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Particle Physics Problems Addressed with Simple Mathematics Related to General Relativity

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Abstract

Existing particle physics models do not account for dark matter and neutrino mass, or explain the three generations of fundamental fermions. This analysis uses simple mathematics, related to general relativity, to address these problems. The paper does *not* address the very difficult problem of quantizing general relativity.

Keywords

Quantum Mechanics, General Relativity, Standard Model

1. Introduction

Particle physics models fail to account for dark matter, and the Standard Model for fundamental particles of ordinary matter faces several problems. First, since neutrinos oscillate between neutrino states when propagating through space, the Standard Model must be modified to accommodate neutrino mass. Second, the Standard Model does not explain why three, and only three, fundamental fermions are in each Standard Model charge state e, $(2/3)e$, and $-(1/3)e$, where e is electron charge. Third, the Standard Model involves point particles with spin angular momentum $\hbar$, or $\hbar/2$. Angular momentum is usually defined for rotating objects extended in space and, regarding point particles with angular momentum, we might ask what is rotating. Fourth, infinite energy density of point particles is a problem. These problems are reframed below using simple mathematics related to general relativity.

2. Dark Matter from Elbaz-Novello Quantized Friedmann Equation

It is often assumed all four forces governing the universe were unified early in the history of the universe. When initial force symmetry broke, the gravitational structure constant $\frac{Gm_P^2}{\hbar c} = 5.9\times10^{-39}$, with $\hbar = 1.05\times10^{-27}\,\text{g}\cdot\text{cm}^2/\text{sec}$, c = 3 × 10^{10} cm/sec, and proton mass $m_p = 1.67\times10^{-24}\,\text{g}$, is the ratio of the strengths of gravity and the strong force after inflation. In flat, homogeneous, and isotropic post-inflationary space with matter density ρ, a strong gravity model for dark matter [1] approximates the strong force as an effective strong gravity acting only on matter, with strength $G_S = \left(\frac{M_P}{m_p}\right)^2 G = 1.7\times10^{38} G$, gravitational constant $G = 6.67\times10^{-8}\,\text{cm}^3/(\text{g}\cdot\text{sec}^2)$, and Planck mass $M_P = \sqrt{\frac{\hbar c}{G}} = 2.18\times10^{-5}\,\text{g}$. The strong gravity Friedmann equation $\left(\frac{\text{d}R}{\text{d}t}\right)^2 - \left(\frac{8}{3}\right)8\pi G_s \rho R^2 = -c^2$ describes local curvature of spaces defining closed massive systems bound by effective strong gravity. Because strong force at short distance is involved, quantum mechanical analysis is necessary. The Schrodinger equation resulting from Elbaz-Novello quantization [2] [3] of the Friedmann equation for closed massive systems bound by effective strong gravity is

$$-\frac{\hbar^2}{2\mu}\frac{\text{d}^2}{\text{d}r^2}\psi - \frac{2G_S\mu M}{3\pi r}\psi = -\frac{\mu c^2}{2}\psi \tag{1}$$

where $M = 2\pi^2 \rho r^3$ is conserved mass of closed systems with radius r and μ an effective mass. Equation (1) is identical in mathematical form to the Schrodinger equation for the hydrogen atom and can be solved immediately. Ground state curvature energy $-\frac{\mu}{2\hbar^2}\left(\frac{2G_S\mu M}{3\pi}\right)^2$ of Equation (1) must equal $-\frac{\mu c^2}{2}$ for consistency with the corresponding Friedmann equation, so effective mass $\mu = \frac{3\pi\hbar c}{2G_s M}$. Ground state solutions of Equation (1) describe stable closed systems bound by effective strong gravity, with zero orbital angular momentum and radius $\langle r\rangle = \frac{G_S M}{\pi c^2} = \frac{\hbar M}{\pi c m_p^2}$. Geodesic paths inside these stable ground state closed systems created by effective strong gravity are all circles with radius $\langle r\rangle = \frac{\hbar M}{\pi c m_p^2}$, so matter within these closed systems is permanently confined within a radius $\langle r\rangle$. No matter can enter or leave them after they form, to increase or decrease the amount of matter in those closed systems, so they constitute rigid impenetrable spheres of dark matter interacting only gravitationally. Assuming velocity-independent rigid sphere scattering [4], (self-interaction collision cross-section)/mass ratio for dark matter particles is $\frac{\sigma}{M} = \frac{4\pi(2r)^2}{M}$.

Consider values of $\frac{\sigma}{M}$ between 0.015 cm^2/g and 0.025 cm^2/g, near the estimated [5] $\frac{\sigma}{M} = 0.015\,\text{cm}^2/\text{g}$. Inserting dark matter particle radius/mass relation $\langle r \rangle = \frac{\hbar M}{\pi c m_p^2}$ into rigid sphere (self-interaction collision cross-section)/mass relation $\frac{\sigma}{M} = \frac{4\pi(2r)^2}{M}$ yields $M = \left[\left(\frac{\sigma}{M}\right)\frac{\pi}{16}\left(\frac{c}{\hbar}\right)^2 m_p^3\right] m_p$. Values of $\frac{\sigma}{M}$ between 0.015 cm^2/g and 0.025 cm^2/g indicate dark matter particle mass between 10.5 GeV and 17.5 GeV, consistent with Kelso/Hooper/Buckley analysis [6]. Estimated nucleon mass equivalent $A = \left(\frac{\sigma}{M}\right)\frac{\pi}{16}\left(\frac{c}{\hbar}\right)^2 m_p^3$ of dark matter particles ranges from 11.2 to 18.7, with radii $r = \left(\frac{A}{\pi}\right)\left(\frac{\hbar}{m_p c}\right)$ ranging from 0.75 × 10^{-13} cm = 0.75 F to 1.25 × 10^{-13} cm = 1.25 F.

If $\frac{\sigma}{M} = 0.02\,\text{cm}^2/\text{g}$, dark matter particles have mass $14\,\text{GeV} = 14.9 m_p$, radius 1.00 F, and density 6 × 10^{15} g/cm^3. Then, if all four forces were unified in the early post-inflationary universe, as the universe continued expanding prior to force symmetry breaking, matter density in the universe steadily dropped. When matter density fell to 6 × 10^{15} g/cm^3, matter could coalesce into close-packed dark matter spheres accounting for 74% of all matter (88% of dark matter).

Impenetrable spheres of dark matter are the ultimate defense against gravitational collapse, suggesting a core of close-packed spheres of dark matter is at the center of black holes, rather than a singularity. Close-packed spheres of n dark matter particles have radius $R_n = \sqrt[3]{n}\frac{\hbar M}{\pi c m_p^2} = \sqrt[3]{n} \times 10^{-13}\,\text{cm} = \sqrt[3]{n}\,\text{F}$ and Schwarzschild radius $R_S = \frac{29.9 G n m_p}{c^2} = 3.7n \times 10^{-51}\,\text{cm}$, smaller than the physical radius of the sphere until $\sqrt[3]{n} \times 10^{-13}\,\text{cm} = 3.7n \times 10^{-51}\,\text{cm}$ or $n = 1.4 \times 10^{56}$, indicating minimum mass for accretionary black holes is $2.1 \times 10^{57} m_p = 3.5 \times 10^{33}\,\text{g}$, or about 1.75 times the solar mass. Surface temperature of black holes with mass near the solar mass is about 10^{-9} K, about a billion times less than the cosmic microwave background temperature, so black hole evaporation will only occur far in the future. Discovery of black holes with mass less than a solar mass would invalidate this analysis.

3. Standard Model Particles Described by Solutions of Einstein Equations

This section describes Standard Model particles with mass as small radius solutions of Einstein's equations.

Fundamental fermions with mass m and Compton wavelength $l = \frac{\hbar c}{mc^2}$ can be treated as spherical shells with radius $\frac{l}{4}$ rotating around an axial core centered on the axis of rotation, with half of any fermion charge on the shell surface at distance of the Planck length $l_P = \sqrt{\frac{\hbar G}{c^3}} = 1.62 \times 10^{-33}$ cm from the axis of rotation. Fundamental fermions can then be represented as Godel solutions of Einstein's equations, with average matter density ρ equal average fermion mass density, pressure $\left(\frac{1}{2}\right)\rho c^2$ from negative vacuum energy density $-\left(\frac{1}{2}\right)\rho c^2$, and effective internal gravitational constant G_f determined by

$$\omega = 2\sqrt{\pi G_f \rho} \tag{2}$$

Rotation axis orientation is unknown until z component of fermion angular momentum is measured, so fermion mass appears sinusoidally distributed on a disk of radius (l/4) perpendicular to the line of sight.

Considered as spheres with radius (1/4) their Compton wavelengths l, fundamental fermions have three associated geometric quantities, volume $\sim l^3$, surface area $\sim l^2$, and diameter $\sim l$. Mass and pressure distribution in fundamental fermions identifies three wavelengths in each charge state as solutions of a cubic equation $Al^3 + Bl^2 + Cl = 0$. Describing mass and pressure distribution in terms of surface and linear elements requires shell thickness and core radius l_P. In each charge state $\frac{ne}{3}$, with $n = 0,1,2$ or 3, total fermion mass is the sum of mass equivalent of pressure, $\frac{m}{2}$, in the volume, mass equivalent of surface pressure $\frac{\pi}{4} S l^2$, and core mass Ll, so

$$\frac{4}{3}\pi\rho\left(\frac{l}{4}\right)^3 = \frac{4}{3}\pi\frac{\rho}{2}\left(\frac{l}{4}\right)^3 + 4\pi S\left(\frac{l}{4}\right)^2 + 2L\left(\frac{l}{2}\right). \tag{3}$$

Writing (3) as

$$Al^3 - Bl^2 - Cl = 0 \tag{4}$$

with $A = \frac{\pi}{96}\rho$, $B = \frac{\pi S}{4}$, and $C = 2L$, the discriminant $B^2C^2 - 4AC^3$ is positive regardless of the sign of B and the equation has three real roots corresponding to three fermion Compton wavelengths in a charge state. Nickalls [7] showed wavelengths l satisfying Equation (4) correspond to projections on the l axis, defined by an angle Θ, of vertices of an equilateral triangle. Θ is the angle between two lines starting at the center of the triangle, one parallel to the l axis and one extending to the rightmost vertex of the equilateral triangle. Nickall's parameters $l_N = -\frac{B}{3A} = -\frac{8S}{\rho}$, $\delta^2 = l_N^2 - \frac{C}{3A} = l_N^2 - \frac{64L}{\rho}$, $3l_N = l_1 + l_2 + l_3$, and $\delta^2 = \frac{(l_1 - l_N)^2}{4} + \frac{(l_2 - l_3)^2}{12}$, identify roots $l_1 = l_N + 2\delta\cos\Theta$, $l_2 = l_N - \delta\left(\cos\Theta - \sqrt{3}\sin\Theta\right)$, and $l_3 = l_N - \delta\left(\cos\Theta + \sqrt{3}\sin\Theta\right)$ corresponding to fermion Compton wavelengths in a charge state. Three positive Compton wavelengths in each charge state require negative surface mass equivalent density

$S = -\frac{\rho l_N}{8}$, $\delta^2 < l_N^2$ in each charge state, and positive mass per unit core length $L = \frac{\rho}{64}\left(l_N^2 - \delta^2\right)$. Negative S results from positive shell vacuum energy density $\left(\frac{1}{2}\right)\rho c^2$, opposite the negative vacuum energy density $-\left(\frac{1}{2}\right)\rho c^2$ in the volume, and negative pressure equivalent mass inside the shell counters positive pressure equivalent mass in the volume. With no net pressure at the fermion surface, no force acts to increase or decrease fermion size, as necessary for stable fundamental fermions identified as Godel solutions within our universe.

Fermion spheres with radius $\frac{l}{4}$ and core radius l_P have moment of inertia $I = \frac{2}{5}\frac{m}{2}\left(\frac{l}{4}\right)^2 + \frac{2}{3}\frac{\pi}{4}Sl^2\left(\frac{l}{4}\right)^2 + \frac{1}{2}Lll_P^2$, with negligible last term because $l_P \ll l$. Angular velocity $\omega = \frac{\hbar}{2I} = \frac{8c}{0.2l - l_N}$ and tangential speed of points on the spherical shell equator as a multiple of the speed of light $\frac{v_T}{c} = \frac{\omega l}{4c} = \frac{2l}{0.2l - l_N}$. $\frac{v_T}{c} > 1$ for lowest mass fermions in each charge state, allowing closed time-like curves within those Godel solutions, is acceptable in fundamental fermions unchanging from creation to annihilation. From Equation (2), $\frac{G_f}{G} = \frac{3l^4}{l_P^2\left(0.2l - l_N\right)^2}$.

Ground state fundamental fermions, the constituents of atoms and molecules, differ from higher mass fundamental fermions in the same charge state by having core mass less than total mass, tangential speeds $\frac{v_T}{c} > 1$, and larger internal gravitational constants. With fine structure constant $\frac{e^2}{\hbar c} = \frac{1}{137}$, electrostatic potential energy of fundamental fermions from repulsion between equal surface charges near the rotation axis is $\left(\frac{ne}{6}\right)^2 \Big/ \left(\frac{l}{2}\right) = n^2\frac{me^2}{9\hbar c} = n^2\frac{m}{1233}$. If electrostatic potential energy is the same for all charged ground state fundamental fermions and electron mass $m_e = 0.511\,\text{MeV}$, up quark mass $m_u = 4m_e = 2.04\,\text{MeV}$ and down quark mass $m_d = 9m_e = 4.60\,\text{MeV}$, well within quark mass error bars. All charged fundamental fermion masses and charges then relate to electron charge and mass.

Treating massive Standard Model bosons as Godel solutions is simpler than for fermions. $W^{\pm}$ and Z bosons can be described as uniform spheres rotating around a core, with radius l_P, surrounding the spin axis. Again, core term contribution is negligible, so the moment of inertia is approximately $I = \frac{2}{5}mr^2$, where r is boson radius $\frac{l}{4}$. Fundamental bosons have angular momentum $\hbar = I\omega$, so their angular velocity is $\omega = \frac{40\hbar}{ml^2} = 40\frac{c}{l}$ and Equation (2) results in

$\frac{G_i}{G} = \frac{25}{3}\left(\frac{l}{l_P}\right)^2$. Higgs bosons can be treated as static Einstein solutions of general relativity with matter energy density ρc^2 and positive vacuum energy density $\frac{1}{2}\rho c^2$, opposite the negative vacuum energy density of Godel solutions. The Friedmann equation for radius of such closed, homogeneous, isotropic systems with internal gravitational constant G_i is $\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - \frac{8\pi G_i}{3}\left[\rho c^2\left(\frac{R_0}{R}\right)^3 + \frac{1}{2}\rho c^2\right]\left(\frac{R}{c}\right)^2 = -c^2$, with $\frac{\mathrm{d}R}{\mathrm{d}t} = 0$, $R = R_0 = \frac{l_H}{4}$, and Higgs Compton wavelength l_H, resulting in $\frac{G_i}{G} = \frac{1}{12}\left(\frac{l_H}{l_P}\right)^2$.

4. Magnetic Moments of Fundamental Particles

Magnetic moments of fundamental particles with fractional charge fe, mass m, and spin $s\hbar$ are approximately $\mu = \frac{fes\hbar}{m}$ [8]. Fractional charge fe circling the spin axis at distance l_P from the axis, with velocity v, produces a current $I = \frac{fev}{2\pi l_P}$. Treating particle cores as solenoids with N turns surrounding the spin axis, charged fundamental fermions and bosons have magnetic moment $\frac{N}{2} fevl_P$ and $N = \frac{2s\hbar}{vml_P}$. So, electrons require $N = \frac{\left|\left(0.2l - l_n\right)\right|\hbar}{8mcl_P^2}$.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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