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Highly Accurate Relations between the Fine Structure Constant and Particle Masses, with Application to Its Cosmological Measurement

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Abstract

Highly accurate algebraic relations between the fine structure constant α and a wide range of particle masses are given, ranging from $\Delta\alpha/\alpha = (2.1 \pm 0.1)\times 10^{-7}$ to $\Delta\alpha/\alpha = (-2.7 \pm 0.3 \pm 0.6)\times 10^{-8}$, and with a very large standard deviation, ranging to $\Delta\alpha/\alpha = -5.5\times 10^{-9}$. The analysis is based on empirical relations that exist among some particle masses, and also on several theoretical assumptions, of which the most significant is that the electromagnetic contribution to the electron's mass is finite, and given by $f\alpha m_{eb}$, where f is a dimensionless parameter that is shown to be equal to 1.032409810 (63), and where m_{eb} is the electron's "bare mass." The relations for α and f are homogeneous degree zero in the particle masses. The relations for f in terms of particle masses are found by trial and error. A quadratic equation is given relating α to f and m_e/m_p. This equation is used in the application to cosmological measurements of α, and $\mu \equiv m_p/m_e$, where it is shown that, to a few percent accuracy, $\delta\alpha/\alpha \approx -\delta\mu/\mu$. This relation can serve to test the validity of measurements of α and μ.

Keywords

Fine Structure Constant, Particle Masses, Proton-Electron Mass Ratio, Cosmological Measurement

1. Introduction

There is a long standing interest in the underlying basis of the dimensionless fine structure constant $\alpha \equiv e^2/\hbar c$ (Gaussian units) following its appearance in Sommerfeld's [1] special relativistic correction to the Bohr orbit model of the hydrogen atom that gave the fine-structure splitting of the energy levels in agreement with experiment. This result, followed by the quantum mechanically and relativistically correct derivation of the fine-structure splitting from the Dirac equation [2] [3] [4] led to numerous efforts to derive α from first principles, of which the attempt by Eddington, summarized in [5], in which the emphasis is on the value of $\alpha^{-1} \approx 137$, is the most well-known. Later considerations to investigate or comment on the significance of the fine structure constant include those of Born [6], Teller [7], Landau [8], Peebles and Dicke [9], Pauli [10], Wyler [11], Peres [12], Isham, *et al.* [13], MacGregor [14], Rozenthal [15], Barrow and Tipler [16], Good [17], and Várlaki *et al.* [18]. As yet another effort to study α, the purpose of this paper is to derive relations of very high accuracy between α and some of the masses of the elementary particles. This approach will be based on two empirical relations, and several theoretical assumptions that are eventually falsifiable. In Section 2, the two empirical observations, based on the work of Nambu [19], and the author [20], are shown to lead to an approximate empirical value for the fine structure constant, denoted by α_{emp} that depends on the ratio of the electron mass m_e to the charged pion mass $m_{\pi^\pm}$, and that results in an agreement given by $\left(\alpha_{emp} - \alpha\right)/\alpha = 3.4 \times 10^{-3}$. Although it is possible to proceed further working with the ratio $m_e/m_{\pi^\pm}$, because the value of the charged pion mass is not known sufficiently to deal with the very high accuracy that will be achieved below for the theoretical value of α denoted by α_{th}, and also because of the interest in the value of the ratio of the electron mass to the proton mass m_e/m_p (although usually expressed in terms of its reciprocal), as well as the fact that m_p is known to very high accuracy, subsequent work makes use of the ratio m_e/m_p. In Section 3, several theoretical assumptions are given that involve introducing the so-called "bare mass" of the electron, *i.e.* the mass of the electron that it would have in the absence of its interaction with the electromagnetic field. In addition, there are the assumptions that the electromagnetic self-energy of the electron is finite and small, and, in terms of mass, is given by $f\alpha m_e$, where f is a dimensionless parameter that, in the course of the analysis, turns out to be slightly greater than unity. These assumptions lead to an expression for the bare mass that is eventually falsifiable. The next assumption involves replacing m_e in the ratio m_e/m_p with the electron's bare mass. This yields a quadratic equation for α_{th} whose solutions are examined, first for $f = 1$, that leads to an agreement given by $\left(\alpha_{th} - \alpha\right)/\alpha = 2.3 \times 10^{-4}$. A further expression for f, that was obtained through trial-and-error, that involves the masses of a suitable number of elementary particles, and that maintains the homogeneous degree zero in the masses behavior of m_e/m_p, yields a substantially greater agreement of $\left(\alpha_{th} - \alpha\right)/\alpha = -2.1^{+0.6}_{-0.5} \times 10^{-8}$. With further trial-and-error in the choice of particle masses used in f, it is possible to improve the agreement for α_{th} still further, albeit with a larger standard deviation. Thus, with a suitable choice of masses in f, one obtained $\left(\alpha_{th} - \alpha\right)/\alpha = 1.4^{+1.6}_{-1.8} \times 10^{-8}$. The standard deviation is due to that in the particle masses used in f. However, the homogeneous degree zero in the masses behavior for f, and more generally that of α_{th}, is still maintained. In Section 4, since the values for the masses in f involved hadrons that only contained the up, down, and strange quarks, expression for f are extended to include the masses of mesons and baryons that contain the charm quark, and masses of mesons that contain the bottom quark. For the case of f containing the mass of the charmed $D^{\pm}$ meson, among the other masses involved, one obtained $(\alpha_{th}-\alpha)/\alpha = 3.3^{+0.6}_{-0.4}\times 10^{-8}$. While, for the case of f containing the mass of the charmed lambda baryon Λ_c^{+}, the result was $(\alpha_{th}-\alpha)/\alpha = -5.5^{+6.9}_{-8.2}\times 10^{-9}$. For f containing masses of the bottom B^0 and $B^{\pm}$ mesons $(\alpha_{th}-\alpha)/\alpha = (-2.7\pm 0.3\pm 0.6)\times 10^{-8}$. For the bottom baryon Λ_b^0, the best value was $(\alpha_{th}-\alpha)/\alpha = 1.8^{+0.5}_{-0.6}{}^{+0.5}_{-0.7}\times 10^{-8}$. In all these cases in which f is a function of the various particle masses, the standard deviations in the value of $(\alpha_{th}-\alpha)/\alpha$ are due to the standard deviations in the particle masses. In Section 5, the analysis is extended to include the gauge bosons W and Z, and the Higgs H^0 boson, with agreements comparable to those found above. In Section 6, there is an application to the cosmological measurement of α, where it is pointed out that, since α_{th} is homogeneous degree zero in the particle masses, any change in the particle masses of the form $m_i \to \phi(z, RA, \delta) m_i$ where z is redshift, RA is right ascension, and δ is declination, would leave α_{th} invariant, and hence α itself, to the level of agreement found in the previous sections. Hence, it is noted that if there were a change in α, some of the masses would have to change differently than the others; a requirement that would be even more difficult to reconcile with the standard model than the above change of the form, $m_i \to \phi m_i$, but it cannot be ruled out on the basis of present knowledge. Since, as pointed out above, that because of the accurate agreements involved, the expression for α_{th} can be regarded as holding for α, and since the above quadratic equation, that under these circumstances holds for α, involves the ratio m_e/m_p, and since the reciprocal of this ratio is defined as μ, it follows that a cosmological variation in α is not independent of a cosmological variation in μ. It is shown that $\delta\alpha/\alpha \approx -\delta\mu/\mu$, the departure from strict equality being of the order of a few percent. This relation provides the possibility of an important test of such measurements that presently does not exist, since α and μ are treated as independent, In Section 7, there are concluding remarks.

2. Empirical Relations

It is well-known that the mass of the muon is related to the mass of the electron in the form

$$m_\mu \approx \frac{3}{2}\alpha^{-1} m_e . \tag{1}$$

This relation leads the empirical list of particle masses given by Nambu [19] that are of the form, integer or half-integer times $\alpha^{-1} m_e$. It is also known empirically, and that also follows from Nambu's list that the mass of the charged pion satisfies

$$m_{\pi^\pm} \approx \frac{4}{3} m_\mu . \tag{2}$$

Such a relation was also found by the author [20] to hold for two classical electron models that were compensated differently, one with the Poincaré pressure, and the other with transverse stress, so that the energy and momentum of these two models transformed properly under a Lorentz transformation. The former was identified with the charged pion, and the latter with the muon. The relation (2) emerged later in the form $m_\mu = (3/4) m_{\pi^\pm}$ in the author's [21] empirical extension of Nambu's list [19] to include particle masses m_i that are approximately of the form, $m_i = (a_i + (b_i/4)) m_{\pi^\pm}$ where a_i are suitable integers, and $b_i = 0,1,2,3$; for example, the proton mass satisfies $m_p \approx 6\frac{3}{4} m_{\pi^\pm}$. However, for the purposes of this work (2) is to be regarded as a purely empirical relation. Upon inserting the value of m_μ from (1) into (2) and rewriting it as a relation for α, one has

$$\alpha \approx 2 m_e / m_{\pi^\pm} . \tag{3}$$

This purely empirical result yields an approximate value for α, denoted by α_{emp}, with the value $\alpha_{emp} = 0.007322$, for $m_e = 0.511$ MeV/c^2 and $m_{\pi^\pm} = 139.57$ MeV/c^2 [22]. More accurate values for these masses from [22] will be used below. Hence an agreement with $\alpha = 0.007297$ given by

$$(\alpha_{emp} - \alpha)/\alpha \approx 3.4 \times 10^{-3} , \tag{4}$$

A more accurate value for α will be used below. Although one can continue working with (3). It turns out that the standard deviation in the mass of the charged pion, as given by $m_{\pi^\pm} = 139.57039 \pm 0.00018$ MeV/c^2, yields too great a standard deviation in the theoretical values for α in some of the examples that will be found below, consequently, it turns out to be desirable to replace (3) with the ratio of the electron mass to the proton mass. From Nambu's second hint empirical mass list in [19], the proton mass obeys the relation.

$m_p \approx 13.5 \alpha^{-1} m_e$, and since the list also gives $m_{\pi^\pm} \approx 2\alpha^{-1} m_e$, one has $m_p \approx 6\frac{3}{4} m_{\pi^\pm}$, as does the neutron mass, and, as was found later, the tauon mass satisfies $m_\tau \approx 12\frac{3}{4} m_{\pi^\pm} = m_n + 6 m_{\pi^\pm}$. Hence, substituting for $m_{\pi^\pm}$ with m_p in (3), one has

$$\alpha_{emp} \approx 13.5 m_e / m_p . \tag{5}$$

With $m_p = 938.272$ MeV/c^2, this yields the value $\alpha_{emp} \approx 0.007352$, and hence an agreement of

$$(\alpha_{emp} - \alpha)/\alpha \approx 7.5 \times 10^{-3} \tag{6}$$

Although this agreement is clearly poorer than in (4), it will turn out from the theoretical work in the next section that considerably more exact agreements will be obtained.

3. Theoretical Assumptions and Improved Agreements

The above expressions for α are of a purely empirical nature: no theoretical assumptions went into obtaining them, However, in the following analysis much more accurate expressions relating α to particle masses will be obtained, that are based on several theoretical assumptions that are all eventually falsifiable. The first assumption is that instead of m_e in the numerators of (3) and (5), one should have the bare mass of the electron m_{eb}, that is defined by

$$m_{eb} = m_e - \Delta m_e , \tag{7}$$

where Δm_e is the addition to the bare mass of the electron due to its interaction with the quantized electromagnetic field. According to current ideas, the bare mass would arise as a consequence of the electron's interaction with the expectation value of the Higgs field [23], although there are no generally accepted results in the literature, and in any case, it is irrelevant to this work as to just how the bare mass arises. As is well-known, Δm_e diverges logarithmically in a first order perturbation expansion, as obtained by Weisskopf [24] using hole-theory, in which he acknowledged it to have been found by Furry as a correction to his previous work, Weisskopf [25]. In contrast, in QED, the logarithmic divergence follows from the one-loop Feynman diagram [26], and is removed by renormalization [27]-[33], that regrettably fails to give a value for Δm_e, since all the final answers are in terms of m_e, and hence the electromagnetic self-mass never appears. Here, in contrast, it will be assumed, secondly, that the self-mass is finite, and thirdly, that it is given by

$$\Delta m_e = f \alpha m_{eb} , \tag{8}$$

where the factor f is of order unity, and will be determined below. A non-rigorous justification for assuming that Δm_e is a small contribution to the mass of the electron can be made by appeal to the quark model. The u and d quarks have masses comparable to that of the electron, but since the electric charge of the u quark is $2e/3$ while that of the d quark is $-e/3$, if the electromagnetic contribution to the masses of these quarks were dominant, then since it would behave as the square of the charges, one would expect $m_u/m_d \approx 4$, but instead it has been found that $m_u/m_d \approx 0.5$ [22]. Further justification follows from Feynman's [34] remark that he suspected, "renormalization is not mathematically legitimate." A possible reason for such a suspicion is that the subtraction of one infinity from another infinity is mathematically ambiguous: the difference need not vanish, it could be a finite quantity such as (8). However, as emphasized above, the basic justification for using (8) is the highly accurate results that are obtained below using it. Upon inserting the value of Δm_e from (8) in (7), and solving for m_{eb}, one obtains

$$m_{eb} = \frac{m_e}{1 + f\alpha} , \tag{9}$$

from which, using (7) one has that $\Delta m_{eb} = f \alpha m_e / (1 + f\alpha)$. Upon replacing m_e in (5) with the value of m_{eb} from (9) one obtains the following quadratic equation

$$\alpha^2 + \frac{\alpha}{f} - \frac{13.5}{f}\frac{m_e}{m_p} = 0. \tag{10}$$

The physically desired solution is the positive root that will be denoted by α_{th}, and is given by

$$\alpha_{th} = \left(\sqrt{1+\left(f\,54\,m_e/m_p\right)} - 1\right)\Big/2f\ . \tag{11}$$

The physical significance of the negative root, if any, is left unresolved at this writing.

As a first approximation for the value of α_{th}, it will be assumed that $f = 1$. With more exact values of the masses from [22], $m_e = 0.51099895000(15)$ MeV/c^2 and $m_p = 938.27208816(29)$ MeV/c^2, one obtains, rounding off, $\alpha_{th} = 0.007299054$, so that in comparison with the present full value of α that will be needed further on, *i.e.* $\alpha = 0.0072973525693(11)$, [22], one has

$$\left(\alpha_{th} - \alpha\right)/\alpha = 2.3\times10^{-4}\,. \tag{12}$$

It is clear that there is already an improvement in accuracy over that given in (4) and (6) by more than an order of magnitude. Equation (10) can alternatively be used to determine m_p/m_e in terms of α and *f*. For $f = 1$, $m_p/m_e = 1836.58$ in comparison with its empirical value of 1836.15, hence with the same accuracy as above, 2.3×10^{-4}. Of interest is the fact that the value for α_{th} given in (11), with $f = 1$, is homogenous, degree zero in the masses, so that $\alpha_{th}\left(\varphi m_e, \varphi m_p\right) = \alpha_{th}\left(m_e, m_p\right)$, and hence

$$\frac{\partial\alpha_{th}}{\partial m_e}m_e + \frac{\partial\alpha_{th}}{\partial m_p}m_p = 0\,. \tag{13}$$

To proceed further, it will be necessary to determine the value of *f* more accurately. It will be assumed that *f* is a function of particle masses, and that it maintains the homogeneous degree zero character of α in (5). In order to assist in the determination of *f*, which will be by trial and error, it is helpful to know the value of *f* that would lead to the current value of the fine structure constant that is given above. One has from (10)

$$f = -\frac{1}{\alpha} + \frac{13.5}{\alpha^2}\frac{m_e}{m_p}, \tag{14}$$

from which one obtains

$$f = 1.032409810(63), \tag{15}$$

where the standard deviation in *f* is due mainly to the standard deviations in the masses of the proton and the electron, and less to that in α. Note also that (14) provides an upper bound on α since on empirical grounds $\alpha > 0$, and on physical grounds $f > 0$, hence one has $\alpha < 13.5\, m_e/m_p$, thus, rounding off, $\alpha < 0.00735233 = 1/136.0113$. In what follows, approximations to *f* will be made that involve a suitable choice of particle masses. As the first example, guided by the fact that m_e/m_p involves a lepton mass divided by a hadron mass, it seemed reasonable to try a ratio involving the muon mass divided by a suitable sum of hadron masses. The following expression for *f* is the result of such a trial and error search

$$f = 1 + \frac{m_\mu}{\Sigma_i m_i} = 1 + \frac{m_\mu}{m_{\Xi^-} + m_n + m_{K^0} + m_{K^\pm}}. \tag{16}$$

From [22], with $m_\mu = 105.6583745(24)$ MeV/c², $m_{\Xi^-} = 1321.71 \pm 0.07$ MeV/c², $m_n = 939.56542052(54)$ MeV/c², $m_{K^0} = 497.611 \pm 0.013$ MeV/c², and $m_{K^\pm} = 493.677 \pm 0.016$ MeV/c², one has

$$\Sigma_i m_i = 3252.563 \pm 0.073\,\mathrm{MeV/c^2}. \tag{17}$$

Hence, after obtaining *f* = 1.03248465 (72), and using (11), one has $\alpha_{th} = 0.00729734864(4)$, and hence an agreement given by

$$(\alpha_{th} - \alpha)/\alpha = (-5.4 \pm 0.1) \times 10^{-7}. \tag{18}$$

Thus, by the above assumption about *f*, one has achieved nearly a thousand-fold improvement over the agreement in (12). One can make a modest improvement to the above agreement in (18) by introducing in the denominator in (16) a different set of masses, consisting of the mass of the τ lepton, that of the proton, and that of the η meson, so that one has

$$f = 1 + \frac{m_\mu}{\Sigma_i m_i} = 1 + \frac{m_\mu}{m_\tau + m_p + m_\eta}. \tag{19}$$

From which, with $m_\tau = 1776.86 \pm 0.12$ MeV/c², $m_\eta = 547.862 \pm 0.017$ MeV/c², and m_p given above, one has

$$\Sigma_i m_i = 3262.994 \pm 0.121\,\mathrm{MeV/c^2}. \tag{20}$$

After obtaining $f = 1.0323808(12)$, and again using (11), one has $\alpha_{th} = 0.00729735409(06)$, and hence an agreement given by

$$(\alpha_{th} - \alpha)/\alpha = (2.1 \pm 0.1) \times 10^{-7}. \tag{21}$$

With a somewhat different choice of masses in the denominator of (16), it is possible to improve the agreement by an order of magnitude over that in (21). Thus with *f* given by

$$f = 1 + \frac{m_\mu}{\Sigma_i m_i} = 1 + \frac{m_\mu}{m_{\Sigma^+} + m_n + m_{K^0} + m_{K^\pm} + m_{\pi^\pm}}, \tag{22}$$

with $m_{\Sigma^+} = 1189.370 \pm 0.07$ MeV/c², $m_{\pi^\pm} = 139.57039$ MeV/c², and the other masses as given above, one has

$$\Sigma_i m_i = 3259.793 \pm 0.073\,\mathrm{MeV/c^2}. \tag{23}$$

After obtaining $f = 1.03241260(72)$, and using (11), one has that $\alpha_{th} = 0.00729735242(4)$, hence an improved agreement given by

$$(\alpha_{th} - \alpha)/\alpha = -2.1^{+0.6}_{-0.5} \times 10^{-8}, \tag{24}$$

where the standard deviation is due primarily to that in the mass of the Σ^+, while the standard deviation in (18) is due primarily to that in the mass of the

Ξ^-.

It is possible to obtain comparable agreement for α_{th}, albeit with a larger standard deviation, if in (16) one replaces m_μ in the numerator by $m_{\pi^\pm}$, and introduces another set of masses, that are found, as before, by trial and error, one has

$$f = 1 + \frac{m_{\pi^\pm}}{\Sigma_i m_i} = 1 + \frac{m_{\pi^\pm}}{m_{\Xi^0} + m_{\Sigma^0} + m_\Lambda + m_\eta + m_{\pi^0}}. \quad (25)$$

With $m_{\Xi^0} = 1314.86 \pm 0.2$ MeV/c², $m_{\Sigma^0} = 1192.642 \pm 0.024$ MeV/c², $m_\Lambda = 1115.683 \pm 0.006$ MeV/c², $m_{\pi^0} = 134.9768 \pm 0.0005$ MeV/c², and with the value of m_η as given above, one has

$$\Sigma_i m_i = 4306.024 \pm 0.202 \text{ MeV/c}^2. \quad (26)$$

Since the values of *f*, when the uncertainties in $\Sigma_i m_i$ are taken into account, are significantly different, they are given separately; for $\Sigma_i m_i = 4306.024$, f = 1.03241282, for $\Sigma_i m_i = 4306.206$, f = 1.03241275, and for $\Sigma_i m_i = 4305.822$, f = 1.03241434, from which $\alpha_{th} = 0.00729735241$, $\alpha_{th} = 0.00729735248$, and $\alpha_{th} = 0.00729735233$, respectively. Hence, one has

$$(\alpha_{th} - \alpha)/\alpha = -2.2^{+1.0}_{-1.1} \times 10^{-8}. \quad (27)$$

In the above analysis, the standard deviation in $m_{\pi^\pm}$ has been ignored. When this is taken into account, and that in $\Sigma_i m_i = 4306.024 \pm 0.202$ MeV/c² is ignored, one finds $f = 1.03241282(04)$, and with this small standard deviation in *f*, the value for α_{th} remains the same as above.

One need not replace m_μ in the numerator in *f* with $m_{\pi^\pm}$, one can instead replace it with, say m_{π^0}, and to be sure, a different set of masses in the denominator, as the following example shows. One has

$$f = 1 + \frac{m_{\pi^0}}{\Sigma_i m_i} = 1 + \frac{m_{\pi^0}}{m_{\Omega^-} + m_{\Sigma^-} + m_{\Sigma^+} + m_\mu}. \quad (28)$$

With $m_{\Omega^-} = 1672.45 \pm 0.29$ MeV/c², $m_{\Sigma^-} = 1197.449 \pm 0.03$ MeV/c², and the other two masses as given previously, one obtains

$$\Sigma_i m_i = 4164.927 \pm 0.30 \text{ MeV/c}^2. \quad (29)$$

Once again the values of *f* are sufficiently different, that they will be presented separately, for the different values of $\Sigma_i m_i$ associated with its standard deviation. For $\Sigma_i m_i = 4164.927$ MeV/c², one has $f = 1.032407963$, for $\Sigma_i m_i = 4165.227$ MeV/c², $f = 1.0324056288$, and for $\Sigma_i m_i = 4164.627$, $f = 1.0324102975$, from which one has $\alpha_{th} = 0.00729735267$, $\alpha_{th} = 0.00729735279$, and $\alpha_{th} = 0.00729735254$, respectively. Hence one has

$$(\alpha_{th} - \alpha)/\alpha = 1.4^{+1.6}_{-1.8} \times 10^{-8}. \quad (30)$$

The above analysis does not include the standard deviation in m_{π^0}. However, it turned out to be negligible, when compared with that for $\Sigma_i m_i$ in (29). Since the hadron masses used above only involved the *u, d,* and *s* quarks, in the next section expressions for α_{th} will be given that involve the charmed *c*, and bottom *b* quarks. In what follows in the next section, one will work with $m_{\pi^\pm}$ in the numerator of the expression for *f*, except for the case involving the bottom particles.

4. Relations for α_{th} That Include Particles Containing Either a Charmed or Bottom Quark

Following the example of the previous section, but in which one now introduces the charmed $D^\pm$ meson, one has

$$f = 1 + \frac{m_{\pi^\pm}}{\Sigma_i m_i} = 1 + \frac{m_{\pi^+}}{m_{D^\pm} + m_{\Xi^-} + m_\Lambda}. \quad (31)$$

From which, with $m_{D^\pm} = 1869.65 \pm 0.05$ MeV/c^2, with m_{Ξ^-} and m_Λ as given previously, one has

$$\Sigma_i m_i = 4307.043 \pm 0.086 \text{ MeV}/c^2. \quad (32)$$

Again, since the values of *f*, when the standard deviation in $\Sigma_i m_i$ is taken into account, are significantly different, they are given separately: for $\Sigma_i m_i = 4307.043$, $f = 1.03240515$, for $\Sigma_i m_i = 4307.139$, $f = 1.03240443$, and for $\Sigma_i m_i = 4306.957$, $f = 1.03240580$, so that one has, $\alpha_{th} = 0.00729735281$, $\alpha_{th} = 0.00729735285$, and $\alpha_{th} = 0.00729735278$, respectively. Hence, since the contribution of the standard deviation thin $m_{\pi^\pm}$ is negligible, one has

$$(\alpha_{th} - \alpha)/\alpha = 3.3^{+0.6}_{-0.4} \times 10^{-8}. \quad (33)$$

As yet another example involving a charmed particle, the charmed baryon Λ_c^+ will be introduced. One has

$$f = 1 + \frac{m_{\pi^+}}{\Sigma_i m_i} = 1 + \frac{m_{\pi^\pm}}{m_{\Lambda_c^+} + m_{\Sigma^-} + m_\eta + m_{\pi^\pm} + m_{\pi^0}}, \quad (34)$$

from which, with $m_{\Lambda_c^+} = 2286.46 \pm 0.14$ MeV/c^2, $m_{\Sigma^-} = 1197.449 \pm 0.0030$ MeV/c^2, and the other masses that have been given previously, one has

$$\Sigma_i m_i = 4306.318 \pm 0.144 \text{ MeV}/c^2. \quad (35)$$

From which $f = 1.03241061(108)$, for the mean one has $\alpha_{th} = 0.00729735253$, and for $\Sigma_i m_i = 4306.462$ MeV/c^2, $\alpha_{th} = 0.00729735258$, and for $\Sigma_i m_i = 4306.174$ MeV/c^2, $\alpha_{th} = 0.00729735247$, so that finally one has

$$(\alpha_{th} - \alpha)/\alpha = -5.5^{+6.9}_{-8.2} \times 10^{-9}. \quad (36)$$

The contribution from the standard deviation of $m_{\pi^\pm}$ is $\pm 0.3 \times 10^{-9}$, and hence negligible.

For the case of a particle containing the b quark, one will work here with the masses of the $B^\pm$ and B^0 mesons, and then with the bottom baryon Λ_b^0. Because of the large mass involved in the denominator of $f - 1$, one has to choose a larger mass in the numerator than $m_{\pi^\pm}$, and m_{K^0} will be chosen., This choice in turn leads to an increased number of masses in the denominator, which will also include the mass of a particle containing the charm quark, and since f will also include the masses of particles containing the three lighter quarks, all the quarks will be represented in the masses in f, except that of the top quark. One has

$$f = 1 + \frac{m_{K^0}}{\Sigma_i m_i} = 1 + \frac{m_{K^0}}{m_{B^0} + m_{B^\pm} + m_{D^\pm} + m_n + m_p + m_\eta + m_{K^0}}. \quad (37)$$

From which, with $m_{B^0} = 5279.65 \pm 0.12$ MeV/c^2, $m_{B^\pm} = 5279.34 \pm 0.12$ MeV/c^2, and the masses of the other particles in the sum given previously, one has

$$15351.95 \pm 0.18 \text{ MeV}/\text{c}^2 . \quad (38)$$

Just taking into account the standard deviation in the above sum, one has $f = 1.03241354(38)$, and hence $\alpha_{th} = 0.00729735237(2)$. Next. just taking the standard deviation into account in m_{K^0} in the numerator, since its contribution to the standard deviation in $\Sigma_i m_i$ is negligible, $f = 1.03241354(85)$, and $\alpha_{th} = 0.00729735237(4)$. One therefore has

$$(\alpha_{th} - \alpha)/\alpha = (-2.7 \pm 0.3 \pm 0.6) \times 10^{-8}, \quad (39)$$

where the first standard deviation in (39) is due to that in $\Sigma_i m_i$, and the second is due to the that of m_{K^0} in the numerator.

Next, one will take up the case when there is the mass of the bottom baryon Λ_b^0 in the denominator of $f - 1$. One has

$$\begin{aligned} f &= 1 + \frac{m_{K^0}}{\Sigma_i m_i} \\ &= 1 + \frac{m_{K^0}}{m_{\Lambda_b^0} + m_\tau + m_{\Xi^-} + m_{\Xi^0} + m_{\Sigma^-} + m_\Lambda + m_n + m_p + m_{K^0} + m_{K^\pm} + m_{\pi^\pm}} \end{aligned} \quad (40)$$

From which, with $\Lambda_b^0 = 5619.60 \pm 0.17$ MeV/c^2, and other masses as given previously, one has

$$\Sigma_i m_i = 15354.853 \pm 0.30 \text{ MeV}/\text{c}^2 . \quad (41)$$

Again, just taking into account the standard deviation in the sum, one finds $f = 1.03240741(63)$. From which one obtains for the mean $\alpha_{th} = 0.00729735270$, for the plus standard deviation, $\alpha_{th} = 0.00729735274$, and for the minus, $\alpha_{th} = 0.00729735266$. The standard deviation from that in m_{K^0} in the numerator is not the same at that above, hence one has

$$(\alpha_{th} - \alpha)/\alpha = 1.8^{+0.5}_{-0.6}{}^{+0.5}_{-0.7} \times 10^{-8} \quad (42)$$

where, as above, the first deviations are due to the standard deviation in the mass sum, and the second is due to that in m_{K^0} in the numerator. For brevity values of f and α_{th} are omitted.

In all the cases considered above, the denominator in $f - 1$ has only involved a sum of masses, based on the possibility that in a future theory, the denominators would arise as a consequence of a quantum theoretical mass sum rule. However, in the present absence of such a theory, there is no obvious objection to having some of masses appear with a minus sign, so that the sum $\Sigma_i m_i$ gets replaced with $\Sigma_i (\pm)_i m_i$. For brevity only one case will be considered involving the baryon Λ_b^0. One has

$$f = 1 + \frac{m_{\pi^\pm}}{\Sigma_i (\pm)_i m_i} = 1 + \frac{m_{\pi^\pm}}{m_{\Lambda_b^0} - m_{\Omega^-} + m_{K^\pm} - m_{\pi^0}}. \quad (43)$$

From which, with $m_{\Lambda_b^0} = 5619.60 \pm 0.17$ MeV/c^2, and the masses of the other particles that have been given previously, one has

$$\Sigma_i (\pm)_i m_i = 4305.85 \pm 0.34 \text{ MeV/c}^2 . \quad (44)$$

Since the values of f and α_{th} are sufficiently different when the standard deviation is taken into account, they will be presented separately. However, the change in α_{th} due to the standard deviation in $m_{\pi^\pm}$ in the numerator is negligible. For the mean, one has f = 1.03241413, for the plus standard deviation, f = 1.03241157, and for the negative, f = 1.03241669, so that one has $\alpha_{th} = 0.00729735234$, $\alpha_{th} = 0.00729735247$, and $\alpha_{th} = 0.00729735221$, respectively. Hence, one has

$$(\alpha_{th} - \alpha)/\alpha = -3.2^{+1.8}_{-1.7} \times 10^{-8}. \quad (45)$$

Another issue is whether one could add or subtract the mass of the electron to the denominators to possible improve the agreement. For example, in the above case, if one adds m_e to the denominator, one gets f = 1.03241029, and hence $\alpha_{th} = 0.007297352544$, and hence an agreement $(\alpha_{th} - \alpha)/\alpha = -3.4 \times 10^{-9}$. But the standard deviation in the result is an order of magnitude greater, and makes the result questionable. But again, in absence of a proper theory, one cannot rule out such a contribution to the denominator.

Although the most accurate agreement for α_{th} up to now has been of order 10^{-9}, albeit with a large standard deviation, and most agreements are of order 10^{-8}, one could actually improve agreements by one to two orders of magnitude were it not for the large standard deviations in many of the particle masses. Therefore, it seems reasonable that with further improvement in particle mass measurements, it will be possible to fit the value of α given in [22] to within its experimental standard deviation of $\pm 1.5 \times 10^{-10}$. Interestingly, as this work was in the process of being completed, Morel *et al.* [35] presented an even more precise value of α, with a standard deviation of $\pm 0.81 \times 10^{-10}$. Since this is only about a factor of two increase in accuracy over α in [22], if with improved particle mass measurements one will be able to fit α in [22], one should also be able to fit this new value in [35]. In any case, the majority of the agreements found here, of the order of several parts in one hundred million, are more than sufficient to deal with the application to the cosmological measurement of α in section 6. However, for completeness, before going on to this application, it will be shown in the next section that the above analysis applies to the gauge W and Z bosons, and the Higgs H^0 boson.

5. Relations for α_{th} with Gauge W and Z, and Higgs H^0 Bosons

With the numerator of $f-1$ given by the bottom meson, B_1^0, and for brevity, rather than dealing with them individually, just the sum of the masses of W and Z, will be utilized in the denominator, along with earlier used particle masses. One has

$$f = 1 + \frac{m_{B_1^0}}{\Sigma_i m_i} = 1 + \frac{m_{B_1^0}}{m_Z + m_W + m_{\Omega^-} + m_n + m_p + m_{K^0} + m_{K^\pm} + m_\eta}. \tag{46}$$

From which, with $m_z = 91.1876 \pm 0.0021$ GeV/c^2 and $m_W = 80.379 \pm 0.012$ GeV/c^2, and with the other masses given previously, one has

$$\Sigma_i m_i = 176.656 \pm 0.012\ \mathrm{GeV/c^2}. \tag{47}$$

With $m_{B_1^0} = 5.7261 \pm 0.0013$ GeV/c^2, one obtains for the mean, $f = 1.03241384$, for the plus standard deviation, $f = 1.03241164$, and for the negative, $f = 1.03241605$, and hence $\alpha_{th} = 0.00729735236$, $\alpha_{th} = 0.00729735247$, and $\alpha_{th} = 0.00729735224$, respectively, so that one has

$$(\alpha_{th} - \alpha)/\alpha = -2.9^{+1.5}_{-1.6} \times 10^{-8}. \tag{48}$$

The standard deviations in the masses of the Z and B_1^0 were negligible compared to that of the W. For the case of the Higgs H^0 boson, the numerator of $f-1$ will be taken to be the mass of the χ_{c1} $c\bar{c}$ meson, so that one has

$$f = 1 + \frac{m_{\chi_{c1}}}{\Sigma_i m_i} = 1 + \frac{m_{\chi_{c1}}}{m_{H^0} + m_{D^0} + m_{K^0} + m_{K^\pm}}. \tag{49}$$

From which, with $m_{H^0} = 125.1 \pm 0.14$ GeV/c^2, $m_{D^0} = 1864.83 \pm 0.05$ MeV/c^2, and $m_{\chi_{c1}} = 4146.8 \pm 2.4$ MeV/c^2, and the values of the other masses given previously, one has

$$\Sigma_i m_i = 127.956 \pm 0.14\ \mathrm{GeV/c^2}. \tag{50}$$

From which one obtains for the mean, $f = 1.03240802$, for the plus standard deviation in (50) (that in $m_{\chi_{c1}}$ will be evaluated separately) one has $f = 1.03240447$, and for the minus, $f = 1.03241156$, and hence, $\alpha_{th} = 0.00729735266$, $\alpha_{th} = 0.00729735285$, and $\alpha_{th} = 0.00729735248$, respectively. Next, the same evaluations will be made for the standard deviation in $m_{\chi_{c1}}$. For the plus, one has, $f = 1.03242677$, and for the minus, $f = 1.03238926$, and hence, one has $\alpha_{th} = 0.00729735168$, and $\alpha_{th} = 0.00729735365$, respectively. From the above one arrives at the following agreement

$$(\alpha_{th} - \alpha)/\alpha = 1.2^{+2.6}_{-2.4}{}^{+14}_{-11} \times 10^{-8}, \tag{51}$$

where the first standard deviation is due to that in the mass of the H^0, and the second, to that in the mass of the χ_{c1}.

6. Application to the Cosmological Measurement of α

Over the years there has been an interest in whether the fundamental constants, such as the fine structure constant, vary with time, so that α would become $\alpha(z)$, where z is the redshift back to an earlier epoch. More generally, there is the question as to whether α would vary spatially as well, because of possible spatial anisotropy, so that one would have $\alpha = \alpha(z, RA, \delta)$, where RA is right ascension, and δ is declination. The following is a reduced list of references to this much researched subject: [7] [36]-[56]. For a detailed discussion, see, e.g., [16] and [50]. The present work has shown the dependency of α_{th} on particle masses has an accuracy given by $(\alpha_{th} - \alpha)/\alpha$ in the range 10^{-7} to 10^{-9}, and since current astronomical determinations α are of the order 10^{-5}, e.g., in [54], Wilczynska, *et al.* give $(\alpha_z - \alpha)/\alpha = (-2.18 \pm 7.27) \times 10^{-5}$, while in [46], Reinhold *et al.* give for a weighted fit, $\Delta\mu/\mu = (2.4 \pm 0.6) \times 10^{-5}$, it follows that statements here, based on the properties of α_{th}, can be taken to hold for the properties of α as they exist physically, and are determined astronomically. Thus, in what follows, the subscript "th" on α can be dropped, and one can assume the properties of α are the same as those of α_{th}. This leads to two interesting consequences. First, since α is homogeneous degree zero in the particle masses, since $\alpha = \alpha(f, \mu)$, and both f and μ are homogeneous degree zero in the particle masses of which they are functions, it follows that any change of the particle masses m_i of the form $m_i \to \phi(z, ra, d) m_i$, necessarily leaves α invariant, so that

$$\alpha(\phi(z, ra, d) m_i) = \alpha(m_i), \tag{52}$$

which entails that

$$\Sigma_i \frac{\partial \alpha}{\partial m_i} m_i = 0. \tag{53}$$

Thus, for there to be a change in α, not all the masses of which α is a function can change in the same way. Consequently, there would have to be one or more changes in particle mass ratios, of which the simplest would be a change in the ratio that is the most accurately determined astronomically, *i.e.*, $\mu \equiv m_p/m_e$. Needless to say, such changes are not expected according to the standard model, since it is based on special relativity for which there is invariance under time and space translations.

To determine how a small change in α is related to a small change in μ it is convenient to rewrite (11) as

$$\alpha = \left(\sqrt{1 + f 54\mu^{-1}} - 1\right) \Big/ 2f, \tag{54}$$

and expand, just keeping the first two terms, which yields

$$\alpha = 13.5\mu^{-1} - f(182.25)\mu^{-2} + \cdots, \tag{55}$$

from which, upon varying α, one has

$$\delta\alpha = -13.5\mu^{-2}\delta\mu + f(364.5)\mu^{-3}\delta\mu - 182.25\mu^{-2}\delta f. \tag{56}$$

Since one is going to divide $\delta\alpha$ by α, it is helpful to determine the relative magnitude of the terms in these two equations, so as to see what can be neglected. With $\mu^{-1} = 5.446\times10^{-4}$ and $f = 1.03241$, (55) becomes

$$\alpha = 7.35\times10^{-3} - 5.58\times10^{-5} . \tag{57}$$

Since the second term is less than one percent of the first term, and since the measurements are not of this accuracy, it will be neglected in what follows. Therefore upon dividing $\delta\alpha$ in (56) by just the first term in (55), one has

$$\frac{\delta\alpha}{\alpha} = -\frac{\delta\mu}{\mu} + f\,27\mu^{-1}\frac{\delta\mu}{\mu} - 13.5\mu^{-1}\delta f . \tag{58}$$

Inserting the values for μ^{-1} and f into (58), it takes the form

$$\frac{\delta\alpha}{\alpha} = -\frac{\delta\mu}{\mu} + 0.015\frac{\delta\mu}{\mu} - 7.4\times10^{-3}\delta f . \tag{59}$$

Since one has already neglected two terms of order one percent, the second term on the right hand side can be neglected. Also, since $\delta f = (\delta f/f)f$, and since $f \approx 1$, $\delta f \approx \delta f/f$. Under the assumption that $\delta f/f$ is no greater in magnitude than $\delta\mu/\mu$ which according to the latest measurements, if it exists at all, would be of the order 10^{-5}, so that this third term is less than one percent of the first term, and hence can also be neglected. Thus, to within a few percent, one has

$$\frac{\delta\alpha}{\alpha} \approx -\frac{\delta\mu}{\mu} . \tag{60}$$

This relation should prove helpful in ruling out false determinations, such as the ones that have been made in the past: since if there is a report of, say, a decrease in α of a certain magnitude for a given cosmological location, then, according to (60), there should be a simultaneous report of an increase in μ of very nearly the same magnitude at the same location, and vice versa.

7. Concluding Remarks

This work shows that it is possible to fit the empirical value of the fine structure constant to several parts per 10^9, albeit with a large standard deviation, and very likely fit it to its current determination, by employing more accurately measured particle masses. These relations, that are homogeneous degree zero in the particle masses, were found partly by empirical considerations, partly by trial and error, and partly by several theoretical assumptions about the mass of the electron that lie outside the current realm of QED. Most importantly, it was assumed that the contribution to the mass of the electron from the electromagnetic field is finite, and of the form: $\Delta m_e = f\alpha m_{eb}$, where m_{eb} is the so-called "bare mass" of the electron, that is defined in Equation (7), and where f is a new parameter that is a homogeneous degree zero function of particle masses that are chosen by trial and error. It is shown to have the value $f = 1.032409810(63)$, where the standard deviation in f is due mainly to the standard deviation in the masses of the proton and electron that are involved in the determination of f according to Equation (14). With Δm_e as above, the bare mass of the electron according to Equation (9) is given by $m_e/(1+f\alpha)$, and this value for m_{eb} should be falsifiable when a generally-accepted determination of the contribution to the mass of the electron from its interaction with the Higgs field becomes available. What would help in this investigation would be the experimental determination of Δm_e. However, a glance at the literature, e.g. [31], shows that although there have been substantial efforts over many years to deal with the logarithmic divergence of the electron's self-energy, of which renormalization is the prime example, little attention has been paid to the issue to how to measure Δm_e, or whether in principle it is measurable at all. Thus it is hoped that this work, as well as an earlier comment by the author [57] will serve to direct attention to this long neglected area.

As indicated in the previous section, this work has significant bearing on the cosmological determination of whether α depends on time and space. It was pointed out that since some of the expressions given here for α_{th}, with improved particle mass determinations, most likely can finally arrive at $\alpha_{th}-\alpha=0$, to within the empirical uncertainty of α itself, then, because the expression for α would be homogeneous degree zero in the particle masses determining it, any variation of the particle masses of the form $m_i \to \phi(z, RA, \delta) m_i$ would necessarily leave α unchanged, so that at least one mass would have to change differently than the other masses in the relation for α. Although this seems highly unlikely on the basis of the standard model, based as it is on special relativity, nevertheless, since the model does not predict the masses of the particles, such behavior cannot be ruled out, and therefore continuing efforts to look for possible cosmological changes in α are fully justified. With regard to such investigations, it was shown that because of the relation between α and μ, as given in Equation (54), such possible changes in α would be accompanied by changes in μ, which would satisfy the relation, $\delta\alpha/\alpha = -\delta\mu/\mu$, to within a few percent. Consequently, when possible changes in α and μ are reported, the determination of whether they satisfied this approximate relation would provide a critical test as to whether the reported changes were true, or whether they were due to some previously unrecognized source of error.

Finally, it is a prediction of this work that a new approach to QED is possible, in which the electron's self-energy is finite, and the value of α emerges from the theory, rather than being inserted from empirical measurement. In view of the relation of α to the particle masses found here, such a theory might shed new light on the particle mass spectrum as well.

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Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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