Journal of Modern Physics, 2021, 12, 353-360
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

Possibility of Geometrical Interpretation of Quantum Mechanics and Geometrical Meaning of "Hidden Variables"

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How to cite this paper: Olkhov, O.A. (2021) Possibility of Geometrical Interpretation of Quantum Mechanics and Geometrical Meaning of "Hidden Variables". *Journal of Modern Physics*, **12**, 353-360.
https://doi.org/10.4236/jmp.2021.123025

Received: January 5, 2021
Accepted: February 23, 2021
Published: February 26, 2021

Abstract

Interpretation of wave function for free particle is suggested as a description of microscopic distortion of the space-time geometry, namely, as some closed topological 4-manifold. Such geometrical object looks in three-dimensional Euclidean space as its topological defect having stochastic and wave-corpuscular properties of quantum particle. All possible deformations (homeomorphisms) of closed topological manifold play the role of "hidden variables", responsible for statistical character of the theory.

Keywords

Quantum Mechanics, Geometrical Interpretation of Quantum Mechanics, "Hidden Variables"

1. Introduction

This investigation started long ago as an attempt to extend Einstein's idea of the geometrization of the theory of gravity to a possible geometrization of quantum theory. In the Einstein's general theory of relativity, gravitation is considered as a result of macroscopic distortion of the space-time geometry [1]; in this work, quantum particles are considered as a microscopic distortion of the space-time geometry. Suggested geometrizatiom of quantum mechanics means new interpretation of its existing mathematical formalism commonly referred to as "copenhagen interpretation". This new interpretation gives possibility to explain known strange features of above mathematical formalism (statistical description, wave-corpuscular dualism) with the help of notions from everyday life (physical model). Attempts to find such explanation started just after the creation of quantum mechanics and this problem is still considered by many physicians as actual. For example, V. Ginsburg considered interpretation of quantum mechanics as the one of three great problems of modern physics (as the problem of appearance of life and the problem of irreversibility of time) [1]. The problem of interpretation of quantum mechanics was investigated for many years by t'Hooft [2] (here is a detailed list of references on the problem). But why any interpretation is needed for mathematical formalism if it is in a good agreement with experiment? One of reasons is the fact that new physical models open new opportunities for development of theories. For example, many attempts (Einstein Weyl, Calutza and others) have been made for this reason to find geometrical interpretation of classical electrodynamics, although it is in a very good agreement with experiment [3] [4]. In addition, the quantum theory cannot be considered as the final one. Another, more concrete, reason—the contradiction between Bohr and Einstein regarding the completeness of quantum mechanics which did not resolved until now [5] [6]. In contrast to Bohr, Einstein thought that the quantum mechanics is not a complete theory because it says nothing about physical reality, responsible for statistical character of the theory (so called "hidden variables" [2] [7] [8]), and the answer to this question is, may be, the main result of this work. As for physical models, author knows two interpretations of quantum mechanics where mathematical formalism of quantum mechanics is not questioned. One is the Everett's "Many Universes Interpretation", where statistical character of quantum theory is explained by existence of infinite number of Universes, corresponding to various realizations of reality [9]. This interpretation has its supporters in spite of exotic character and serious criticism [10]. Another interpretation is the t'Hooft "The Cellular Automaton Interpretation of Quantum Mechanics", where a very special set of mutually orthogonal states in Hilbert space is considered [2]. This approach is now under development. Among the works where the apparatus of quantum physics is undergoing serious changes we can mention the string theory (see, e.g. [11]) and Santini's investigations [12]. The possibility is shown in this work to interpret the quantum mechanical wave function for free particle as a description of microscopic distortion of the space-time geometry. Some characteristics of this geometrical object play the role of "hidden variables" responsible for stochastic behavior of quantum particle, and these characteristics are the physical reality that exists before measurement. Other characteristics explain wave-corpuscular properties of the particle. It may be said that quantum mechanics within suggested interpretation satisfies the completeness criterion formulated by Einstein. Preliminary results see [13] [14] [15] [16] [17].

2. Quantum Particle as the Microscopic Distortion of the Space-Time Geometry

Let's consider the free neutral particle with mass m and spin 0. It will be shown that wave function of such particle can be interpreted as a mathematical description of some geometrical object. This scalar wave function is the solution of the

Klein-Fock-Gordon equation, and it has the form [18] [19]

$$\Psi = \text{const} \cdot \exp\left(-\frac{i}{\hbar}\left(Et - \boldsymbol{pr}\right)\right). \tag{1}$$

This function describes within existing interpretation the particle's state with definite energy E and definite momentum $\boldsymbol{p}$. The particle's position before measurements is unknown—it may be observed in any point with equal probability. This fact reflects statistical character of quantum mechanics—unusual property within classical representations. Another unusual property—wave-corpuscular dualism of quantum particles that is defined by phase of the wave function and by wave length and frequency, connecting with the particle's energy and momentum by known relations [18] [19]

$$\lambda_i = \frac{\hbar}{p_i}, \omega = \frac{E}{\hbar}, i = x, y, z. \tag{2}$$

Substituting (2) in (1), we have

$$\Psi = \text{const} \cdot \exp\left(-i\omega t + i\boldsymbol{kr}\right), k_i = 2\pi\lambda_i \tag{3}$$

This type of functions (plane wave) is often used in classical physics (for example, for description of plane running sound wave). Within existing interpretation of quantum mechanics the origin of periodical dependence of wave function is not discussing.

Let us rewrite the function (1) not with space coordinates x, y, z and, separately with time coordinate t, but with only space coordinates x^1, x^2, x^3, x^4 of the specific space—the space of events of the special theory of relativity—four dimensional pseudo Euclidean space of index 1 (the Minkowski space [20]). Time, multiplied by light velocity, plays in this space the role of fourth space coordinate ($ct = x^4$). Using in (1) instead of E the relativistic 4-momentum $p_4 = E/c$, the wave function can be written in symmetric form as

$$\Psi = \text{const} \cdot \exp\left(-ix^{\mu} p_{\mu}\right). \tag{4}$$

Here and later relativistic units are used where $\hbar = c = 1$. Summation over repeating indexes is suggested in (4) with signature $(+---)$. In relativistic case [18] [19]

$$p_1^2 - p_2^2 - p_3^2 - p_4^2 = m^2 \tag{5}$$

where m—the particle's mass. Let's write down (4) in such a way that it contains only values with dimensionality of length

$$\Psi = \text{const} \cdot \exp\left(-2\pi i x^{\mu} \lambda_{\mu}^{-1}\right), \tag{6}$$

where

$$\lambda_1^{-2} - \lambda_2^{-2} - \lambda_3^{-2} - \lambda_4^{-2} = \lambda_m^{-2},\ \lambda_{\mu} = 2\pi p_{\mu}^{-1},\ \lambda_m = 2\pi m^{-1}. \tag{7}$$

In contrast to (1, 3) function (6) does not look as a plane wave—it represents periodical function of four space coordinates in the Minkowsri space.

Function (6) may be considered as a function realizing representation of the group whose elements are discrete translations along four coordinates axes in the Minkowski space. Indeed, function (6) goes into itself at translations

$$x^{\mu} \to x^{\mu'} + n_{\mu}\lambda_{\mu}, \tag{8}$$

where n_{μ}—integers (μ = 1, 2, 3, 4). This group is isomorphic to the group $\mathbb{Z}^4$, whose elements are products of integers n_{μ} In turn, the group $\mathbb{Z}^4$ is isomorphic to the fundamental group of closed 4-manifold that is homeomorphic to the foir dimensional torus T^4 [21] [22]. Now we nay formulate the main hypothesis: quantum particle, described by the wave function (6), can be considered as a closed space-time manifold that is homeomorphic to the four dimensional torus imbedded into five dimensional pseudo Euclidean space of index 1. Relation (7) imposes a metric restriction on the acceptable under deformations path lengths $\lambda_i\left(i=1,2,3,4\right)$. Thus, the relation (7) defines also the geometrical interpretation of the particle's mass and 4-momentum. It will be shown in the next Section that such geometrical object looks in three dimensional Euclidean space as moving topological defect of this space having stochastic and wave-corpuscular properties of quantum particle.

Representation of particle as a closed manifold means that this particle before measurement may be considered as a "mixture" of its all possible geometrical representations (homeomorphisms), and only interaction with device fixes one of them. This means that wave function describes not an individual particle, but statistical ensemble of all its possible geometrical representations, and this explains statistical character of quantum mechanics. Thus, ensemble of all possible homeomorphisms plays the role of "hidden variables," responsible for stochastic behavior of particles.

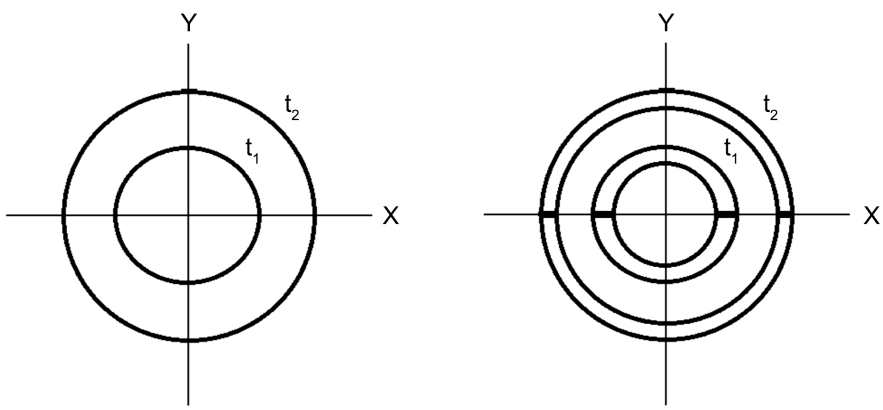
3. Quantum Particle as a Topological Defect of Euclidean Space

Let's proceed to decoding of the representation of quantum particle as a closed 4-manifold, that is let's show how such object looks from the point of view of the observer in Euclidean space. But the important notice should be made before going to this problem. The geometry of four dimensional closed manifolds is now under development: the full recognition algorithm is not now known even for three dimensional closed manifolds [22]. Therefore the only way to establish what the representation of quantum particle as a closed 4-manifold means from the point of view of the observer in Euclidean space is to use low dimensional analogies. Having this in mind let's consider closed manifold homeomorphic to the two dimensional torus embedded into three dimensional pseudo Euclidean space of index 1. To obtain concrete result only one of infinite number of possible homeomorphisms of this manifold will be considered, namely usual two dimensional torus $T^2 = S^1 \times S^1$, where S^1—a circle. Such torus may be considered in three dimensional Euclidean space as a surface obtained by rotation of a circle around vertical axis lying in the plain of this circle (**Figure 1(a)**). In pseudo Euclidean three dimensional space this circle is located in pseudo Euclidean plane and it looks on Eucldean plane of Figure 1(b) as a isosceles hyperbola [23]. That is two dimensional torus, representing particle, looks in three dimensional Euclidean space as a hyperboloid (Figure 1(b)). Within considered low dimensional analogy physical space-time (space of events) is a two dimensional pseudo Euclidean space, and the particle's positions in different moments of time in the Euclidean (one dimensional) space are defined by points of intersection with this space of the projections of the hyperboloid's temporary cross-sections. These cross-sections look as expanding circles in two-dimensional Euclidean plane XY (Figure 2(a)). These circles can be considered as moving topological defect of one dimensional physical space. It is the fact that intersection point belongs to topological defect that distinguishes this point at Figure 2(a) from neighboring points of one dimensional Euclidean space, turning it into a physical "material point.

The particle's positions in Euclidean (one dimensional) space are defined by pounts of its intersection with the circle, corresponding to the only one of the torus possible homeomorphisms. Accounting for all possible homeomorphisms leads, obviously, to "blurring" of this circle and so leads to transformation of the one intersection point in finite region of Euclidean space (this region is indicated at Figure 2(b) by a bold line segment on X-axis). This region has at every moment of time a finite size because the range of all possible homeomorphisms is limited by metric condition (7) that restrict the maximum possible dimensions of closed manifold. As a result, the observer in Euclidean space will detect the

(a) (b)

Figure 1. Two-dimensional torus embedded into three-dimensional Euclidean and pseudo Euclidean spaces.



(a) (b)

Figure 2. Topological defect of one dimensional Euclidean space (X-axis).

particle with equal probability in one of points of above mentioned region. This means that wave function describes not a position of separate particle but the ensemble of its possible positions, and this explains statistical character of quantum mechanics. It is obvious that all possible homeomorphisms of the closed manifold, representing this particle, play the role of "hidden variables", responsible for the particle's stochastic behavior: each homeomorphism corresponds to the one particle's possible position in Euclidean space. The points of the intersection region have different velocities. This means that the intersection region at **Figure 2(b)** are moving expanding, and finally it will fill all Euclidean (one dimensional) space. In result the probability to observe the particle in any point of space will be the same, as it should be according to laws of quantum mechanics for free particle, described by wave function (1).

The fact that the particle can be represented in physical Euclidean space as a part of topological defect allows to explain the particle's wave properties. It is sufficient for this to suppose that the defect's position in the external five dimensional Euclidean space relative to the three dimensional space changes according to periodical low described by wave function (1) (a rigorous proof of this assumption is not possible within the framework of low dimensional analogy). It can be said that the phase of the defect's periodical movement is an additional degree of freedom on which the effect of the particle on the device depends. The particle's corpuscular properties (4-momentum) are defined through parameters of above periodical movement of defect by relations

$$p_\mu = 2\pi\lambda^{-1}. \tag{9}$$

These relations are identical to the definition (2) of the particle's wave length through its momentum within existing interpretation [19], but now they have the "reverse" meaning of definition of momentum through the wave length, as it should be in the consistent theory where less general concepts (classical momentum) are defined through more fundamental ones (wave length of the defect's periodical movement).

4. Conclusion

The wave function plays a dual role within suggested interpretation. First, it is a function, realizing the representation of the fundamental group for a closed 4-manifold, representing a free particle. Second, this function describes periodical movement of topological defect in the external space, and intersection of this defect with physical space defines the possible particle's positions. These properties of the wave function make it possible to explain the stochastic behavior of the particle and its wave-corpuscular dualism. The role of "hidden variables", responsible for the particle's stochastic behavior, is played by all possible homeomorphisms of the closed 4-manifold, representing the particle. Notice in conclusion that relation (7) defines geometrical interpretation of the particle's mass as a characteristic of some fundamental length λ_m. Geometrical interpretation of elementary electrical charge and the particle's spin and possibility of application of geometrical approach to the quantum field theory is now under consideration. After that, the advantages of the proposed approach will become clear.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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