A Potentially Unifying Constant of Nature (Brief Note)

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Abstract

This brief note describes a method by which numerous empirically-determined quantum constants of nature can be substituted into Einstein's field equation (EFE) for general relativity. This method involves treating the ratio $G/\hbar$ as an empirical constant of nature in its own right. This ratio is represented by a new symbol, N_T. It turns out that the value of N_T (which is $6.32891937 \times 10^{23}$ m·kg^{-2}·s^{-1}) is within 5% of Avogadro's number N_A, although the units are clearly different. Nevertheless, substitutions of N_T or N_A into the EFE, as shown, should yield an absolute value similar in magnitude to that calculated by the conventional EFE. The method described allows for quantum term EFE substitutions into Einstein's gravitational constant κ. These terms include $\hbar$, α, m_e, m_P, R, k_B, F, e, M_U, and m_U. More importantly, perhaps, one or more of the many new expressions given for κ may provide a more accurate result than κ incorporating G. If so, this may have important implications for additional forward progress towards unification. Whether any of these new expressions for Einstein's field equation can move us closer to quantizing gravity remains to be determined.

Keywords

Unification, General Relativity, Quantum Theory, Einstein's Gravitational Constant, Tatum's Number, Avogadro's Number

1. Introduction and Background

There are a myriad of difficulties in attempting to unite gravity with the other fundamental forces of nature. Not the least of these is that our best gravity theory, general relativity, has classical deterministic features, whereas quantum theory is anti-deterministic, probabilistic, and built upon the foundation of Hei-

senberg's uncertainty principle. To unite gravity with the other forces using these two theories, without some modifications to one or both theories, is a bit like mixing oil and water. It simply won't work. *An "emulsifier" approach, successfully combining certain features of both theories, is needed.*

One possible approach towards unification is to work towards quantizing gravity (*i.e.*, "quantum gravity"), as we see with the work of string theorists. The other possible approach is to work towards gravitizing quantum theory [1], although there seems to be less progress from this direction.

This brief note points to a potentially useful way of bringing elements of both theories together by harmonizing two of their most fundamental constants (G and $\hbar$) in the form of a ratio ($G/\hbar$). One can then insert this ratio into Einstein's most fundamental gravity and quantum equations as shown herein.

2. Results

The analytic process in this brief note primarily involves substituting various constants of nature into previously-established relativity and quantum equations, in order to better see their relationships. Only simple algebra is required. No figures or tables are necessary to elucidate these relationships.

The latest available values of G and $\hbar$ are [2]:

$$G = 6.67430 \times 10^{-11}\, \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} \tag{1}$$

$$\hbar = 1.054571817 \times 10^{-34}\, \text{m}^2 \cdot \text{kg} \cdot \text{s}^{-1} \tag{2}$$

Their ratio, which we represent as N_T, is:

$$G/\hbar = N_T = 6.32891937 \times 10^{23}\, \text{m} \cdot \text{kg}^{-2} \cdot \text{s}^{-1} \tag{3}$$

Interestingly, this ratio *approximates* (within 5%) Avogadro's number N_A:

$$N_A = 6.02214076 \times 10^{23}\, \text{mol}^{-1} \tag{4}$$

We will take advantage of this similarity ("near-equivalence") below.

Einstein's field equation (EFE) of general relativity can be expressed as follows:

$$G_{\mu v} + g_{\mu v}\Lambda = \left[\frac{8\pi G}{c^4}\right]T_{\mu v} \tag{5}$$

wherein $G_{\mu v}$ is the Einstein tensor, $g_{\mu v}$ is the metric tensor, Λ is the cosmological constant, G is Newton's gravitational constant, c is speed of light, and $T_{\mu v}$ is the stress-energy tensor.

Notice that the *bracketed coefficient* of the stress-energy tensor, sometimes referred to as κ or Einstein's gravitational constant, contains G. Thus, a rearrangement of relation (3) can be given as follows:

$$G = 6.32891937 \times 10^{23}\, \hbar \tag{6}$$

And substituted in relation (5) as follows:

$$G_{\mu v} + g_{\mu v}\Lambda = \left[\frac{8\pi N_T \hbar}{c^4}\right]T_{\mu v} \tag{7}$$

wherein N_T stands for $6.32891937 \times 10^{23}$ m·kg^{-2}·s^{-1} [see relation (3)].

Thus, Newton's gravitational constant G has been removed from the κ term of the EFE and substituted by the $N_T\hbar$ factor. This allows Planck's reduced constant to become a part of the EFE, without sacrificing any accuracy of the mathematical expression. Furthermore, it has been shown by Seshavatharam & Lakshminarayana [3] that the *magnitude* of G can be expressed as:

$$G = \frac{16\pi^4 m_e^{14} \hbar c}{\alpha^2 m_p^{16}} \tag{8}$$

wherein m_e is mass of the electron, α is the fine structure constant, and m_p is mass of the proton. Therefore, one can rearrange this relation as follows:

$$\frac{G}{\hbar} = N_T = \frac{16\pi^4 m_e^{14} c}{\alpha^2 m_p^{16}} \tag{9}$$

And substitute for N_T in relation (7) as follows:

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = \left[\frac{128\pi^5 m_e^{14} \hbar}{\alpha^2 c^3 m_p^{16}}\right] T_{\mu\nu} \tag{10}$$

This should also be an accurate expression of the EFE, but now with additional quantum terms integrated to express the magnitude of κ.

Furthermore, the near-equivalence of the *magnitude* of N_T and N_A allows for the magnitude of Einstein's gravitational constant to be expressed (approximately) as:

$$\kappa = \left[\frac{8\pi N_A \hbar}{c^4}\right] \tag{11}$$

Additionally, we know of several equivalent expressions for N_A as follows:

$$N_A = \frac{R}{k_B} \tag{12}$$

wherein R is the molar gas constant and k_B is Boltzmann's constant.

$$N_A = \frac{F}{e} \tag{13}$$

wherein F is the Faraday constant and e is the elementary charge.

$$N_A = \frac{M_U}{m_U} \tag{14}$$

wherein M_U is the molar mass constant and m_U is the atomic mass constant.

Relations (12) thru (14) are well-known [4]. These relations are mentioned here in order to provide for additional relations (15) thru (17). Thus, Einstein's gravitational constant term κ can also be expressed as follows:

$$\kappa = \left[\frac{8\pi R\hbar}{k_B c^4}\right] \tag{15}$$

wherein the EFE can now incorporate the molar gas constant, Boltzmann's constant, and Planck's reduced constant.

$$\kappa = \left[\frac{8\pi F \hbar}{ec^4} \right] \qquad (16)$$

wherein the EFE can now incorporate the Faraday constant, elementary charge e, and Planck's reduced constant.

$$\kappa = \left[\frac{8\pi M_U \hbar}{m_U c^4} \right] \qquad (17)$$

wherein the EFE can now incorporate the molar mass constant, the atomic mass constant, and Planck's reduced constant.

Furthermore, if one chooses to insert Newton's gravitational constant G into Einstein's quantum equation for photon energy, $E = h\nu$, one can re-express this relation with Planck's reduced constant, by $E = 2\pi \hbar \nu$, and then substitute G/N_T for $\hbar$ as follows:

$$E = \frac{2\pi G \nu}{N_T} \qquad (18)$$

Thus, Newton's gravitational constant can be worked into Einstein's most famous quantum equation. Of course, it is trivial to continue further substitutions for N_T along similar lines as given above. Unfortunately, nothing would be gained by this approach, as this would introduce into Einstein's (and Planck's) original *precise* quantum formula the roughly 5% absolute magnitude error of using N_A as a substitute for N_T. This would be an unacceptably large error in many quantum applications.

3. Discussion

The approach taken above with respect to substituting various empirically-determined quantum terms into the EFE may have some value, given the relative imprecision in measuring G to more than 3 or 4 decimal places. It is conceivable that one or more of the κ substitutions introduced herein [*i.e.*, in relations (10), (11), (15), (16) and (17)], when integrated into the EFE κ term of relation (5), could potentially *improve* upon the accuracy of the EFE employing G alone. Of course, this remains to be determined.

4. Summary and Conclusion

This brief note has described a method by which numerous empirically-determined quantum constants of nature can be substituted into Einstein's field equation for general relativity. This method involves treating the ratio $G/\hbar$ as an empirical constant of nature in its own right. This ratio is represented by a new symbol, N_T. It turns out that the value of N_T (6.32891937 × 10^{23} m·kg^{-2}·s^{-1}) is within 5% of Avogadro's number N_A, although the units are clearly different. Nevertheless, substitutions of N_A into the EFE, as shown, should yield an absolute value of a similar magnitude to that calculated by the conventional EFE [*i.e.*, relation (5)]. More importantly, perhaps, one or more of these new expressions given for Einstein's gravitational constant term κ may provide a more accurate

result than κ incorporating G. If so, this may have important implications for additional forward progress towards unification. Whether any of these new expressions for Einstein's field equations can move us closer to quantizing gravity remains to be determined.

Data Availability

Due to the purely theoretical nature of this undertaking, no new data were generated or analyzed in support of this research.

Acknowledgements and Dedications

This paper is dedicated to Drs. Stephen Hawking and Roger Penrose for their groundbreaking work on black holes and cosmology. Dr. Tatum also thanks Dr. Rudolph Schild of the Harvard-Smithsonian Center for Astrophysics for his past support and encouragement. Author Seshavatharam is indebted to professors Shri M. Nagaphani Sarma, Chairman, Shri K.V. Krishna Murthy, founder Chairman, Institute of Scientific Research in Vedas (I-SERVE), Hyderabad, India and Shri K.V.R.S. Murthy, former scientist IICT (CSIR), Govt. of India, Director, Research and Development, I-SERVE, for their valuable guidance and great support in developing this subject.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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