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# Empirical Equation for a Fine-Structure Constant with Very High Accuracy

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## Abstract

We proposed several empirical equations about the electromagnetic force and gravity. The main three equations were connected mathematically. However, these equations have small errors of approximately $10^{-3}$. Therefore, we attempted to improve the accuracy. Regarding the factors of 9/2 and π, we used 4.48870 and 3.13189, respectively. Then, the errors become smaller than $10^{-5}$. However, we could not show any reasons for these compensations. We noticed the following equations. $4.488855 = \sqrt{\frac{136.0113 \times 4}{27}}$, $3.131777 = \frac{Rk}{(3 \times 136.0113)^{1.5}}$. Then, we can explain the von Klitzing constant $Rk = 3.131777037 \times 4.488855463 \times 13.5 \times 136.0113077$. It is well known that the von Klitzing constant can be measured with very high accuracy. We examined this equation for the von Klitzing constant in detail. Then, we noticed that 136.0113 should be uniquely determined. The von Klitzing constant is highly related to the fine-structure constant. After the examination of the numerical connections, we can explain the value of 137.035999081 as a fine-structure constant with very high accuracy. Then, we attempt to explain this value from Wagner's equation.

## Keywords

Fine-Structure Constant, Wagner's Equation

## 1. Introduction

We discovered Equation (1) [1] [2], which was seemed to be very simple.

$$\frac{Gm_p}{\frac{\lambda_p}{2}} \times 1\,\text{kg} = \frac{9}{2} kT_c \tag{1}$$

Next, we discovered Equations (2) and (3) [3], which were seemed to be simple, too.

$$\frac{Gm_p^2}{\left(\frac{e^2}{4\pi\varepsilon_0}\right)} = \frac{4.5}{2\pi}\times\frac{m_e}{e}\times hc\times\left(1\frac{\text{C}}{\text{J}\cdot\text{m}}\times\frac{1}{1\text{ kg}}\right) \tag{2}$$

$$\frac{m_e c^2}{e}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right) = \pi k T_c\times\left(1\frac{\text{J}\cdot\text{m}}{\text{C}}\right) \tag{3}$$

Equations (1), (2) and (3) were connected mathematically [3]. However, we could not establish the background theory. Then, we abandoned theoretical explanations and tried to reduce the error.

Our three equations have small errors of approximately $10^{-3}$ and $10^{-4}$ [3]. We attempted to reduce the error in previous reports by changing Factors 4.5, π and $T_c$ [4] [5]. Regarding the value of $T_c$, 2.72642 K was used instead of 2.72548 K. Regarding the factors of 9/2 and π, we used 4.48870 and 3.13189, respectively. Then, the errors become smaller than $10^{-5}$. Then,

$$\frac{Gm_p^2}{hc} = \frac{4.4887}{2}\frac{kT_c}{1\text{ kg}\times c^2} \tag{4}$$

Equation (4) is deduced from Equation (1).

$$\frac{Gm_p^2}{\left(\frac{e^2}{4\pi\varepsilon_0}\right)} = \frac{4.4887}{2\times 3.13189}\times\frac{m_e}{e}\times hc\times\left(1\frac{\text{C}}{\text{J}\cdot\text{m}}\times\frac{1}{1\text{ kg}}\right) \tag{5}$$

$$\frac{m_e c^2}{e}\times\left(\frac{e^2}{4\pi\varepsilon_0}\right) = 3.13189\times kT_c\times\left(1\frac{\text{J}\cdot\text{m}}{\text{C}}\right) \tag{6}$$

4.48870 and 3.13189 are connected with the following equations.

$$\frac{m_p}{m_e} = \frac{1}{4.4887\times 3.13189}\frac{q_m}{e} \tag{7}$$

However, we could not provide any reasons behind 4.48870 and 3.13189. In this report, we proposed the following two values for compensation.

$$3.131777037(\Omega) = \frac{Rk}{(3\times 136.0113077)^{1.5}} \tag{8}$$

$$4.488855463 = \sqrt{\frac{136.0113077\times 4}{27}} \tag{9}$$

The rest of the paper is organized as follows. In Section 2, we present an empirical equation for the fine-structure constant with very high accuracy. In Section 3, we explained the value of the von Klitzing constant using these values to support our empirical equation. In Section 4, we propose and refine our three equations. For the value of $T_c$, we use 2.72652 K instead of 2.72548 K. The significant figures of the measured $T_c$ are only 3 or 4 in the present study. Therefore, this assumption is valid. For the value of $G$, we use $6.674777\times 10^{-11}$

$m^3 \cdot kg^{-1} \cdot s^{-2}$ instead of $6.6743 \times 10^{-11}$ $m^3 \cdot kg^{-1} \cdot s^{-2}$. In Section 5, we discuss the meaning of our empirical equations.

## 2. Empirical Equation for a Fine-Structure Constant with Very High Accuracy

### 2.1. Symbol List

These Values Were Obtained from Wikipedia.

$G$: gravitational constant: $6.6743 \times 10^{-11}$ ($m^3 \cdot kg^{-1} \cdot s^{-2}$)

(we used the compensated value $6.674777 \times 10^{-11}$ in this report)

$T_c$: temperature of the cosmic microwave background: 2.72548 (K)

(we used the compensated value 2.72652 K in this report)

$k$: Boltzmann constant: $1.380649 \times 10^{-23}$ ($J \cdot K^{-1}$)

$c$: speed of light: 299792458 (m/s)

$h$: Planck constant: $6.62607015 \times 10^{-34}$ (J·s)

$\varepsilon_0$: electric constant: $8.8541878128 \times 10^{-12}$ ($N \cdot m^2 \cdot C^{-2}$)

$\mu_0$: magnetic constant: $1.25663706212 \times 10^{-6}$ ($N \cdot A^{-2}$)

$e$: electric charge of one electron: $-1.602176634 \times 10^{-19}$ (C)

$q_m$: magnetic charge of one magnetic monopole: $4.13566770 \times 10^{-15}$ (Wb)

(this value is only a theoretical value, $q_m = h/e$)

$m_p$: rest mass of a proton: $1.6726219059 \times 10^{-27}$ (kg)

(we used the compensated value $1.672621923 \times 10^{-27}$ kg in this report)

$m_e$: rest mass of an electron: $9.1093837 \times 10^{-31}$ (kg)

$\lambda_p$: Compton wavelength of a proton: $1.32141 \times 10^{-15}$ (m)

$Rk$: von Klitzing constant: 25812.80745 (Ω)

$Z_0$: wave impedance in free space: 376.730313668 (Ω)

$\alpha$: fine-structure constant: 1/137.035999081

### 2.2. Empirical Equation for a Fine-Structure Constant

Our empirical equation for the fine-structure constant is

$$\alpha = \frac{1}{\left(1 + \frac{1}{1836.152654 \times 3}\right) \times \frac{1836.152654}{13.5} + 1} = \frac{1}{137.0359991} \tag{10}$$

where 1836.152654 is explained as follows:

$$\frac{m_p}{m_e} = 1836.152654 = 13.5 \times 136.0113077 \tag{11}$$

where the measured $m_p$ is $1.6726219059 \times 10^{-27}$ (kg). Therefore, the precise value is

$$\frac{m_p}{m_e} = 1836.152673 \tag{12}$$

Therefore, the error is

$$\text{Error} = \frac{1836.152673}{1836.152654} - 1 = 1.01977 \times 10^{-8} \tag{13}$$

This value is sufficiently small. When compensation is needed, the value of $m_p$ should be 1.672621923 × $10^{-27}$ kg. This small error will be explained by a very difficult theory, such as the explanation for the Lamb shift, in the future. In the next section, we attempt to explain Equation (10) from the examination of the value of the von Klitzing constant.

## 3. Examination of the Von Klitzing Constant

### 3.1. Two Important Coefficients

Previously, we proposed the two coefficients 4.48870 and 3.13189 [4] [5]. Then, we thought that 4.48870 is the deviation from 4.5. 3.13189 is the deviation from π.

Then, we noticed the following equations.

$$4.48867 = \sqrt{\frac{136 \times 4}{27}} \tag{14}$$

In Equation (14), when we believe that 4.48867 should be a deviation from 4.5,

$$4.48867 = 4.5 \times \sqrt{\frac{17}{\left(\frac{3}{2}\right)^7}} \tag{15}$$

Next, we persevered for half a year and discovered the following equation.

$$3.13217(\Omega) = \frac{Rk}{(3 \times 136)^{1.5}} \tag{16}$$

In Equation (16), when we believe that 3.13217 should be a deviation from π,

$$\pi = 2\left(1+\frac{1}{3}\left(1+\frac{2}{5}\left(1+\frac{3}{7}\left(1+\frac{4}{9}\left(1+\frac{5}{11}\left(1+\frac{6}{13}\right)\right)\right)\right)\right)\right) = 3.132156732 \tag{17}$$

For convenience, Equation (7) is rewritten as follows:

$$\frac{m_p}{m_e} = \frac{1}{4.4887 \times 3.13189}\frac{q_m}{e} = \frac{1}{14.05809432}Rk \tag{18}$$

where 14.5809432 (Ω) is the resistance. Using new values of 4.48867 and 3.13217,

$$3.13217(\Omega) \times 4.48867 = 14.05926332(\Omega) \tag{19}$$

Therefore, the error between Equations (18) and (19) is

$$\text{Error} = \frac{14.05926332}{14.05809432} - 1 = 8.31552 \times 10^{-5} \tag{20}$$

In Equation (20), the error is too large. In Equation (18), we use 14.05926332 instead of 14.05809432.

$$\frac{m_p}{m_e} = \frac{1}{14.05926332}\frac{q_m}{e} = \frac{25812.80746}{14.05926332} = 1836 \neq 1836.1526 \tag{21}$$

The ratio between $m_p$ and $m_e$ becomes smaller than the measured value. This problem can be solved using any other values instead of 136 in Equations (14) and (16), which will be explained in a later section.

### 3.2. Examination of the Relationship among 4.48866, 3.13217 and *Rk*

The relationship among 4.48866, 3.13217 and *Rk* is examined in this section. For convenience, Equations (14) and (16) are rewritten below:

$$4.48867 = \sqrt{\frac{136 \times 4}{27}} \tag{22}$$

$$3.13217(\Omega) = \frac{Rk}{(3 \times 136)^{1.5}} \tag{23}$$

From Equations (22) and (23),

$$Rk = 3.13217 \times 4.48867 \times 13.5 \times 136 \tag{24}$$

Equation (24) can be deduced from Equation (21), directly. Again, from Equations (22) and (23),

$$\frac{3.13217}{4.48867} = \frac{\dfrac{Rk}{(3 \times 136)^{1.5}}}{\sqrt{\dfrac{136 \times 4}{27}}} = \frac{Rk}{(136)^2} \times \frac{1}{2} = 0.697794 \tag{25}$$

Therefore,

$$Rk = 2 \times \frac{3.13217}{4.48867} \times (136)^2 \tag{26}$$

In Equation (26), *Rk* can be explained by 136. From Equation (22),

$$136 = \frac{13.5}{2} \times (4.48867)^2 \tag{27}$$

In Equation (27), 136 can be explained by 13.5. From Equation (23),

$$Rk = 3.13217 \times (3 \times 136)^{1.5} \tag{28}$$

In Equation (28), *Rk* can be explained by 3 and 136. From Equations (27) and (28),

$$Rk = 3.13217 \times \left(3 \times \frac{13.5}{2} \times (4.48867)^2\right)^{1.5} \tag{29}$$

In Equation (29), *Rk* can be explained by 3 and 13.5. Therefore,

$$Rk = \left(\frac{9}{2}\right)^3 \times 3.13217 \times (4.48867)^3 \tag{30}$$

Consequently, we proposed the four equations (Equations (24), (26), (28) and (30)) to explain *Rk*. The key numbers are 3, 13.5 and 136.

### 3.3. Equations (22) and (23) as a Function of the Value 136

We believe that Equations (22) and (23) are functions of 136. Then, we propose the following equations. The significant digits should be more than ten to explain the following equations.

$$4.488855463 = \sqrt{\frac{136.0113077 \times 4}{27}} \tag{31}$$

$$3.131777037 = \frac{Rk}{(3 \times 136.0113077)^{1.5}} \tag{32}$$

From Equations (31) and (32),

$$Rk = 3.131777037 \times 4.488855463 \times 13.5 \times 136.0113077 \tag{33}$$

Again, from Equations (31) and (32),

$$Rk = 2 \times \frac{3.131777037}{4.488855463} \times (136.0113077)^2 \tag{34}$$

From Equation (31),

$$136.0113077 = \frac{13.5}{2} \times (4.488855463)^2 \tag{35}$$

From Equation (32),

$$Rk = 3.131777037 \times (3 \times 136.0113077)^{1.5} \tag{36}$$

From Equations (35) and (36),

$$Rk = \left(\frac{9}{2}\right)^3 \times 3.131777037 \times (4.488855463)^3 \tag{37}$$

Consequently, we propose four equations (Equations (33), (34), (36) and (37)) to calculate $Rk$. Furthermore, 1836.152654 in Equation (11) can be explained. The key numbers are 3, 13.5 and 136.0113077. Thus, Equation (10) can be explained.

### 3.4. Relationship between $Rk$ and $Z_0$

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 2\alpha \times Rk \tag{38}$$

For example, from Equations (34) and (38),

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{4 \times 3.131777037}{4.488855463} \times \frac{(136.0113077)^2}{137.0359991} = 376.7303137 \tag{39}$$

However, Equation (10) explaining $\alpha$ is too complex to calculate $Z_0$. Therefore, an explanation for the electric constant and the magnetic constant will be published in future reports.

## 4. Explanation for Our Three Refined Empirical Equations

### 4.1. The Problem for the Measured Values of $G$ and $T_c$

The significant figures of the measured $T_c$ are only 3 or 4 in the present study. For the value of $T_c$, we use 2.72652 K instead of 2.72548 K.

https://en.wikipedia.org/wiki/Cosmic_microwave_background.

The measured public value of $G$ was often changed. We strongly propose that the value of $G$ should be measured in space. On the ground, the value of $G$ is influenced by the centrifugal force due to the rotation of the earth. We use $6.674777 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$ instead of $6.6743 \times 10^{-11}\ \mathrm{m^3 \cdot kg^{-1} \cdot s^{-2}}$.

https://en.wikipedia.org/wiki/Gravitational_constant.

### 4.2. Our Three Refined Empirical Equations

Equation (1) is refined as follows:

$$\frac{Gm_p^2}{hc} = \frac{4.488855}{2}\frac{kT_c}{1\,\mathrm{kg} \times c^2} \tag{40}$$

$$\frac{Gm_p^2}{hc} = \frac{6.674777 \times 10^{-11} \times \left(1.6726219 \times 10^{-27}\right)^2}{6.626070 \times 10^{-34} \times 2.9979246 \times 10^8} = 9.400600 \times 10^{-40} \tag{41}$$

$$\frac{4.488855}{2}\frac{kT_c}{1\,\mathrm{kg} \times c^2} = \frac{4.488855}{2} \times \frac{1.3806490 \times 10^{-23} \times 2.726516}{\left(2.9979246 \times 10^8\right)^2} = 9.400600 \times 10^{-40} \tag{42}$$

Equation (41) is equal to Equation (42). Therefore, the compensation method is perfect.

Next, Equation (2) is refined as follows:

$$\frac{Gm_p^2}{\left(\frac{e^2}{4\pi\varepsilon_0}\right)} = \frac{4.488855}{2 \times 3.131777} \times \frac{m_e}{e} \times hc \times \left(1\frac{\mathrm{C}}{\mathrm{J \cdot m}} \times \frac{1}{1\,\mathrm{kg}}\right) \tag{43}$$

$$\frac{Gm_p^2}{\frac{e^2}{4\pi\varepsilon_0}} = \frac{6.674777 \times 10^{-11} \times \left(1.672621923 \times 10^{-27}\right)^2}{\frac{\left(1.60217663 \times 10^{-19}\right)^2}{4\pi \times 8.8541878 \times 10^{-12}}} = 8.094129 \times 10^{-37} \tag{44}$$

$$hc = 6.626070 \times 10^{-34} \times 2.9979246 \times 10^8 = 1.986446 \times 10^{-25} \tag{45}$$

$$\frac{4.488855}{2 \times 3.131777} \times \frac{m_e}{e} \times hc = \frac{4.488855 \times 9.10938 \times 10^{-31} \times 1.986446 \times 10^{-25}}{2 \times 3.131777 \times 1.602177 \times 10^{-19}} = 8.094129 \times 10^{-37} \tag{46}$$

Equation (44) is equal to Equation (46). Therefore, the compensation method is perfect.

Next, Equation (3) is refined as follows:

$$\frac{m_e c^2}{e} \times \left(\frac{e^2}{4\pi\varepsilon_0}\right) = 3.131777 \times kT_c \times \left(1\frac{\mathrm{J \cdot m}}{\mathrm{C}}\right) \tag{47}$$

$$\frac{m_e c^2}{e} \times \left(\frac{e^2}{4\pi\varepsilon_0}\right) = \frac{9.10938 \times 10^{-31} \times \left(2.9979246 \times 10^8\right)^2 \times \left(1.60217663 \times 10^{-19}\right)^2}{1.60217663 \times 10^{-19} \times 4\pi \times 8.8541878 \times 10^{-12}} = 1.1789142 \times 10^{-22} \tag{48}$$

$$\begin{aligned}3.131777\times kT_c &= 3.131777\times 1.3806490\times 10^{-23}\times 2.726516\\ &= 1.1789142\times 10^{-22}\end{aligned} \tag{49}$$

Equation (48) is equal to Equation (49). Therefore, our refined empirical Equations (40), (43) and (47) are perfect. However, we use the compensated value for $T_c$ and $G$, which is slightly different from the measured value.

## 5. Discussion

The main purpose of this report is to show empirical equations. However, without any theories, this report can appear to be nothing more than numerology. Therefore, we wrote this section. Our arguments strongly depend on numerical connections rather than logical connections. Therefore, our discussions appear to be different from the majority's opinion. This does not mean that we ignore the majority's opinions. Using simple unknown ideas, our opinions may be compatible with the majority's opinion.

### 5.1. Consider the Value of 136.0113077

The value of 136.0113077 is related to the mass ratio between some particle and an electron. The rest mass of a pion ($\pi^+$) is 139.57 MeV. The mass of an electron is 0.51100 MeV. Therefore,

$$\frac{139.57\ \text{MeV}}{0.51100\ \text{MeV}} = 2\times 136.57 \approx 2\times 136.011+1 \tag{50}$$

When the pion consists of two quarks and an electron, the value of 136.011 is the mass ratio between a quark and an electron. This conclusion seems to disprove the concept of gluon. However, we notice that the concept of a gluon is useful for the mass ratio between a proton and an electron.

### 5.2. Consideration for the Value of 1836.152654

The value of 1836.152654 is related to the mass ratio between a proton and an electron.

$$\frac{m_p}{m_e} = 1836.152654 = 4.5\times 3\times 136.0113077 \tag{51}$$

When a proton consists of three quarks and the value of 136.011 is the mass ratio between a quark and an electron, there remains the mystery about the factor of 4.5. We believe that the factor of 4.5 derives from the degrees of freedom of the thermal energy due to the combined three quarks. Then, we must explain the value of 3.5 (=4.5 − 1).

According to Albert Einstein, $E = mc^2$. Therefore, the thermal energy should be related to the weightness. A hot mass should be heavier than a cold mass. Thus, we must consider what happens concerning the degree of freedom.

When the boundary between thermal energy and the rest mass energy becomes unclear, 77% (=3.5/4.5) of the rest mass energy related to the thermal energy from the degree of freedom seems to be binding energy. This concept of binding energy may be compatible with the concept of gluons.

### 5.3. Consideration of the Theoretical Meaning of Equation (10)

The theory of loop quantum gravity is related to the theory of the lattice structure in space time. In the area of electrochemistry, the theory of vacancies in the lattice structure is governed by Wagner's equation. The fine-structure constant is the interference constant. When the interference constant for electrons is $\alpha$, the transference number for electrons should be $1 - \alpha$. Then, the transference of quarks is $\alpha$, which explains the strong interaction [6]. According to the advanced Wagner model, the diffusion time for mixed electronic and ionic currents should decrease exponentially with distance [7]. When the diffusion time for mixed electrons and quark flux should decrease exponentially with distance, the Yukawa potential can be explained.

For convenience, Equation (10) is rewritten as follows:

$$\alpha = \frac{1}{\left(1+\frac{1}{1836.152654\times 3}\right)\times\frac{1836.152654}{13.5}+1} = \frac{1}{137.0359991} \tag{52}$$

Therefore,

$$\frac{1}{\alpha} = \left(1+\frac{1}{\frac{m_p}{m_e}\times 3}\right)\times\frac{\frac{m_p}{m_e}}{13.5}+1 = \frac{\left(\frac{m_p}{m_e}\times 3+1\right)\times\frac{m_p}{m_e}}{\frac{m_p}{m_e}\times 3\times 13.5}+1 = \frac{\left(\frac{m_p}{m_e}\times 3+1\right)}{3\times 13.5}+1 \tag{53}$$

So,

$$\frac{1}{\alpha} = \frac{1}{13.5}\times\frac{m_p}{m_e}+\frac{1}{3\times 13.5}+1 \tag{54}$$

From Equations (11) and (35),

$$\frac{1}{13.5}\times\frac{m_p}{m_e} = \frac{1836.152673}{13.5} = 136.0113077 = \frac{13.5}{2}\times(4.488855463)^2 \tag{55}$$

Therefore, from Equations (54) and (55),

$$\frac{1}{\alpha} = \frac{13.5}{2}\times(4.488855463)^2+\frac{1}{3\times 13.5}+1 \tag{56}$$

Equation (56) is one of the variations of Equation (52). From Equation (56), we can justify that the value of 4.488855463 is not coincidental.

### 5.4. Consideration of the Degree of Freedom inside Electrons

In Equation (47), the number 3.131777037 is the deviation from π. The degree of freedom inside an electron seems to be 2π. We believe that 2π is related to the spin of the electron. Angrick *et al.* confirmed that the spin of electrons cannot be ignored thermodynamically [8]. Furthermore, Aquino *et al.* discovered a new method for vector analysis [9]. We hope that the degrees of freedom in electrons will be clarified experimentally in detail.

## 6. Conclusions

Previously, we proposed several empirical equations to describe the relationship between an electromagnetic force and $T_c$ [1] [2] [3]. Three equations were explained by the factors of 9/2 and π. However, these equations have small errors of approximately $10^{-3}$ and $10^{-4}$ [4] [5].

We tried to improve the accuracies by changing the value of 9/2 and π. Then, we persevered and discovered the following two equations.

$$4.48867 = \sqrt{\frac{136 \times 4}{27}}$$

$$3.13217(\Omega) = \frac{Rk}{(3 \times 136)^{1.5}}$$

Then, the above two equations were the function of 136. Using the value of 136.0113007,

$$3.131777037(\Omega) = \frac{Rk}{(3 \times 136.0113077)^{1.5}}$$

$$4.488855463 = \sqrt{\frac{136.0113077 \times 4}{27}}$$

Then, $Rk$ can be calculated as follows.

$$Rk = 3.131777037 \times 4.488855463 \times 13.5 \times 136.0113077$$

$$\frac{m_p}{m_e} = 1836.152654 = 13.5 \times 136.0113077$$

Then, the empirical equation for the fine-structure constant with very high accuracy is

$$\alpha = \frac{1}{\left(1 + \frac{1}{1836.152654 \times 3}\right) \times \frac{1836.152654}{13.5} + 1} = \frac{1}{137.0359991}.$$

Avoiding numerology, we discuss this equation. Then, we can justify that the value of 4.488855463 is not coincidental. The explanation for the electric constant and the magnetic constant will be published in future reports.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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