Scientific Research Publishing

Journal of Modern Physics, 2021, 12, 1656-1707
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

# A New Physics Would Explain What Looks Like an Irreconcilable Tension between the Values of Hubble Constants and Allows $H_0$ to Be Calculated Theoretically Several Ways

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**How to cite this paper:** Mercier, C. (2021) A New Physics Would Explain What Looks Like an Irreconcilable Tension between the Values of Hubble Constants and Allows $H_0$ to Be Calculated Theoretically Several Ways. *Journal of Modern Physics*, **12**, 1656-1707.
https://doi.org/10.4236/jmp.2021.1212098

**Received:** September 1, 2021
**Accepted:** October 23, 2021
**Published:** October 26, 2021



## Abstract

Observing galaxies receding from each other, Hubble found the universe's expansion in 1929. His law that gives the receding speed as a function of distance implies a factor called Hubble constant $H_0$. We want to validate our theoretical value of $H_0 \approx 72.09548580(32)$ km·s$^{-1}$·MParsec$^{-1}$ with a new cosmological model found in 2019. This model predicts what may look like two possible values of $H_0$. According to this model, the correct equation of the apparent age of the universe gives ~ 14.14 billion years. In approximation, we get the well-known equation $1/H_0 \approx 13.56$ billion years. When we force these ages to fit the $1/H_0$ formula, it gives two different Hubble constant values of ~69.2 and 72.1 km·s$^{-1}$·MParsec$^{-1}$. When we apply a theoretical correction factor of $\eta \approx 1.042516951$ on the first value, both target the second one. We found 42 equations of $H_0$ linking different physics constants. Some are used to measure $H_0$ as a function of the average temperature $T$ of the Cosmological Microwave Background and the universal gravitational constant $G$:

$H_0 \approx 72.06(90)$ km·s$^{-1}$·MParsec$^{-1}$ from $T$ by Cobra probe & Equation (16)
$H_0 \approx 71.95(50)$ km·s$^{-1}$·MParsec$^{-1}$ from $T$ by Partridge & Equation (16)
$H_0 \approx 72.086(36)$ km·s$^{-1}$·MParsec$^{-1}$ from $G$ & Equation (34)
$H_0 \approx 72.105(36)$ km·s$^{-1}$·MParsec$^{-1}$ from $G$ & Equations (74), (75), or (76).

With 508 published values, $H_0 \approx 72.0957 \pm 0.33$ km·s$^{-1}$·MParsec$^{-1}$ seems to be the "ideal" statistical result. It validates our model and our theoretical $H_0$ value which are useful to find various interactions with the different constants. Our model also explains the ambiguity between the different universe's age measurements and seems to unlock a tension between two $H_0$ values.

## Keywords

Hubble Constant $H_0$, Hubble Tension, Age of the Universe

## 1. Introduction

In astrophysics, the Hubble constant $H_0$ [1] is a parameter to analyze the universe. Nevertheless, it is also one of the lesser-known values.

In 1916, Einstein found the general relativity laws [2]. His equations expect that the universe is either expanding or in a Big Crunch. He could have been the first to predict the universe's expansion, but influenced by the popular idea, Einstein forced his model to be static with a cosmological constant Λ. In 1922, Friedmann showed from relativity that the universe expands at a calculable rate [3]. In 1927, Georges Lemaitre published independent research [4], giving what is now known as Hubble's law. In 1929, Hubble discovered the universe's expansion [1]. Equation (1) gives Hubble's law, with $v$ being the receding speed in $km \cdot s^{-1}$, $D$ being the distance between the observed object and the observer, and $H_0$ being the Hubble constant. He measured about $H_0 \approx 500\ km \cdot s^{-1} \cdot MParsec^{-1}$. His high value was due to a wrong calibration of the cepheids used to evaluate distances. Hubble's law was correct, but $H_0$ was remaining to be found with accuracy.

$$v = DH_0 \tag{1}$$

Physicists get $H_0$ based on far cosmic objects (Cepheids, supernovae, red giants, etc.) or local measurements (CMB, universal gravitational constant $G$, etc.). Including error margins of published values (see the software in **Annex A**), $H_0$ is between 19 to 174 $km \cdot s^{-1} \cdot MParsec^{-1}$. However, two values are often measured ~69.2 and ~72.1 $km \cdot s^{-1} \cdot MParsec^{-1}$. An irreconcilable tension between some $H_0$ values shows up [5]. Even with good accuracies, their error margins do not always overlap. It may let us think that only one of these values is right. No one considered it possible that both values may be in some way correct.

In 2019, we wrote an article [6] explaining what may look like two values for $H_0$. We calculated the universe age, obtained a result of complex type, and an apparent age of the universe of ~14.4 billion years. The complete equation may be approximated by $1/H_0$, giving ~13.56 billion years. We notice that there is a difference of ~4.25% between the approximated and the non-approximated values.

Cosmologists use $1/H_0$ to calculate the universe's age. Thus, if we could measure the apparent age of the universe with no approximation, we would conclude wrongly that the Hubble constant is ~4.25% lower than it should be.

We hypothesize that two values of $H_0$ are somehow obtained from an approximated and non-approximated equation of the apparent age of the universe. The confusion leads to a tension between two values when there should be only one.

We summarize our cosmological model [6] to get $H_0$ as a function of $a$, $c$, and $r_e$. We found ways to measure $H_0$ locally by using the Cosmological Microwave Background (CMB) temperature $T$ and by using the universal gravitational constant $G$ [6] [7]. Based on our model, we found a theoretical equation to calculate $H_0$ from CODATA values (Committee of Data for Science and Technology) [8].

$$H_0 = \frac{c\alpha^{19}\sqrt{\beta}}{r_e} \approx 72.09548580(32)\,\mathrm{km}\cdot\mathrm{s}^{-1}\cdot\mathrm{MParsec}^{-1} \quad \text{where } \beta = 3-\sqrt{5} \approx 0.76 \tag{2}$$

We want to validate this theoretical value of $H_0$ and highlight the tension between two measured values of $H_0$. We list the results of the most recent measures of $H_0$ and build a graph showing someway the popularity of each $H_0$ value range.

We list 42 $H_0$ equations. Certain overcome the difficulties to do experimental measurements. We use one of them as a third measurement of $H_0$. Our cosmological model shows that $H_0$ and the speed of light are not constant.

## 2. Physics Parameters

A compact form of notation is used to display tolerances (*i.e.*, 2.734(10) K means 2.734 ± 0.010 K). The CODATA 2014 [8] is used to compare the results of our new equations with the articles published in 2019 and 2020.

| | |
|---|---|
| Light speed in a vacuum | $c = 299792458\ \mathrm{m}\cdot\mathrm{s}^{-1}$ |
| Permeability of free space | $\mu_0 = 4\pi\times10^{-7}\ \mathrm{NA}^{-2}$ |
| Permittivity of free space | $\varepsilon_0 \approx 8.854187817\times10^{-12}\ \mathrm{F}\cdot\mathrm{m}^{-1}$ |
| Universal gravitational constant | $G \approx 6.67408(31)\times10^{-11}\ \mathrm{m}^3\cdot\mathrm{kg}^{-1}\cdot\mathrm{s}^{-2}$ |
| Electron rest mass | $m_e \approx 9.10938356(11)\times10^{-31}\,\mathrm{kg}$ |
| Classical electron radius | $r_e \approx 2.8179403227(19)\times10^{-15}\,\mathrm{m}$ |
| Electron charge | $q_e \approx -1.6021766208(98)\times10^{-19}\ \mathrm{C}$ |
| Planck length | $L_p \approx 1.616229(38)\times10^{-35}\,\mathrm{m}$ |
| Planck time | $t_p \approx 5.39116(13)\times10^{-44}\,\mathrm{s}$ |
| Planck mass | $m_p \approx 2.176470(51)\times10^{-8}\ \mathrm{kg}$ |
| Planck constant | $h \approx 6.626070040(81)\times10^{-34}\ \mathrm{J}\cdot\mathrm{s}$ |
| Fine-structure constant | $\alpha \approx 7.2973525664(17)\times10^{-3}$ |
| Boltzmann constant | $k_b \approx 1.38064852(79)\times10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}$ |
| Rydberg constant | $R_\infty \approx 10973731.568508(65)\ \mathrm{m}^{-1}$ |

## 3. Summary of our Theory

Our theory is based on a cosmological model officially shown in 2019 [6], but it summarizes papers we wrote at www.pragtec.com/physique since 2011. First, we outline some main milestones as we did in 2020 [7].

### 3.1. Our Cosmological Model

We hypothesize that there was one expanding sphere containing all matter at the Big Bang. There was no light. After ~360000 years [9], electrons became free to move because of a lower density universe, and the light appeared and began to travel through space, creating a 4-D expanding sphere called the "luminous universe". As the matter cannot travel as fast as light [10], it created a smaller 4-D expanding sphere, the "material universe", imbricated in the "luminous universe".

Einstein found that the presence of a massive object reduces the speed of light $v_L$ [11]. Schwarzschild calculated $v_L$ in a context of a weak gravitational field $\Phi$ using general relativity [12]. With $|\Phi| << c^2$ around a spherical mass, Equation (3) gives $v_L$ as a function of $c$ and a local refractive index $n_0$ (function of $G$ [13]).

$$v_L(r) = \frac{c}{n_0} \quad \text{where} \quad n_0 = \sqrt{\frac{1-2\Phi/c^2}{1+2\Phi/c^2}} \quad \text{and} \quad \Phi = \frac{-Gm}{r} \le 0 \tag{3}$$

From an observer on Earth, $c$ seems constant. However, the knowledge of a precise value of $c$ dates only from 19 century [14]. In 1929, Edwin Hubble found that the universe is expanding [1]. As the apparent universe radius increases, the density of this latest must decrease over time, causing the refractive index of the vacuum to drop. As a result, it causes light to accelerate slowly.

In Equation (3), $c$ is the local speed limit for light in a vacuum in our universe area. Admitting that light accelerates while the universe expands, it will tend towards another asymptotical speed limit $k$ affected by a local refractive index $n$. For now, $k$ is unknown. Let us build Equation (4), which is analog to Equation (3) for the universe [2]. Our universe parcel is at a distance $r_u$ from the universe's apparent mass center $m_u$. The local speed of light $c$ results from Equation (4).

$$c = \frac{k}{n} \quad \text{where} \quad n = \sqrt{\frac{1-2\Theta/k^2}{1+2\Theta/k^2}} \quad \text{and} \quad \Theta = \frac{-Gm_u}{r_u} \le 0 \tag{4}$$

Similarly to $r_u$, the $R_u$ value is the apparent radius of curvature of the luminous universe [6] [15] (also called Hubble radius [16]). It is a function of $c$ and $H_0$. It is "apparent" since Equation (5) assumes $c$ constant for a time equal to the universe's age. Now, its speed is $c$, but it is not constant in our model [6]. It was lower in the past and will increase while the universe expands. The $H_0$ value represents the expansion rate of the material universe in km·s$^{-1}$·MParsec$^{-1}$ [1]. It is the local derivative of the velocity of matter $v_m$ with respect to the element of distance d$r$.

$$H_0 = \left.\frac{\mathrm{d}v_m}{\mathrm{d}r}\right|_{r=r_u} = \frac{\beta c}{r_u} = \frac{\beta c}{\beta R_u} \Rightarrow R_u = \frac{c}{H_0} \tag{5}$$

Locally, at a distance $r = r_u$, matter recedes radially from the center of mass of the universe at a rate $\beta$ times slower than the speed of light $c$.

$$r_u = \beta R_u = \frac{\beta c}{H_0} \tag{6}$$

The apparent mass $m_u$ of the universe is given by Equation (7) [15] [17]:

$$m_u = \frac{c^3}{GH_0} = \frac{R_u c^2}{G} \tag{7}$$

Our universe parcel is at a distance $r_u$ from the center of the mass $m_u$. It travels at a speed $v_m$ relative to this latest. The ratio $\beta$ is the asymptotical speed of light $k$ in a vacuum (when $R_u$ tends towards infinity) influenced by a refractive index $n$ that is itself influenced by a gravitational potential $\Theta$.

$$v_m = \frac{\beta k}{n} \quad \text{where} \quad n = \sqrt{\frac{1-2\Theta/k^2}{1+2\Theta/k^2}} \quad \text{and} \quad \Theta = \frac{-Gm_u}{r_u} \tag{8}$$

Hubble measured $H_0$ from the global movement of galaxies at our location [1], at $r_u$. They have their own movement. As the universe expands, they are generally moving away from each other. The derivative of the material universe speed $v_m$ according to the element of distance d$r$ evaluated at $r = r_u$ is $H_0$ [6].

$$H_0 = \left.\frac{\mathrm{d}v_m}{\mathrm{d}r}\right|_{r=r_u} = \frac{k\beta y}{r_u}\left(\frac{1}{(1+y)\sqrt{1-y^2}}\right) \quad \text{where} \quad y = \frac{2Gm_u}{k^2 r_u} \tag{9}$$

Solving Equations (4) to (7), and (9) gives Equations (10) to (14) [6]. The expending speed ratio $\beta$ between the material and the luminous universes is geometric. It is also the ratio between $r_u$ and $R_u$. It is unique to our model and essential to depict many constants and make links between the infinitely large and small in the Dirac hypothesis on large numbers [18] [19].

$$k = c\sqrt{2+\sqrt{5}} \approx 6.17\times10^8 \text{ m}\cdot\text{s}^{-1} \tag{10}$$

$$\beta = 3-\sqrt{5} \approx 0.764 \tag{11}$$

$$R_u \approx 1.28\times10^{26} \text{ m} \tag{12}$$

$$r_u \approx 9.80\times10^{25} \text{ m} \tag{13}$$

$$m_u \approx 1.73\times10^{53} \text{ kg} \tag{14}$$

### 3.2. Our First Method to Measure $H_0$ as a Function of $T$ (from CMB)

The accuracies of $m_u$, $r_u$, and $R_u$ widely depend on $H_0$ which could be between 19 and 174 km·s$^{-1}$·MParsec$^{-1}$ (listed in the software in **Annex A**). Therefore, a better method of measuring $H_0$ is required to know $m_u$, $r_u$, and $R_u$ more accurately.

We calculated the CMB temperature $T$ as a function of $H_0$ and $G$ [6]. This equation considers the universe as an ideal black body since it would absorb any incident radiation coming from outside, and it does not reflect or transmit any form of energy outside of the luminous universe (since it expands at the speed of light).

$$T = \frac{\beta}{k_b}\left(\frac{15\alpha^2 h^3 c^5 H_0^2}{8\pi^6 G}\right)^{1/4} \tag{15}$$

Let us isolate $H_0$ from Equation (15). The accuracy mainly depends on the CMB temperature $T$. Using $T \approx 2.736(17)$ K (from Cobra probe [20]), we get.

$$H_0(T) = \frac{\pi^3 T^2 k_b^2}{\beta^2 \alpha}\sqrt{\frac{8G}{15c^5 h^3}} \approx 72.06(90) \text{ km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1} \tag{16}$$

with Partridge $T \approx 2.734(10)$ K, and $H_0 \approx 71.95(50)$ km·s$^{-1}$·MParsec$^{-1}$ [21]. As the least accurate value is $T$, Equation (16) measures $H_0$ from the CMB temperature. These values lead to new links and are in our software in **Annex A**.

### 3.3. Dirac Hypothesis about Large Numbers

Dirac found (inaccurately) that large numbers come into a few orders of magnitude with same dimensions quantities ratios [18] [19]. All ratios come from $N$, via certain factors [22]. It represents the maximum number of photons in the universe. We get the highest number when the associated mass $m_{ph}$ of a photon is the smallest. This happens when the energy of the photons is at its lowest and with a wavelength of the same length as the circumference of the luminous universe (*i.e.*, $2\pi R_u$) [6]. Let us calculate $m_{ph}$ by equating its corpuscular and wave energies.

$$m_{ph}c^2 = \frac{hc}{2\pi R_u} \quad \Rightarrow \quad m_{ph} = \frac{h}{2\pi R_u c} \approx 2.74\times 10^{-69}\ \text{kg} \tag{17}$$

We get $N$ by dividing the apparent mass $m_u$ of the universe (Equation (7)) by the mass $m_{ph}$ associated with a photon of $2\pi R_u$ wavelength (Equation (17)).

$$N = \frac{m_u}{m_{ph}} = \frac{2\pi c^5}{hGH_0^2} \approx 6.3018(62)\times 10^{121} \tag{18}$$

If we try to make a precise evaluation of $N$ by using the Equations (6), (7), (16), and (17), we obtain Equation (19) which is dependent mainly on $T$. We evaluate the result by using the CODATA 2014 [8] and the average CMB temperature from Cobra probe [20]. Finally, we note that $N$ is dimensionless as $\alpha$.

$$N = \frac{15h^2\alpha^2\beta^4c^{10}}{4\pi^5G^2k_b^4T^4} \approx 6.31(15)\times 10^{121} \tag{19}$$

Assuming $\alpha$ used as a scale factor applied a few times, we postulate Equation (20). It seems impossible to get this equation from standard physics [2].

$$\text{POSTULATE \#1:} \qquad N = 1/\alpha^{57} \approx 6.303419702(84)\times 10^{121} \tag{20}$$

In the next formulas, Planck temperature is $T_p \approx 1.42\times 10^{32}$ K. This is the highest temperature reached at the Big Bang. It happens when we put the entire mass $m_u$ in a point-like pellet of Planck length radius $L_p$. Planck charge is given by $q_p \approx 1.88\times 10^{-18}$ C.

"Large" numbers are obtained with $N$ exponent a fraction, such as $N^{1/2}$, $N^{1/3}$, $N^{1/4}$, … $N^{1/57}$, etc. We get these in different ways by using various parameters of the universe [2]. They are always unitless. Some come from Dirac's hypothesis on large numbers [18] [19]. Some links will be used later [6].

$$N^{2/3} = \frac{m_u\alpha}{m_e\beta^{1/2}} = \frac{R_u^2\beta}{r_e^2} = \frac{m_p^4\alpha^4}{m_e^4\beta^2} = \frac{m_e^2\beta}{m_{ph}^2\alpha^2} \approx 1.58\times 10^{81} \tag{21}$$

$$N^{1/2} = \frac{m_p}{m_{ph}} = \frac{R_u}{L_p} = \frac{1}{t_pH_0} = \frac{2\pi T_pk_b}{hH_0} = \frac{-1}{q_e}\sqrt{\frac{4\pi m_uR_u\alpha}{\mu_0}} \approx 7.94\times 10^{60} \tag{22}$$

$$N^{1/3} = \frac{m_ur_e\alpha}{m_eR_u\beta} = \frac{m_e\sqrt{\beta}}{m_{ph}\alpha} = \frac{R_u\sqrt{\beta}}{r_e} = \frac{\alpha q_e^2}{4\pi\varepsilon_0G\beta m_e^2} \approx 3.99\times 10^{40} \tag{23}$$

$$N^{1/4} = \frac{T_p}{T}\left(\frac{15\beta^4\alpha^2}{\pi^3}\right)^{1/4} = \frac{k_bT}{m_{ph}c^2}\left(\frac{\pi^3}{15\beta^4\alpha^2}\right)^{1/4} \approx 2.82\times 10^{30} \tag{24}$$

$$N^{1/6}=\frac{r_e}{L_p\sqrt{\beta}}=\frac{m_p\alpha}{m_e\sqrt{\beta}}=\frac{\alpha^3}{4\pi R_\infty L_p\sqrt{\beta}}=\frac{2\pi r_e k_b T_p}{hc\sqrt{\beta}}\approx 1.99\times10^{20} \tag{25}$$

$$N^{1/16}=\left(\frac{4\pi c R_\infty\sqrt{\beta}}{H_0}\right)^{57/256}=\left(\frac{\beta T_p}{T}\right)^{1/4}\left(\frac{15\alpha^2}{\pi^3}\right)^{1/16}\approx 4.10\times10^{7} \tag{26}$$

$$N^{1/19}=\frac{1}{4\pi R_\infty r_e}=\left(\frac{\beta m_e^2}{m_{ph}^2}\right)^{1/12}=16\pi^2\beta L_p R_u R_\infty^2\sqrt{\alpha}\approx 2.57\times10^{6} \tag{27}$$

$$N^{1/57}=\frac{q_p^2}{q_e^2}=\left(\frac{m_p^2}{m_e^2\beta}\right)^{1/21}=\left(\frac{q_e^2}{4\pi\beta\varepsilon_0 G m_e^2}\right)^{1/20}=\frac{1}{\alpha}\approx 137 \tag{28}$$

In a non-published document [22], we show over 150 links that give $N$ with various parameters. The universe is well-linked between the infinitely large and the infinitely small. Almost everything changes while the universe is expanding.

### 3.4. Precise Calculation of $H_0$

Unlike Equation (16), we look for an equation that does not use $G$ and $T$ to get $H_0$ since they do not have good accuracies. Usually, $G$ intervenes in the calculations of gravitational force and energy. Without any details (see [6] [7]), let us calculate the electrical energy $E_e$ between two electrons separated by a space equal to the classical electron radius $r_e$. The electrical energy $E_e$ is not linked to the distance since we get $E_e=m_ec^2$. We evaluate the gravitational energy for the same conditions, finding $E_g=Gm_e^2/r_e$. If these experiments are done at the luminous universe periphery, we get an electrical energy $E'_e=E_e$ and a gravitational energy $E'_g=E_g/\beta$. The ratio between $E'_e$ and $E'_g$ gives Equation (29).

$$\frac{E'_e}{E'_g}=\frac{m_ec^2}{\left(\frac{Gm_e^2\beta}{r_e}\right)}=\frac{c^2r_e}{Gm_e\beta}\approx 5.45\times10^{42} \tag{29}$$

As in Equation (20), we realize that the fine-structure constant $\alpha$ plays a role in determining orders of magnitude. By adjusting the exponent of the fine-structure constant $\alpha$, we obtain a result identical to Equation (29).

$$\frac{1}{\alpha^{20}}\approx 5.45\times10^{42} \tag{30}$$

Equations (29) and (30) seam equal. By isolating $G$, we get an equation that becomes postulate #2. We cannot deduce this equation from standard physics.

$$\text{POSTULATE \#2:}\quad G=\frac{c^2r_e\alpha^{20}}{m_e\beta}\approx 6.673229809(86)\times10^{-11}\ \mathrm{m^3\cdot kg^{-1}\cdot s^{-2}} \quad \text{where } \beta=3-\sqrt{5} \tag{31}$$

We associate the wave energy with the energy of the electron mass $m_e$.

$$m_ec^2=\frac{hc\alpha}{2\pi r_e} \tag{32}$$

with Equations (20), (31), and (32), we get Equation (33).

$$H_0 = \frac{c\alpha^{19}\sqrt{\beta}}{r_e} \approx 72.09548580(32)\ \mathrm{km\cdot s^{-1}\cdot MParsec^{-1}} \tag{33}$$

This value is like Soltis with 72.1 ± 2.0 km·s$^{-1}$·MParsec$^{-1}$, Martinelli's with $72.1^{+2.1}_{-1.8}$ km·s$^{-1}$·MParsec$^{-1}$, and Salvatelli's with $72.1^{+3.2}_{-2.3}$ km·s$^{-1}$·MParsec$^{-1}$ (see the list of value in our software in **Annex A**). Our theoretical value seems to make sense.

### 3.5. Our Second Method to Measure $H_0$ as a Function of $G$

We want to find a second way to measure $H_0$ as a function of $G$. We must use accurate parameters, such as $\alpha$ and the characteristics of the electron ($m_e$ and $r_e$). We look for an equation dependent on $G$ without any rational exponent that reduces the sensitivity. We can use Equations (31) and (33). From each of them, we isolate $r_e$, and we make both equal to get $H_0$. Since $G$ is the least precise value, Equation (34) evaluates $H_0$ as a function of $G$. We used CODATA 2014 values.

$$H_0 = \frac{c^3\alpha^{39}}{Gm_e\sqrt{\beta}} \approx 72.086(36)\ \mathrm{km\cdot s^{-1}\cdot MParsec^{-1}} \tag{34}$$

This result is about 25 times more precise than Equation (16) that uses the average CMB temperature $T$. We included this result in our software in **Annex A**.

## 4. Hubble Constant versus the Age of the Universe

We calculate the universe's age with our cosmological model to understand what seems to be two potential values of $H_0$. We measure $H_0$ by observing cosmological objects. Universe's age $\Delta t_u$ is of complex type and results from the integral of the inverse of the expanding speed of the material universe $v_m$ with the element of distance $dr$ evaluated between the universe's center of mass (at $r = 0$) and the apparent material universe radius of curvature at our location $r_u$.

$$\Delta t_u = \int_{r=0}^{r=r_u} \frac{1}{v_m(r)}\mathrm{d}r = \Delta t_{hu} + \Delta t_{0h} \tag{35}$$

The $\Delta t_{hu}$ is the time elapsed between the horizon ($r = r_h$) and here ($r = r_u$):

$$\Delta t_{hu} = \int_{r=r_h}^{r=r_u} \frac{1}{v_m(r)}\mathrm{d}r \quad (\text{Real type result}) \tag{36}$$

The $\Delta t_{0h}$ is the elapsed time between $r = 0$ and the horizon $r = r_h$:

$$\Delta t_{0h} = \int_{r=0}^{r=r_h} \frac{1}{v_m(r)}\mathrm{d}r \quad (\text{Imaginary type result}) \tag{37}$$

At the universe horizon $r = r_h$, the speed of light is zero. We cannot see beyond the horizon. The delay $\Delta t_{hu}$ is the time elapsed between the horizon $h$ and our actual position $r_u$ in the universe. The delay $\Delta t_{0h}$ is the time elapsed between the center of mass of the universe and the horizon $r_h$ (given by Equation (38)).

$$r_h = \frac{2Gm_u}{k^2} \tag{38}$$

Performing the integral calculation of Equation (35), we get Equation (39).

$$\int \frac{1}{v_m(r)} \mathrm{d}r = \frac{\left(z(r) + 2G \cdot m_u \ln\left(2\left[k^2 r + z(r)\right]\right)\right)}{\beta k^3} \quad \text{where } z(r) = \sqrt{k^4 r^2 - 4G^2 m_u^2} \tag{39}$$

We can decompose the age of the universe $\Delta t_u$ into two parts, $\Delta t_{hu}$ and $\Delta t_{0h}$. The value $\Delta t_{hu}$ represents the time elapsed between $r = r_h$ (at the horizon) and our actual position $r = r_u$ in the universe. The value $\Delta t_{0h}$ gives the time elapsed between $r = 0$ (at the Big Bang) and $r = r_h$ (at the horizon).

$$\Delta t_u = \Delta t_{hu} + \Delta t_{0h} \approx (9.50 + 10.47i) \times 10^9 \text{ years} \quad \text{where } i = \sqrt{-1} \tag{40}$$

The imaginary time $\Delta t_{0h}$ means that it elapses independently of our time. We cannot see an event between $r = 0$ and $r = r_h$, and an observer located between $r = 0$ and $r_h$ could not see us. The $\Delta t_{hu}$ equation is:

$$\Delta t_{hu} = \frac{1}{H_0}\left(\frac{\left(\omega + 2\ln\left[\omega + \beta\left(2+\sqrt{5}\right)\right] - \ln(4)\right)^2}{\sqrt{22+10\sqrt{5}}}\right) \approx \frac{7}{10H_0} \quad \text{where } \omega = \sqrt{\beta^2\left(9+4\sqrt{5}\right) - 4} \tag{41}$$

The precise equation for $\Delta t_{0h}$ is:

$$\Delta t_{0h} = \frac{-(2+\pi)}{H_0\sqrt{22+10\sqrt{5}}} i \approx \frac{-77}{100H_0} i \tag{42}$$

The modulus of the complex age $\Delta t_u$ gives the universe's apparent age $T_u$.

$$T_u = \left|\Delta t_u\right| = \left|\Delta_{hu} + \Delta t_{0h}\right| = \sqrt{\left(\Delta t_{hu}\right)^2 + \left(\Delta t_{0h} i\right)^2} \approx 14.14 \times 10^9 \text{ years} \tag{43}$$

$$T_u = \frac{1}{H_0} \underbrace{\sqrt{\left(\frac{\left(\omega + 2\ln\left[\omega + \beta\left(2+\sqrt{5}\right)\right] - \ln(4)\right)^2}{\sqrt{22+10\sqrt{5}}}\right)^2 + \left(\left[\frac{-(2+\pi)}{\sqrt{22+10\sqrt{5}}}\right] i\right)^2}}_{\eta \approx 1 \ \text{(with about 4.25 \% of error)}} \tag{44}$$

As the square root over the accolade is approximatively equal to 1, we get:

$$T_u \approx \frac{1}{H_0} \approx 13.56 \times 10^9 \text{ years} \tag{45}$$

The value of the correction factor between Equations (43) and (45) is $\eta$.

$$\eta = H_0 T_u = H_0 \sqrt{\left(\Delta t_{hu}\right)^2 + \left(\Delta t_{0h} i\right)^2} \approx 1.042516951 \tag{46}$$

This $\eta$ explains why scientists currently measure two values of $H_0$. Scientists can only size the apparent age of the universe with different techniques. They cannot measure the real part and the imaginary part of the universe's age.

There is no "local" or "far" value of $H_0$. There is only one $H_0$. Some techniques give $H_0$ directly, and others need a correction factor. There is no need for any correction factor when $H_0$ is calculated from Equation (33), measured with the

CMB temperature with Equation (16), or with the universal gravitational constant $G$ with Equation (34). Other techniques may get similar results than Equation (43), and if we impose that value to fit with Equation (45), we get $H_0'$.

$$H_0' = \frac{1}{|\Delta t_u|} \approx \frac{1}{14.14\times10^9 \text{ years}} \approx \frac{H_0}{\eta} \approx 69.2 \text{ km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1} \tag{47}$$

However, Equation (45) gives the actual $H_0$ value:

$$H_0 \approx \frac{1}{13.56\times10^9 \text{ years}} \approx 72.1 \text{ km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1} \tag{48}$$

If scientists could measure the real part of the universe's age and associate this value with $1/H_0$, they would obtain the following value.

$$H_0 \approx \frac{1}{9.50\times10^9 \text{ years}} \approx 102.94 \text{ km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1} \tag{49}$$

If scientists could measure the imaginary part somehow, the association of this value with $1/H_0$ (like in Equation (45)) would give the following $H_0$ value.

$$H_0 \approx \frac{1}{10.47\times10^9 \text{ years}} \approx 93.39 \text{ km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1} \tag{50}$$

with different types of experiments to measure the apparent age of the universe, scientists usually get either $\sim H_0 \approx 69.2$ or $\sim 72.1$ km·s$^{-1}$·Mparsec$^{-1}$. We assume that all calibration factors are used. New techniques could require other unknown corrective factors that have nothing to do with the related phenomenon.

The articles rarely give enough details to check if the process used needs $\eta$. Scientists must verify if the $\eta$ factor is required for their approach.

## 5. Other Experimental Measurements of Hubble Constant $H_0$

In 1929, Hubble made the first observational-based measurements with cepheids and got $H_0 \approx 500$ km·s$^{-1}$·MParsec$^{-1}$ [1]. Sadly, even with a correct principle, his value is higher than the typical value due to errors in distance calibrations.

Let us validate our theoretical $H_0$ with an adequate interpretation of 508 measurements found on the Internet. The ends of their tolerance ranges give 1016 values. To find $H_0$ that has the highest probability to be measured, we compile the number of crossings with the tolerance ranges for each value of $H_0$. It generates a curve with two tips (**Figure 1**). The higher it is, the greater the chances are that this value of $H_0$ may be part of many tolerance ranges among the collected data.

A simple statistical phenomenon may be described with a Gaussian function. For fitting a wavy curve, it is necessary to make the sum of many Gaussians. A simpler model with fewer degrees of liberty must always be privileged.

A curve fit is done by summing different Gaussians (shown in **Figure 2**). A better gap fitting reduces the risk of finding other results. Thus, we gave a heavier weight (×10) to all data located between 69.2 and 72.1 km·s$^{-1}$·MParsec$^{-1}$ (from our theory). We tried with and without this approach, and it gives about the same result. As it improves the gap fitting, we kept this approach.

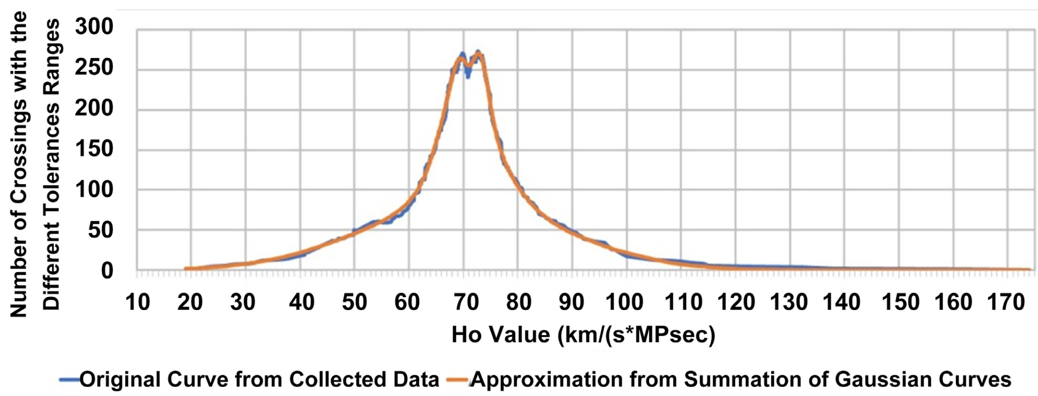


Figure 1. Number of crossings with tolerance ranges as a function of the $H_0$.

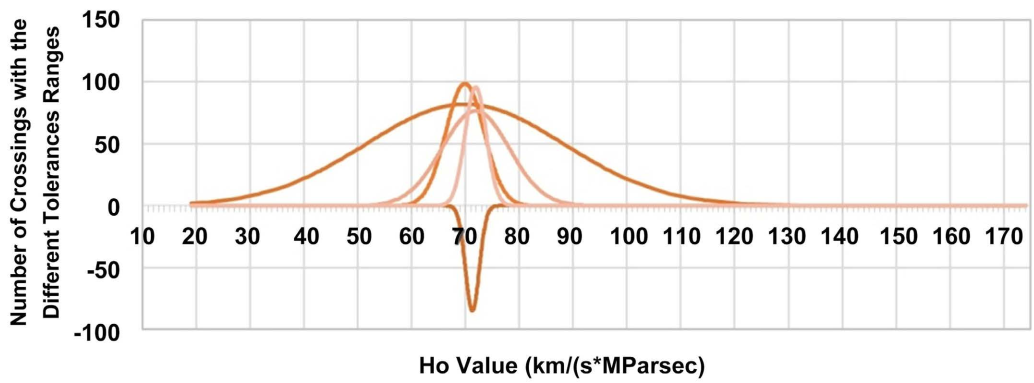


Figure 2. Gaussian curves used to approximate the original curve in Figure 1.

Each tip in Figure 1 is approximated in Figure 2 with two positive Gaussians. We force these curves to be around two means, even though there are four positive curves. It removes two degrees of liberty. We must add a negative Gaussian to model the gap between the two mean values. We must elaborate on this negative Gaussian. Our theory predicts "two close values" of $H_0$. On the curve, a deep gap shows up. It is impossible to get such a gap by only adding positives Gaussians which give two little bumps without any gap. To get a real gap, we must add a negative Gaussian. Let us see in Figure 3 what would look like a curve fit without any negative Gaussian. Since the tips are close, they mix up to build only one tip.

The Gaussian sum in Figure 3 peaks around $H_0 \approx 71.11$ km·s$^{-1}$·MParsec$^{-1}$. The result is not close to our theoretical $H_0 \approx 72.09548580$ km·s$^{-1}$·MParsec$^{-1}$ (Equation (33)), but it is about what is found if statistics were used through the whole data set, thinking they should see only one tip. Moreover, Jang & Lee showed a similar value of $H_0 \approx 71.17$ km·s$^{-1}$·MParsec$^{-1}$ (listed in our software in **Annex A**) that supposedly reduces the tension between the values obtained by cepheids (calibrated on SNe Ia) and CMB.

In Figure 1, we find two groups around $H_0 \approx 69.7$ and 71.8 km·s$^{-1}$·MParsec$^{-1}$. It is known that there is currently a tension between two groups [5]. A significant gap appears between the two tips. The only way to create such a gap is to

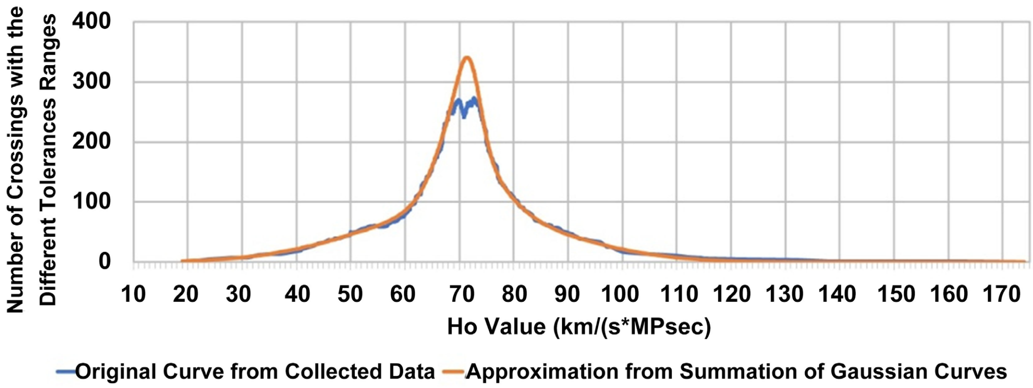


Figure 3. Approximated curve (orange tip) without negative Gaussian curves.

withdraw values nearby a specific value. It would then create a negative Gaussian, such as in Figure 2. It is delicate to debate why some values may have been withdrawn. It could be intentional or not. In the past, it was difficult to see a difference between these groups. Now, the tolerances are small enough to clearly see two groups. With recent growing tensions between these two clans, some may be inclined to shrink or shift some tolerance ranges when it overlaps with neighbor values.

In Figure 4, we apply $\eta$ to the curves around $H_0 \approx 69.882$ km·s$^{-1}$·MParsec$^{-1}$. Then, all curves stand around $H_0 \approx 72.36$ km·s$^{-1}$·MParsec$^{-1}$. Then, with the curves of Figure 4, we build the curve in Figure 5. Figure 6 is a zoom of its tip.

We want to know the precise value of $H_0$ for which the derivative of the Gaussian summation is 0. It corresponds to the highest probability of getting the true $H_0$ value. Unfortunately, the derivative of a Gaussian summation is not an easy equation to get in a software. We rather use a numerical technic to get it. In Figure 6, we show a zoom of the quadratic curve fit around the tip value. Using the equation, we take the derivative and find its maximum. The quadratic equation has the following form:

$$y(x) = Ax^2 + Bx + C$$

$$\text{At the tip, the slope is}: y'(x) = \frac{\mathrm{d}x}{\mathrm{d}y} = 2Ax + B = 0 \rightarrow x = \frac{-B}{2A} \tag{51}$$

$$\text{where } x = H_0,\ y = \text{number of crossings with different tolerance ranges}$$

$$H_0 = \frac{-B}{2A} \approx \frac{2423.2459592464}{2 \times 16.8057572117} \approx 72.0957088907\ \mathrm{km} \cdot \mathrm{s}^{-1} \cdot \mathrm{MParsec}^{-1} \tag{52}$$

This result is well centered on our theoretical value within 3 parts per million. Our approach considers that both clans are someway right. Indeed, their different approaches and results also highlight a new phenomenon. It gives credit to our theory of the universe's complex age that predicts a few possible fake $H_0$ values.

We have 508 data. Each has a tolerance range (that may be symmetrical or not) that generates two $H_0$ values. Therefore, there are a total of $i_{\max} = 1016$ data at the end. The following equation depicts the statistical error $e_i$:

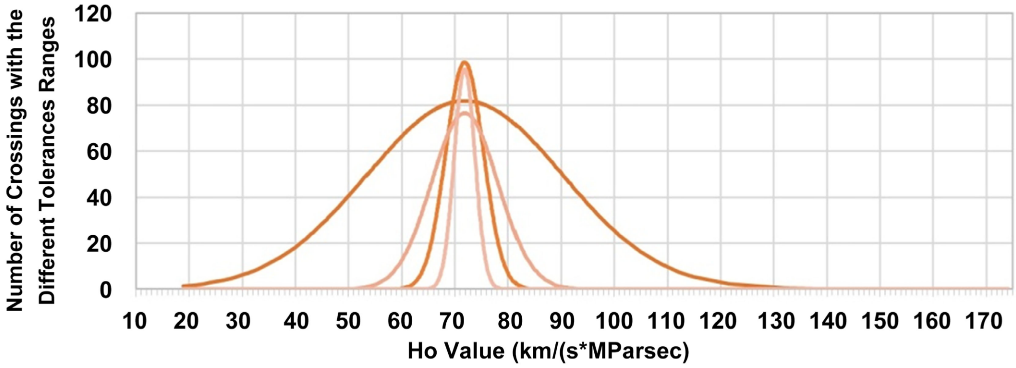


**Figure 4.** Gaussian curves modified with a correction factor $\eta$.

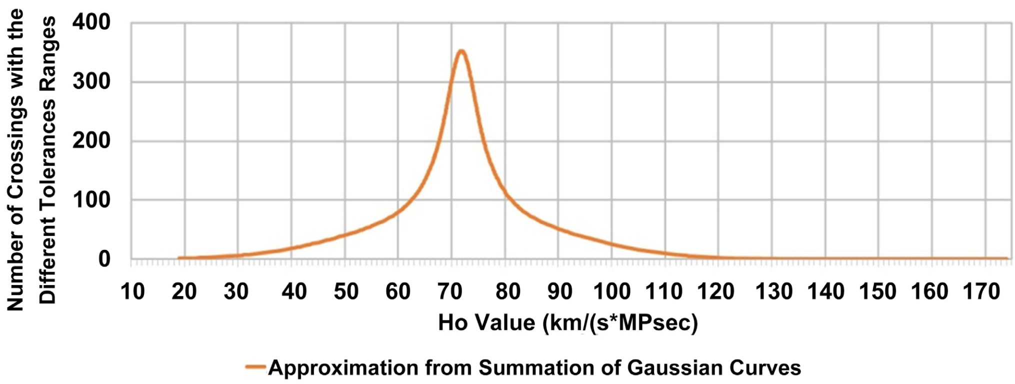


**Figure 5.** Result of the summation of 4 Gaussian curves from **Figure 4**.

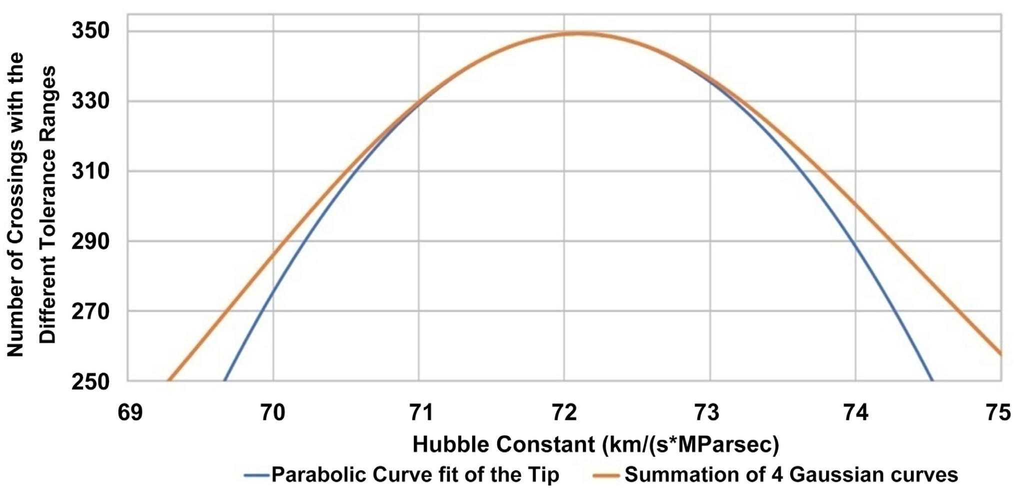


**Figure 6.** Zoom of the tip shown in **Figure 5** + parabolic curve fit.

$$e_t \approx \frac{\sqrt{\sum_{i=1}^{i=i_{\max}} e_i^2}}{i_{\max}} \tag{53}$$

We mention that 16 $H_0$ values in our software in **Annex A** come from statistics. We kept them since some are mixed up with new valuable data information. So, we modify Equation (53) to remove them to reduce their impact on the total

$e_t$ error. We use the following equation where $n = 2 \times 16 = 32$ (each data generates two $H_0$ values) is the number of elements to exclude from our sample. The total $e_t$ error reduces with the square root of the number of elements included in our sample.

$$e_t \approx \frac{\sqrt{\sum_{i=1}^{i=i_{\max}} e_i^2}}{i_{\max}} \cdot \left[ \frac{\sqrt{i_{\max}}}{\sqrt{i_{\max} - n}} \right] \tag{54}$$

If $n = 0$, we fall back on Equation (53). With $i_{\max} = 1016$, Equation (53) gives $e_t \approx \pm 0.32\ \text{km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1}$. With $i_{\max} = 1016$ and $n = 32$, Equation (54) rounds up to $e_t \pm 0.33\ \text{km}\cdot\text{s}^{-1}\cdot\text{MParsec}^{-1}$. The impact of these $n$ elements has a very little impact.

$$H_0 \approx 72.0957 \pm 0.33\ \text{km} \cdot \text{s}^{-1} \cdot \text{MParsec}^{-1} \tag{55}$$

In **Annex A**, we supply the software used to get this result. All the main steps enumerated in this article are clearly shown. The software uses starting values (found via Excel) to fit the original curve with 5 Gaussian curves (#0 to #4 to use the same numbers as the software). Each Gaussian uses three parameters: $\mu$ is the mean value, $\sigma$ represents the variance, and $m$ is a multiplication factor.

$$f(H_0) \approx \frac{m}{\sigma\sqrt{2\pi}} e^{\frac{-1}{2}\left(\frac{H_0 - \mu}{\sigma}\right)^2} \tag{56}$$

Here are the values for the 5 Gaussian curves used to fit the original curve:

| Gaussian # | $\mu$ | $\sigma$ | $m$ |
|---|---|---|---|
| 0 | 71.271 | 1.286 | −272.7 |
| 1 | 69.882 | 18.422 | 3777.7 |
| 2 | | 3.554 | 877.9 |
| 3 | 71.870 | 6.259 | 1199.2 |
| 4 | | 1.963 | 470.4 |

(57)

For Gaussians #1 and #2, we force the software to use the same mean value. We do the same thing for Gaussians #3 and #4. We also note that the multiplication factor $m$ of Gaussian #0 is negative. With these values, we stopped iterating when the sum of squares of errors was lower than 22000. We see in **Figure 1** that the obtained approximated curve fits well the original curve. In our software (**Annex A**), the iterations start with values close to what they should be.

The specificity of our approach is to say that the two clans are someway right. However, we must apply a correction factor to one of them. Indirectly, it gives credit to a complex universe age that predicts a few possible fake values of $H_0$.

After reading this article, scientists should continue their work as they were doing, without applying any correction factor to their raw data. The correction factor should only be used on the final Gaussian curve to analyze data.

## 6. A Reminder of Different Useful Identities

To avoid repeating everything unnecessarily, we recall different identities that will be used later to determine $H_0$. Planck units are commonly defined as follows.

$$\text{Planck mass}: \ m_p = \sqrt{\frac{hc}{2\pi G}} \approx 2.18\times10^{-8}\ \text{kg} \tag{58}$$

$$\text{Planck time}: \ t_p = \sqrt{\frac{hG}{2\pi c^5}} = \frac{L_p}{c} \approx 5.91\times10^{-44}\ \text{s} \tag{59}$$

$$\text{Planck length}: \ L_p = \sqrt{\frac{hG}{2\pi c^3}} = ct_p \approx 1.61\times10^{-35}\ \text{m} \tag{60}$$

$$\text{Planck Temperature}: \ T_p = \sqrt{\frac{hc^3}{2\pi G k_b^2}} = \frac{m_p c^2}{k_b} \approx 1.42\times10^{32}\ \text{K} \tag{61}$$

$$\text{Planck charge}: \ q_p = \sqrt{2ch\varepsilon_0} = \frac{-q_e}{\sqrt{\alpha}} \approx 1.88\times10^{-18}\ \text{C} \tag{62}$$

The fine-structure constant $\alpha$ is linked to Rydberg constant $R_\infty$ and the electron mass $m_e$ by the following equation:

$$R_\infty = \frac{c m_e \alpha^2}{2h} \tag{63}$$

The speed of light $c$ is given as a function of $\mu_0$ and $\varepsilon_0$.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \tag{64}$$

Associating the mass-energy of a Planck particle with its wave energy and then, using Equations (31), (32), (64), and (62), we get Planck charge $q_p$ defined several ways and as a function of $c$, $G$, and $h$ like the other Planck units.

$$q_p = \sqrt{\frac{2h}{c\mu_0}} = \sqrt{\frac{4\pi m_p L_p}{\mu_0}} = \sqrt{\frac{\beta G h^2}{\pi \mu_0 r_e^2 c^4 \alpha^{19}}} \tag{65}$$

The electron's charge is determined from the mass of the electron $m_e$, the classical electron radius $r_e$, and the vacuum permeability $\mu_0$.

$$q_e = \sqrt{\frac{4\pi m_e r_e}{\mu_0}} \approx -1.60\times10^{-19}\ \text{C} \tag{66}$$

Let us calculate the precise value of the average temperature $T$ of the CMB. We first make equal Equations (16) and (33). Then, we replace $G$ by Equation (31), and we get rid of Planck constant $h$ by its value from Equation (32).

$$T = \frac{m_e c^2}{k_b}\left(\frac{15\beta^6\alpha^{17}}{\pi^3}\right)^{1/4} \approx 2.7367958(16)\ \text{K} \tag{67}$$

This CMB temperature is like Kimura with 2.737 K [23].

## 7. Different Equations to Calculate $H_0$

For an academic purpose and to show the interdependence of $H_0$ with the other "constants", we will enumerate equations using various universe parameters. Some overcome the inherent difficulties in measuring $H_0$ and show a roundabout way of obtaining an accurate value of it. We also find some others which depend on interesting values, or more precise ones. Using the constants $c$, $k_b$, $T$, $m_e$, $r_e$, $h$, $G$, $\mu_0$, $\varepsilon_0$, $m_u$, $R_u$, $R_\infty$, $q_e$, $q_p$, $t_p$, $l_p$, $T_p$, $m_p$, $m_{ph}$, and $\beta$, we find many equations.

The $H_0$ parameter is not constant since $1/H_0$ represents an approximation of the apparent universe's age, and $H_0$ get smaller over time. Since the universe is old, $H_0$ changes slowly. If the constancy of all the universe's parameters is maintained as it is currently done in metrology, the universe's age and $H_0$ will seem constant.

Results of 508 different experiences reduce the error by $508^{1/2} \approx 22.5$. It may look like a significant number, but it is nothing besides what has been done to measure the electron characteristics accurately. Particle accelerators use millions of electrons at each experiment, and they repeat these many times to find something new. Computers analyze the collisions' results to make the electron's characteristics more and more accurate. It is why there is no manner to get better results than that of Equation (33), as it is based on well-known characteristics of the electron. We will see further many other equations that give precise results.

Replacing $G$ by Equation (31) in Equation (16), we get Equation (68).

$$H_0 = \pi^3 k_b^2 T^2 \alpha^9 \sqrt{\frac{8r_e}{15 m_e c^3 h^3 \beta^5}} \tag{68}$$

Replacing $h$ in Equation (68) by using Equation (32), we get Equation (69).

$$H_0 = k_b^2 T^2 \sqrt{\frac{\pi^3 \alpha^{21}}{15 r_e^2 m_e^4 \beta^5 c^6}} \tag{69}$$

Replacing $h$ in Equation (16) by using Equation (32), we get Equation (70).

$$H_0 = \frac{k_b^2 T^2}{\beta^2} \sqrt{\frac{\pi^3 G \alpha}{15 r_e^3 m_e^3 c^8}} \tag{70}$$

Replacing $T$ in Equation (70) by using Equation (67), we get Equation (71).

$$H_0 = \beta \alpha^9 \sqrt{\frac{G m_e}{r_e^3}} \tag{71}$$

Using Equation (32) in Equation (33), we get Equation (72).

$$H_0 = \frac{2\pi m_e c^2 \alpha^{18} \sqrt{\beta}}{h} \tag{72}$$

With Equations (63) and (72), we get the most accurate equation.

$$H_0 = 4\pi c R_\infty \alpha^{16} \sqrt{\beta} \tag{73}$$

Using Equation (31) in Equation (73), we get Equation (74).

$$H_0 = \frac{4\pi G m_e R_\infty \beta^{3/2}}{c r_e \alpha^4} \tag{74}$$

Equation (74) gives $H_0$ with $G_{2014}$ (from CODATA 2014). The measurement of $H_0(G_{2014}) \approx 72.105(36)$ km·s$^{-1}$·MParsec$^{-1}$ is our fourth way to measure $H_0$.

Using Equation (63) in Equation (74), we get Equation (75). This equation is also a good candidate for measuring $H_0$ as a function of $G$.

$$H_0 = \frac{8\pi G h R_\infty^2 \beta^{3/2}}{r_e c^2 \alpha^4} \quad (75)$$

Using Equation (32) in Equation (75), we get Equation (76).

$$H_0 = \frac{16\pi^2 G m_e R_\infty^2 \beta^{3/2}}{c\alpha^7} \quad (76)$$

This equation is another good candidate for measuring $H_0$ as a function of $G$. The measure still gives the same result as Equation (74).

We will enumerate other equations without making all the rather fastidious demonstrations. However, all these may be found from previous equations.

$$H_0 = \frac{2\pi m_{ph} c^2}{h} \quad (77)$$

$$H_0 = 2c^2 \sqrt{\frac{\pi R_\infty \alpha^{55}}{G m_e}} \quad (78)$$

$$H_0 = \beta \alpha^9 \sqrt{\frac{m_e c}{r_e^3}} \quad (79)$$

$$H_0 = 8\beta \sqrt{G m_e \left(\pi R_\infty\right)^3 \alpha^9} \quad (80)$$

$$H_0 = 8c \sqrt{\beta r_e \left(\pi R_\infty\right)^3 \alpha^{29}} \quad (81)$$

$$H_0 = \sqrt{\frac{2\pi c^5 \alpha^{57}}{hG}} \quad (82)$$

$$H_0 = 2c \sqrt{\frac{\pi \beta R_\infty \alpha^{25}}{r_e}} \quad (83)$$

$$H_0 = \sqrt{\frac{2\pi \beta m_e c^3 \alpha^{37}}{h r_e}} \quad (84)$$

$$H_0 = \frac{2\pi^2}{h} \left( \frac{k_b^4 T^4}{15 m_u \alpha^2 c^2 \beta^4} \right)^{1/3} \quad (85)$$

$$H_0 = \frac{2\pi k_b T_p}{h} \sqrt{\alpha^{57}} \quad (86)$$

$$H_0 = \frac{c \mu_0 q_e^2 \alpha^{19} \sqrt{\beta}}{4\pi m_e r_e^2} \quad (87)$$

$$H_0 = \frac{q_e^2 \alpha^{19} \sqrt{\beta}}{4\pi c \varepsilon_0 m_e r_e^2} \quad (88)$$

$$H_0 = \frac{q_p^2 \alpha^{18} \sqrt{\beta}}{2h \varepsilon_0 r_e} \quad (89)$$

$$H_0 = \frac{c \mu_0 q_p^2 \alpha^{20} \sqrt{\beta}}{4\pi m_e r_e^2} \quad (90)$$

$$H_0 = \frac{q_p^2 \alpha^{20} \sqrt{\beta}}{4\pi c \varepsilon_0 m_e r_e^2} \tag{91}$$

$$H_0 = \frac{c\beta q_p^2 \sqrt{\alpha^{19}}}{4\pi \varepsilon_0 k_b T_p r_e^2} \tag{92}$$

$$H_0 = \frac{c\beta q_e^2 \sqrt{\alpha^{17}}}{4\pi \varepsilon_0 k_b T_p r_e^2} \tag{93}$$

$$H_0 = \frac{c q_e^2 \beta^2}{4\varepsilon_0 k_b T r_e^2} \left( \frac{15\alpha^{97}}{\pi^7} \right)^{1/4} \tag{94}$$

$$H_0 = \frac{2k_b T}{\beta h} \left( \frac{\pi^7 \alpha^{55}}{15} \right)^{1/4} \tag{95}$$

$$H_0 = \frac{k_b T}{\beta r_e m_e c} \left( \frac{\pi^3 \alpha^{59}}{15} \right)^{1/4} \tag{96}$$

$$H_0 = \frac{4\pi k_b T}{\beta c \mu_0 q_e^2} \left( \frac{\pi^3 \alpha^{59}}{15} \right)^{1/4} \tag{97}$$

$$H_0 = \frac{4\pi k_b T}{\beta c \mu_0 q_p^2} \left( \frac{\pi^3 \alpha^{55}}{15} \right)^{1/4} \tag{98}$$

$$H_0 = \frac{G k_b T}{c^3 r_e^2} \left( \frac{\pi^3}{15\alpha^{21}} \right)^{1/4} \tag{99}$$

$$H_0 = \frac{2G m_e k_b T}{h r_e c^2} \left( \frac{\pi^7}{15\alpha^{25}} \right)^{1/4} \tag{100}$$

$$H_0 = \frac{G m_p k_b T}{m_e c^3 r_e^2} \left( \frac{\pi^3 \alpha^{21}}{15\beta^2} \right)^{1/4} \tag{101}$$

$$H_0 = \frac{G m_u k_b T}{m_e c^3 r_e^2} \left( \frac{\pi^3 \alpha^{135}}{15\beta^2} \right)^{1/4} \tag{102}$$

$$H_0 = \frac{G m_e k_b T}{m_p c^3 r_e^2} \left( \frac{\pi^3 \beta^2}{15\alpha^{63}} \right)^{1/4} \tag{103}$$

$$H_0 = \frac{16 c G k_b T \varepsilon_0 m_e^2}{q_e^4} \left( \frac{\pi^{11}}{15\alpha^{21}} \right)^{1/4} \tag{104}$$

$$H_0 = \beta m_e^2 \sqrt{\frac{8\pi^3 G c^3 \alpha^{15}}{h^3}} \tag{105}$$

$$H_0 = \frac{c^3 \alpha^{39}}{G m_e \sqrt{\beta}} \tag{106}$$

The last equation measures $H_0$ from $G$ since all other constants are accurate. Many equations are excellent candidates for measuring $H_0$ as a function of $G$ or

*T*. These equations could represent valuable tools for cosmologists.

This document gives 42 equations of $H_0$ as a function of various universe parameters. Since $H_0$ may be defined using different parameters, we suggest that some of the most critical universe parameters are well linked, as much in the infinitely small as in the infinitely large, and $H_0$ is part of these.

## 8. Why Is $H_0$ Not Really a Constant?

We want to explain why Hubble parameter $H_0$ cannot be constant over time. As simple as it is, the reverse of Hubble parameter $H_0$ is related to the apparent age of the universe (see Equation (45)). Consequently, the $H_0$ parameter is changing over time. It is, therefore, by abuse of language that we call $H_0$ the Hubble "constant". To be more precise, we should say the Hubble "parameter".

When $H_0$ is expressed in $km \cdot s^{-1} \cdot MParsec^{-1}$, the ninth digit after the dot changes every year. It goes completely unnoticed. More than that, even if we could achieve this precision in our measurements of $H_0$, it would still go unnoticed since we forced $c$ to be constant in 1983. In metrology, scientists choose the speed of light as a standard. Even though $c$ changes every year, if we force it to be constant, we willfully readjust all other constants and units (distance, time, and mass) as a function of $c$ to keep it constant. Then, $H_0$ looks constant as other parameters.

## 9. Conclusions

This article aimed to show that our theoretical value from Equation (33) (giving $H_0 \approx 72.09548580(32)\ km \cdot s^{-1} \cdot MParsec^{-1}$) [6] is the right one, despite a growing tension [5] between values around 69.2 and 72.1 $km \cdot s^{-1} \cdot MParsec^{-1}$.

With 508 data (from [24] to [310] shown in our software in **Annex A**), a graph showing the actual tension [5] between two values is shown. We decomposed the curve into Gaussians. A negative one is required to explain the large gap between the two $H_0$ values, and it is due to withdrawn values. So, we restored them by removing that curve. Then, we applied a $\eta \approx 1.042516951$ correction factor (from our theory) to the curves located at $\sim H_0 \approx 69.2\ km \cdot s^{-1} \cdot MParsec^{-1}$. Our theory highlights a misunderstanding of the link between $1/H_0$ and the universe's apparent age. With the proper correction factor applied, we get a statistical value of $H_0 \approx 72.0957 \pm 0.33\ km \cdot s^{-1} \cdot MParsec^{-1}$, which is close to our theoretical value. Our discovery of the $\eta$ factor may help to reduce the tension between scientists. Someway we show that even if two $H_0$ values seem to be commonly found with various techniques, both are accurate if a proper correction factor is used.

With a new cosmological model, we get an apparent age of the universe of about 14.14 billion years. The exact formula is approximated from an elaborate integral result by the well-known $1/H_0$ equation that gives 13.56 billion years. Different techniques may lead to either value. It depends if it is an attempt to measure the universe's age locally or far away. There is no "local" or "distant" value of $H_0$, as some may pretend [46] [47]. Sticking their measurement of the apparent age of the universe to $1/H_0$, most cosmologists get results that stand around 69.2 or 72.1 km·s$^{-1}$·MParsec$^{-1}$. Our hypothesis may explain the actual tension [5] relative to these two values. However, there is only one true $H_0$ value, and the other one is just misinterpreted as being the Hubble constant without quite being so.

Even if many theoretical equations of $H_0$ are shown in this article, we highlight that we also found a few interesting ways to measure the $H_0$ accurately using the CMB temperature $T$ and the value of the universal gravitational constant $G$ from CODATA 2014. These results confirm our theoretical value.

$H_0 \approx 72.06(90)$ km·s$^{-1}$·MParsec$^{-1}$ and 71.95(50) km·s$^{-1}$·MParsec$^{-1}$ from $T$,

$H_0 \approx 72.086(36)$ km·s$^{-1}$·MParsec$^{-1}$ and 72.105(36) km·s$^{-1}$·MParsec$^{-1}$ from $G$.

For an academic purpose, we enumerated 42 equations of $H_0$ using different parameters. These equations showed that $H_0$ is intricated with all other "constants". For metrology purposes, the speed of light in a vacuum is forced to be constant to be an unchanging standard. If this situation is considered valid in a metrology context, $H_0$ should also be considered constant and become part of the CODATA. However, if $1/H_0$ represents an approximation of the universe's age, it would also make sense to say that $H_0$ is changing over time.

Einstein's and Schwarzschild's equations show that massive objects such as the universe influence the speed of light. As the universe expands, its density diminishes, and the local speed of light increases over time.

The fine-structure constant $\alpha$ is unitless and may be described as a ratio where the variation rate at the numerator counterbalances the variation rate at the denominator. Apart from $\alpha$ and $\beta$, all "constants" used to describe $H_0$ in our equations somehow emanate from fundamental units such as the meter, the second, and the kilogram. These units are now defined by the speed of light. As $H_0$ describes the universe's age and depends on many unit-dependent "constants" based on $c$, we should consider $c$ and all universe's unit-dependent parameters as changing over time. Forcing $c$ to be constant is necessary for metrology purposes, but it is not in the interest of physicists for explaining phenomena. An accurate value of $H_0$ has a great interest in deepening our understanding of the universe.

## Conflicts of Interest

The author claims that he has no conflict of interest in connection with the publication of this article.

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## Annex A (C++ Software)

```
// This software finds "the best" experimental value of H0 with a set of 508 data
//Compiled on Dev-C++ 5.11 available for free at:
//              https://sourceforge.net/projects/orwelldevcpp/
#include<stdio.h>
#include<stdbool.h>
#include<math.h>
#define printf __mingw_printf
#define nbH0 508 //Number of measurements of H0 analyzed
#define Pi 3.141592654 //Definition of Pi
double Mean[5],Sigma[5],Multiplier[5];//Caracteristics of Gaussian curves
double A,B,C; //Coefficients of the quadratic equation of the tip
double LMSTip; //Least mean square for the approximation tip curve
double BEH; //Best estimate of H0
double H0[2*nbH0-1],NbCrossings[2*nbH0-1];
double nbCrossings[2*nbH0-1]; //Vector of number of crossings
double Accuracy_ppm; //Accuracy of H0 compared to the theoretical value
double TVH = 72.09548580; //Theoretical value of H0 (in km/(s*MParsec)
unsigned int PosTipIndex; //Index corresponding to the tip of nbCrossings array
int n = 20; //Sample before & after PosTipIndex to build the tip equation

double Sqr(double value) {return value*value;} //***Returns the square value

//***Function that returns the square root value
double Sqrt(double Value) {
  double D; /*Dummy value*/   double V; /*Returned value*/   int i;
/*Counter*/
  V=0;  D=Value;
  for (i=0;i<=50;i++) {  //Gives 50 bits of precision
    D=-D/2;
    if (D<0) while (V*V>Value) V=V+D; else while (V*V<Value) V=V+D;}
  return V;
  }  //End of Sqrt

double Exp(double Value) { return pow(2.718281828,Value);} //***e^Value

//***This procedure creates a table of 2*nbH increasing values
//***of H0[i] with the tolerances T[i] and the signs +/- Variation[i].
void CreateTableOfCrossingH0Ranges(void) {
  double PT[nbH0],NT[nbH0]; //Positive tolerance and negative tolerance
  double Variation[2*nbH0]; //Variation from the tolerance range
  double DH0,DSgn; //Dummy H0 and Sgn used to put H in increasing order
  int DeltaCrossings; //Variation of on the number of crossings
```

```
int i,j,k; //Counters
i=0;
//We enumerate all H0 values and their tolerance range found on Internet
//In brackets, we add the bibliographic references
H0[i]=69;        PT[i]=+16;     NT[i++]=-8; //[24]Abbott et al.
H0[i]=70;        PT[i]=+2.7;    NT[i++]=-2.7; //[25]Addisson
H0[i]=72.4;      PT[i]=+3.9;    NT[i++]=-4.8; //[25]Addisson
H0[i]=73.1;      PT[i]=+3.3;    NT[i++]=-3.9; //[25]Addisson
H0[i]=73.2;      PT[i]=+1.3;    NT[i++]=-1.3; //[25]Addisson
H0[i]=68.7;      PT[i]=+1.3;    NT[i++]=-1.3; //[25]Addisson
H0[i]=73.5;      PT[i]=+5.3;    NT[i++]=-5.3; //[26]Baxter & Sherwin
H0[i]=73.3;      PT[i]=+0.7;    NT[i++]=-0.7; //[27]Blakeslee et al.
H0[i]=73.78;     PT[i]=+0.84;   NT[i++]=-0.84; //[28]Bonilla
H0[i]=73.577;    PT[i]=+0.106;  NT[i++]=-0.106; //[29]Dainotti et al.
H0[i]=73.493;    PT[i]=+0.144;  NT[i++]=-0.144; //[29]Dainotti et al.
H0[i]=73.222;    PT[i]=+0.262;  NT[i++]=-0.262; //[29]Dainotti et al.
H0[i]=73.664;    PT[i]=+0.223;  NT[i++]=-0.223; //[29]Dainotti et al.
H0[i]=73.576;    PT[i]=+0.105;  NT[i++]=-0.105; //[29]Dainotti et al.
H0[i]=73.513;    PT[i]=+0.142;  NT[i++]=-0.142; //[29]Dainotti et al.
H0[i]=73.192;    PT[i]=+0.265;  NT[i++]=-0.265; //[29]Dainotti et al.
H0[i]=73.678;    PT[i]=+0.223;  NT[i++]=-0.223; //[29]Dainotti et al.
H0[i]=71.8;      PT[i]=+3.9;    NT[i++]=-3.3; //[30]Denzel et al.
H0[i]=72.94;     PT[i]=+0.75;   NT[i++]=-0.75; //[31]Di Valentino
H0[i]=72.7;      PT[i]=+1.1;    NT[i++]=-1.1; //[31]Di Valentino
H0[i]=68.8;      PT[i]=+45.7;   NT[i++]=-25.5; //[32]Gayathri et al.
H0[i]=62.3;      PT[i]=+9.1;    NT[i++]=-9.1; //[33]Hagstotz et al.
H0[i]=70.5;      PT[i]=+2.37;   NT[i++]=-2.37; //[34]Kethan et al.
H0[i]=72.86;     PT[i]=+0.036;  NT[i++]=-0.036; //Mercier (this document)
H0[i]=72.105;    PT[i]=+0.036;  NT[i++]=-0.036; //Mercier (this document)
H0[i]=68.3;      PT[i]=+4.6;    NT[i++]=-4.6; //[35]Mukherjee et al.
H0[i]=70;        PT[i]=+0.5;    NT[i++]=-0.5; //[36]Park et al.
H0[i]=65.1;      PT[i]=+3;      NT[i++]=-5.4; //[37]Philcox et al.
H0[i]=65.6;      PT[i]=+3.4;    NT[i++]=-3.5; //[37]Philcox et al.
H0[i]=70.6;      PT[i]=+3.7;    NT[i++]=-5; //[37]Philcox et al.
H0[i]=78.3;      PT[i]=+2.9;    NT[i++]=-2.9; //[38]Qi et al.
H0[i]=73.6;      PT[i]=+1.8;    NT[i++]=-1.6; //[38]Qi et al.
H0[i]=73;        PT[i]=+1.4;    NT[i++]=-1.4; //[39]Riess et al.
H0[i]=73.2;      PT[i]=+1.3;    NT[i++]=-1.3; //[39]Riess et al.
H0[i]=72.1;      PT[i]=+2;      NT[i++]=-2; //[40]Soltis et al.
H0[i]=69.5;      PT[i]=+4;      NT[i++]=-4; //[41]Wang & Giannios
H0[i]=71;        PT[i]=+20;     NT[i++]=-20; //[42]Zhang et al.
H0[i]=67.4;      PT[i]=+0.5;    NT[i++]=-0.5; //[43]Aghanim et al.
H0[i]=67.73;     PT[i]=+0.41;   NT[i++]=-0.41; //[44]Benevento
```

```
H0[i]=68.22;    PT[i]=+0.39;    NT[i++]=-0.39; //[44]Benevento
H0[i]=72.5;     PT[i]=+1.85;    NT[i++]=-1.85; //[44]Benevento
H0[i]=69.17;    PT[i]=+1.09;    NT[i++]=-1.09; //[44]Benevento
H0[i]=74.5;     PT[i]=+5.6;     NT[i++]=-6.1; //[45]Birrer et al.
H0[i]=67.4;     PT[i]=+4.1;     NT[i++]=-3.2; //[45]Birrer et al.
H0[i]=75.35;    PT[i]=+1.68;    NT[i++]=-1.68; //[46]Camarena & Marra
H0[i]=74;       PT[i]=+0.625;   NT[i++]=-0.625; //[47]Chang & Zhu
H0[i]=73.8;     PT[i]=+6.3;     NT[i++]=-5.8; //[48]Coughlin et al.
H0[i]=71.2;     PT[i]=+3.2;     NT[i++]=-3.1; //[48]Coughlin et al.
H0[i]=72.4;     PT[i]=+1.4;     NT[i++]=-1.4; //[49]D'Agostino
H0[i]=71.5;     PT[i]=+1.3;     NT[i++]=-1.3; //[49]D'Agostino
H0[i]=71.54;    PT[i]=+1.78;    NT[i++]=-1.78; //[50]Dai WM et al.
H0[i]=73.12;    PT[i]=+1.14;    NT[i++]=-1.14; //[50]Dai WM et al.
H0[i]=66.2;     PT[i]=+4.4;     NT[i++]=-4.2; //[51]Dietrich et al.
H0[i]=69.9;     PT[i]=+0.84;    NT[i++]=-0.86; //[52]Gonzalez et al.
H0[i]=71;       PT[i]=+4;       NT[i++]=-4; //[53]González-Serrena et al.
H0[i]=74.62;    PT[i]=+12.35;   NT[i++]=-11.34; //[54]Haboury
H0[i]=71.89;    PT[i]=+11.02;   NT[i++]=-10.17; //[54]Haboury
H0[i]=76.44;    PT[i]=+55.76;   NT[i++]=-50.17; //[54]Haboury
H0[i]=50.9;     PT[i]=+31.1;    NT[i++]=-31.9; //[54]Haboury
H0[i]=50.81;    PT[i]=+28.19;   NT[i++]=-27.81; //[54]Haboury
H0[i]=71;       PT[i]=+2;       NT[i++]=-3; //[55]Harvey
H0[i]=65.9;     PT[i]=+1.5;     NT[i++]=-1.5; //[56]Holanda et al.
H0[i]=65.9;     PT[i]=+4.4;     NT[i++]=-4; //[56]Holanda et al.
H0[i]=64.3;     PT[i]=+4.5;     NT[i++]=-4.4; //[56]Holanda et al.
H0[i]=66.8;     PT[i]=+13.4;    NT[i++]=-9.2; //[57]Howlett & Davis
H0[i]=64.8;     PT[i]=+7.3;     NT[i++]=-7.2; //[57]Howlett & Davis
H0[i]=75.8;     PT[i]=+5.2;     NT[i++]=-4.9; //[58]Jaeger et al.
H0[i]=65.8;     PT[i]=+3.5;     NT[i++]=-3.5; //[59]Kim et al.
H0[i]=72.3;     PT[i]=+1.4;     NT[i++]=-1.4; //[60]Kreisch et al.
H0[i]=71.5;     PT[i]=+11.9;    NT[i++]=-10.6; //[61]Li & Zhang
H0[i]=74.7;     PT[i]=+5.8;     NT[i++]=-5.8; //[62]Lombriser
H0[i]=72.06;    PT[i]=+0.09;    NT[i++]=-0.09; //[7]Mercier
H0[i]=74;       PT[i]=+1.6;     NT[i++]=-1.6; //[63]Millon et al.
H0[i]=74.2;     PT[i]=+1.7;     NT[i++]=-1.8; //[63]Millon et al.
H0[i]=50.4;     PT[i]=+28.1;    NT[i++]=-19.5; //[64]Mukherjee et al.
H0[i]=62.2;     PT[i]=+29.5;    NT[i++]=-19.7; //[64]Mukherjee et al.
H0[i]=43.1;     PT[i]=+24.6;    NT[i++]=-11.4; //[64]Mukherjee et al.
H0[i]=67.6;     PT[i]=+4.3;     NT[i++]=-4.2; //[64]Mukherjee et al.
H0[i]=68.6;     PT[i]=+14;      NT[i++]=-8.5; //[65]Nicolaou et al.
H0[i]=69.6;     PT[i]=+1;       NT[i++]=-1.3; //[66]Niedermann and Sloth
H0[i]=71.4;     PT[i]=+1;       NT[i++]=-1; //[66]Niedermann and Sloth
H0[i]=72;       PT[i]=+12;      NT[i++]=-8.2; //[67]Palmese et al.
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H0[i]=69.03;    PT[i]=+0.87;   NT[i++]=-0.87; //[68]Pandey et al.
H0[i]=70.6;     PT[i]=+1.1;    NT[i++]=-1.1; //[68]Pandey et al.
H0[i]=68.44;    PT[i]=+0.52;   NT[i++]=-0.52; //[68]Pandey et al.
H0[i]=68.1;     PT[i]=+0.58;   NT[i++]=-0.58; //[68]Pandey et al.
H0[i]=73.9;     PT[i]=+3;      NT[i++]=-3; //[69]Pesce et al.
H0[i]=68.6;     PT[i]=+1.8;    NT[i++]=-1.8; //[70]Pogosian et al.
H0[i]=74.03;    PT[i]=+1.42;   NT[i++]=-1.42; //[71]Rui-Yun et al.
H0[i]=75.1;     PT[i]=+2.3;    NT[i++]=-2.3; //[72]Schombert et al.
H0[i]=74.2;     PT[i]=+2.7;    NT[i++]=-3; //[73]Shajib et al.
H0[i]=67.52;    PT[i]=+0.96;   NT[i++]=-0.95; //[74]Sharov & Sinyakov
H0[i]=70.87;    PT[i]=+1.63;   NT[i++]=-1.62; //[74]Sharov & Sinyakov
H0[i]=69;       PT[i]=+29;     NT[i++]=-14; //[75]Vasylyev & Filippenko
H0[i]=67;       PT[i]=+41;     NT[i++]=-26; //[75]Vasylyev & Filippenko
H0[i]=71;       PT[i]=+34;     NT[i++]=-30; //[75]Vasylyev & Filippenko
H0[i]=70;       PT[i]=+29;     NT[i++]=-18; //[75]Vasylyev & Filippenko
H0[i]=72.3;     PT[i]=+2.9;    NT[i++]=-2.8; //[76]Vogl
H0[i]=75.3;     PT[i]=+3;      NT[i++]=-2.9; //[77]Wei & Melia
H0[i]=75.3;     PT[i]=+1.9;    NT[i++]=-1.9; //[77]Wei & Melia
H0[i]=67.9;     PT[i]=+1.1;    NT[i++]=-1.3; //[78]Wu et al.
H0[i]=72;       PT[i]=+2.1;    NT[i++]=-2.5; //[78]Wu et al.
H0[i]=73.65;    PT[i]=+1.95;   NT[i++]=-2.26; //[79]Yang et al.
H0[i]=67.95;    PT[i]=+0.78;   NT[i++]=-1.03; //[80]Zhang & Huang
H0[i]=69.81;    PT[i]=+2.22;   NT[i++]=-2.7; //[80]Zhang & Huang
H0[i]=66.75;    PT[i]=+3.42;   NT[i++]=-4.23; //[80]Zhang & Huang
H0[i]=70.75;    PT[i]=+1.55;   NT[i++]=-1.55; //[81]Agrawal
H0[i]=73.7;     PT[i]=+1.4;    NT[i++]=-1.4; //[82]Anderson
H0[i]=72.5;     PT[i]=+2.1;    NT[i++]=-2.3; //[83]Birrer
H0[i]=67.4;     PT[i]=+0.5;    NT[i++]=-0.5; //[84]Chang et al.
H0[i]=82.8;     PT[i]=+9.4;    NT[i++]=-8.3; //[85]Chen et al.
H0[i]=70.1;     PT[i]=+5.3;    NT[i++]=-4.5; //[85]Chen et al.
H0[i]=77;       PT[i]=+4;      NT[i++]=-4.6; //[85]Chen et al.
H0[i]=75.6;     PT[i]=+3.2;    NT[i++]=-3; //[85]Chen et al.
H0[i]=76.8;     PT[i]=+2.6;    NT[i++]=-2.6; //[85]Chen et al.
H0[i]=75.7;     PT[i]=+4.5;    NT[i++]=-4.4; //[86]Collett
H0[i]=76.8;     PT[i]=+4.2;    NT[i++]=-3.8; //[86]Collett
H0[i]=74.2;     PT[i]=+3;      NT[i++]=-2.9; //[86]Collett
H0[i]=67.6;     PT[i]=+1.1;    NT[i++]=-1.1; //[87]Cuceu et al.
H0[i]=67.4;     PT[i]=+6;      NT[i++]=-6.2; //[88]Domínguez
H0[i]=66.6;     PT[i]=+1.6;    NT[i++]=-1.6; //[88]Domínguez
H0[i]=70.3;     PT[i]=+1.36;   NT[i++]=-1.35; //[89]Dutta et al.
H0[i]=77;       PT[i]=+37;     NT[i++]=-18; //[90]Fishbach et al.
H0[i]=76;       PT[i]=+19;     NT[i++]=-13; //[90]Fishbach et al.
H0[i]=69.8;     PT[i]=+0.8;    NT[i++]=-0.8; //[91]Freedman et al.
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H0[i]=68.09;   PT[i]=+0.45;  NT[i++]=-0.45; //[92]Guo et al.
H0[i]=69.34;   PT[i]=+0.93;  NT[i++]=-0.93; //[92]Guo et al.
H0[i]=69.67;   PT[i]=+0.95;  NT[i++]=-0.94; //[92]Guo et al.
H0[i]=69.36;   PT[i]=+0.82;  NT[i++]=-0.82; //[92]Guo et al.
H0[i]=69.25;   PT[i]=+0.99;  NT[i++]=-0.99; //[92]Guo et al.
H0[i]=74;      PT[i]=+16;    NT[i++]=-8; //[93]Hotokezaka et al.
H0[i]=70.3;    PT[i]=+5.3;   NT[i++]=-5; //[93]Hotokesaka et al.
H0[i]=82.4;    PT[i]=+8.4;   NT[i++]=-8.3; //[94]Jee et al.
H0[i]=67;      PT[i]=+3;     NT[i++]=-3; //[95]Kozmanyan et al.
H0[i]=72.2;    PT[i]=+2.1;   NT[i++]=-2.1; //[96]Liao et al.
H0[i]=73;      PT[i]=+2.8;   NT[i++]=-3; //[96]Liao et al.
H0[i]=67.8;    PT[i]=+1.3;   NT[i++]=-1.3; //[97]MacAulay et al.
H0[i]=67.37;   PT[i]=+0.62;  NT[i++]=-0.62; //[98]Martinelli
H0[i]=68.8;    PT[i]=+1.6;   NT[i++]=-1.6; //[98]Martinelli
H0[i]=73.9;    PT[i]=+2.5;   NT[i++]=-2.5; //[98]Martinelli
H0[i]=67.68;   PT[i]=+0.46;  NT[i++]=-0.46; //[98]Martinelli
H0[i]=68.4;    PT[i]=+1;     NT[i++]=-1; //[98]Martinelli
H0[i]=69.2;    PT[i]=+1.5;   NT[i++]=-1.5; //[98]Martinelli
H0[i]=67.51;   PT[i]=+0.61;  NT[i++]=-0.61; //[98]Martinelli
H0[i]=68.9;    PT[i]=+1.1;   NT[i++]=-1.1; //[98]Martinelli
H0[i]=72.1;    PT[i]=+2.1;   NT[i++]=-1.8; //[98]Martinelli
H0[i]=67.75;   PT[i]=+0.46;  NT[i++]=-0.46; //[98]Martinelli
H0[i]=68.59;   PT[i]=+0.86;  NT[i++]=-0.86; //[98]Martinelli
H0[i]=69.6;    PT[i]=+1.3;   NT[i++]=-1.3; //[98]Martinelli
H0[i]=71.505;  PT[i]=+0.03;  NT[i++]=-0.03; //[2]Mercier
H0[i]=69;      PT[i]=+1.7;   NT[i++]=-1.7; //[99]Park & Ratra
H0[i]=69.8;    PT[i]=+1.8;   NT[i++]=-1.8; //[99]Park & Ratra
H0[i]=68.9;    PT[i]=+1.7;   NT[i++]=-1.7; //[99]Park & Ratra
H0[i]=70.1;    PT[i]=+1.9;   NT[i++]=-1.9; //[99]Park & Ratra
H0[i]=68.5;    PT[i]=+1.8;   NT[i++]=-1.8; //[99]Park & Ratra
H0[i]=69.6;    PT[i]=+1.9;   NT[i++]=-1.9; //[99]Park & Ratra
H0[i]=72;      PT[i]=+1.9;   NT[i++]=-1.9; //[100]Reid
H0[i]=73.5;    PT[i]=+1.4;   NT[i++]=-1.4; //[100]Reid
H0[i]=74.22;   PT[i]=+1.82;  NT[i++]=-1.82; //[101]Riess et al.
H0[i]=74.03;   PT[i]=+1.42;  NT[i++]=-1.42; //[101]Riess et al.
H0[i]=72.8;    PT[i]=+1.1;   NT[i++]=-1.1; //[102]Riess
H0[i]=74.3;    PT[i]=+1;     NT[i++]=-1; //[102]Riess
H0[i]=71.6;    PT[i]=+3.8;   NT[i++]=-4.9; //[103]Rusu et al.
H0[i]=67.99;   PT[i]=+0.91;  NT[i++]=-0.88; //[104]Ryan
H0[i]=68.24;   PT[i]=+2.39;  NT[i++]=-2.33; //[104]Ryan
H0[i]=66.79;   PT[i]=+2.6;   NT[i++]=-2.32; //[104]Ryan
H0[i]=66.8;    PT[i]=+2.5;   NT[i++]=-2.3; //[104]Ryan
H0[i]=66.13;   PT[i]=+1.38;  NT[i++]=-2.09; //[104]Ryan
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H0[i]=67.1;     PT[i]=+2.4;     NT[i++]=-2.3; //[104]Ryan
H0[i]=68.44;    PT[i]=+0.7;     NT[i++]=-0.69; //[104]Ryan
H0[i]=69.32;    PT[i]=+1.42;    NT[i++]=-1.42; //[104]Ryan
H0[i]=68;       PT[i]=+2.27;    NT[i++]=-1.94; //[104]Ryan
H0[i]=66.6;     PT[i]=+2.2;     NT[i++]=-1.9; //[104]Ryan
H0[i]=67.19;    PT[i]=+1;       NT[i++]=-1.6; //[104]Ryan
H0[i]=66.8;     PT[i]=+1.8;     NT[i++]=-1.7; //[104]Ryan
H0[i]=63.13;    PT[i]=+6.48;    NT[i++]=-6.48; //[105]Saha & Sahoo
H0[i]=74.2;     PT[i]=+2.7;     NT[i++]=-3; //[106]Shajib et al.
H0[i]=75;       PT[i]=+40;      NT[i++]=-32; //[107]Soares-Santos
H0[i]=78;       PT[i]=+96;      NT[i++]=-24; //[107]Soares-Santos
H0[i]=73.1;     PT[i]=+0.7;     NT[i++]=-0.7; //[108]Taubenberger et al.
H0[i]=68;       PT[i]=+14;      NT[i++]=-7; //[109]Tiwari et al.
H0[i]=68;       PT[i]=+18;      NT[i++]=-8; //[109]Tiwari et al.
H0[i]=73.9;     PT[i]=+1;       NT[i++]=-1; //[110]Verde et al.
H0[i]=72.5;     PT[i]=+1.2;     NT[i++]=-1.2; //[110]Verde et al.
H0[i]=73.3;     PT[i]=+1.7;     NT[i++]=-1.8; //[111]Wong et al.
H0[i]=72.4;     PT[i]=+2;       NT[i++]=-2; //[112]Yuan et al.
H0[i]=68.36;    PT[i]=+0.53;    NT[i++]=-0.52; //[113]Zhang & Huang
H0[i]=64.9;     PT[i]=+4.6;     NT[i++]=-4.3; //[114]Zeng and Yan
H0[i]=67.4;     PT[i]=+1.1;     NT[i++]=-1.2; //[115]Abbott et al.
H0[i]=69.3;     PT[i]=+0.4;     NT[i++]=-0.6; //[115]Abbott et al.
H0[i]=73.24;    PT[i]=+1.74;    NT[i++]=-1.74; //[116]Benetti et al.
H0[i]=72.5;     PT[i]=+2.1;     NT[i++]=-2.1; //[117]Bolejko
H0[i]=68.1;     PT[i]=+2;       NT[i++]=-2; //[117]Bolejko
H0[i]=76;       PT[i]=+8;       NT[i++]=-8; //[118]Braatz
H0[i]=69.3;     PT[i]=+4.2;     NT[i++]=-4.2; //[118]Braatz
H0[i]=71.9;     PT[i]=+7.1;     NT[i++]=-7.1; //[119]Cantiello et al.
H0[i]=73.24;    PT[i]=+1.74;    NT[i++]=-1.74; //[120]Chen
H0[i]=67.4;     PT[i]=+0.5;     NT[i++]=-0.5; //[120]Chen
H0[i]=73.24;    PT[i]=+1.74;    NT[i++]=-1.74; //[121]Choudhury & Choubey
H0[i]=72.8;     PT[i]=+1.6;     NT[i++]=-1.6; //[122]Dhawan et al.
H0[i]=55;       PT[i]=+7;       NT[i++]=-20;//[123]Di Valentino & Melchi-
orri
H0[i]=67.06;    PT[i]=+1.68;    NT[i++]=-1.68; //[124]Gomez-Valent
H0[i]=68.9;     PT[i]=+1.96;    NT[i++]=-1.96; //[124]Gomez-Valent
H0[i]=68.45;    PT[i]=+2;       NT[i++]=-2; //[124]Gomez-Valent
H0[i]=73.5;     PT[i]=+4.6;     NT[i++]=-4.7; //[125]Grillo
H0[i]=72.8;     PT[i]=+4.3;     NT[i++]=-4.1; //[125]Grillo
H0[i]=69.8;     PT[i]=+5.3;     NT[i++]=-4.1; //[125]Grillo
H0[i]=70.38;    PT[i]=+0.6;     NT[i++]=-0.6; //[126]Hoeneisen et al.
H0[i]=71.17;    PT[i]=+1.66;    NT[i++]=-1.66; //[127]Lee & Jang
H0[i]=73.52;    PT[i]=+1.62;    NT[i++]=-1.62; //[128]Riess et al.
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H0[i]=73.83;    PT[i]=+1.48;   NT[i++]=-1.48; //[128]Riess et al.
H0[i]=73.48;    PT[i]=+1.66;   NT[i++]=-1.66; //[129]Riess et al.
H0[i]=74.4;     PT[i]=+4.9;    NT[i++]=-4.9; //[130]Van Putten
H0[i]=74.5;     PT[i]=+7.3;    NT[i++]=-7.3; //[130]Van Putten
H0[i]=74.9;     PT[i]=+2.6;    NT[i++]=-2.6; //[130]Van Putten
H0[i]=66.8;     PT[i]=+1.9;    NT[i++]=-1.9; //[130]Van Putten
H0[i]=73.75;    PT[i]=+1.44;   NT[i++]=-1.44; //[130]Van Putten
H0[i]=70;       PT[i]=+12;     NT[i++]=-8; //[131]Vitale
H0[i]=67;       PT[i]=+4;      NT[i++]=-4; //[132]Yu et al.
H0[i]=67.498;   PT[i]=+7.97;   NT[i++]=-3.278; //[133]Zhang
H0[i]=70;       PT[i]=+12;     NT[i++]=-8; //[134]Abbott
H0[i]=72.5;     PT[i]=+2.5;    NT[i++]=-8; //[135]Bethapudi & Desai
H0[i]=71.9;     PT[i]=+2.4;    NT[i++]=-3; //[136]Bonvin et al.
H0[i]=69.2;     PT[i]=+1.4;    NT[i++]=-2.2; //[136]Bonvin et al.
H0[i]=79;       PT[i]=+4.4;    NT[i++]=-4.2; //[136]Bonvin et al.
H0[i]=73.75;    PT[i]=+2.11;   NT[i++]=-2.11; //[137]Cardona
H0[i]=67.81;    PT[i]=+0.92;   NT[i++]=-0.92; //[137]Cardona
H0[i]=66.93;    PT[i]=+0.62;   NT[i++]=-0.62; //[137]Cardona
H0[i]=73.46;    PT[i]=+1.4;    NT[i++]=-1.4; //[137]Cardona
H0[i]=68.3;     PT[i]=+2.7;    NT[i++]=-2.6; //[138]Chen Yun et al.
H0[i]=68.4;     PT[i]=+2.9;    NT[i++]=-3.3; //[138]Chen Yun et al.
H0[i]=65;       PT[i]=+6.6;    NT[i++]=-6.6; //[138]Chen Yun et al.
H0[i]=67.9;     PT[i]=+2.4;    NT[i++]=-2.4; //[138]Chen Yun et al.
H0[i]=68;       PT[i]=+2.8;    NT[i++]=-2.8; //[139]Farooq
H0[i]=73.24;    PT[i]=+1.74;   NT[i++]=-1.74; //[139]Farooq
H0[i]=72.72;    PT[i]=+1.67;   NT[i++]=-1.67; //[140]Feeney et al.
H0[i]=73.15;    PT[i]=+1.78;   NT[i++]=-1.78; //[140]Feeney et al.
H0[i]=67.6;     PT[i]=+0.7;    NT[i++]=-0.6; //[141]Grieb et al.
H0[i]=73;       PT[i]=+1.75;   NT[i++]=-1.75; //[142]Guo & Zhang
H0[i]=73.24;    PT[i]=+1.74;   NT[i++]=-1.74; //[143]Hjorth et al.
H0[i]=69.13;    PT[i]=+0.24;   NT[i++]=-0.24; //[144]Huang and Huang
H0[i]=71.66;    PT[i]=+1.8;    NT[i++]=-1.8; //[145]Jang & Lee
H0[i]=73.72;    PT[i]=+2.03;   NT[i++]=-2.03; //[145]Jang & Lee
H0[i]=71.17;    PT[i]=+1.66;   NT[i++]=-1.66; //[145]Jang & Lee
H0[i]=66.2;     PT[i]=+8.9;    NT[i++]=-8.9; //[146]Pritychenko
H0[i]=67.2;     PT[i]=+6.9;    NT[i++]=-6.9; //[146]Pritychenko
H0[i]=69.13;    PT[i]=+2.34;   NT[i++]=-2.34; //[147]Wang et al.
H0[i]=73.24;    PT[i]=+1.74;   NT[i++]=-1.74; //[148]Wei & Wu
H0[i]=69.6;     PT[i]=+0.7;    NT[i++]=-0.7; //[148]Wei & Wu
H0[i]=73.1;     PT[i]=+5.7;    NT[i++]=-6; //[149]Wong et al.
H0[i]=72.5;     PT[i]=+3.1;    NT[i++]=-3.1; //[150]Zhang et al.
H0[i]=67.8;     PT[i]=+0.9;    NT[i++]=-0.9; //[151]Ade et al.
H0[i]=66;       PT[i]=+6;      NT[i++]=-6; //[152]Gao et al.
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H0[i]=70.1;    PT[i]=+0.34;   NT[i++]=-0.34; //[153]Ichiki et al.
H0[i]=66.5;    PT[i]=+1.8;    NT[i++]=-1.8; //[154]Ludovic et al.
H0[i]=64.2;    PT[i]=+1.9;    NT[i++]=-1.9; //[154]Ludovic et al.
H0[i]=91.8;    PT[i]=+5.3;    NT[i++]=-5.3; //[155]Moresco et al.
H0[i]=72.25;   PT[i]=+2.51;   NT[i++]=-2.51; //[156]Riess et al.
H0[i]=72.04;   PT[i]=+2.67;   NT[i++]=-2.67; //[156]Riess et al.
H0[i]=76.18;   PT[i]=+2.37;   NT[i++]=-2.37; //[156]Riess et al.
H0[i]=74.5;    PT[i]=+3.27;   NT[i++]=-3.27; //[156]Riess et al.
H0[i]=73.24;   PT[i]=+1.74;   NT[i++]=-1.74; //[156]Riess et al.
H0[i]=76.2;    PT[i]=+3.4;    NT[i++]=-3.4; //[157]Tully et al.
H0[i]=75;      PT[i]=+2;      NT[i++]=-2; //[157]Tully et al.
H0[i]=68.17;   PT[i]=+1.55;   NT[i++]=-1.56; //[158]Cheng & Qing Guo
H0[i]=68.11;   PT[i]=+1.69;   NT[i++]=-1.69; //[158]Cheng & Qing Guo
H0[i]=68.11;   PT[i]=+0.86;   NT[i++]=-0.86; //[158]Cheng & Qing Guo
H0[i]=67.7;    PT[i]=+1.1;    NT[i++]=-1.1; //[159]Cuesta et al.
H0[i]=69.8;    PT[i]=+2.6;    NT[i++]=-2.6; //[160]Jang & Lee
H0[i]=72.2;    PT[i]=+3.3;    NT[i++]=-3.3; //[160]Jang & Lee
H0[i]=68.1;    PT[i]=+5.9;    NT[i++]=-5.9; //[161]Kumar et al.
H0[i]=73;      PT[i]=+26;     NT[i++]=-22; //[162]Kuo et al.
H0[i]=70.6;    PT[i]=+2.6;    NT[i++]=-2.6; //[163]Rigault et al.
H0[i]=68.8;    PT[i]=+3.3;    NT[i++]=-3.3; //[163]Rigault et al.
H0[i]=67.3;    PT[i]=+1.2;    NT[i++]=-1.2; //[164]Ade et al.
H0[i]=70.8;    PT[i]=+2.4;    NT[i++]=-2.4; //[165]Ben-Dayan et al.
H0[i]=69.6;    PT[i]=+0.7;    NT[i++]=-0.7; //[166]Bennett et al.
H0[i]=64.9;    PT[i]=+4.2;    NT[i++]=-4.2; //[167]Busti et al.
H0[i]=72.5;    PT[i]=+2.5;    NT[i++]=-2.5; //[168]Efstathiou
H0[i]=70.6;    PT[i]=+3.3;    NT[i++]=-3.3; //[168]Efstathiou
H0[i]=74.1;    PT[i]=+2.2;    NT[i++]=-2.2; //[169]Lima & Cunha
H0[i]=70;      PT[i]=+2.2;    NT[i++]=-2.2; //[170]Bennett et al.
H0[i]=69.32;   PT[i]=+0.8;    NT[i++]=-0.8; //[170]Bennett et al.
H0[i]=68;      PT[i]=+4.8;    NT[i++]=-4.8; //[171]Braatz et al.
H0[i]=68;      PT[i]=+2.8;    NT[i++]=-2.8; //[172]Farooq & Bathra
H0[i]=73.8;    PT[i]=+2.4;    NT[i++]=-2.4; //[172]Farooq & Bathra
H0[i]=69.7;    PT[i]=+2.4;    NT[i++]=-2.4; //[173]Hinshaw et al.
H0[i]=70.4;    PT[i]=+2.5;    NT[i++]=-2.5; //[173]Hinshaw et al.
H0[i]=69.33;   PT[i]=+0.88;   NT[i++]=-0.88; //[173]Hinshaw et al.
H0[i]=70.2;    PT[i]=+1.4;    NT[i++]=-1.4; //[173]Hinshaw et al.
H0[i]=70;      PT[i]=+3;      NT[i++]=-3; //[174]Humphreys et al.
H0[i]=68;      PT[i]=+9;      NT[i++]=-9; //[175]Kuo et al.
H0[i]=49.97;   PT[i]=+0.19;   NT[i++]=-0.19; //[176]Pietrzynski et al.
H0[i]=68.9;    PT[i]=+7.1;    NT[i++]=-7.1; //[177]Reid et al.
H0[i]=72.1;    PT[i]=+3.2;    NT[i++]=-2.3; //[178]Salvatelli et al.
H0[i]=74.1;    PT[i]=+2.1;    NT[i++]=-2.1; //[179]Scowcroft et al.
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H0[i]=69;    PT[i]=+6;    NT[i++]=-6; //[180]Sereno and Pacificz
H0[i]=80;    PT[i]=+5.8;  NT[i++]=-5.7; //[181]Suyu et al.
H0[i]=75.2;  PT[i]=+4.4;  NT[i++]=-4.2; //[181]Suyu et al.
H0[i]=73.1;  PT[i]=+2.4;  NT[i++]=-3.6; //[181]Suyu et al.
H0[i]=74.4;  PT[i]=+3;    NT[i++]=-3; //[182]Tully et al.
H0[i]=71.3;  PT[i]=+2;    NT[i++]=-2; //[183]Xia et al.
H0[i]=73.8;  PT[i]=+2.4;  NT[i++]=-2.4; //[184]Calabrese et al.
H0[i]=68;    PT[i]=+2.8;  NT[i++]=-2.8; //[184]Calabrese et al.
H0[i]=69.7;  PT[i]=+2.5;  NT[i++]=-2.5; //[184]Calabrese et al.
H0[i]=74.3;  PT[i]=+3.1;  NT[i++]=-3.1; //[185]Chavez
H0[i]=67;    PT[i]=+3.2;  NT[i++]=-3.2; //[186]Colless et al.
H0[i]=74.3;  PT[i]=+3;    NT[i++]=-3; //[187]Freedman et al.
H0[i]=70.2;  PT[i]=+0.14; NT[i++]=-0.14; //[188]Pozzo
H0[i]=75.4;  PT[i]=+2.9;  NT[i++]=-2.9; //[189]Riess et al.
H0[i]=56;    PT[i]=+2;    NT[i++]=-2; //[190]Wang
H0[i]=68;    PT[i]=+5.5;  NT[i++]=-5.5; //[191]Chen & Ratra
H0[i]=67;    PT[i]=+3.2;  NT[i++]=-3.2; //[192]Beutler et al.
H0[i]=71;    PT[i]=+2.5;  NT[i++]=-2.5; //[193]Jarosik et al.
H0[i]=70.4;  PT[i]=+1.3;  NT[i++]=-1.4; //[193]Jarosik et al.
H0[i]=74.8;  PT[i]=+3.1;  NT[i++]=-3.1; //[194]Riess et al.
H0[i]=74.4;  PT[i]=+2.5;  NT[i++]=-2.5; //[194]Riess et al.
H0[i]=73.8;  PT[i]=+2.4;  NT[i++]=-2.4; //[194]Riess et al.
H0[i]=73;    PT[i]=+2;    NT[i++]=-2; //[195]Freedman & Madore
H0[i]=66;    PT[i]=+6;    NT[i++]=-4; //[196]Paraficz et Hjorth
H0[i]=76;    PT[i]=+3;    NT[i++]=-3; //[196]Paraficz et Hjorth
H0[i]=70.6;  PT[i]=+3.1;  NT[i++]=-3.1; //[197]Suyu et al.
H0[i]=69.7;  PT[i]=+4.9;  NT[i++]=-5; //[197]Suyu et al.
H0[i]=70.5;  PT[i]=+1.3;  NT[i++]=-1.3; //[198]Hinshaw et al.
H0[i]=71.9;  PT[i]=+2.6;  NT[i++]=-2.7; //[198]Hinshaw et al.
H0[i]=70.5;  PT[i]=+1.3;  NT[i++]=-1.3; //[199]Komatsu et al.
H0[i]=70.4;  PT[i]=+1.4;  NT[i++]=-1.4; //[199]Komatsu et al.
H0[i]=70.9;  PT[i]=+1.3;  NT[i++]=-1.3; //[199]Komatsu et al.
H0[i]=70.1;  PT[i]=+1.3;  NT[i++]=-1.3; //[199]Komatsu et al.
H0[i]=74.2;  PT[i]=+3.6;  NT[i++]=-3.6; //[200]Riess et al.
H0[i]=84.2;  PT[i]=+6;    NT[i++]=-6; //[201]Russell
H0[i]=83.4;  PT[i]=+8;    NT[i++]=-8; //[201]Russell
H0[i]=88;    PT[i]=+6;    NT[i++]=-6; //[201]Russell
H0[i]=61.7;  PT[i]=+1.2;  NT[i++]=-1.1; //[202]Leith et al.
H0[i]=67;    PT[i]=+13;   NT[i++]=-10; //[203]Vuissoz et al.
H0[i]=63;    PT[i]=+7;    NT[i++]=-3; //[203]Vuissoz et al.
H0[i]=70;    PT[i]=+6;    NT[i++]=-6; //[204]Oguri
H0[i]=68;    PT[i]=+6;    NT[i++]=-6; //[204]Oguri
H0[i]=73.5;  PT[i]=+3.2;  NT[i++]=-3.2; //[205]Spergel et al.
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H0[i]=73.2;   PT[i]=+3.1;   NT[i++]=-3.2; //[205]Spergel et al.
H0[i]=70.4;   PT[i]=+1.5;   NT[i++]=-1.6; //[205]Spergel et al.
H0[i]=76.9;   PT[i]=+3.9;   NT[i++]=-3.4; //[206]Bonamente et al.
H0[i]=73.7;   PT[i]=+4.6;   NT[i++]=-3.8; //[206]Bonamente et al.
H0[i]=77.6;   PT[i]=+4.8;   NT[i++]=-4.3; //[206]Bonamente et al.
H0[i]=70.8;   PT[i]=+1.9;   NT[i++]=-1.8; //[207]Hütsi
H0[i]=74.92;  PT[i]=+2.28;  NT[i++]=-2.28; //[208]Ngeow and Kanbur
H0[i]=74.37;  PT[i]=+2.27;  NT[i++]=-2.27; //[208]Ngeow and Kanbur
H0[i]=62.3;   PT[i]=+1.3;   NT[i++]=-1.3; //[209]Sandage et al.
H0[i]=60.9;   PT[i]=+1.3;   NT[i++]=-1.3; //[209]Sandage et al.
H0[i]=60.7;   PT[i]=+1.5;   NT[i++]=-1.5; //[209]Sandage et al.
H0[i]=72;     PT[i]=+6;     NT[i++]=-6; //[210]Wang et al.
H0[i]=73.2;   PT[i]=+7;     NT[i++]=-7; //[211]Gibson & Brook
H0[i]=75;     PT[i]=+7;     NT[i++]=-7; //[212]Hamuy
H0[i]=65;     PT[i]=+12;    NT[i++]=-12; //[212]Hamuy
H0[i]=58;     PT[i]=+2;     NT[i++]=-2; //[213]Magain
H0[i]=58;     PT[i]=+2;     NT[i++]=-2; //[214]Olivares et al.
H0[i]=73;     PT[i]=+4;     NT[i++]=-4; //[215]Riess
H0[i]=69;     PT[i]=+8;     NT[i++]=-8; //[216]Schmidt et al.
H0[i]=66;     PT[i]=+8;     NT[i++]=-8; //[217]Stritzinger et al.
H0[i]=78;     PT[i]=+9;     NT[i++]=-9; //[217]Stritzinger et al.
H0[i]=67;     PT[i]=+30;    NT[i++]=-18; //[218]Udomprasert et al.
H0[i]=64;     PT[i]=+7;     NT[i++]=-4; //[219]Boffi & Riess
H0[i]=33;     PT[i]=+5;     NT[i++]=-5; //[220]Dumin
H0[i]=69;     PT[i]=+12;    NT[i++]=-12; //[221]Jimenez et al.
H0[i]=75;     PT[i]=+7;     NT[i++]=-6; //[222]Koopmans
H0[i]=70;     PT[i]=+7;     NT[i++]=-7; //[223]Mei et al.
H0[i]=68;     PT[i]=+6;     NT[i++]=-6; //[223]Mei et al.
H0[i]=68;     PT[i]=+5;     NT[i++]=-5; //[223]Mei et al.
H0[i]=71;     PT[i]=+4;     NT[i++]=-4; //[223]Mei et al.
H0[i]=77;     PT[i]=+19;    NT[i++]=-15; //[224]Saunders et al.
H0[i]=85;     PT[i]=+20;    NT[i++]=-17; //[224]Saunders et al.
H0[i]=72;     PT[i]=+5;     NT[i++]=-5; //[225]Spergel et al.
H0[i]=71;     PT[i]=+4;     NT[i++]=-3; //[225]Spergel et al.
H0[i]=63;     PT[i]=+2;     NT[i++]=-2; //[226]Fassnacht et al.
H0[i]=72;     PT[i]=+8;     NT[i++]=-8; //[227]Freedman
H0[i]=57;     PT[i]=+23;    NT[i++]=-16; //[228]Grainge et al.
H0[i]=48;     PT[i]=+7;     NT[i++]=-4; //[229]Kochanek
H0[i]=71;     PT[i]=+6;     NT[i++]=-6; //[229]Kochanek
H0[i]=72;     PT[i]=+8;     NT[i++]=-8; //[229]Kochanek
H0[i]=62;     PT[i]=+7;     NT[i++]=-7; //[229]Kochanek
H0[i]=75;     PT[i]=+8;     NT[i++]=-8;//[230]Tikhonov & Galazoutdi-
nova
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H0[i]=81;    PT[i]=+5;    NT[i++]=-5;//[230]Tikhonov & Galazoutdi-
nova
H0[i]=59;    PT[i]=+15;   NT[i++]=-10; //[231]Treu & Koopmans
H0[i]=71;    PT[i]=+2;    NT[i++]=-2; //[232]Freedman et al.
H0[i]=71;    PT[i]=+3;    NT[i++]=-3; //[232]Freedman et al.
H0[i]=70;    PT[i]=+5;    NT[i++]=-5; //[232]Freedman et al.
H0[i]=72;    PT[i]=+9;    NT[i++]=-9; //[232]Freedman et al.
H0[i]=82;    PT[i]=+6;    NT[i++]=-6; //[232]Freedman et al.
H0[i]=72;    PT[i]=+8;    NT[i++]=-8; //[232]Freedman et al.
H0[i]=65;    PT[i]=+5;    NT[i++]=-5; //[233]Itoh
H0[i]=76;    PT[i]=+1.3;  NT[i++]=-1.3; //[234]Jensen et al.
H0[i]=72;    PT[i]=+2.3;  NT[i++]=-2.3; //[234]Jensen et al.
H0[i]=65;    PT[i]=+5;    NT[i++]=-5; //[235]Koopmans et al.
H0[i]=71;    PT[i]=+8;    NT[i++]=-8; //[236]Liu & Graham
H0[i]=64;    PT[i]=+14;   NT[i++]=-18; //[237]Mason et al.
H0[i]=66;    PT[i]=+14;   NT[i++]=-11; //[237]Mason et al.
H0[i]=70;    PT[i]=+7;    NT[i++]=-7; //[238]Mei et al.
H0[i]=69;    PT[i]=+4;    NT[i++]=-4; //[239]Tonry
H0[i]=71;    PT[i]=+6;    NT[i++]=-6; //[240]Willick & Puneet
H0[i]=63;    PT[i]=+4.3;  NT[i++]=-4.3; //[241]Xiao-Feng et al.
H0[i]=69;    PT[i]=+4;    NT[i++]=-4; //[242]Ferrarese et al.
H0[i]=68;    PT[i]=+2;    NT[i++]=-2; //[243]Gibson et al.
H0[i]=71;    PT[i]=+6;    NT[i++]=-6; //[244]Mould et al.
H0[i]=68;    PT[i]=+6;    NT[i++]=-6; //[244]Mould et al.
H0[i]=71;    PT[i]=+4;    NT[i++]=-4; //[245]Sakai et al.
H0[i]=77;    PT[i]=+7;    NT[i++]=-7; //[246]Tikhonov et al.
H0[i]=69;    PT[i]=+12;   NT[i++]=-19; //[247]Biggs et al.
H0[i]=69;    PT[i]=+18;   NT[i++]=-12; //[248]Chae KH
H0[i]=74;    PT[i]=+18;   NT[i++]=-17; //[248]Chae KH
H0[i]=42;    PT[i]=+9;    NT[i++]=-9; //[249]Collier et al.
H0[i]=73;    PT[i]=+6;    NT[i++]=-6; //[250]Freedman et al.
H0[i]=64;    PT[i]=+8;    NT[i++]=-6; //[251]Jha et al.
H0[i]=85;    PT[i]=+27;   NT[i++]=-23; //[252]Mason & Myers
H0[i]=61;    PT[i]=+15;   NT[i++]=-14; //[252]Mason & Myers
H0[i]=61;    PT[i]=+23;   NT[i++]=-21; //[252]Mason & Myers
H0[i]=80;    PT[i]=+19;   NT[i++]=-17; //[252]Mason & Myers
H0[i]=68;    PT[i]=+21;   NT[i++]=-19; //[252]Mason & Myers
H0[i]=71;    PT[i]=+5;    NT[i++]=-5; //[252]Mason & Myers
H0[i]=86;    PT[i]=+24;   NT[i++]=-24; //[253]Mazumdar & Narasimba
H0[i]=67;    PT[i]=+7;    NT[i++]=-7; //[254]Tanvir et al.
H0[i]=62.9;  PT[i]=+1.6;  NT[i++]=-1.6; //[255]Tripp & Branch
H0[i]=62;    PT[i]=+2;    NT[i++]=-2; //[255]Tripp & Branch
H0[i]=60;    PT[i]=+10;   NT[i++]=-10; //[256]Branch
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H0[i]=66;     PT[i]=+15;   NT[i++]=-14; //[257]Goicoechea et al.
H0[i]=77;     PT[i]=+8;    NT[i++]=-8; //[258]Harris et al.
H0[i]=47;     PT[i]=+23;   NT[i++]=-15; //[259]Hughes & Birkinshaw
H0[i]=82;     PT[i]=+8;    NT[i++]=-8; //[260]Lauer et al.
H0[i]=89;     PT[i]=+10;   NT[i++]=-10; //[260]Lauer et al.
H0[i]=65.2;   PT[i]=+1.3;  NT[i++]=-1.3; //[261]Riess et al.
H0[i]=63.8;   PT[i]=+1.3;  NT[i++]=-1.3; //[261]Riess et al.
H0[i]=55;     PT[i]=+8;    NT[i++]=-8; //[262]Tammann & Labhardt
H0[i]=60;     PT[i]=+6;    NT[i++]=-6; //[263]Tripp
H0[i]=70;     PT[i]=+5;    NT[i++]=-5; //[264]Giovanelli
H0[i]=76;     PT[i]=+8;    NT[i++]=-8; //[264]Giovanelli
H0[i]=67;     PT[i]=+8;    NT[i++]=-8; //[264]Giovanelli
H0[i]=75;     PT[i]=+6;    NT[i++]=-6; //[265]Gregg
H0[i]=67;     PT[i]=+8;    NT[i++]=-8; //[266]Hjorth & Tanvir
H0[i]=70;     PT[i]=+7;    NT[i++]=-7; //[266]Hjorth & Tanvir
H0[i]=60;     PT[i]=+40;   NT[i++]=-23; //[267]Holzapfel et al.
H0[i]=78;     PT[i]=+34;   NT[i++]=-28; //[267]Holzapfel et al.
H0[i]=78;     PT[i]=+60;   NT[i++]=-40; //[267]Holzapfel et al.
H0[i]=58;     PT[i]=+10;   NT[i++]=-5; //[268]Hoyle et al.
H0[i]=74;     PT[i]=+10;   NT[i++]=-10; //[269]Schechter
H0[i]=52.5;   PT[i]=+2.5;  NT[i++]=-2.5; //[270]Sciama
H0[i]=54.8;   PT[i]=+0.3;  NT[i++]=-0.3; //[270]Sciama
H0[i]=81;     PT[i]=+6;    NT[i++]=-6; //[271]Tonry et al.
H0[i]=69;     PT[i]=+8;    NT[i++]=-8; //[272]Amendola
H0[i]=80;     PT[i]=+17;   NT[i++]=-17; //[272]Amendola
H0[i]=49.5;   PT[i]=+4.5;  NT[i++]=-4.5; //[273]Biesiada
H0[i]=65;     PT[i]=+8;    NT[i++]=-8; //[274]Forbes et al.
H0[i]=103;    PT[i]=+59;   NT[i++]=-28; //[275]Kobayashi
H0[i]=82;     PT[i]=+56;   NT[i++]=-24; //[275]Kobayashi
H0[i]=60;     PT[i]=+24;   NT[i++]=-13; //[275]Kobayashi
H0[i]=51;     PT[i]=+10;   NT[i++]=-7; //[275]Kobayashi
H0[i]=33;     PT[i]=+22;   NT[i++]=-9; //[275]Kobayashi
H0[i]=74;     PT[i]=+26;   NT[i++]=-15; //[275]Kobayashi
H0[i]=63;     PT[i]=+28;   NT[i++]=-15; //[275]Kobayashi
H0[i]=80;     PT[i]=+17;   NT[i++]=-17; //[276]Mallik
H0[i]=87;     PT[i]=+7;    NT[i++]=-7; //[276]Mallik
H0[i]=55;     PT[i]=+3;    NT[i++]=-3; //[277]Schaefer
H0[i]=56;     PT[i]=+3;    NT[i++]=-3; //[277]Schaefer
H0[i]=82.5;   PT[i]=+5.9;  NT[i++]=-3; //[278]Grogin & Narayan
H0[i]=82.5;   PT[i]=+8.7;  NT[i++]=-5.6; //[278]Grogin & Narayan
H0[i]=71;     PT[i]=+30;   NT[i++]=-25; //[279]Herbig
H0[i]=74.6;   PT[i]=+47;   NT[i++]=-33; //[280]Holzapfel et al.
H0[i]=38;     PT[i]=+18;   NT[i++]=-16; //[281]Jones
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H0[i]=80;    PT[i]=+17;  NT[i++]=-17; //[282]Kennicutt Jr et al.
H0[i]=73;    PT[i]=+11;  NT[i++]=-11; //[283]Mould
H0[i]=81;    PT[i]=+11;  NT[i++]=-11; //[283]Mould
H0[i]=84;    PT[i]=+16;  NT[i++]=-16; //[283]Mould
H0[i]=76;    PT[i]=+10;  NT[i++]=-10; //[283]Mould
H0[i]=82;    PT[i]=+11;  NT[i++]=-11; //[283]Mould
H0[i]=71;    PT[i]=+10;  NT[i++]=-10; //[283]Mould
H0[i]=80;    PT[i]=+17;  NT[i++]=-17; //[283]Mould
H0[i]=80;    PT[i]=+17;  NT[i++]=-17; //[284]Nakamura & Suto
H0[i]=58;    PT[i]=+6;   NT[i++]=-6; //[285]Rephaeli
H0[i]=51;    PT[i]=+7;   NT[i++]=-7; //[286]Schaefer
H0[i]=61;    PT[i]=+12;  NT[i++]=-12; //[286]Schaefer
H0[i]=26;    PT[i]=+5;   NT[i++]=-5; //[286]Schaefer
H0[i]=69;    PT[i]=+8;   NT[i++]=-8; //[287]Tanvir et al.
H0[i]=78;    PT[i]=+11;  NT[i++]=-11; //[288]Whitmore & Schweizer
H0[i]=65;    PT[i]=+25;  NT[i++]=-25; //[289]Birkinshaw & Hughes
H0[i]=55;    PT[i]=+17;  NT[i++]=-17; //[289]Birkinshaw & Hughes
H0[i]=80;    PT[i]=+17;  NT[i++]=-17; //[290]Freedman
H0[i]=84;    PT[i]=+5;   NT[i++]=-5; //[291]Lu et al.
H0[i]=73;    PT[i]=+6;   NT[i++]=-6; //[292]Schmidt & Kirshner
H0[i]=90;    PT[i]=+10;  NT[i++]=-10; //[293]Tully
H0[i]=43.5;  PT[i]=+2.7; NT[i++]=-2.7; //[294]Duemmler
H0[i]=77;    PT[i]=+8;   NT[i++]=-8; //[295]Lauer & Postman
H0[i]=51;    PT[i]=+5;   NT[i++]=-5; //[295]Lauer & Postman
H0[i]=75;    PT[i]=+30;  NT[i++]=-30; //[296]Leibundgut & Pinto
H0[i]=40;    PT[i]=+9;   NT[i++]=-9; //[297]Birkinshaw
H0[i]=45;    PT[i]=+12;  NT[i++]=-12; //[297]Birkinshaw
H0[i]=82;    PT[i]=+7;   NT[i++]=-7; //[298]Tonry
H0[i]=52;    PT[i]=+2;   NT[i++]=-2; //[299]Sandage & Tammann
H0[i]=45;    PT[i]=+3;   NT[i++]=-3; //[299]Sandage & Tammann
H0[i]=73;    PT[i]=+10;  NT[i++]=-10; //[300]Visvanathan
H0[i]=50;    PT[i]=+10;  NT[i++]=-10; //[301]Sandage & Tammann
H0[i]=52;    PT[i]=+2;   NT[i++]=-2; //[301]Sandage & Tammann
H0[i]=50;    PT[i]=+7;   NT[i++]=-7; //[301]Sandage & Tammann
H0[i]=67;    PT[i]=+10;  NT[i++]=-10; //[302]Dressler
H0[i]=74.3;  PT[i]=+11;  NT[i++]=-11; //[304]Visvanathan
H0[i]=74.3;  PT[i]=+11;  NT[i++]=-11; //[305]Hanes
H0[i]=76;    PT[i]=+8;   NT[i++]=-8; //[306]Bottinelli & Gouguenheim
H0[i]=50.3;  PT[i]=+4.3; NT[i++]=-4.3; //[307]Sandage & Tammann
H0[i]=56.9;  PT[i]=+3.4; NT[i++]=-3.4; //[308]Sandage & Tammann
H0[i]=57;    PT[i]=+6;   NT[i++]=-6; //[309]Sandage & Tammann
H0[i]=55.5;  PT[i]=+8.7; NT[i]=-8.7; //[310]Sandage & Tammann
//Creates an H0 array that contains all the extremities of the tolerance ranges
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```
  for (i=0;i<=nbH0-1;i++) {
    H0[i+nbH0]=H0[i]+PT[i]; Variation[i+nbH0]=PT[i];
    H0[i]=H0[i]+NT[i]; Variation[i]=NT[i]; }
  //Sorts H0 array in ascending order with corresponding Variation of tolerance
  for (j=0;j<=2*nbH0-2;j++) {
    for (i=j+1;i<=2*nbH0-1;i++) {
      if (H0[i]<H0[j]) {
        DH0=H0[j]; DSgn=Variation[j]; H0[j]=H0[i];
        Variation[j]=Variation[i]; H0[i]=DH0; Variation[i]=DSgn; } } }
  for (i=0;i<=2*nbH0-1;i++) {     //Builds the nbCrossings array
    if (i==0) { nbCrossings[i]=1; }
    else {
     if (Variation[i]<0) {nbCrossings[i]=nbCrossings[i-1]+1;}
     if (Variation[i]>0) {nbCrossings[i]=nbCrossings[i-1]-1;}
     if (H0[i]==H0[i-1]) {
       j=i; DeltaCrossings=0;
       do {
         if (Variation[j]<0) {DeltaCrossings=DeltaCrossings+1;}
         if (Variation[j]>0) {DeltaCrossings=DeltaCrossings-1;}
         j=j-1; } while (H0[j]==H0[i]);
       for(k=i;k>j;k--) {nbCrossings[k]=nbCrossings[j]+DeltaCrossings;}
  } } } } //End of CreateTableOfCrossingH0Ranges

//***Function that returns the y coordinate corresponding to x for non
//***centered Gaussian curve
double GaussianCurve(double x, double Mean, double Sigma,
     double Multiplier) {
  double y; //Coordinate y corresponding to x for a non centered Gaussian
  y=(Multiplier/(Sigma*Sqrt(2*Pi)))*exp(-0.5*Sqr((x-Mean)/Sigma));
  return y; }    //End of CreateApproximativeCurve

//***These are the best Gaussians to fit the nbCrossing array as a function of H0***
double FindsGaussianCurvesLS(double Mean[5],double Sigma[5],
     double Multiplier[5]) {
  int i,j; /*Counters*/       double LS = 0; //Least square
  double Sum; //Sum of the 5 Gaussien curve for a specific H0 value
  for (j=0;j<=2*nbH0-1;j++) {
    Sum=0;
for (i=0;i<=4;i++) {
  Sum=Sum+GaussianCurve(H0[j],Mean[i],Sigma[i],Multiplier[i]); }
//We give a heavier weight to any error between 69.2 and 72.1 to
// model the gap between these values
    if ((H0[i]>=69.2)&&(H0[i]<=72.1)) {
```

```
            LS=LS+10*(Sqr(nbCrossings[j]-Sum));}
        else { LS=LS+Sqr(nbCrossings[j]-Sum); }
        } return LS; }    //End of FindsGaussianCurvesLS

//***This function finds the best Gaussians to fit the real curve
void FindsBestGaussiansToFitRealCurve(void) {
    int i; //Counter
    double DMean[5],DSigma[5],DMultiplier[5]; //Dummy arrays
    double LS, PLS; //Least Square and Previous Least Square
    double StepMean = 0.1, StepSigma = 0.1, StepMultiplier = 0.1; //Variations
    int nbMeanNotImproved = 0; //Tells how many times not improved
    int nbSigmaNotImproved = 0; //Tells how many times not improved
    int nbMultiplierNotImproved = 0; //Tells how many times not improved
    //Starting values (approximative values only)
    Mean[0]=71;          Sigma[0]=1;    Multiplier[0]=-280;
    Mean[1]=68;          Sigma[1]=17;   Multiplier[1]=3800;
    Mean[2]=Mean[1];     Sigma[2]=3;    Multiplier[2]=880;
    Mean[3]=73;          Sigma[3]=7;    Multiplier[3]=1200;
    Mean[4]=Mean[3];     Sigma[4]=2;    Multiplier[4]=470;
    //Fills the 3 dummy arrays DMean, DSigma and DMultiplier
    //with the same values than the arrays Mean, Sigma and Multiplier
    for(i=0;i<=4;i++) {
        DMean[i]=Mean[i]; DSigma[i]=Sigma[i]; DMultiplier[i]=Multiplier[i]; }
        //Tries to find the 5 best Gaussians to fit the curve
        do {
            for (i=0;i<=4;i++) {
                //We improve Mean[i], but we force
                //Mean[2] = Mean[1] & Mean[4] = Mean[3]
                if ((i!=2)&&(i!=4)) {
                    PLS=FindsGaussianCurvesLS(Mean,Sigma,Multiplier);
                    DMean[i]=Mean[i]+StepMean;
                    if (i==1) {DMean[2]=DMean[i];}
                    if (i==3) {DMean[4]=DMean[i];}
                    LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
                    if (LS<PLS) {
                        Mean[i]=DMean[i];
                        if (i==1) {Mean[2]=DMean[i];}
                        if (i==3) {Mean[4]=DMean[i];}
                        nbMeanNotImproved=0; }
                    else {
                        DMean[i]=Mean[i]-StepMean;
                        if (i==1) {DMean[2]=DMean[i];}
                        if (i==3) {DMean[4]=DMean[i];}
```

```
        LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
        if (LS<PLS) {
          Mean[i]=DMean[i];
          if (i==1) {Mean[2]=DMean[i];}
          if (i==3) {Mean[4]=DMean[i];}
          nbMeanNotImproved=0; }
        else {
          DMean[i]=Mean[i];
          if (i==1) {DMean[2]=DMean[i];}
          if (i==3) {DMean[4]=DMean[i];}
          nbMeanNotImproved++;
          if (nbMeanNotImproved>=100) {
            nbMeanNotImproved=0; StepMean=StepMean/10; } } } }
//We try to improve Sigma[i]
PLS=FindsGaussianCurvesLS(Mean,Sigma,Multiplier);
DSigma[i]=Sigma[i]+StepSigma;
LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
if (LS<PLS) { Sigma[i]=DSigma[i]; nbSigmaNotImproved=0; }
else {
  DSigma[i]=Sigma[i]-StepSigma;
  LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
  if (LS<PLS) {
    Sigma[i]=DSigma[i]; nbSigmaNotImproved=0; }
  else {
    DSigma[i]=Sigma[i]; nbSigmaNotImproved++;
    if (nbSigmaNotImproved>=100) {
      nbSigmaNotImproved=0; StepSigma=StepSigma/10; } } }
//We try to improve Multiplier[i]
PLS=FindsGaussianCurvesLS(Mean,Sigma,Multiplier);
DMultiplier[i]=Multiplier[i]+StepMultiplier;
LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
if (LS<PLS) {
  Multiplier[i]=DMultiplier[i]; nbMultiplierNotImproved=0; }
else {
  DMultiplier[i]=Multiplier[i]-StepMultiplier;
  LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
  if (LS<PLS) {
    Multiplier[i]=DMultiplier[i]; nbMultiplierNotImproved=0; }
  else {
    DMultiplier[i]=Multiplier[i]; nbMultiplierNotImproved++;
    if (nbMultiplierNotImproved>=100) {
      nbMultiplierNotImproved=0;
        StepMultiplier=StepMultiplier/10; } } } }
```

```
        LS=FindsGaussianCurvesLS(DMean,DSigma,DMultiplier);
        } while (LS>=22000); //Sets a stop point
      for(i=0;i<=4;i++) {
    printf("\n Mean[%i]=%10lf    Sigma[%i]=%10lf    Multiplier[%i]=%10lf",
      i,Mean[i],i,Sigma[i],i,Multiplier[i]);
    } } //End of FindsBestGaussiansToFitRealCurve

//***The function begins by shifting Mean[1] and Mean[2] of the two
//***Gaussian curves that are around H0=69.2km/(s*MParsec) to
//***H0=72.1km/(s*MParsec) with a theoretical factor of 1.042516951. Then
//***the function that modifies the global H0 array builds the summation of
//***the 4 positive Gaussian curves.
void CreatesFinalGaussianCurve(void) {
    int i; //Dummy index value
    //We shift Mean[1] and Mean[2] with a theoretical factor of 1.042516951
    Mean[1]=Mean[1]*1.042516951; Mean[2]=Mean[1];
    //We omit i=0 to remove the negative Gaussian curve
    for(i=1;i<=nbH0*2-1;i++) {
      nbCrossings[i]=GaussianCurve(H0[i],Mean[1],Sigma[1],Multiplier[1]);
      nbCrossings[i]=nbCrossings[i]+GaussianCurve(H0[i],Mean[2],Sigma[2],
        Multiplier[2]);
      nbCrossings[i]=nbCrossings[i]+GaussianCurve(H0[i],Mean[3],Sigma[3],
        Multiplier[3]);
      nbCrossings[i]=nbCrossings[i]+GaussianCurve(H0[i],Mean[4],Sigma[4],
        Multiplier[4]); } } //End of CreatesFinalGaussianCurve

//***Returns the Least Mean Square of the equation DA*x^2+DB*x+DC.
double FindsLMS(double DA, double DB, double DC) {
    int i; /*Dummy index value*/      double LMS = 0; //Least Mean Square
    for(i=PosTipIndex-n;i<=PosTipIndex+n;i++){
      LMS=LMS+Sqr(nbCrossings[i]-(DA*H0[i]*H0[i]+DB*H0[i]+DC));
      } return LMS; }     //End of FindsLMS

//***Returns A, B, and C coefficients of the quadratic equation of the tip
void ApproximatesTipEquation(void) {
    double LMSTipMin; //Reminds the lowest value of least mean square
    double PLMSTip, NLMSTip; //LMSTip for a forward and backward step
    double DA,DB,DC; //Dummy values of A, B and C coefficients
    double StepA,StepB,StepC; //Step variation of the coefficients
    double nbCrossingsMax = 0; //Maximum number crossings at the tip
    double xa,xb,xc,ya,yb,yc; /*3 coordinates*/    int i; //Dummy index value
    for(i=0;i<=nbH0*2-1;i++) { //Finds the index of the approximated tip
      if (nbCrossings[i]>nbCrossingsMax) {
```

```
        nbCrossingsMax=nbCrossings[i]; PosTipIndex=i; } }
    i=PosTipIndex; xa=H0[i-n]; xb=H0[i]; xc=H0[i+n];
    ya=nbCrossings[i-n]; yb=nbCrossings[i]; yc=nbCrossings[i+n];
    //Sets coefficients ABC
    A=((yc-ya)/((xc-xa)*(xc-xb)))-((yb-ya)/((xb-xa)*(xc-xb)));
    B=((yb-ya)/(xb-xa))-A*(xb+xa);
    C=ya-A*xa*xa-B*xa; DA=A;DB=B;DC=C;
    }   //End of ApproximatesTipEquation

//***Function that returns the Best estimate of H0
void BestEstimateOfH0(void) {
    CreateTableOfCrossingH0Ranges(); FindsBestGaussiansToFitRealCurve();
    CreatesFinalGaussianCurve(); ApproximatesTipEquation();
    BEH=-B/(2*A); Accuracy_ppm = (BEH-TVH)/(TVH*1E-6);
    LMSTip=FindsLMS(A,B,C); }   //End of BesEstimateOfH0

int main(void) {
    BestEstimateOfH0();
    printf("\n\n Equation of the tip: y = %.10lfx^2+ %.10lfx+ %.10lf",A,B,C);
    printf("\n Best estimate of H0 = %.10lf km/(s*MParsec)",BEH);
    printf("\n Theoretical H0       = %.10lf km/(s*MParsec)",TVH);
    printf("\n Relative accuracy versus theoretical value = %.10lf ppm",
    Accuracy_ppm);
    getchar();   return 0;
    } //End of main
```