

On Spontaneous Quantum-Events and the Emergence of Space-Time

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Abstract

We show that the real existence of quantum-events, resulting from spontaneously broken unitary-evolution by quantum-transactions, can explain the dynamic metric of space-time, governed by Einstein's equation, if light-clocks are being used to measure the rhythm of events. In the derivation of Einstein's equation there naturally arises a term for a cosmological constant Λ .

Keywords

Quantum-Events, Transactional Interpretation, Einstein Equation, Cosmological Constant

1. Introduction

There has been a long-standing quest for a theory of quantum-gravity and promising candidates like string-theory or loop-quantum gravity are widely discussed today. The straightforward way of treating the metric tensor as a spin-2 quantum field, however, has led to technical difficulties early. There are other routes towards harmonizing relativity with quantum theory, where gravity is not being treated as a fundamental but an emerging force. All these attempts are closely linked to the metaphysical question, what space, time and matter actually are, even if this question does normally not stand at the first place in developing mathematical models of nature [1]. In this paper we proceed the reverse way. Starting from a few foundational assumptions in quantum physics, we develop a theory of space-time in the flavor of emerging gravity.

One of the main differences between relativity and quantum theory is locality. While relativity is per design a local theory, quantum physics definitely shows non-local features, which are not easily reconciled [2]¹. A key notion of relativity theory is an "event" in space-time, which we define to be an (idealized) physical

¹In fact, there are non-local phenomena in GR too, like the non-localizability of gravitational energy.

system of mass m occurring at a point in space-time $(t, x) \in \mathbb{R}^4$. Einstein's equation encapsulates the local, metric relationship between events under the influence of gravity. By the collapse-postulate also quantum-fields can be localized in space-time and we call these localizations quantum-events. It is therefore a straightforward idea to link quantum-events to gravity. In order to do that, we must consequently take a quantum-event and the corresponding collapse for a real physical process. This is the foundational assumption, on which we are going to base our arguments. There has been extensive work done in the field of collapse-theories and we chose ideas from the transactional theory of quantum mechanics [3] [4] [5] [6] as the technical basis of our work.

The paper is organized as follows: in paragraph 2 we briefly introduce the transactional interpretation and some of its key tenets, which are important for our work. We do this without entering into the details of the theory and the reader is referred to [6] for a comprehensive exposition. In the main paragraph 3 we derive local gravitational acceleration and the Einstein equation as a result of transactions between quantum-systems. Finally, we give a summary and draw some further conclusions in paragraph 4. In the appendices we prove some technical results.

2. Quantum Events and Light Clocks

2.1. Quantum Events

Quantum states of closed, isolated physical systems are represented as unit-elements of a complex Hilbert space $|\psi\rangle \in H_c$, $\langle\psi|\psi\rangle = 1$, and can under some realistic assumptions be uniquely assigned to the respective physical systems [7]. In the transactional interpretation [3] a quantum state $|\psi\rangle \in H_c$ is launched as an "offer-wave" by an emitter and gets possible responses by "confirmation-waves" $\langle\psi_i|$, $i \in I$, which are the projections of the dual vector $\langle\psi| \in H_c^*$ onto absorbers \mathcal{A}_i . The indices $i \in I$ denote a range of values, which the system can assume in a measurement of some physical operator². The "selection" of a specific confirmation $i_0 \in I$ leads to a "transaction", which is the actualization of emission and absorption as real events in space-time. The specific probability for a particular transaction $i_0 \in I$ is $|\langle\psi_{i_0}|\psi\rangle|^2$ and the "selection" is purely random. The relativistic transactional interpretation [4] [5] additionally offers an explanation, why offer-waves (and confirmation-waves) are actually created. Quantum-fields are elements of abstract mathematical spaces and their components are indexed through space-time points. Relativistic interactions can be thought of as the mutual exchange of virtual bosons between fields, creating possibilities in a pre-space-time process. Transactions, in turn, are triggered by the exchange of real bosons and their four-momentum. The amplitude for emission or absorption of real bosons is the coupling amplitude between the matter-and gauge-fields and a specific transaction can happen spontaneously, if the conservation laws are satisfied. By the exchange of four-momentum the quan-

²They actually index the spectrum of a Hermitian operator.

tum states of the emitter and absorber collapse and the physical systems are found at the corresponding space-time points (regions). We will sometimes use the term “event-radiation” for the four-momentum transfer in transactions. Space-time thus becomes the connected set of emission-and absorption events corresponding to transactions, which define, by the four-momentum of the exchanged bosons, time-like (or null-like) space-time intervals, whose end points are these emission and absorption events. It is here, where the transactional view touches causal-set theory [8] [9], in which events spread in space-time by a stochastic Poisson-process. Boson-exchange, understood as a decay-process, is then a special case in this model³. Note that the actualization of a space-time interval amounts to spontaneously breaking the unitary evolution of the quantum states. At the same time the four-momentum, which is exchanged, determines a time direction, since positive energy is transmitted, and selects a space-direction. In this sense spatio-temporal symmetry is also spontaneously broken. We will in the sequel focus on the electromagnetic force and the related exchange of photons.

2.2. Light Clocks

It takes a (closed and isolated) quantum-system, represented by a vector in Hilbert space, $|\psi_0\rangle \in H_{\mathbb{C}}$, with average energy above ground state $(\bar{E} - E_0)$, minimally a time of

$$\overline{\Delta t} = \frac{h}{4(\bar{E} - E_0)}, \quad (1)$$

in order to unitarily evolve to an orthogonal state $|\psi_1\rangle$, $\langle\psi_0|\psi_1\rangle = 0$, ($h \stackrel{\text{def}}{=} \text{Planck's constant}$) [10]. We can use such a system as a clock⁴ with period $\overline{\Delta t}$. Special interest will lie on the case, where the system is a photon of energy (above the ground-state) $(E - E_0) = h\nu$. The corresponding light-clock then has the period

$$\overline{\Delta t} = \frac{1}{4\nu}. \quad (2)$$

We will encounter the situation, where there is not a single photon but many of them over a range of frequencies in thermal equilibrium, and where the energy is given by a temperature T . For oscillators with $\bar{E}_\nu \approx k_B T$ ($h\nu \ll k_B T$, $k_B \stackrel{\text{def}}{=} \text{Boltzmann constant}$) we get a corresponding clock with period

$$\overline{\Delta t} = \frac{h}{4k_B T}. \quad (3)$$

We call the special light-clock (3) a thermal clock.

3. Space-Time

There is an intricate interplay between space-time and quantum-fields, which we

³The transactional interpretation thinks of space-time slightly different than the causal-set-approach does. This has no impact on our mathematical result.

⁴We actually use it as the “core” of a clock, *i.e.* as an abstract periodic process without ability to “indicate” time.

will now start to explore.

3.1. Minkowski Space-Time

For any photon in vacuum the ratio between its energy E and its 3-momentum $p = |\mathbf{p}|$ is a constant, namely the speed of light c

$$\frac{E}{p} = c. \quad (4)$$

Equation (4) is a quantum-identity and, if expressed in space-time, must hold in every inertial reference frame. If we write energy and momentum in space-time coordinates, we get $E = h\Delta\nu = \frac{h}{\Delta t}$ and, by the de Broglie-relation, $p = \frac{h}{\Delta x}$.

Therefore (4) takes the form

$$\frac{E}{p} = \frac{\Delta x}{\Delta t} = c. \quad (5)$$

Since Equation (5) must hold in every inertial reference frame $\bar{x} = (t, x) \in \mathbb{R}^4$, it constrains the metric in \mathbb{R}^4 and the result is Minkowski space-time \mathbb{M}^4 with its metric tensor $\eta_{ab} = -\delta_{ab}, 1 \leq a, b \leq 3, \eta_{0a} = \delta_{0a}, 0 \leq a \leq 3$, and its linear isometries $O(1,3)$, the Lorentz transformations. As indicated in paragraph 2.1, we take the ontological standpoint that quantum-systems spontaneously break the unitary time-evolution through the exchange of real bosons and thus become manifest in space-time. This is what we call “quantum-events” or synonymously “actualizations”. The kind of bosons depends upon the force in action. So we treat space and time as distinct attributes of matter, represented by a four-dimensional continuum, which adopts its metric structure by the “sprinkling” of matter through quantum-events. On the other hand the symmetries of \mathbb{M}^4 influence the structure of quantum states, which transform under suitable representations of the Lorentz group⁵. So the influence between space-time and quantum states is bidirectional.

The concept of a thermal clock (3) unfolds its power, if we consider multiple events of interacting quantum-systems. Multiple events manifest themselves in space-time by acceleration. In \mathbb{M}^4 physical systems of constant acceleration κ in x -direction, say, can be expressed in Rindler-coordinates. This happens by choosing a co-moving coordinate system, defined in the wedge limited by $|x| = t$, and given by the transformations

$$x = \varrho \cosh(\kappa\mathcal{G}), \quad t = \varrho \sinh(\kappa\mathcal{G}), \quad \varrho \geq 0, \quad -\infty < \mathcal{G} < \infty. \quad (6)$$

The corresponding line-element is

$$ds^2 = \left(\frac{\kappa\varrho}{c^2}\right)^2 c^2 d\mathcal{G}^2 - d\varrho^2 - dy^2 - dz^2. \quad (7)$$

Contrary to velocity, acceleration is not purely perspectival and cannot be transformed away by a Lorentz transformation. But there are local inertial reference-frames at $t = \mathcal{G} = 0$, where systems are instantaneously at rest. By the

⁵The spin-number serves to classify these representations.

Tolman-Ehrenfest effect [11] we have in thermal equilibrium for systems being instantaneously at rest and located at arbitrary ϱ_1, ϱ_2

$$T_{\varrho_1} \frac{\kappa \varrho_1}{c} d\mathcal{G} = T_{\varrho_2} \frac{\kappa \varrho_2}{c} d\mathcal{G}. \tag{8}$$

For a system at the origin $\varrho_1 = \frac{c^2}{\kappa}$ and an arbitrary one at ϱ_2 we get with $T_{\kappa} = T_{\varrho_1}$

$$T_{\kappa} \frac{c^2}{\kappa} = T_{\varrho_2} \varrho_2 = \text{const}. \tag{9}$$

The constant on the right does no longer depend on κ . Assume that in this chart (coordinate system) there is a thermal bath of temperature T_{κ} , and we want to gauge proper time by a corresponding thermal clock. By (3), (6) and (9) we get for a system instantaneously at rest at the origin and with $d\hat{s} = \frac{c}{\kappa} ds$

$$d\tau = \frac{d\hat{s}}{\Delta t} = \frac{4}{h} k_B T_{\kappa} \frac{c^2}{\kappa} d\mathcal{G}. \tag{10}$$

We want to fix the constant in (9) and for this purpose synchronize⁶ (10) with a quantum-clock, defined by a matter-wave with rest mass m_0 , frequency $\omega = 2\pi\nu$ and corresponding acceleration κ_{ω} . In its respective oscillatory rest-frame and for $m_0 \ll \frac{\hbar\omega}{c^2}$, the matter-clock measures time in analogy to (10) in units of

$$d\tau_{\omega} = \frac{4}{h} E_{\omega} \frac{c^2}{\kappa_{\omega}} d\mathcal{G}. \tag{11}$$

By the de Broglie-relation there holds with $k = |\mathbf{k}|$ denoting the wave number

$$E_{\omega}^2 = \hbar^2 \omega^2 = c^2 \hbar^2 k^2 + m_0^2 c^4. \tag{12}$$

Further with $u_{\omega} = \frac{\omega}{k}$ and $v_{\omega} = c^2 \frac{k}{\omega}$ denoting the phase- and group velocity, respectively, we have

$$\kappa_{\omega} = 2\pi u_{\omega} \omega. \tag{13}$$

By (12) and (13) Equation (11) turns into

$$d\tau_{\omega} = \frac{4}{h} \hbar \omega \frac{c^2 k}{2\pi \omega^2} d\mathcal{G} = \frac{c^2 k}{\pi^2 \omega} d\mathcal{G} = \frac{v_{\omega}}{\pi^2} d\mathcal{G}. \tag{14}$$

If we synchronize the two clocks, $d\tau = d\tau_{\omega}$, we therefore get

$$\frac{4}{h} k_B T_{\kappa} \frac{c^2}{\kappa} = \frac{v_{\omega}}{\pi^2}. \tag{15}$$

For the temperature T_{κ} this implies

$$T_{\kappa} = \frac{\hbar \kappa v_{\omega}}{2\pi k_B c^2}. \tag{16}$$

⁶By "synchronization" we just understand equality of periods.

Expression (16) is a generalized Davies-Unruh temperature. If we choose a massless wave ($m_0 = 0$), then we are in the situation $u_\omega = v_\omega = c$ and (16) turns into the familiar Davies-Unruh temperature formula [12] [13]

$$T_\kappa = \frac{\hbar\kappa}{2\pi k_B c}. \quad (17)$$

We will use formula (17) in paragraph 3.2.3 in a concrete physical situation.

3.2. Lorentz Space-Time

We now want to generalize our approach by assuming that space-time is just locally flat⁷. In order to apply formula (17) we must have an appropriate acceleration. Of course, we want it to be gravitational acceleration. In the next paragraph we will show how transactions can give rise to local gravitational acceleration g_R .

3.2.1. Gravitational Acceleration

The following argument bases on an exposition in [14]. Let a quantum-event be given by two physical systems of mass m and M , respectively, which come into being by a photon-transaction in locally flat space-time at relative rest and distance R to each other. Let further an elementary bit of information be connected to the existence or non-existence of a physical system in space-time. Since a photon offer-wave is a priori emitted symmetrically in all space-directions, we find that the information about the spatial existence of (actualized) systems at time $\Delta t = \frac{1}{\nu} = \frac{R}{c}$ is located on the surface of the sphere with radius R around M ⁸. This is a kind of holographic principle. We may also assume that a bit of information is part of the surface-information, once it is at a distance of its Compton length $\lambda = \frac{\hbar}{mc}$ from the sphere, and that this information changes linearly with the distance $0 \leq \Delta x \leq \frac{\hbar}{mc}$ [14] [15]⁹. This holds because structureless systems can reasonably be supposed to have the size of their Compton-length. So the quantum-event causes an entropy change of

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x. \quad (18)$$

For the total energy within the ball of radius R we have by the holographic principle

$$E = Mc^2 = \frac{1}{2} k_B NT. \quad (19)$$

The number T is the surface-temperature on the sphere of radius R and N

⁷“Locally flat” means approximately flat in small regions around a point x_0 . Technically this amounts to $g_{ab}(x_0) = \eta_{ab}$, $\Gamma_{ij}^k(x_0) = 0$, but generally $\Gamma_{ij,\nu}^k(x_0) \neq 0$, $0 \leq i, j, k, \nu \leq 3$.

⁸Since transactions can go either way, there is a priori a symmetry regarding the question, which of the two masses is actually in the center. This is why all masses mutually gravitate.

⁹This assumption implies $R > \lambda$.

denotes the number of bits on the surface, for which we have with the Planck-length $l_p = \sqrt{\frac{G\hbar}{c^3}}$ ¹⁰

$$N = \frac{A_R}{l_p^2} = \frac{4\pi R^2 c^3}{G\hbar}. \tag{20}$$

By (19) and (20) we get for the surface-temperature T

$$T = \frac{MG\hbar}{2\pi R^2 k_B c}. \tag{21}$$

For the total energy-change on the surface we have the entropic-force equation

$$\Delta S \cdot T = F \cdot \Delta x. \tag{22}$$

By plugging (18) and (21) into (22), we arrive at

$$F = G \cdot \frac{Mm}{R^2} = m \cdot g_R. \tag{23}$$

Therefore we can think of local gravitational acceleration as the result of a kind of “osmotic pressure” towards the other emerging parts of space-time. Local gravity is a consequence of light-induced quantum-events and the second law.

3.2.2. Einstein Equation

Let a test-system at small distance R be actualized by exchanging photons with M and consequently feel the acceleration g_R . The energy-emission by the photons must appear in the local rest-frame of the accelerated system as a spontaneous emission from a heat bath in the environment. The temperature is T_{g_R} (17), since the period of the corresponding thermal clock must be synchronous with the one of the underlying light-clock (11). This synchronization amounts by (15) to the equation

$$\frac{4}{h} k_B T_{g_R} \frac{c}{g_R} = \frac{1}{\pi^2}. \tag{24}$$

with $E = Mc^2, l_p = \sqrt{\frac{G\hbar}{c^3}}$ and $A_R = 4\pi R^2$ we derive from (24)

$$k_B T_{g_R} A_R = 4l_p^2 E. \tag{25}$$

By using (17), this leads to

$$g_R A_R = \frac{4\pi G}{c^2} E. \tag{26}$$

Note that Equation (26) is interesting per se, since it encapsulates the Gauss-Bonnet theorem for compact orientable surfaces in \mathbb{R}^3 of genus 2 (*i.e.* without handles) (see **Appendix B**). We are interested, however, in a dynamic development of (26). In the sequel we will continue to work in the local inertial coordinate-chart around the origin (M) and develop Einstein’s equation for the $oo(tt)$ -

¹⁰The Planck-length can be understood as a minimal Schwarzschild radius, as shown in **Appendix A**.

component of the metric-tensor. This will suffice to reveal the structure of the equation. With $V_R(t)$ denoting the volume of a small ball of test-systems at radius $R(t)$ around the origin, with $R(0) = R$, $\dot{R}(0) = 0$ and $\ddot{R}(0) = g_R$, we can rewrite (26) as [16] [17]

$$\left. \frac{d^2}{dt^2} \right|_{t=0} V_R = \frac{4\pi G}{c^2} E. \tag{27}$$

If we introduce the energy-momentum tensor $T_{ab}, 0 \leq a, b \leq 3$, with zero-component $T_{00} = \lim_{R \rightarrow 0} \frac{E_R}{V_R}$ ¹¹, denoting the energy density at the origin, and use the local properties of the Ricci tensor $R_{ab}, 0 \leq a, b \leq 3$, we have at the origin [16] [17] (see **Appendix C**)

$$\left. \frac{\ddot{V}_R}{V_R} \right|_{t=0} \xrightarrow{R \rightarrow 0} c^2 R_{00}. \tag{28}$$

Hence (26) turns in the limit $R \rightarrow 0$ into

$$R_{00} = \frac{4\pi G}{c^4} T_{00}. \tag{29}$$

3.2.3. Momentum-Flow

In a transaction there is a transfer of four-momentum through photons connected to a quantum-event. In paragraph 3.1 we called this momentum-transfer “event-radiation”. In order to synchronize local light-clocks (24) we have so far only made use of the energy (zero)-component of event-radiation. Let $A_i, i = 1, 2, 3$, be small surface elements with $\langle \mathbf{n}_{A_i} | \mathbf{e}_j \rangle = 0, i \neq j$. From the 3-momentum there arises pressure in the spatial-directions, which defines the Laue-scalar at the origin

$$T = \sum_{i=1}^3 T_{ii} = \lim_{A_i \rightarrow 0} \sum_{i=1}^3 \frac{F_i}{A_i} = \lim_{A_i \rightarrow 0} \sum_{i=1}^3 \frac{1}{A_i} \frac{dp_i}{dt}. \tag{30}$$

This quantity also contributes to the energy density in (29). Let $N_R(t)$ be the number of actualizations within volume V_R at time t . We have with $x_0 = ct, \tilde{N}_R(x_0) = N_R\left(\frac{x_0}{c}\right)$ and the de Broglie-relation $p = \frac{h}{R}$

$$T = 3 \frac{dN_R(t)}{dt A_R} \cdot \frac{h}{R} = 3 \frac{c \cdot h}{3} \cdot \frac{d\tilde{N}_R(x_0)}{dx_0 V_R} = c \cdot h \cdot \frac{d\lambda(x_0)}{dx_0}. \tag{31}$$

The function $\lambda(x_0) = \frac{\tilde{N}_R(x_0)}{V_R}$ denotes the number of events per 3-volume at time x_0 and therefore $\frac{d\lambda(x_0)}{dx_0}$ is the change-rate of actualizations per

3-volume. We assumed in (31) that $\lambda(x_0)$ is constant over 3-space (*i.e.* in particular independent of R), which amounts to the homogeneity and isotropy of space with respect to actualizations. We have also tacitly assumed that $\lambda(x_0)$ is

¹¹We assume E to be homogeneously distributed over V_R .

a differentiable function in x_0 . This is an assumption, which cannot hold in the quantum-realm, since events represent discrete sets and are not deterministic, but obey a random-process. The only known Lorentz-invariant stochastic law for the spreading of events in \mathbb{M}^4 , such that $N \sim V$, is a Poisson-process with constant average (photon) transaction-rate ϱ_γ [18]. The homogeneity and isotropy of space-time are thus an immediate consequence of this law. Hence, in the above terminology we have for the averages (expectation values) and $\Delta x_0 > 0$ ¹²

$$\bar{\lambda}(x_0 + \Delta x_0) = \bar{\lambda}(x_0) + \varrho_\gamma \cdot \Delta x_0. \tag{32}$$

So by (32) we can define in analogy to (31)

$$\bar{T}_\gamma = 3 \frac{c \cdot h}{3} \cdot \frac{\Delta \bar{\lambda}(x_0)}{\Delta x_0} = c \cdot h \cdot \varrho_\gamma. \tag{33}$$

If we set $T = (T_{00} - \bar{T}_\gamma)$ we can complete the right hand side of (29) to

$$\frac{4\pi G}{c^4} T_{00} \rightarrow \frac{8\pi G}{c^4} \left(T_{00} - \frac{1}{2} T \delta_{00} \right). \tag{34}$$

We may alternatively shift the added amount \bar{T}_γ to the left of (29). We have by (33)

$$\frac{4\pi G}{c^4} \bar{T}_\gamma = \frac{4\pi G h}{c^3} \varrho_\gamma = 8\pi^2 l_p^2 \varrho_\gamma. \tag{35}$$

Therefore, with

$$\Lambda = 8\pi^2 l_p^2 \varrho_\gamma, \tag{36}$$

the synchronization-equation takes the form

$$R_{00} - \Lambda \delta_{00} = \frac{4\pi G}{c^4} T_{00}. \tag{37}$$

Note that Λ has the dimension of $\frac{1}{(\text{length})^2}$. If matter-energy does not only

stem from a static mass M , but from more complicated material systems, which also exercise pressure T , we finally get our main result by repeating the procedure in (34)

$$R_{00} - \Lambda \delta_{00} = \frac{8\pi G}{c^4} \left(T_{00} - \frac{1}{2} T \delta_{00} \right). \tag{38}$$

Under the assumption of known transformation rules, the full Einstein equations are equivalent to the fact that (38) holds in every local inertial coordinate system around every point in space-time [17].

4. Summary

To derive Equation (38) we have used three ideas. The first one is that quantum-events are real actualizations of quantum-systems in space-time and are accompanied by the transfer of four-momentum through bosons, so called

¹²We can expect that there is a lower bound $0 < ct_0 \leq \Delta x_0$.

event-radiation. The number of events follows a Poisson-process, and the type of bosons depends on the respective force in action [3] [4] [5] [6]. The second idea is that quantum-systems can serve as (abstract) clocks and that the rhythm of actualizations induced by the electromagnetic force is best measured by the light-clocks, naturally given by the transferred photons. The third idea is that quantum-events induce an “osmotic” force, which locally leads to gravitational acceleration and that clock-periods from the perspective of unequally accelerated systems need to be synchronized, in order to define the same rhythm of time. If the acceleration is of gravitational origin, then the full synchronization-equation turns out to be (38).

The dynamic and expanding space-time of general relativity is hence a consequence of quantum-events and their corresponding event-radiation together with a fixed “yardstick”, namely the locally constant speed of light c , implicit in the light-clocks used to measure time. There is in particular no direct connection of the constant Λ to the energy of the quantum-vacuum. This is a fundamentally different picture to the one we get by trying to attribute fundamental reality to the metric field and quantize it. It furthermore explains quite naturally, why gravitational influence spreads with the speed of light.

Our result was derived under the assumption of a constant cosmological term Λ (*i.e.* ρ_γ). It is well possible that the value of Λ is in fact varying with cosmic time and only appears to be constant over the time periods, which we can possibly oversee. This allows the connection to the Hubble “constant” $\Lambda \sim H^2$, which seems to hold, given the empirical data and the theoretical models at our disposal today [19].

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A

We follow the exposition in [20]. It is well known that an amount of energy $E \sim M$ has an associated Schwarzschild radius $r_s = r_s(M) = \frac{2GM}{c^2}$, which screens it off from the rest of space-time, if it is concentrated within a sphere of radius $r \leq r_s$. The question is, whether there is some lower bound r_s^{\min} on the radii, below which no mass can concentrate. Indeed such a minimum exists due to quantum considerations. Assume that there is a system of energy E with corresponding mass $M = \frac{E}{c^2}$. By the uncertainty relation we know that, if the object is localized within a range $\sim L$, then its momentum satisfies $p \geq \frac{\hbar}{L}$. We further assume that classically $r_s \sim \frac{GM}{c^2}$, because the horizon-radius varies for spinning black holes, and we are looking for a minimum¹³. Further, since we want to localize very precisely, we will be in the relativistic limit and $E \sim pc$. If we take L arbitrarily small, then M will grow so much that r_s becomes larger than L and we lose localization. So we can lower L only until we have $r_s = L$. Hence there is the following sequence for a minimal radius L_{\min}

$$L_{\min} = \frac{GM}{c^2} = \frac{GE}{c^4} = \frac{Gp}{c^3} = \frac{G\hbar}{c^3 L_{\min}}. \quad (\text{A1})$$

Therefore we get the Planck-length $L_{\min} = l_p$

$$l_p = \sqrt{\frac{\hbar G}{c^3}}. \quad (\text{A2})$$

No object in space-time can be concentrated below the Planck-length and l_p becomes an absolute “edge” of space-time. Therefore the Planck-length can consistently be thought of as a kind of minimum Schwarzschild radius

$$r_s^{\min} = l_p. \quad (\text{A3})$$

Appendix B

Let $K_S = \frac{1}{R^2}$ denote the Gaussian curvature of the 2-sphere $S \subset \mathbb{R}^3$ of radius R . With a suitable constant α we can rewrite Equation (26) in integral form

$$\alpha \int_S K_S dA = \frac{4\pi G}{c^2} E. \quad (\text{B1})$$

Let \mathcal{S} denote a diffeomorphic surface without any additional energy enclosed. We then have

$$\alpha \int_{\mathcal{S}} K_S dA = \frac{4\pi G}{c^2} E. \quad (\text{B2})$$

Since $\alpha = MG$ we finally get with the Euler characteristic $\chi(\mathcal{S}) = 2$

¹³The radius lies between $r_s = \frac{2GM}{c^2}$ for non-rotating black holes and $r_s = \frac{GM}{c^2}$ for maximally spinning ones.

$$\int_S K_S dA = 4\pi = 2\pi\chi(S). \tag{B3}$$

Appendix C

For the sake of completeness, we want to indicate how to derive the key relation (28). For this we follow the exposition in [17]. Let two nearby particles at relative rest to each other start to fall freely. If the initial velocity of particle one was v , then the one of the second particle follows from parallel transport along the connecting vector εu . If we compare the two velocities after some small time ε , then the first one moved along εv and we have to again parallel transport it to v_1 in order to compare it to the corresponding v_2 . Over the passage of time the average relative acceleration of the two particles a_ε is $a_\varepsilon = \frac{v_2 - v_1}{\varepsilon}$. By the definition of the curvature tensor R there holds

$$\lim_{\varepsilon \rightarrow 0} \frac{v_2 - v_1}{\varepsilon^2} = R(u, v)v. \tag{C1}$$

Hence, by the symmetries of the tensor R ,

$$\lim_{\varepsilon \rightarrow 0} \frac{a_\varepsilon}{\varepsilon} = -R(v, u)v, \tag{C2}$$

or in coordinate-components

$$\lim_{\varepsilon \rightarrow 0} \frac{a_\varepsilon^j}{\varepsilon} = -R_{klm}^j v^k u^l v^m. \tag{C3}$$

A small ball V_R of test particles, starting at relative rest and moving geodesically, changes in second order to an ellipsoid whose axes initially don't rotate. We can therefore choose local inertial coordinates in which (to second order) the center of the ball doesn't move and the principal axes of the ellipsoid stay aligned with the coordinate axes. If the ball's initial radius is ε , then

$$r^j(t) = \varepsilon + \frac{1}{2} a^j t^2 + O(t^3). \text{ Hence}$$

$$\lim_{t \rightarrow 0} \frac{\ddot{r}^j}{r^j} = \frac{a^j}{\varepsilon}. \tag{C4}$$

with u denoting the unit-vector in j -direction and v the one in time-direction we have by C3 without summation over j

$$\lim_{\varepsilon \rightarrow 0} \lim_{t \rightarrow 0} \frac{\ddot{r}^j(t)}{r^j(t)} = -R_{jt}^j. \tag{C5}$$

Since the volume of our ball is proportionate to the radii,

$$\lim_{t \rightarrow 0} \left(\frac{\ddot{V}}{V} \Big|_t \right) = \lim_{t \rightarrow 0} \sum_j \frac{\ddot{r}^j(t)}{r^j(t)}, \text{ so with summation over all four } j \text{ (since } R_{tt}^t = 0 \text{)}$$

$$\lim_{V \rightarrow 0} \frac{\ddot{V}}{V} \Big|_{t=0} = -R_{jt}^j = -R_{00}. \tag{C6}$$

We get the reverse sign in (28) because in (26) we are actually working with $-g_R$, and the factor c^2 in front of the Ricci-curvature stems from the line-element $d\tau = c dt$.