Journal of Modern Physics, 2022, 13, 368-384
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Qubit or Qudet: Projections onto Reality

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How to cite this paper: Klingman, E.E. (2022) Qubit or Qudet: Projections onto Reality. *Journal of Modern Physics*, **13**, 368-384.
https://doi.org/10.4236/jmp.2022.134026

Received: March 4, 2022
Accepted: April 8, 2022
Published: April 11, 2022

Abstract

2022 is the centennial of an event which many consider to be a basis from which quantum mechanics can be derived—the Stern-Gerlach experiment of 1922—despite that the meaning of quantum theory is today an open question. Key is "the measurement problem", the need to measure quantum phenomena with classical equipment while the boundary separating quantum from classical is unknown. The mechanism of the SG-experiment is analyzed, and the *Qubit* nature normally projected onto the data is traced to *quantization of the detector*, labelled a *Qudet*. This novel interpretation should have downstream consequences, such as the SG-based interpretation of Bell's Theorem.

Keywords

Qubit, Qudet, Stern-Gerlach, Inhomogeneous Field, Measurement Problem, Ontology, Quantum Mechanics

1. Introduction

The experiment performed by Stern and Gerlach in 1922 can be used to derive the *theory of quantum mechanics* [1] [2], but the experiment was performed a year before de Broglie proposed his particle-wave model, $p = \hbar/\lambda$, that is the basis of quantum theory. Therefore, it was *not* understood in the way quantum theory is understood today. It was performed three years before Goudsmit and Uhlenbeck, based on spectroscopic studies, proposed ½-integral spin existed. Goudsmit's interpretation that every spin component, $s_i = \pm 1$, became embedded in quantum mechanics as the *projection postulate*, now known as the *qubit*. However, in 1928 Gordon established that the magnetic moment of the electron is due to the circulating flow of charge in the electron wave field; a 3D flow compatible with the Maxwell-Hertz field equations of classical electrodynamics; the same electromagnetic field as is the basis of quantum electrodynamics

(QED).

There is, a century later, still no complete quantum theoretic treatment of Stern-Gerlach, complicated further by *Jarzynski's Equality* relating work on a non-equilibrium thermodynamics system to the free energy of the system [3] [4]. In these terms Deissler investigated the fundamental question of whether a magnetic field does work on an atom [5]. Work is not an observable in the standard sense [6]; it is not represented by a Hermitian operator, thus is not an ordinary quantum observable: the number of possible values of work $W = E_f - E_i$ is typically larger than the dimension of the space of states; hence a Hermitian operator representing work cannot exist [7].

The "qubit" nature of spin is neither derived nor proved; it is assumed. The radical nature of this assumption, deriving largely from Goudsmit's *Projection Postulate*, is re-analyzed. We begin with derivation of constant classical spin from Poisson brackets, which spin will serve as the basis of a work-based analysis of Stern-Gerlach, leading to an *Energy-Exchange Theorem* underlying the dynamics. It is shown that dynamic spin stability in an inhomogeneous field leads to beam-splitting only if a gradient threshold is exceeded, associating quantization with the detector, hence *Qudet*. This *Qudet* interpretation is then contrasted with the standard *Qubit* interpretation and implications for Goudsmit's postulate are discussed.

2. Derivation of Constant Classical Spin from Poisson Brackets

The current state of understanding of particle dynamics is that the spin that physicists focus on is generally *not observable*. If spin is a *rotation* characterizing a particle, the rotating charge on the particle conceptually forms a current loop that induces a local magnetic moment which is the basis of indirect observation of particle spin. A magnetic moment traversing a nonuniform field is typically treated as a particle in a field. The dynamic equations of the field, expressed in geometric algebra, are $(\nabla + \partial_t)\boldsymbol{F} = \boldsymbol{J}$ where $\boldsymbol{F} = \boldsymbol{E} + i\boldsymbol{B}$ and $\boldsymbol{J} = \rho + \boldsymbol{j}$, the single equation equivalent to the four Maxwell electrodynamic equations in standard form [8]. Particle dynamics are prescribed by Poisson brackets: $\dot{\boldsymbol{x}} = \{\boldsymbol{x}, H\}$ where $\boldsymbol{x} \in \{\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{s}\}$ and H is the Hamiltonian: $H = \frac{p^2}{2m} - \boldsymbol{\mu}\cdot\boldsymbol{B}$, $\boldsymbol{\mu} = \mu_0 \boldsymbol{s}$. We analyze inhomogeneous magnetic field $\boldsymbol{B}(\boldsymbol{r})$ in the standard Stern-Gerlach model (Griffiths, [9]) $\boldsymbol{B} = (B_0 + \alpha x)\hat{i} + (-\alpha y)\hat{j}$ and in the quadrupole form $\boldsymbol{B} = (y)\hat{i} + (x)\hat{j}$. *Only precession can be solved for exactly.* In $\{\boldsymbol{r}, \boldsymbol{p}, \boldsymbol{s}\}$-representation the particle at position $\boldsymbol{r}$ has momentum $\boldsymbol{p}$ and spin $\boldsymbol{s}$ and magnetic moment proportional to spin $\boldsymbol{s}$: $\boldsymbol{\mu} = \mu_0 \boldsymbol{s}$. Point-based classical physics led to the Poisson bracket formalism in which the time derivative of the physical parameters is given by the generic

$$\dot{\boldsymbol{x}} = \{\boldsymbol{x}, H\} \tag{1}$$

where $\boldsymbol{x}$ is the physical entity/aspect of reality and H is the Hamiltonian of the system. As suggested by Kauffman we solve the equations deriving from the Hamiltonian in magnetic field $\boldsymbol{B}(\boldsymbol{r})$:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t}=\{\boldsymbol{r},H\}\Rightarrow\frac{\boldsymbol{p}}{m} \quad \text{velocity predicts position at a moment in time} \tag{2}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t}=\{\boldsymbol{p},H\}\Rightarrow-\mu_0\nabla(\boldsymbol{s}\cdot\boldsymbol{B}(\boldsymbol{r})) \quad \textit{momentum}\text{ force} \tag{3}$$

$$\frac{\mathrm{d}\boldsymbol{s}}{\mathrm{d}t}=\{\boldsymbol{s},H\}\Rightarrow-\mu_0\boldsymbol{B}(\boldsymbol{r})\times\boldsymbol{s} \quad \textit{precession}\text{ force (torque)} \tag{4}$$

Given the *specified quadrupole B-field*, $\mathbf{B}(\boldsymbol{r})=\alpha(y,x,0)$, $\beta=B_0/r_0$ we calculate

$$\boldsymbol{s}\cdot\boldsymbol{B}=\beta(s_x,s_y,s_z)\cdot(y,x,0)=\beta(ys_x+xs_y)$$

and

$$\nabla(\boldsymbol{s}\cdot\boldsymbol{B})=\beta(\partial_x,\partial_y,\partial_z)(ys_x+xs_y)=\beta(s_y,s_x,0).$$

From the *Poisson brackets* we obtain the equation of motion:

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}=-\frac{\mu_0}{m}\nabla(\boldsymbol{s}\cdot\boldsymbol{B})=-\frac{\mu_0}{m}\beta(s_y,s_x,0) \tag{5}$$

i.e.

$$\ddot{x}=-\frac{\mu_0}{m}\beta s_y,\quad \ddot{y}=-\frac{\mu_0}{m}\beta s_x,\quad \ddot{z}=0 \tag{6}$$

while the *first order spin equation* is

$$\dot{\boldsymbol{s}}=\boldsymbol{B}(\boldsymbol{r})\times\boldsymbol{s} \tag{7}$$

which, for the specified quadrupole field yields

$$\boldsymbol{B}\times\boldsymbol{s}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ y & x & 0\\ s_x & s_y & s_z\end{vmatrix}=(xs_z,-ys_z,(ys_y-xs_x)). \tag{8}$$

The most salient fact that emerges follows from:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}(s_x^2+s_y^2+s_z^2)&=2(s_x\dot{s}_x+s_y\dot{s}_y+s_z\dot{s}_z)\\ &=2\mu_0\beta(xs_zs_x-ys_zs_y+ys_zs_y-xs_zs_x)\equiv0\end{aligned} \tag{9}$$

which implies that *the magnitude of spin is conserved*:

$$\frac{\mathrm{d}}{\mathrm{d}t}|\boldsymbol{s}|^2=0\Rightarrow|\boldsymbol{s}|^2=\text{constant} \tag{10}$$

This result derives from *classical* equations of motion, not *quantum*, although it implies that the spin is quantized. In addition, we see from the third spin precession Equation (6c) that s_z is conserved. To relate spin physics to reality, we consider the Stern-Gerlach experiment.

3. Physics of Stern-Gerlach and Analysis of Work

Angular momentum of particles at the quantum level is notoriously hard to measure. *Spinning charge* induces a magnetic dipole proportional to angular momentum; as this dipole interacts with magnetic fields, spin experiments are based on measuring magnetic dipoles as a surrogate for spin. Angular momentum $\boldsymbol{\lambda}$ of an atom is often called an observable, but is generally unobservable; a magnetic dipole moment associated with charged particle motion $\boldsymbol{\mu} = \mu_B \boldsymbol{\lambda}$ where μ_B is the Bohr magneton, 0.927 × 10^{-20} erg/gauss. Erg-per-gauss relates to energy of a dipole in a magnetic field, and this energy, often indirectly observable via photon emission and absorption, is the physical observable. To observe $\boldsymbol{\lambda}$, one must establish a reference frame, via a magnetic field, $\boldsymbol{B}$. Jared Stenson, focused on representation, [10] notes that: "…*the Stern-Gerlach experiment has become axiomatic in modern physics… it ties together classical and quantum systems of thought. It seems to explain a clearly quantum result in terms of almost purely classical concepts.*" The experiment sends neutral particles with a spin-based magnetic moment through magnetic fields. Two cases are significant: particles in a *constant field* precess—a spin $\boldsymbol{\lambda}$, moving in a constant field simply precesses and is not deflected. But the same particles moving through an inhomogeneous field are deflected by the gradient force $\boldsymbol{F} = \nabla(\boldsymbol{\mu} \cdot \boldsymbol{B})$ with $\boldsymbol{\mu}$, the magnetic moment (proportional to λ), represented by quantum operator $\hat{\sigma}$, and $\boldsymbol{B}(\boldsymbol{x})$ the magnetic field in direction $\boldsymbol{b}$. The data are from the iconic postcard Stern and Gerlach sent to Bohr announcing their discovery of the spin-dependent "splitting", shown in **Figure 1(b)**. Per Messiah [11]:

"*The appearance on the screen of a more or less spread-out distribution of impacts indicates that the atoms are not all in the same initial condition and that the dynamical variables defining the initial states are statistically distributed over a somewhat extended domain.*"

For vertical x and horizontal z, the x-component of the magnetic field is B_x at the south end of the dipole. The field component at the north end depends on the gradient and the projections of $\boldsymbol{\lambda}$ on respective axes. The force exerted on the magnet is the sum of forces acting on each end:

$$F_x = \hat{m}(\boldsymbol{\lambda} \cdot \nabla B_x) = \boldsymbol{\mu} \cdot \nabla B_x \Rightarrow \boldsymbol{F} = \nabla(\boldsymbol{\mu} \cdot \boldsymbol{B}). \tag{11}$$

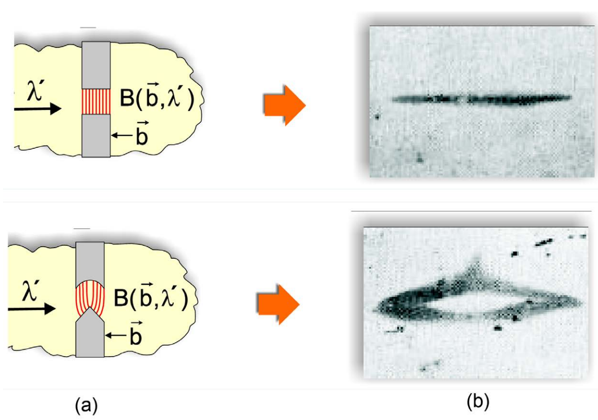

Figure 1. (a) Apparatus with no gradient. (b) Stern-Gerlach apparatus with gradient.

The force on the dipole is proportional to magnetic dipole moment, $\boldsymbol{\mu}$, the gradient of B_x and the cosine of the angle between $\boldsymbol{\mu}$ and ∇B_x. In addition to this force on the magnetic moment due to the inhomogeneous field, the moment precesses in a homogeneous field. Magnetic force on a charged particle is orthogonal to the path, and hence does no work, but this is not the case for an uncharged magnetic dipole. The field gradient force acting on the magnetic dipole is exerted over a finite distance, $F \cdot x$, yet no work is done on the system (*KE* = kinetic energy): Normally

$$\int_0^x \mathrm{d}(KE) = \text{'work'}, \tag{12}$$

but no work is done if precession energy (*PE*) is exchanged with (changed into) kinetic energy (*KE*) as described in the following *Energy-Exchange theorem*. Assume the particle is deflected from the *z*-axis (*x = 0*) to a distance *x* from the *z*-axis and precession energy *PE decreases* to compensate the increase in kinetic energy of translation until precession energy is exhausted at *q*.

$$\int_0^x \mathrm{d}(KE) + \int_0^q \mathrm{d}(PE) = \sum \Delta E = \text{'work'} \tag{13}$$

To show the energy exchange explicitly we break the kinetic energy integral from 0 to *x* into two integrals, from 0 to *q* and from *q* to *x*:

$$\int_0^x \mathrm{d}(KE) + \int_0^q \mathrm{d}(PE) = \underbrace{\int_0^q \mathrm{d}(KE) + \int_0^q \mathrm{d}(PE)}_{\text{NO WORK}} + \underbrace{\int_q^x \mathrm{d}(KE)}_{\text{WORK}} \equiv \Delta E \tag{14}$$

Work ($W = \Delta E$) is defined as the *change in energy*; we observe that no work is done from zero to q as the total energy change is zero. After the moment aligns with the local magnetic field at position q (where precession has ended) work is done from q to *x*. In this model no work is done *while the spin is in process of alignment* with the local field, but once aligned, the field performs work on the particle. The dynamics (which caused Bell to despair [12]) are even more complicated than Deissler assumed as there are now *two* phases: a phase in which no work is done by the field, due to internal compensation, followed by a phase in which the field performs work on the particle, as seen in **Figure 2**.

In an SG-apparatus, a magnetic moment traverses a non-uniform magnetic field, experiencing a gradient-based deflecting force. Uniform magnetic fields do *not* do work on particles, and inhomogeneous magnetic fields do not do work on a *spin-less atom*, but analysis of a quasi-particle defined by electron spin, local magnetic field, and precession energy shows that spin models of particles in an inhomogeneous field will *exchange energy between local energy modes* to compensate for changes in the local field. We are thus motivated to describe the energy transfers between eigenmodes, in which case an *Energy-Exchange theorem* is basic.

4. The Energy-Exchange Theorem

In a physical system possessing two energy modes, M_0, M_1, *coupled to a*

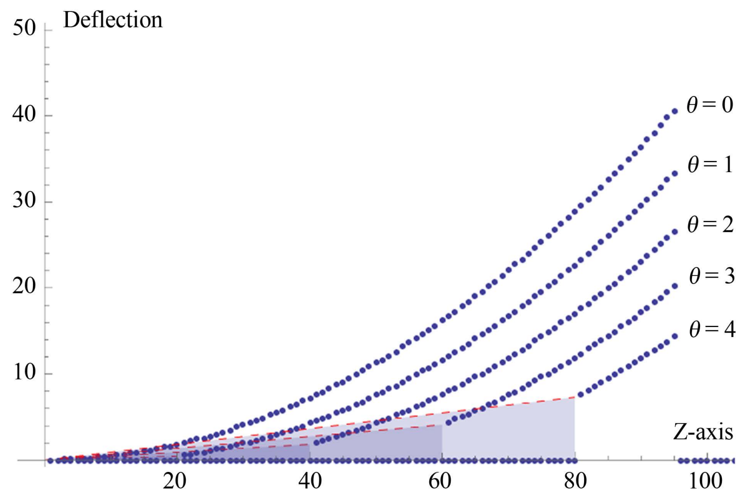

Figure 2. The θ-dependence for small angles is shown. Shaded areas below the dashed red lines represent the alignment process during which the initial spin enters the field with angle θ and proceeds to align with the local field. After alignment the gradient force accelerates all particles equally, seen as dotted blue lines. Scales have been chosen to enhance the different deflections. Actual scales depend upon field strength, field gradient, length of travel, and geometry.

common variable θ, *if energies of the modes are not separated by a quantum gap* $\Delta\varepsilon > 0$, *then if the common variable changes,* $\mathrm{d}\theta/\mathrm{d}t \neq 0$, *the modes will exchange energy.*

Assume that total energy is $\varepsilon = \varepsilon_0 + \varepsilon_1$ when $H_i|\psi\rangle = \varepsilon_i|\psi\rangle$ and that total energy $H = H_0 + H_1$ is conserved:

$$\begin{aligned} &\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \Rightarrow \frac{\mathrm{d}H_0}{\mathrm{d}t} + \frac{\mathrm{d}H_1}{\mathrm{d}t} = 0 \Rightarrow \frac{\mathrm{d}H_0}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\mathrm{d}H_1}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0 \\ &\Rightarrow \left(\frac{\mathrm{d}H_0}{\mathrm{d}\theta} + \frac{\mathrm{d}H_1}{\mathrm{d}\theta}\right)\frac{\mathrm{d}\theta}{\mathrm{d}t} = 0 \end{aligned} \tag{15}$$

Since $\mathrm{d}\theta/\mathrm{d}t \neq 0$ then $\frac{\mathrm{d}H_0}{\mathrm{d}\theta} = -\frac{\mathrm{d}H_1}{\mathrm{d}\theta}$ and energy flows between mode M_0 and M_1. QED

In the Stern-Gerlach experiment a neutral silver atom enters the inhomogeneous field with *z*-axis momentum and its intrinsic magnetic moment $\boldsymbol{\mu}$ at angle θ to the local B-field in the *x*-direction. The force of the *x*-directed field gradient $\boldsymbol{F} = \nabla(\boldsymbol{\mu}\cdot\boldsymbol{B})$ accelerates the particle in the *x*-direction while making no change to the initial *z*-momentum. The change in kinetic energy due to particle acceleration is compensated by the precession energy to conserve local energy: $E_{in} = E_{out}$

$$-\boldsymbol{\mu}\cdot\boldsymbol{B} = mv_x^2/2 - |\boldsymbol{\mu}||\boldsymbol{B}| \tag{16}$$

where *z*-momentum energy has been canceled for both sides. The change in translational kinetic energy, $mv_x^2/2$ is thus $mv_x^2/2 = \mu B(1-\cos\theta)$, where $\mu = |\boldsymbol{\mu}|$, $B = |\boldsymbol{B}|$. Applying the constant acceleration formula, $v_f^2 = v_i^2 + 2ax$, as an approximation, yields. $m(ax) \cong \mu B(1-\cos\theta)$ and hence an approximate θ

-dependent component of deflection

$$x = \frac{\mu B}{ma}(1 - \cos\theta). \tag{17}$$

The distance x is the amount of deflection from the z-axis that a particle with initial angle of precession θ experiences when the particle becomes aligned with the local field. The relation $x = f(\theta)$ is such that deflection x is determined by *initial* angle θ as well as field strength and field gradient. The *Energy-Exchange Principle* constrains local dynamics until locally available energy is exhausted. A quasi-particle with internal degrees of freedom traversing a non-uniform field locally conserves energy over its N-degrees-of-freedom. A precessing, translating, magnetic moment has two such degrees of freedom, but finite compensation mechanisms, when exhausted, can no longer accommodate changes in the local field. At such time the local gradient begins delivering power. So, a local field gradient will not impart energy as work to the quasi-particle, either atom or precessing spin, *until* precession energy has been converted to deflection energy. If precession energy vanishes ($\theta = 0$) the local gradient drives the translation of the particle.

Rate of change of precession

An unanswered question concerning a dipole in an inhomogeneous magnetic field is the rate at which the angle of precession changes; how long it takes for the angle of precession to reach zero, *i.e.*, for the dipole to align with the local field. If it's not instantaneous, the rate is a matter of geometry and field strength and strength of gradient. If it were instantaneous, the entire travel through the Stern-Gerlach device would experience the maximum force and the deflection would be to one spot, regardless of the initial angle; corresponding to Bell's model, deflection measurements would yield ±1. A finite decay time is angle dependent and the spread of deflection is a function of the initial angle. Absent means of calculating the decay rate of the precession angle to determine the time of alignment, the question *is* answerable experimentally; the original Stern-Gerlach data on the iconic postcard exhibits the spread of deflections expected from energy exchange. From *the Energy-Exchange theorem* we conclude that precession energy is exchanged with kinetic energy of translation; the particle is accelerated by force $\boldsymbol{F} = \nabla(\boldsymbol{\mu} \cdot \boldsymbol{B})$. While the rate of precession may be independent of the particle mass, the rate of acceleration is not. Since $\boldsymbol{F} = m\boldsymbol{a} = \nabla(\boldsymbol{\mu} \cdot \boldsymbol{B})$ and $\boldsymbol{a} = \nabla(\boldsymbol{\mu} \cdot \boldsymbol{B})/m$ acceleration is a function of the gradient of the B-field, the angle of precession, and the mass of the particle. Based on these dependencies we take the derivatives of $mv^2/2 = \mu B(1 - \cos\theta)$. Maximum energy of translation occurs when $\theta = 0$, corresponding to maximum velocity v, thus maximum velocities occur for small angles θ. From $\cos\theta \approx 1 - \theta^2/2 + \cdots$ in (17):

$$mv^2/2 = \mu B(1 - \cos\theta) \xrightarrow{\theta \to 0} \underbrace{mv^2}_{\text{FINI}} = \underbrace{\mu B\theta^2}_{\text{INIT}}. \tag{18}$$

With $k = \sqrt{\mu/m}$ we obtain $v \approx k\sqrt{B}\theta$ hence $\frac{\mathrm{d}\theta}{\mathrm{d}t} \approx \frac{1}{k\sqrt{B}} \frac{\mathrm{d}v}{\mathrm{d}t}$ where from

Equation (11) $\frac{\mathrm{d}v}{\mathrm{d}t} \sim \frac{\mu}{m}\frac{\partial B}{\partial x}$, so:

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} \approx \frac{k}{\sqrt{B}}\frac{\partial B}{\partial x} \quad . \tag{19}$$

The Asymmetric Approximation

A finite decay rate implies a θ-dependent spread of SG deflections, in contrast to the single data point expected if alignment were instantaneous. The asymmetric treatment displays the second-order effects of the stronger and weaker regions of gradient. The combination of these classical effects is shown in **Figure 3** overlapping the real Stern-Gerlach data from the iconic postcard, on which we have overlaid the gradient-producing magnetic field geometry. We consider only $\partial B/\partial x$, *i.e.*, vertical deflection; incompatible with Maxwell's $\nabla \cdot \boldsymbol{B} = 0$. Inclusion of a y-axis term provides the left/right displacements shown in the data but adds no new physics insight to the model.

A typical asymmetric Stern-Gerlach *x*-deflection for random initial angle θ is next calculated by rewriting Equation (17) to accommodate the asymmetry via β and $\beta/3$:

$$x = +1 + \beta\cos\theta \qquad \theta < \pi/2 \tag{20}$$

$$x = -1 - (\beta/3)\cos\theta \qquad \theta > \pi/2 \tag{21}$$

The scaling value $\beta = 4$ is chosen to yield a best match to the iconic postcard data. Actual deflection depends on the strength of the field, the strength of the gradient, the initial angle, the velocity, the length of the SG-magnet and the distance from the magnet to the detection screen. We effectively normalize the deflection by choosing the deflection due to the magnet-to-screen travel to be +1 or −1. The values can be approximated via appropriate strengths and geometries. If we vary the spin angle θ randomly and plot the vertical deflections based on Equations (20) and (21) we obtain the asymmetric distribution; the (red) calculated random data points are scaled and overlaid on the SG iconic data. The match between calculated data and experimental data is extremely good.

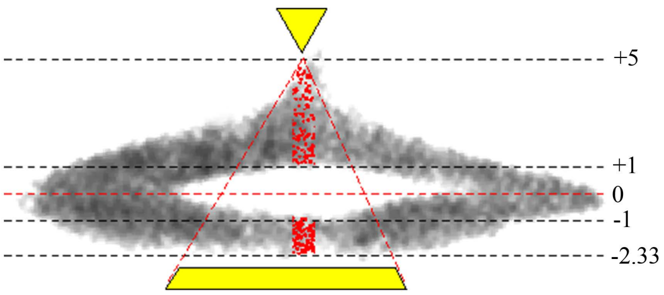

Figure 3. The θ-based model of spin deflection in the Stern-Gerlach experiment is driven with random spin vectors and trajectories are calculated. The red dots representing individual particles are overlaid on the gray SG data from the iconic postcard. The scale has been chosen to facilitate the overlay. The Stern-Gerlach magnets are diagrammed in yellow. Horizontal spreading of the red data points is for illustrative clarification and is not physically meaningful.

5. Analysis of Dynamic Spin Stability

To address Bell's statement about "*compass needles pointing in the wrong direction*" being "*not dynamically sound*" requires making use of Deissler's observation about spin and precessional rotation and requires an asymmetry foreign to Bell's analysis, best understood by comparison with the assumptions of first-order energy exchange, namely

$$\begin{array}{ll} \text{Symmetric} & \text{Asymmetric} \\ E_{in} = E_{out} & E_{in} = E_{out} \\ B_{in} = B_{out} & B_{out} = B_{in} + \Delta B \end{array} \tag{22}$$

First-order approximation energy-exchange model with instantaneous decay of precession leads to Bell-like measurement spectrum of ±1. Our finite decay model leads to a symmetric spread spectrum. A second-order gradient-based approximation yields an asymmetric energy-exchange model. To develop the asymmetric model, we explicitly consider "spin-up" and "spin-down" cases.

Spin down $\boldsymbol{\theta} \le \pi/2$

Dipole moment $\boldsymbol{\mu}$ is the product of electronic charge and spin, so the negative electron implies $\boldsymbol{\mu} = -\boldsymbol{s}$ as shown in **Figure 4** and spin-down precession (CCW) is opposite the rotation of the spin (CW). As the force on the dipole accelerates the particle to the region of stronger B-field, $B \to B + \Delta B$, the (CCW) precession frequency increases, opposing the intrinsic CW spin and decreasing kinetic rotational energy:

$$\begin{Bmatrix} \text{cw} \\ \text{spin} \end{Bmatrix} + \begin{Bmatrix} \text{ccw} + \Delta\text{ccw} \\ \text{precession} \end{Bmatrix} = \begin{Bmatrix} \text{rot. energy} \\ \Downarrow \end{Bmatrix}$$

As $\boldsymbol{B}$ increases, the $-\boldsymbol{\mu} \cdot \boldsymbol{B}$ energy becomes more negative. If energy is conserved, translational kinetic energy $mv_x^2/2$ becomes more positive as the particle is accelerated upward. This agrees with the decrease in rotational kinetic energy as shown. We next consider the opposite spin.

Spin up $\boldsymbol{\theta} \ge \pi/2$

A particle with spin up is accelerated into a region of weaker local magnetic field so the precession frequency (energy) decreases. Since the (CCW) spin and (CCW) precession are in the same direction, as shown in **Figure 5**, the net rotational energy decreases:

$$\begin{Bmatrix} \text{ccw} \\ \text{spin} \end{Bmatrix} + \begin{Bmatrix} \text{ccw} - \Delta\text{ccw} \\ \text{precession} \end{Bmatrix} = \begin{Bmatrix} \text{rot. energy} \\ \Downarrow \end{Bmatrix}$$

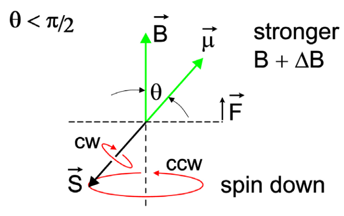

Figure 4. Spin $\boldsymbol{s}$ and magnetic moment $\boldsymbol{\mu}$ in inhomogeneous magnetic field $\boldsymbol{B}$ with spin down.

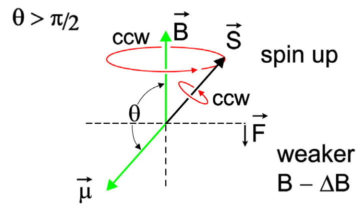

Figure 5. Spin $\boldsymbol{s}$ and magnetic moment $\boldsymbol{\mu}$ in inhomogeneous magnetic field $\boldsymbol{B}$ with spin up.

Thus as $\boldsymbol{B}$ decreases the $\boldsymbol{\mu}\cdot\boldsymbol{B}$ energy becomes smaller (less positive) and the translational energy $mv_x^2/2$ grows as the rotational energy decreases. Hence for all angles of the initial dipole with respect to the magnetic field

$$\underset{\text{rotational}}{\begin{Bmatrix} I\omega^2/2 \\ \Downarrow \end{Bmatrix}} + \underset{\text{translational}}{\begin{Bmatrix} mv_x^2/2 \\ \Uparrow \end{Bmatrix}} = \begin{matrix}\text{energy}\\ \text{balance}\end{matrix}$$

With this detailed analysis of the spin dynamics we now reanalyze the energy balance equation $E_{in} = E_{out}$. Since there is no change in the z-component of velocity, we know that it cancels and we also assume that the incoming x-velocity is zero, thus we retain our energy exchange equation $-\boldsymbol{\mu}\cdot\boldsymbol{B}_{in} = mv_x^2/2 - |\boldsymbol{\mu}||\boldsymbol{B}_{out}|$ adapted from Equation (6). We again consider two cases $\theta \le \pi/2$ and $\theta \ge \pi/2$. Our initial energy exchange analysis explicitly conserved energy, $E_{in} = E_{out}$, but simplistically assumed $B_{in} = B_{out}$. A better approximation, $B_{out} = B_{in} \pm \Delta B$ takes a necessary gradient into account. We can now explore the energy exchange spin dynamics in an explicit gradient formulation. It is again necessary to treat the spin up and spin down cases separately, as the $\cos\theta$ term yields a sign change between these two cases.

For $\theta < \pi/2$ the dipole experiences a positive force toward a region where the field is stronger. The initial angle between the spin and the local field is θ and the final aligned state has zero angle. Begin with local conservation of energy: $E_{in} = E_{out}$ and $-\boldsymbol{\mu}\cdot\boldsymbol{B}_{in} = mv_x^2/2 - |\mu||B_{out}|$. Let

$$B_{out} = B_{in} + \Delta B \quad \text{where} \quad \Delta B = \int_0^x \frac{\mathrm{d}B}{\mathrm{d}x}\mathrm{d}x \tag{23}$$

$$mv_x^2/2 = |\mu||B_{out}| - \boldsymbol{\mu}\cdot\boldsymbol{B}_{in} = |\mu|(B_{in} + \Delta B) - \boldsymbol{\mu}\cdot\boldsymbol{B}_{in} \tag{24}$$

$$mv_x^2/2 = |\mu||B_{in}|(1-\cos\theta) + \mu\Delta B. \tag{25}$$

All terms are positive and thus the kinetic energy is positive as required. The $\mu\Delta B$ additive energy represents greater kinetic energy (and correspondingly greater deflection) than the simpler symmetric approximation.

For $\theta > \pi/2$ the dipole experiences a force toward a weaker region of the field. Here the cosine is negative, so $\boldsymbol{\mu}\cdot\boldsymbol{B}$ is negative and energy $-\boldsymbol{\mu}\cdot\boldsymbol{B}$ is therefore positive. Again $E_{in} = E_{out}$ but now $-\boldsymbol{\mu}\cdot\boldsymbol{B}_{in} = mv_x^2/2 - |\mu||B_{out}|(\cos\pi)$. Taking signs into account, this becomes:

$$|\boldsymbol{\mu}||\boldsymbol{B}_{in}||\cos\theta| = mv_x^2/2 + |\mu||B_{out}| \tag{26}$$

$$mv_x^2/2 = |\boldsymbol{\mu}||\boldsymbol{B}_{in}||\cos\theta| - |\mu||B_{out}| \tag{27}$$

But, due to the gradient, $B_{out} < B_{in}$ so $B_{out} = B_{in} - \Delta B$ and thus,

$$\begin{aligned} &mv_x^2/2 = |\boldsymbol{\mu}||\boldsymbol{B}_{in}|\left(|\cos\theta| - 1\right) + \mu\Delta B \\ &\Rightarrow \Delta B > \left|B\left(|\cos\theta| - 1\right)\right| \end{aligned} \tag{28}$$

The term $\left(|\cos\theta| - 1\right)$ is *always* negative, so the requirement of positive kinetic energy of translation $mv_x^2/2$ demands the $\mu\Delta B$ term be sufficiently positive to overcome the negative term. Thus Equation (28) implies a threshold gradient, *below which the Stern-Gerlach apparatus will not work*. Equations (28) show the (integral of) the gradient to be a function of θ and of translational velocity *v*:

$$\Delta B = B\left(1 - |\cos\theta|\right) + \frac{1}{2}\left(\frac{v^2}{k^2}\right) \Rightarrow \frac{mv^2}{2} > 0 \tag{29}$$

The above analysis enables *a new physical conclusion*, which is the existence of a velocity-dependent threshold of inhomogeneity required for Stern-Gerlach apparatus. Below this threshold the atomic beam will not split, in accord with Bell's statement that "the compass needles" would be pointing in the wrong direction and hence "not dynamically sound". Below threshold the apparatus only detects one beam distribution for random θ input atoms. It becomes a *quantum detector* or *Qudet* only when the gradient threshold is exceeded. Above the threshold the classical atomic beam is quantized or split into spin-dependent beams.

6. The Qubit Interpretation of Stern-Gerlach

The above analysis of Stern-Gerlach in terms of magnetic moments traversing inhomogeneous magnetic fields forces particles with random spin orientation to align with the 3D field and makes intuitive physical sense. The approximate equations derived from the analysis produce an output distribution that readily maps onto the iconic SG "post card" results. We compare this interpretation of SG with the quantum interpretation, yet according to *Quantum Theory at the Crossroads*, [13]:

"*The interpretation of quantum theory is probably as controversial as it has ever been … the meaning of quantum theory is today an open question.*"

We adopt the Stanford interpretation of the meaning of quantum mechanics presented by *Stanford Physics Department head*, Leonard Susskind, in his video lecture. Texts by Feynman, Sakurai, and Townsend say that spin in a Stern-Gerlach experiment leads to the quantum formalism. For Susskind *qubits* are fundamental and spin is incidental; he introduces *a "qubit" or quantum bit rather than physical spin and defines it as a two-state system or "a bit"*. In his qubit-based intro to quantum mechanics [14] he claims we should not expect to "understand" the qubit in any classical sense: At *minute 11*, *second 17*, written [11:17], he states:

"*Quantum mechanics is about things that your evolutionarily developed neural structure is not prepared to deal with.*"

At [21:00] "*the simplest possible system in quantum mechanics… I'm going to call it a qubit—a system that can have two states.*" It has a mathematical degree of freedom, σ, and an equivalent arrow symbolism shown in **Figure 6**.

At [24:05] "*in addition to the system, it has an apparatus. The apparatus is a black box oriented with a "This side up" label, containing a window in which measurement data is displayed (+1 or −1) connected to a detector that senses the qubit. In QM, we cannot ignore the apparatus, but we don't care how it works. The experiment is to measure the state of the qubit:* $\uparrow$ or $\downarrow$."

But from the preceding sections it is clear that "*we should care how it works.*" The physics of the Stern-Gerlach *Qudet* experiment is the theory of "*how it works*" and teaches us that any random spin measured by the system will align itself with the *Qudet*, or *quantized detector*.

"*A device that measures is also a device to prepare a system in a given state.*" [29:10] After measurements confirm that the state is $\uparrow$ we turn the detector "upside down" and we find the state $\downarrow$. We can do so repeatedly. [32:00] "*by turning the detector over, we get the opposite answer.*" What is this telling us about the nature of the system? "*… we're learning that whatever the system is, this particular qubit has a sense of directionality to it. … The qubit has spatial directionality associated with it. … a sense of orientation … We might begin to suspect that it's a vector.*"

Thus, the idea is that we can, as stated at [34:00], "*detect the value of a component of a (spin) vector along the (vertical) direction*" and then [by rotating the detector] "*measure not the vertical component of the vector but the horizontal component of the vector*". But the idea that a *constant* spin vector with components s_x, s_y and s_z can be measured sequentially and components s_x, s_y and s_z determined is *counterfactual to reality*; the SG detector requires a non-uniform magnetic field or else the result of the experiment is null; and in a non-uniform magnetic field all three spin components s_x, s_y and s_z will *change*! The SG physics is such that spin will align *with* or *against* the local B-field, thus yielding ±1 results *regardless* of the original orientation of the spin.

Key to the qubit interpretation is the implication that the same *component* can be detected multiple times. If the component in the x-direction is $\uparrow$, then

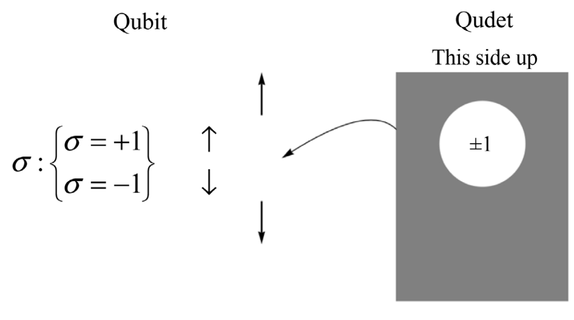

Figure 6. Schematic diagrams of (a) *qubit* and (b) *qudet*.

when measured again with the apparatus upside-down, we detect the x-component as $\downarrow$. Finally, Susskind lays the *Qudet* on its side, and explains that, if we prepare an up-state $\uparrow$ and test it using the apparatus on its side, we expect to get zero, but we always get ±1, in agreement with Goudsmit's *projection postulate*. But if the apparatus causes the physical system represented by the qubit to *align* or *anti-align with* the detector, then the quantum (±1 on *any* axis) argument vanishes; the classical (±1 on *one* axis) argument prevails; the *one* axis being the current B-field direction of the SG apparatus.

This counterfactual reasoning, based on the physically nonsensible idea that we can actually measure *each* of the spin components s_x, s_y and s_z via *three sequential measurements*, is the basis of the quantum *qubit*. Goudsmit interpreted Stern-Gerlach *splitting* of atomic beams as based on half-integral spin. If Goudsmit had simply proclaimed that

"*The projection of spin on one axis is* ±1," $\langle$the $\boldsymbol{B}$ axis$\rangle$ this would have been compatible with classical physics *and* with the facts of the SG experiment. Instead, his counterfactual projection postulate "*The projection of spin on any axis is* ±1", has led to countless apologies to the effect that viewing spin as a "*little arrow pointing in some direction*" is naïve, and "*any attempt to visualize it classically will badly miss the point.*" SG-measurement of any axis affects *other axes*, rendering meaningless the "preservation of $\boldsymbol{s}$-components" and subsequent comparison of such with another 3D direction. Only *one axis* is relevant: the physical B-field local axis. Irrespective of original spin, measured spin is parallel or anti-parallel to the local axis after the measurement. After exiting the magnetic field, spin is preserved until a next measurement, at which time the spin is rotated to the current local *x*-axis. The orthogonal axes vanish.

7. Analysis of Classical Qudet vs. Quantum Qubit

At [57:30] I ask Susskind about the logic of his presentation:

"*You've postulated a two-state system and a detector, but what would be the difference in logic in saying your system was a variable because your detector is a two-state device. It seems like you get the same output.*"

Susskind asks me to repeat the question.

"Y*ou've postulated a two-state system: +1 and −1. But then you have a detector that always produces plus or minus one. If you postulated a variable* [system] *to start with, your detector still seems like it would still give you +1 or −1.*"

Susskind: (hand to ear) "*Seems like what?*"

"*You're really saying your detector is a two-state system that you can orient, and that doesn't seem to imply* (*that your system is a two-state system*)."

Susskind: "*you're absolutely right, that at some point we have to come back to this and say the detector is a system and it has states and understand the combined system as a quantum system composed of two quantum systems.*

The standard approach is that the universe is quantum, but all measurement devices are treated as classical; we do not know where the boundary between quantum and classical resides. Yet, from the above, we can alternatively understand the entire thing as *two classical systems*: the classical spin and the classical SG apparatus. We know the *apparatus* to be a 1D system, the *Qudet*; we do not know that the *system* is 1D, a *qubit*; this is an *assumption*, *not a conclusion based on the facts.* The *qubit* nature of the particle is an artifact, imposed by the 1D apparatus.

Susskind has published a book derived from the video lecture [15]. On page 6: a measurement results in $\sigma_z = +1$ based on the (*Qudet*) apparatus pointing in the *z*-direction. When the apparatus is flipped without disturbing the previously measured spin the new measurement is $\sigma_z = -1$, thus "*turning the system over changes* $\sigma = +1$ to $\sigma = -1$. *If we are convinced that spin is a vector*, *it has 3 components* $\sigma_x, \sigma_y, \sigma_z$. *When the apparatus measures the x-component*, *we expect* $\sigma_x = 0$ *but we find that* $\sigma_x = \pm 1$." Since the apparatus measures only one direction, "*one simply cannot know the components of spin along the two different axes*…" Hence, "*there is something different about the state of a quantum system and the state of a classical system.*"

Let us examine the state of a classical system. Assume we have a gyroscope pointing in an arbitrary direction in space. The gyroscope is spinning. If we overlay an (*x*,*y*,*z*)-coordinate system on the gyroscope, and assume (for simplicity) that the gyro spin $\boldsymbol{s}$ is in the *xz*-plane at angle θ with respect to the *z*-axis, there is no *y*-component of the gyro-spin, $s_y = 0$, $s_z = |\boldsymbol{s}|\cos\theta$, $s_x = |\boldsymbol{s}|\sin\theta$. The magnitude of spin is $|\boldsymbol{s}| = \sqrt{\boldsymbol{s}\cdot\boldsymbol{s}} = \sqrt{s_x^2 + s_y^2 + s_z^2} = \sqrt{|\boldsymbol{s}|^2 \sin^2\theta + 0 + |\boldsymbol{s}|^2 \cos^2\theta} = |\boldsymbol{s}|$. Why is this different from electron spin? It is different because our measurement device is the eyeball measuring photons reflected from the gyro-axis. The photons are quantum objects, but they do *not* disturb the gyro-spin axis; they are too weak. As we have seen the Qudet *does* disturb all axes, aligning the spin with the quantized apparatus. So, the *Qudet* is responsible for the quantum aspect of the spin of the atom. Yet, per Susskind, "… *knowing a quantum state means knowing as much as can be known about how the system was prepared.*" However, if the system was prepared by a *Qudet*, then all we can know about the quantum state is that *it is the state of the Qudet*!

Many authors are quite clear about the central role of the measurement apparatus: [16] Amri *et al.* state: "*In the quantum world*, *the measurement process plays a central role*, *as it leads to an unavoidable and strong modification of the system which is measured.*" Similarly, Gondran and Gondran, [17] state: "*The result of the Stern-Gerlach experiment is not the measure of the spin projection along the z-axis*, *but the orientation of the spin either in the direction of the magnetic field gradient*, *or in the opposite direction.*" Devereaux (2015) argues that "*a comparison with double slit experiments of photons or electrons is misleading because in the case of SG a real energy transfer takes place which destroys any superposition.*" Franca (2009) *proposed an explanation of the alignment of the magnetic moment as a purely classical phenomenon.* Kauffmann

(2015) remarked that "*… your view of measurement as a gross transformation of the quantum system rather than as a mere sampling of it that leaves it essentially undisturbed is helpful in dispelling some of that confusion.*"

Yet according to Schmidt-Bocking *et al.* [18], "*for most physicists this was a* "*miraculous interaction*" *between moving atoms and the SG apparatus. … the physical mechanism responsible for the alignment of the silver atoms remained and remains a mystery.* […] *the trajectories of the atoms can be calculated using equations of motion from classical physics, but the rotation of the magnetic moments into well-defined orientation remain a puzzle.*" Even for physicists of the caliber of Julian Schwinger: "*it is as though the atoms emerging from the oven have already sensed the direction of the field of the magnet and have lined up accordingly… we must learn to live with it.*" And Feynman: "*That a beam of atoms whose spins would apparently be randomly oriented gets split up into separate beams is most miraculous… instead of trying to give you a theoretical explanation, we will just say that you are stuck with the result.*" Bell noted that phenomena of the kind exhibited by Stern-Gerlach "*made physicists despair of finding any consistent space-time picture of what goes on on the atomic and subatomic scale.*"

From the literature it seems that most physicists expect the atomic spin to exhibit Larmor precession with fixed precession angle θ and expect the atom to move under the force of the inhomogeneous magnetic field. Yet per Carrega *et al.* [19] "*A deep understanding … of energy exchange in strongly coupled systems … may have a profound impact … from a fundamental point of view.*" Of this aspect, Navascues and Popescu [20] state: "*Traditional descriptions of the measurement process and quantum mechanics typically overlook the energy exchange between the system under consideration and the measurement device carrying it.*"

8. Conclusions: Projections onto Reality

Pusey, Barrett, and Rudolph [21] quote Jaynes:

"*Our* [quantum] *formalism* [was] *all scrambled up by Heisenberg and Bohr into an omelet that nobody has seen how to unscramble. …if we cannot separate the subjective and objective aspects of the formalism, we cannot know what we're talking about; it is that simple.*"

A major conceptual part of the omelet is due to Goudsmit's 1925 statement that "*The projection of spin on any axis is* ±1." We cannot geometrically *picture*, hence not *imagine*, a physical spin or quantized entity whose projection on *any* axis is ±1. However [22]: "*…when we measure a particle's component of spin in a* [any] *particular direction in a Stern-Gerlach experiment, it is the general belief that we are not measuring a pre-existing property.*"

There are physical situations, such as electron spin in magnetic domains, in which broken symmetry causes spins to align with each other. In such cases the two-state formalism is useful. But if the iconic postcard data represents actual physics occurring in an inhomogeneous magnetic field, our model accurately depicts the spin dynamics expected from the SG experiment. The *energy-exchange model* treats free energy that is *not* accounted for in standard quantum treatments.

Another popular approach is to interpret qubits as "quantum information" which can exist as both a 0 and a 1. Treatment of such is beyond the scope of this paper, however Siegfried ends an article on this approach by saying "*So it seems likely that the meaning and power of quantum information, represented by the qubit, has yet to be fully understood and realized.*" [23]

Quantum theory is probabilistic, not predictive. What does *testable by experiment* mean? Generally, it means that a theory *can fit the data*. But if our experiment must yield at least two states, and the theory provides four extra states that only theoretically exist, how do we *fit the data*? Recall Fermi: "*with 5 points I can fit an elephant.*" The assumption that three spin axes can be measured, when, in fact, only one axis, the *Qudet* axis, can be measured (prepared) implies imaginary data that must be handled properly. This is achieved in quantum mechanics by probabilistically forcing the "average" measurement of the other axes to zero, or to $\cos\theta$ if the *Qudet* is rotated an angle θ with respect to the prepared spin. In short, quantum theory projects non-existent spin axes onto reality and formalistically "fits the data", at the cost of ending up with a theory of reality believed to be "*beyond our evolutionarily developed neural structure*".

Acknowledgements

The author acknowledges helpful discussions with Steven Kenneth Kauffman, Otis Monty Frost, and Richard Zacher during the development of this model. It was also helpful that Leonard Susskind presented on video a development of the qubit in such a manner that it becomes obvious that it is the detector that is quantized, as shown herein.

Conflicts of Interest

The author declares no conflicts of interest regarding publication of this paper.

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