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Selection Rules in Weak Interaction and Conservation of Fermion Quantum Number

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Abstract

Traditionally, in weak interaction, I_3, Y and four flavour quantum numbers are not conserved but several empirical selection rules work well. Recently, it was found that, in weak interaction, there are three levels of conservation of additive quantum numbers, and fermion quantum number F is conserved in all kinds of interactions. It is known that weak interaction has three types: fermionic, pure hadronic and pure leptonic, corresponding to the first and the second level of conservation of additive quantum numbers respectively. It is demonstrated in this paper that the selection rules in all types of weak interaction can be interpreted by conservation of F, and the formula of relation between Q/e, F and F_0 is more general than Gell-Mann-Nishijima formula. Description of weak interaction becomes simpler, If only we take Q, F_0 and F, based on the conserved physical quantities.

Keywords

Weak Interaction, Selection Rules, Fermion Quantum Number

1. Introduction

At the most elementary level, at present, fermions include quarks and leptons. Traditionally, isospin (I) with isospin projection (I_3) is assigned to u and d quarks, but the other four quarks have the flavour quantum numbers: S, C, B^* and T for s, c, b and t quarks respectively. All quarks have the baryon quantum number B and electric charge Q. Hypercharge Y is defined from I_3 and Q. Leptons, much simpler than quarks, have Q and lepton quantum number L. Q, B, L, I, I_3, Y and four flavour quantum numbers are additive quantum numbers. Q, B and L are conserved in strong interaction, electromagnetic interaction and weak interaction. I_3, Y and four flavour quantum numbers are conserved in strong interaction and electromagnetic interaction, but not conserved in weak interaction. I is conserved in strong interaction, but not conserved in either electromagnetic interaction or weak interaction. On the other hand, in weak interaction, the systematic way in which symmetry related to conservation is broken leads to several empirical selection rules which may serve to check observations and to put constraint on models. It is conventional to refer to weak interaction events as being pure leptonic, pure hadronic (or nonleptonic) or fermionic (or semi-leptonic, semi-hadronic) depending on whether they involve leptons only, hadrons only, or both leptons and hadrons. Knowledge above is described in textbooks, e.g., [1] [2] [3] [4].

Recently, it was found that, in weak interaction, there are three levels of conservation of additive quantum numbers [5]: at the first (the highest) level, Q, B, L and fermion quantum number F are conserved including all kinds of fermions (both quarks and leptons). At the second level, quark quantum number H is only conserved including pure hadrons, and lepton quark-like quantum number H_L is only conserved including pure leptons. At the third (the lowest) level, I, I_3, Y and flavour quantum numbers are not conserved. Because realized that the types of weak interaction correspond to the first and the second level of conservation of the additive quantum numbers respectively, it is natural to consider if there is any relation between conservation of the additive quantum numbers and the selection rules in weak interaction.

2. Additive Quantum Numbers

Characteristics of additive quantum numbers are described in [5] and here are summarized in brief. Flavor quantum number D and U for d and u quark respectively are related to I_3 in

$$I_3 = (D+U)/2. \tag{1}$$

H is sum of all six flavor quantum numbers:

$$H = D+U+S+C+B^*+T = \begin{cases} -1: \text{quark is ‘d’ type} \\ +1: \text{quark is ‘u’ type} \end{cases} \tag{2}$$

Antiquarks have additive quantum numbers with the same absolute values as quarks but the opposite sign of quarks, except that antiquarks have same I as quarks. H of each lepton is zero.

H_L of leptons is similar with H of quarks:

$$H_L = \begin{cases} -1: \text{lepton is ‘d’ type} \\ +1: \text{lepton is ‘u’ type} \end{cases} \tag{3}$$

Antileptons have additive quantum numbers with the same absolute values as leptons but opposite sign of leptons. H_L of each quark is zero.

F for all fermions is combined from H and H_L:

$$F = H + H_L = \begin{cases} -1: \text{fermion is ‘d’ type} \\ +1: \text{fermion is ‘u’ type} \end{cases} \tag{4}$$

The formula of relation between electronic charge Q/e, F and F_0 is

$$Q/e = (F_0 + F)/2, \tag{5}$$

where F_0 is:

$$F_0 = \begin{cases} B = +1/3 : \text{fermion is quark} \\ -L = -1 : \text{fermion is lepton} \end{cases} \tag{6}$$

Values of additive quantum numbers of quarks and leptons are listed in Table 1 and Table 2 respectively. Comparing Equation (5) with Gell-Mann-Nishijima formula [6] [7] [8] (in extended form)

$$Q/e = I_3 + Y/2, \tag{7}$$

we can see that Equation (5) only includes conserved additive quantum numbers so as to express more general and more profound connotation than Equation (7) including not-conserved additive quantum numbers in weak interaction.

3. Selection Rules and Conservation of *F*

In the following paragraphs we check all three types of weak interaction one by one to verify if the selection rules are related to conservation of F. Firstly, hadrons composed by only u, d and s quarks are considered, so from Equation (2),

$$H = D + U + S. \tag{8}$$

After it, the relation is generalized to heavier quarks (Section 3.4).

3.1. Pure Hadronic Weak Interaction

From Equation (1) and (8),

Table 1. Additive quantum numbers of quarks.

quark	Q/e	B	I	I_3	D	U	S	C	B^*	T	Y	H	F
d	−1/3	+1/3	+1/2	−1/2	−1	0	0	0	0	0	+1/3	−1	−1
u	+2/3	+1/3	+1/2	+1/2	0	+1	0	0	0	0	+1/3	+1	+1
s	−1/3	+1/3	0	0	0	0	−1	0	0	0	−2/3	−1	−1
c	+2/3	+1/3	0	0	0	0	0	+1	0	0	+4/3	+1	+1
b	−1/3	+1/3	0	0	0	0	0	0	−1	0	−2/3	−1	−1
t	+2/3	+1/3	0	0	0	0	0	0	0	+1	+4/3	+1	+1

Table 2. Additive quantum numbers of leptons.

Lepton	Q/e	L	H_L	F
e^-	−1	+1	−1	−1
ν_e	0	+1	+1	+1
μ^-	−1	+1	−1	−1
ν_μ	0	+1	+1	+1
τ^-	−1	+1	−1	−1
ν_τ	0	+1	+1	+1

$$\Delta H = \Delta(S + D + U) = \Delta S + 2\Delta I_3 . \quad (9)$$

The pure hadronic weak interaction obeys selection rule

$$\Delta S = \mp 1, \ \Delta I = \pm 1/2, \ \Delta I_3 = \pm 1/2 . \quad (10)$$

Obviously, the selection rule (Equation (10)) can be deduced from $\Delta H = 0$ (Equation (9)), *i.e.*, conservation of H as well as conservation of F because in pure hadronic weak interaction $H_L = 0$ and then $F = H_L$ (Equation (4)). For example, in K^0 decay

$$K^0 \to \pi^+ + \pi^- , \quad (11)$$

or in quark terms

$$\bar{s}d \to \bar{d}u + \bar{u}d , \quad (12)$$

we can see that $\Delta H = 0$ as well as $\Delta F = 0$ which leads to $\Delta S = -1$ and $\Delta I_3 = 1/2$.

Although lower probability than $\Delta S = \pm 1$, there exists the pure hadronic weak interaction with $\Delta S = \pm 2$. For example, in the decay

$$\Xi^- \to n + \pi^- , \quad (13)$$

or in quark terms

$$ssd \to ddu + \bar{u}d , \quad (14)$$

the selection $\Delta S = 2$ and $\Delta I_3 = -1$ can still be deduced from $\Delta H = 0$ (Equation (9)) as well as $\Delta F = 0$.

In consequence, in the pure hadronic weak interaction including u, d and s quarks, the selection rules can be deduced from conservation of H as well as conservation of F.

3.2. Pure Leptonic Weak Interaction

In the pure leptonic weak interaction, because $S = 0$ and $H = 0$, there is no selection rule, but H_L as well as F is conserved. For example, in muon decay

$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu , \quad (15)$$

$\Delta H_L = 0$ as well as $\Delta F = 0$ is satisfied.

3.3. Fermionic Weak Interaction

The fermionic weak interaction, in which both hadrons and leptons are involved, obeys two selection rules:

1) The first selection rule is

$$\Delta S = 0, \ \Delta I = \pm 1, \ \Delta I_3 = \pm 1 . \quad (16)$$

From Equation (4) and Equation (9),

$$\Delta F = \Delta H_L + 2\Delta I_3 . \quad (17)$$

For example, in β decay

$$n \to p + e^- + \bar{\nu}_e , \quad (18)$$

or in quark terms

$$\mathrm{ddu} \rightarrow \mathrm{uud} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}\,, \tag{19}$$

where s quark does not appear, we can see that $\Delta F = 0$ (Equation (17)) which leads to $\Delta I_3 = 1$ and $\Delta H_L = -2$.

Another example is that in Σ^+ decay

$$\Sigma^{+} \rightarrow \Lambda^{0} + \mathrm{e}^{+} + \nu_{\mathrm{e}}\,, \tag{20}$$

or in quark terms

$$\mathrm{suu} \rightarrow \mathrm{sud} + \mathrm{e}^{+} + \nu_{\mathrm{e}}\,, \tag{21}$$

where S is not changed as s quark so called "spectator" does not participate in the reaction, we can see that $\Delta F = 0$ (Equation (17)) which causes $\Delta I_3 = -1$ and $\Delta H_L = 2$.

2) The second selection rule is

$$\Delta S = \Delta Q_h = \pm 1,\ \ \Delta I = \pm 1/2,\ \ \Delta I_3 = \pm 1/2\,, \tag{22}$$

where ΔQ_h is change of electric charge of hadrons. From Equation (4) and Equation (9),

$$\Delta F = \Delta H_L + \Delta S + 2\Delta I_3\,. \tag{23}$$

For example, in decay

$$\Sigma^{-} \rightarrow \mathrm{n} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}\,, \tag{24}$$

or in quark terms

$$\mathrm{sdd} \rightarrow \mathrm{ddu} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}}\,, \tag{25}$$

$\Delta S = \Delta Q_h = 1$, $\Delta I_3 = 1/2$, and $\Delta H_L = -2$. The selection can be deduced from $\Delta F = 0$ (Equation (23)).

A reverse case is that a reaction with $\Delta S = -\Delta Q_h$ is consistent with Gell-Mann-Nishijima formula Equation (7), but has not been observed. For example,

$$\Sigma^{+} \rightarrow \mathrm{n} + \mathrm{e}^{+} + \bar{\nu}_{\mathrm{e}}\,, \tag{26}$$

or in quark terms

$$\mathrm{suu} \rightarrow \mathrm{ddu} + \mathrm{e}^{+} + \bar{\nu}_{\mathrm{e}}\,, \tag{27}$$

where $\Delta S = -\Delta Q_h = 1$ and $\Delta I_3 = -3/2$, but $\Delta H_L = 0$, so that $\Delta F = -2$. So, the reason why a reaction with selection $\Delta S = -\Delta Q_h$ has not been observed is clear: F is not conserved in the case.

In consequence, in the fermionic weak interaction including u, d and s quarks, the selection rules can be deduced from conservation of F.

3.4. Heavier Quarks

Relation between selection rules in weak interaction and F conservation can be extended to the other quarks including c, b and t quarks. For example, including c quark, from Equation (2), $H = U + D + S + C$ and then from Equation (1),

$$\Delta F = \Delta S + 2\Delta I_3 + \Delta C + \Delta H_L \tag{28}$$

D^+ decay is a fermionic weak interaction:

Table 3. Conserved additive quantum numbers and selection rules in different types of weak interaction. Hadrons composed by only u, d and s quarks are listed.

type of weak interaction	conserved additive quantum number	selection rule in weak interaction
pure leptonic	F, H_L	null
pure hadronic	F, H	1) $\Delta S = \mp 1$, $\Delta I = \pm 1/2$, $\Delta I_3 = \pm 1/2$.
		2) $\Delta S = \mp 2$, $\Delta I_3 = \pm 1$
fermionic	F	1) $\Delta S = \Delta Q_h = \pm 1$, $\Delta I = \pm 1/2$, $\Delta I_3 = \pm 1/2$
		2) $\Delta S = 0$, $\Delta I = \pm 1$, $\Delta I_3 = \pm 1$

$$D^+ \to \bar{K}^0 + e^+ + \nu_e \,, \tag{29}$$

or in quark terms

$$c\bar{d} \to \bar{d}s + e^+ + \nu_e \,, \tag{30}$$

we can see that $\Delta F = 0$ (conservation of F) leads to selection $\Delta S = -1$, $\Delta C = -1$, $\Delta I_3 = 0$ (Equation (28)). In consequence, conservation of F determines the selection rules of weak interaction including heavier quarks.

4. Conclusions

This Conservation of F determines the selection rules in all types of weak interaction. Especially, conservation of H, as the hadronic part of F, determines the selection rules in pure hadronic weak interaction, and conservation of H_L, as the leptonic part of F, determines the selection rules in pure leptonic weak interaction (Table 3). Compared with miscellaneous selection rules, conservation of F is more distinct and rather simpler for judgment on how fermions react in all types of weak interaction. Moreover, the reason why some selections, e.g., $\Delta S = -\Delta Q_h$ have not been observed can be explained by not conservation of F in the case, but cannot be explained by Gell-Mann-Nishijima formula Equation (7). Equation (5) is more general than Equation (7).

Conservation of F indicates that both hadrons and leptons must be considered together in weak interaction, but not separately. In fact, F due to its conservation gives a unified concept about both hadrons and leptons in weak interaction. If we only take Q, F_0 and F, description of weak interaction becomes simpler, only based on the conserved physical quantities. Conservation of F in weak interaction, strong interaction and electromagnetic interaction and a tight correlation among Q, F_0 and F should be utilized in quantum field theory.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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