Olszewski demonstrated the invariance of several new component electromagnetic-field vectors with respect to the Lorentz transformation [25].

The Lorentz transformation is the space-time transformation of special relativity. Logically and physically, it should be inferred not only from relativity of space, but also from relativity of time. That is, logically and physically, there should exist two equivalent approaches to derivation of the Lorentz transformation. However, Einstein's simple derivation of the Lorentz transformation took only one approach: approach from relativity of space. Though his derivation was full of wisdom and skill, the methodology of simple derivation of the Lorentz transformation was not complete.

In this paper we review Einstein's simple derivation of the Lorentz transformation, and develop, under the two principles of special relativity, a new simple derivation of the Lorentz transformation, by a new approach of light-pulse observation or relativity of time. This work tends to fill the blank of the methodology of derivation of the Lorentz transformation and make the theory of special relativity complete or perfect.

2. The Principles of Special Relativity

The principles of special relativity published by Einstein in 1905 are [4] [5]:

1) The laws by which the states of physical systems undergo change are not affected, whether these changes of state be referred to the one or the other of two systems of co-ordinates in uniform translatory motion. (The Principle of Relativity)

2) Any ray of light moves in the "stationary" system of co-ordinates with the determined velocity c, whether the ray be emitted by a stationary or by a moving body. (The Principle of the Constancy of the Velocity of Light)

3. Reviewing Einstein's Simple Derivation of the Lorentz Transformation

Following the two principles of special relativity, Einstein derived the Lorentz Transformation [4] [5].

Then he derived the Lorentz transformation in a simple way [13] [14]. We comment on his simple derivation of the Lorentz transformation by the two points:

1) Einstein's simple derivation of the Lorentz transformation is based on the Principles of Special Relativity (the Principle of Relativity and the Principle of the Constancy of the Velocity of Light).

2) Einstein's simple derivation of the Lorentz transformation uses relativity of space, by observation of length-contraction, to determine the constants in the equations for the Lorentz transformation.

4. Our New Simple Derivation Developed for the Lorentz Transformation

Two Cartesian coordinate systems, K and K', are so constructed that the X-axis and the X'-axis permanently coincide, other axes are parallel respectively, $OY /\!/ O'Y'$, $OZ /\!/ O'Z'$. The coordinate system K' ($O'X'Y'Z'$) moves with speed v relative to K ($OXYZ$) along the X-axis. The origins of the coordinate systems, O and O', coincide at the moment ($t = t' = 0$). Now, we are going to establish the Lorentz transformation between the inertia systems K and K' by observation of time-dilation.

We consider first only events located on the X-axis (and the X'-axis). Any such event is expressed by the abscissa x and the time t, relative to the coordinate system K, and the abscissa x' and the time t', relative to the coordinate system K'.

Suppose that generation and transmission of a light-signal, a light pulse of an infinitesimal pulse-duration δ ($\delta \to 0$), are started at the space-time points ($x = 0$, $t = 0$) and ($x' = 0$, $t' = 0$). The light-signal is transmitted proceeding along the positive or negative X-axis and X'-axis. Following the analysis of Einstein [13] [14], the transmission equation of the light-signal proceeding along the positive X-axis is

$$x = ct$$

or

$$x - ct = 0\,. \tag{1}$$

Obeying the principles of special relativity, the same light-signal, proceeding along the positive X'-axis, is transmitted with the velocity c, relative to the coordinate system K', following

$$x' - ct' = 0\,. \tag{2}$$

The transmission satisfying Equation (1) must satisfy Equation (2), because both the equations are of the same transmission of the same light-signal. Then we have

$$\left(x' - ct'\right) = s\left(x - ct\right), \tag{3}$$

where s is a constant.

Considering the transmission of the light-signal proceeding along the negative X-axis and the negative X'-axis, we have a similar equation

$$\left(x' + ct'\right) = r\left(x + ct\right), \tag{4}$$

where r is a constant.

Introducing the constants α and β to replace the constants s and r as

$$\alpha = \frac{s+r}{2}, \quad \beta = \frac{s-r}{2},$$

adding Equation (3) and Equation (4), subtracting Equation (3) from Equation (4), we obtain the equations

$$x' = \alpha x - \beta ct\,, \tag{5a}$$

$$ct' = \alpha ct - \beta x . \tag{5b}$$

The coordinate of the origin of the system K' is permanently

$$x' = 0 ,$$

then, from Equation (5a),

$$x = \frac{\beta c}{\alpha} t ,$$

then the velocity of the origin of K' moving relative to K is

$$v = \frac{\beta c}{\alpha} . \tag{6}$$

We have established Equation (5a) and Equation (5b), referring to Einstein's inference [13] [14]. Now we determine the constants in Equation (5a) and Equation (5b) by using relativity of time, instead of relativity of space used by Einstein [13] [14]:

Observation 1:

The pulse-duration of the light-signal at

$$x = 0 \tag{7a}$$

$$\text{is} \quad \Delta t = \delta, \quad (\delta \to 0) . \tag{7b}$$

From Equation (5b)

$$\Delta t' = \alpha \Delta t - \frac{\beta}{c} \Delta x = \alpha \delta \tag{8}$$

by virtue of Equation (7a) and Equation (7b).

An observer, Observer 1, makes a "video" of K from K' and measures the pulse-duration $\Delta t'$ indicated by Equation (8).

Observation 2:

The pulse-duration of the light-signal at

$$x' = 0 \tag{9a}$$

$$\text{is} \quad \Delta t' = \delta, \quad (\delta \to 0) . \tag{9b}$$

From Equation (5a)

$$x = \frac{\beta ct}{\alpha} + \frac{x'}{\alpha} . \tag{10}$$

Substituting Equation (10) into Equation (5b) leads to

$$ct' = \alpha ct - \frac{\beta^2 c}{\alpha} t - \frac{\beta x'}{\alpha} . \tag{11}$$

From Equation (11) we have

$$t' = \left(\alpha - \frac{\beta^2}{\alpha} \right) t - \frac{\beta x'}{\alpha c} = \alpha \left(1 - \frac{\beta^2}{\alpha^2} \right) t - \frac{\beta x'}{\alpha c} . \tag{12}$$

Substituting Equation (6) into Equation (12) leads to

$$t' = \alpha\left(1-\frac{v^2}{c^2}\right)t - \frac{vx'}{c^2}. \tag{13}$$

We have, from Equation (13),

$$\Delta t = \frac{\Delta t'}{\alpha\left(1-\frac{v^2}{c^2}\right)} + \frac{v\Delta x'}{\alpha\left(c^2-v^2\right)} = \frac{\delta}{\alpha\left(1-\frac{v^2}{c^2}\right)} \tag{14}$$

by virtue of Equation (9a) and Equation (9b).

An observer, Observer 2, makes a "video" of K' from K and measures the pulse-duration Δt indicated by Equation (14).

The same light-signal is transmitted relative to both the system K and the system K'. The principle of relativity requires that the pulse-duration, observed or judged from K', of the light-signal transmitted relative to K, should be equal to the pulse-duration, observed or judged from K, of the light-signal transmitted relative to K'. Therefore, $\Delta t'$ in Equation (8) must be equal to Δt in Equation (14). And so we have

$$\alpha\delta = \frac{\delta}{\alpha\left(1-\frac{v^2}{c^2}\right)}. \tag{15}$$

From Equation (15)

$$\alpha^2 = \frac{1}{1-\frac{v^2}{c^2}},$$

then

$$\alpha = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{16}$$

From Equation (6) and Equation (16)

$$\beta = \frac{\alpha v}{c} = \frac{v}{c\sqrt{1-\frac{v^2}{c^2}}}. \tag{17}$$

Substituting Equation (16) and Equation (17) into Equation (5a) and Equation (5b) results in

$$x' = \alpha x - \beta ct = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \tag{18}$$

and

$$t' = \alpha t - \frac{\beta}{c}x = \frac{t-\frac{v}{c^2}x}{\sqrt{1-\frac{v^2}{c^2}}}. \tag{19}$$

Thus we have deduced the Lorentz transformation for events on the X-axis and the X'-axis, Equation (18) and Equation (19).

Following Einstein and supplementing Equation (18) and Equation (19) with

$$y' = y \tag{20a}$$

$$\text{and} \quad z' = z \tag{20b}$$

to incorporate the events which happen outside the X-axis and the X'-axis, we finally obtain the Lorentz transformation, Equation (18), Equation (19), Equation (20a) and Equation (20b) [13] [14].

5. Conclusions

1) Based on the two principles of special relativity, we have derived the Lorentz transformation from relativity of time.

2) Under the two principles of special relativity, there exist two equivalent simple derivations of and two equivalent approaches to the Lorentz transformation: Einstein's simple derivation and his measuring-rod observation approach, and the simple derivation and the light-pulse observation approach developed in this paper.

3) The two simple derivation approaches, Einstein's simple derivation approach and our simple derivation approach in this paper, show physical equivalence of relativity of space and relativity of time, deepen, in a special way, our understanding of the relationship between relativity of space and relativity of time, or our understanding of relativity of space-time. The two simple derivation approaches also suggest methodological completeness of deriving the Lorentz space-time transformation for special relativity.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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