Scientific Research Publishing

Journal of Modern Physics, 2020, 11, 1215-1235
https://www.scirp.org/journal/jmp
ISSN Online: 2153-120X
ISSN Print: 2153-1196

A Possible Solution to the Disagreement about the Hubble Constant II

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How to cite this paper: Tangherlini, F.R. (2020) A Possible Solution to the Disagreement about the Hubble Constant II. *Journal of Modern Physics*, **11**, 1215-1235.
https://doi.org/10.4236/jmp.2020.118076

Received: July 22, 2020
Accepted: August 18, 2020
Published: August 21, 2020

Abstract

This work continues the previous study (2018) *Journal of Modern Physics*. 9, 1827-1837, that proposes that the disagreement arises because the cosmic microwave background (CMB) value for the Hubble constant H_0 is actually for a universe which is decelerating rather than accelerating. It is shown that when H_0 of Freedman *et al.* (2019) *Astrophysical Journal*, 882: 34 (24 pp.) is re-determined for redshift $z = 0.07$, by replacing $q_0 = -0.53$ with $q_0 = 0.50$, the new lower value is in excellent agreement (0.1%) with the CMB H_0. The model is modified to include the clustering of galaxies, and the recognition that there are clusters that do not experience the Hubble expansion, such as the Local Group, and hence, in accordance with the model, within the Local Group the speed of light is c. The bearing of this result on the neutrino and light time delay from SN1987a is discussed. It is suggested that the possible emission of a neutrino from the blazar TXS-0506+56, that was flaring at the time, as well as possible neutrino emission earlier, may arise instead from a more distant source that happens to be, angle-wise, near the blazar, and hence the correlation is accidental. The model is further modified to allow for a variable index of refraction, and a comparison with the ΛCDM model is given. The age of the universe for different values of H_0 is studied, and comparison with the ages of the oldest stars in the Milky Way is discussed. Also, gravitational wave determination of H_0 is briefly discussed.

Keywords

Hubble Constant Disagreement, Decelerating Universe, Galactic Clusters, Age of Universe

1. Introduction

In the previous work [1] that dealt with the disagreement about the Hubble constant H_0, it was proposed that the cosmic microwave background (CMB) [2] and the baryon acoustic oscillation [BAO] [3] values for H_0, although based on the flat ΛCDM model of an accelerating universe, with the dark energy possessing a negative pressure supplied by the cosmological term [4] [5] [6], could alternatively be interpreted as being for a decelerating universe, with the dark energy possessing rather an index of refraction, as described in previous works [7] [8] [9] [10]. It was pointed out that this alternative interpretation was possible because the CMB and closely matched BAO determinations do not involve the deceleration parameter q_0, and hence they are distance determinations of H_0, as was shown explicitly in [1] for the BAO determination. Consequently, if an alternative model based on a decelerating universe can get the same distances, to within experimental error, it follows that the CMB and BAO determinations can alternatively be thought as for a decelerating universe. Thus, as was noted in [1], *there is an ambiguity in the CMB and BAO determinations of* H_0. To resolve this ambiguity, it was noted that the SHoES local distance ladder (LDL) determination (Note: Formerly described as cosmic distance ladder (CDL) determination.) of H_0 of Riess *et al.* [11] does in fact make use of the deceleration parameter q_0, with $q_0 = -0.55$, and hence this determination *unambiguously* describes an accelerating universe. It was further noted that the empirical inequality $H_0(\text{LDL}) > H_0(\text{CMB})$ can be readily understood if the Hubble constant $H_0(\text{CMB})$ is actually for a decelerating universe, rather than an accelerating universe, as is customarily assumed. This follows from the fact that since $H_0 = \dot{a}_0/a_0$, at the present epoch, where $a(t)$ is the expansion parameter for the Friedmann, LeMaître, Robertson, Walker (FLRW) line-element, and since $a_0(\text{LDL}) = a_0(\text{CMB})$, because the diminished brightness of the Type Ia supernovae (SNe Ia) has to be the same for both models, it follows $\dot{a}_0(\text{LDL}) > \dot{a}_0(\text{CMB})$. This latter inequality is what one expects if $H_0(\text{CMB})$ is for a decelerating universe, assuming that for a sufficiently large redshift, $\dot{a}(t)$ was essentially the same for both models. Then, because of the acceleration of the one model, and the deceleration of the other, at the present epoch, $\dot{a}_0$ will be greater for the accelerating model than for the decelerating model, and hence the above inequality. Thus, in effect, *the decelerating model predicts the sign of the inequality between the two competing values of the Hubble constant.* It was also shown in [1] that if in the determination of $H_0(\text{LDL})$ a positive deceleration parameter was employed rather than a negative one, $H_0(\text{LDL})$ could be brought down towards $H_0(\text{CMB})$, to within a correction involving the ratio of the distances for the two models for a given redshift, with the decelerating model requiring a greater distance. It was therefore suggested in [1] that astronomers should re-determine the distances in the LDL work, to see whether, when this was done, and the decelerating parameter changed from one for an accelerating universe to one for a decelerating universe, the resulting new value of H_0 would agree with $H_0(\text{CMB})$. This suggestion was further described in a contributed talk [12] delivered at the American Physical Society April 2019 meeting, as will be discussed further below. A few months later, the author learned that a new

LDL determination of H_0 had been made by the Carnegie-Chicago collaboration of Friedman *et al.* [13] that employed the Tip of the Red Giant Branch (TRGB) for local distance determinations, rather than the Cepheids that were used in the SHoES team of Riess *et al.* [11]. Importantly, the Carnegie-Chicago work gave a lower value of H_0, corresponding to a greater distance than SHoES for a given redshift. This finding can be taken as support for the view expressed above that the redshift distances are greater for the proposed model than for SHoES; this will be discussed in greater detail in Section 2. In Section 3, the clustering of galaxies, that was not taken up previously, will be taken up, and its importance for the model described. This discussion will include a less *ad hoc* explanation for the relatively short arrival time difference between photons and neutrinos from SN1987a than what was given in [10]. Also, in Section 2, a possible objection to the model (not referenced in [1], but in [12]) will be taken up that is based on the 3σ correlation of gamma rays with a neutrino that was possibly emitted from a blazar that was flaring at the time [14]. *Importantly, because of the cosmological distances involved, such a correlation, if confirmed by later work, would falsify the proposed model.* In Section 4, the index of refraction n that was taken to be a constant in the model for the redshift range $0 \le z \le 1.0$ originally, and that was later in [10] restricted to the range $0 \le z \le 0.6$, will now be assumed to vary linearly for the range $0 \le z \le 1.7$, with $n = 1$ for $z \ge 1.7$. The choice of the rounded-off value $z = 1.7$ goes back to the finding of Riess *et al.* [15] that at $z = 1.65 \pm 0.15$ there was no acceleration. A comparison of the revised model with the ΛCDM is given in **Table 1**, and its implication is discussed. In Section 5, the age of the universe from the standpoint of the flat ΛCDM model, and the various values of H_0 associated with this model will be examined, and compared with the age of the oldest stars in the Milky Way. In Section 6, there are concluding remarks.

2. Possible Support for the Model from the TRGB Determination of H_0

As was pointed out in the Introduction, if the value found for $H_0(\text{CMB})$ is interpreted as being for a decelerating universe, rather than for an accelerating universe, the proposed model predicts the sign of the inequality that has been found from observation to be $H_0(\text{LDL}) > H_0(\text{CMB})$. In order to clarify this interpretation further, assume: 1) that the decelerating model gives the correct distances as a function of redshift, as will be justified below from the excellent agreement that is obtained with $H_0(\text{CMB})$, and 2) that the distances from the ΛCDM model provide a good approximation to the distances from the decelerating model that results in the ambiguity described above in the Introduction. Then, in obtaining the value of H_0 from the distances associated with the ΛCDM model, it would be quite reasonable to assume, as the Planck collaboration does, that one is determining the Hubble constant for an accelerating universe. This would also be the case for the BAO determinations. Importantly, it is possible to verify the above re-interpretation of $H_0(\text{CMB})$ by re-determining the value for $H_0(\text{LDL})$ by replacing the negative deceleration parameter currently used, $q_0 = -0.55$, with the positive deceleration parameter, $q_0 = 0.50$, appropriate to the Einstein de Sitter (EdS) decelerating universe, to see whether the subsequently revised value of the Hubble constant comes down reasonably close to the lower CMB value. In Equation (10) in [1] it was shown that the logarithms for the ratio of the Hubble constants for the two different values of the deceleration parameter q_0 could be written as

$$\log\left(\frac{H_0(-0.55)}{H_0(0.50)}\right) = M - M' + \log\left(\frac{1+0.775z-0.274z^2}{1+0.25z-0.125z^2}\right), \quad (1)$$

where M and M' are the redshift-dependent distance moduli for the accelerating and decelerating universes, respectively, and the next term, in which z is the redshift, involves the kinematic corrections to the lowest order Hubble-LeMaître relation

$$k(z, q_0, j_0) = \left\{1 + \frac{1}{2}[1-q_0]z - \frac{1}{6}\left[1 - q_0 - 3q_0^2 + j_0\right]z^2\right\}, \quad (2)$$

where j_0 is the jerk given by $\dddot{a}_0 a_0^2 / \dot{a}_0^3$ [16] [17] [18] [19], and is taken to be unity in [11], while for the decelerating EdS universe, with $a(t) = Kt^{2/3}$, j_0 is also unity, independently of epoch. If, as was done in [12], one now introduces luminosity distances corresponding to the distance moduli in (10), defined by $-\log D_L \equiv M$, and $-\log D_L' \equiv M'$, and removes the logarithms in (1), and solves for $H_0(0.50)$, one obtains

$$H_0(0.50) = H_0(-0.55)\frac{D_L}{D_L'}\left(\frac{1+0.25z-0.125z^2}{1+0.775z-0.274z^2}\right). \quad (3)$$

Since there is going to be a comparison with the work in [13] further below, instead of the values for $H_0(-0.55)$ and z that were used in [1] and [12], the values used here will be $H_0(-0.55) = 74.03 \pm 1.42\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, as given in [20], and in order to stay within the range of range of redshift values used in [13] that satisfied $z \le 0.08$, the value of redshift used in (3) for comparison will be $z = 0.07$ so that $k(0.07, 0.50, 1)/k(0.07, -0.55, 1) = 0.966$ and hence (3) yields

$$H_0 = (D_L/D_L')\,71.51 \pm 1.37\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}. \quad (4)$$

One sees that the SHoES value of $H_0(\text{LDL})$ does come down towards the 2018 Planck CMB value of $H_0 = 67.36 \pm 0.54\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ [21], but to fully attain it, it must be the case that the distance predicted by the model D_L' is greater than that given by SHoES. It was therefore suggested in [12] that the astronomical community re-determine the LDL distances to see if this would turn out to be the case. Quite surprisingly, almost as if to answer this suggestion, as remarked in the Introduction, there appeared the Carnegie-Chicago work of Freedman *et al.* [13], that found $H_0 = 69.8 \pm 0.8(\text{stat}) \pm 1.7(\text{sys})$, based on obtaining distances from the observation of the TRGB, that lends support to the model's requirement that the correct distances are larger than those obtained from the Cepheid measurements used in the SHoES determination of $H_0(\text{LDL})$. To show that the TRGB finding does indeed lend support, account must be taken of the fact that in [13] $q_0 = -0.53$ was used, since later determination led to $\Omega_m = 0.315 \pm 0.007$, that was simplified to $\Omega_m = 0.315$, rather than $\Omega_m = 0.308$ that was used in [11], and since $q_0 = \frac{1}{2}\Omega_m - \Omega_\Lambda$, this led to the above change in the q_0 used in [13]. Also, one must take into account that, since in [13] lower values of redshift were used than in [11], their form for the kinematic factor $k(z, q_0, j_0)$ omitted the z^2 term. However, if one omits it in obtaining (4), but keeps the other terms the same, one finds the ratio of the kinematic factors is still 0.966, so the omission of the z^2 term in the kinematic factor is not significant for this low value of redshift. Next, without changing the luminosity distances found in [13], so that $D'_L = D_L$, but changing only the deceleration parameter from $q_0 = -0.53$ to $q_0 = 0.50$, one has that (3) reduces to

$$H_0(0.50) = H_0(-0.53)\left(\frac{1+0.25z}{1+0.765z}\right). \tag{5}$$

Then, with $H_0(-0.53) = 69.8 \pm 2.0\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ from [13], where the uncertainties have been rounded to ±2.0 $\text{kms}^{-1}\cdot\text{Mpc}^{-1}$, and with $z = 0.07$, one has

$$H_0(0.50) = 67.43 \pm 1.93\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}, \tag{6}$$

in excellent agreement (0.1%) with the above CMB value [21]. However, *this agreement is for only one redshift*, and hence it was suggested to the Carnegie-Chicago collaboration that their value for H_0 be recalculated for the entire range or redshifts they used, upon replacing $q_0 = -0.53$, with $q_0 = 0.50$, in order to compare with the CMB value of H_0; but at this writing, this has not been done. The value in (6) is also in excellent agreement with the latest BAO results, as given in Addison *et al.* [22] for which $H_0 = 66.98 \pm 1.18\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, Macaulay *et al.* [23], for which $H_0 = 67.8 \pm 1.3\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, and that of Ryan *et al.* [24], for which $H_0 = 67.78^{+0.91}_{-0.87}\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$: Since, if one takes the weighted average of the three BAO measurements of H_0, with the fractional weights w_i given by $\sigma_i^{-2}/\sum_i \sigma_i^{-2}$, $i = 1, 2, 3$, one obtains

$$\langle H_0 \rangle = \sum_i w_i H_0(i) = 67.57 \pm 0.52\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}. \tag{7}$$

Hence the agreement of $\langle H_0 \rangle$ with (6), apart from the uncertainties, is again excellent (0.3%). (Note: The uncertainties for $H_0 = 67.78^{+0.91}_{-0.87}$ have been rounded to $H_0 = 67.78 \pm 0.90$). As remarked earlier, the CMB and BAO determinations are based on distance determinations that do not explicitly involve the deceleration parameter, unlike the LDL determinations that do, and hence they can be alternatively attributed to a decelerating model that gets the same distance as the ΛCDM model to within experimental error. However, this simple observation seems to fail, in view of the recent HOLiCOW XIII result of Wong *et al.* [25], that finds for a flat ΛCDM model, the value, $H_0 = 73.3^{+1.7}_{-1.8}\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, based on a joint analysis of six gravitationally lensed quasars with measured time delays. Since the different time delays result from different distances traversed by light rays following different paths from the source to the observer, one is dealing with what is essentially a distance measurement, and therefore, from the above argument, the results should have agreed with the CMB and BAO values, instead of agreeing with the LDL value, as they do. This apparent contradiction with the proposed model will be taken up in Section 5, where the implications of the various models for the age of the universe are discussed.

Finally, it should be mentioned that from the gravitational wave (GW) standard siren measurement of the distance of the binary neutron star merger [26] [27] that was the source of GW170817, combined with optical identification of the host galaxy NGC 4993 that provided the redshift determination, it was possible to determine a value of the Hubble constant given by $H_0 = 70^{+12}_{-8}\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, as described in Abbot *et al.* [28]. In determining H_0, they used $v_H = H_0 D$, where v_H is the Hubble flow velocity, D is the distance to the source, and $v_H = cz$. Since for this case $z \approx 0.01$, they could ignore higher order corrections involving $k(z, q_0, j_0)$, that would have contributed of the order of one percent. More recently, a less uncertain determination of H_0 has been made by Hotokezaka *et al.* [29] by using the high angular resolution imaging of the radio counterpart of GW170817, and combining this with previous GW and electromagnetic (EM) data. They found that $H_0 = 70.3^{+5.3}_{-5.0}\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$. These two results for H_0 are too uncertain to decide between the CMB and BAO lower values, and the LDL higher values of H_0, although, as H.-Y. Chan *et al.* [30] pointed out, future GW detection by LIGO and VIRGO can be expected to improve the determination of H_0 to a precision of approximately two percent in five years, and approximately one percent within ten years, and this should enable a judgment between the two discordant values to be made. According to the decelerating model, since one is dealing with a distance measurement, the result should agree with the CMB and BAO values, or the model would be wrong. However, it should be noted that the low velocity relation $v_H = H_0 D$, that they used to determine H_0 depended on the fact that the redshift of the source galaxy was so small that the higher order corrections involving $k(z, q_0, j_0)$ could be neglected. But if some of the future GWs are from binary neutron star mergers that are at higher redshifts, say, $z \approx 0.07$, so that it would be necessary to include the next order correction, $k(z, q_0) = \left(1 + \frac{1}{2}(1 - q_0) z\right)$, then the determination of H_0 would have to include q_0, and if they set, $q_0 = -0.53$, in accordance with the current accelerating universe paradigm, the result should agree with the value of H_0 found in [13], allowing for the uncertainties in both determinations. But if instead it should agree with the SHoES value, then it would also be another indication that the decelerating model proposed here is wrong. Thus, standard siren measurements can play a fundamental role, not only in determining H_0, but in determining whether a major paradigm shift from the current accelerating universe to a decelerating universe is needed. This view is in contrast with alternative explanations for the Hubble constant disagreement, such as in [31] [32] [33] [34], and many others, that rely on *ad hoc* corrections to the current accelerating model, and hence do not predict that the present disagreement about the Hubble constant requires a major paradigm shift to a decelerating universe.

3. Consequences of Galactic Clusters for the Model

In the model that has been developed up until now galactic clusters were neglected for simplicity. An additional purpose of this work is to revise the model by taking this clustering into account, and to examine some of the resulting consequences. One such consequence will be that it explains why the difference of arrival times between neutrinos and photons from SN1987a [35] [36] [37] was only three hours, see ref.1 in [35], rather than thousands of years, as would have been the case if all intergalactic space (such as that between the Milky Way (MW) and the Large Magellanic Cloud (LMC) in which SN1987a was located) necessarily contained dark energy with an index of refraction of $n \approx 1.5$, for $z \leq 0.6$. Indeed, a main consequence of this revised model is that it incorporates the fact that some galactic clusters have so much dark matter holding them together that they do not partake of the Hubble expansion. In this regard, these clusters behave like normal galaxies. To be sure, this recognition traces back to the pioneering work of Zwicky [38] with his well-known studies of galactic peculiar velocities in the Coma cluster, and his recognition that the cluster contained more gravitating mass than one could infer from the observed light, as well as to subsequent numerous large-scale studies, well beyond the scope of this work. Galactic clusters that have such a strong gravitational binding because of their dark matter content that they do not experience the Hubble expansion will be called here "tight clusters." In contrast, superclusters of galaxies, that are so spread out that they experience the Hubble expansion, might be referred to as "loose clusters." It follows that, in accordance with the original model, the intergalactic space within tight clusters does not have any dark energy, for which $n > 1$, since, according to the model, it is the Hubble expansion after the epoch corresponding to $z = 1.65 \pm 0.15$ [15] that caused the dark matter that was in the expanding space to undergo a phase transition into dark energy, with its associated increase of the index of refraction. A relevant case of a tight cluster for the considerations here is the Local Group that contains the MW. That the Local Group is a tight cluster follows, for example, from the well-known fact that the great spiral galaxy in Andromeda, M31, at a distance of ~2.5 Mlyr, is approaching the MW rather than receding, as would be the case if it were experiencing the Hubble expansion. It follows that the LMC, at ~1.5×10^5 lyr from the MW, since it is well within the Local Group, is not within a sea of dark energy, but only within the tight cluster's dark matter, for which $n = 1$. Therefore, the photons that came to the earth from the explosion associated with SN1987a traveled with speed c, and hence the enormous time delay did not occur, that would have occurred if they had traveled through dark energy from the LMC to the MW at a speed of approximately $2c/3$. Although this was the explanation for the absence of a lengthy time delay that was given previously in [19], it was *ad hoc*, since it was not justified, as is done here, on the general grounds that the LMC and MW are located in a tight cluster, the Local Group. If this model is correct, tight clusters must have a boundary region where n varies from unity within the cluster to $n > 1$ outside the cluster, depending on the redshift of the cluster. The shape and depth of this boundary where dark matter is undergoing a phase transition into dark energy, with the associated increase in the index of refraction, is of course unknown. Superclusters would not have such a boundary region. For $z >\sim 1.7$, since the phase transition of the dark matter into dark energy has not yet occurred, it follows that at these higher redshifts the speed of light within superclusters is the vacuum speed of light *c*, as it is in tight clusters. Consequently, the speed of light is then *c* throughout all space, since, in accordance with the model, there is no dark energy at all, and the speed of light through the dark matter that would then be filling space in its place is *c*.

As was first noted in [8], and further discussed in [39] (which contains numerous references to the literature), as is well-known, because of their very low masses, for the energies under consideration, neutrinos travel exceedingly close to *c*, while according to the model, gamma ray bursts (GRBs), when traveling over distances with redshifts $z < 1.7$, but outside of tight clusters through which they might pass on their journey to the MW, travel at speeds less than *c*, and hence there should not be any coincidences between the arrival of neutrinos and GRBs. Until fairly recently, no such coincidences had been observed, as referenced in [39], when, as was noted in [12], on 22 September 2017, as reported in Science (18 July, 2018), by Aartsen *et al.* [14], there was observed at IceCube a neutrino 170922A at 290 TeV that came to within 0.1 degrees from the direction of the blazar TXS-0506+056, at a redshift $z = 0.34$. The blazar was reported as flaring at the time with enhanced activity in the GeV range. The correlation between the neutrino and the flaring blazar was given as 3σ. If there had been a correlation of 5σ, the model proposed here would have been proven wrong. Thus, there is still the possibility that the model can survive this observation, since to date, no new examples of possible coincidences of neutrinos with GRBs have been observed. However, interestingly, a study of earlier possible coincidences with the same blazar has been found by the IceCube Collaboration [40]. They reported that two years prior to the above discovery, by examining 9.5 years of archival data, there was an independent 3.5σ excess of neutrino flux from the direction of the same blazar. Once again, if these neutrinos did indeed actually come from this blazar, the proposed model would be wrong. However, there is a difficulty with drawing such a conclusion: Active galactic nuclei are widely distributed across the sky [41], so that if blazars are the sources of coincidental GRBS and neutrinos, it is difficult to understand why the observation of possible coincidences has only shown up for the blazar TXS-0506+056. On the other hand, if the observed neutrinos, both from the 2017 detection, and from the archival data, were actually from a more distant source that by chance happened to be located, angle-wise, near to blazar TXS-0506+056, so that there were accidental correlations, this would explain why one has not seen possible coincidences of neutrinos and GRBSs associated with other blazars. Thus, until such possible coincidences with other blazars are observed, it is reasonable to conclude that the proposed model is still viable. Finally, it should be noted that galactic clusters impact on the previous discussion of a supplementary source of discordant redshifts given in [10] [39]. Thus, if the main discordant redshift galaxy is in a tight cluster, the refractive explanation given previously no longer holds. However, if the main discordant redshift galaxy is outside of a tight cluster, or within a supercluster, so that it is surrounded by dark energy, the previous discussion is still valid, although, since there is now in the revised model a variable index of refraction, as will be discussed in the next section, this supplemental source of discordant redshifts should diminish as $z \to 1.7$, since the index of refraction approaches unity at that value.

4. A Variable Index of Refraction

In the previous work it was assumed that for the redshift range $0 \le z \le 0.6$, the index of refraction of the dark energy n is a constant, and it was found that a least squares fit to the ΛCDM model for various values of Ω_m yielded the following range of values for n given by $1.47 \le n \le 1.54$. For the values $1.47 \le n \le 1.50$, the values of the distances predicted by the model for the redshift range $0.1 \le z \le 0.5$ were less than that for the ΛCDM model, whereas, as discussed in Section 2, the model requires that the distances should be greater than that for the ΛCDM model, when the latter is combined with the LDL determinations of H_0. On the other hand, for $n = 1.54$, as shown in Table 2 in [9] at $z = 0.1$, $z = 0.2$ (note the range of redshifts for LDL in [11] was $0.023 < z < 0.15$), the model predicts a greater distance than the ΛCDM model, so that the choice of a constant value of $n = 1.54$ would seem to be favored. However, this is ruled out, since it predicts progressively even greater distances than ΛCDM at higher redshifts, whereas ΛCDM fits very well at these higher redshifts, particularly at $z = 0.5$, the value of redshift where the majority of SNe Ia were located that led to the accelerating ΛCDM model [4] [5] [6]. In fact it was found that for $\Omega_m = 0.315$, the value of the constant index of refraction model needed to fit ΛCDM at $z = 0.5$ is $n = 1.46$. Thus it is clear that to satisfy the above requirements, a constant index of refraction will not suffice. This is further emphasized by the fact that n must go to unity at $z = 1.7$, where Riess *et al.* [15] found no evidence of acceleration, as noted above, and this is interpreted in the model as being where the Hubble expansion caused the dark matter outside the tight clusters and cluster-free galaxies to start to undergo a phase transition into dark energy. Note that, for simplicity, the value of $z = 1.65 \pm 0.15$ in [15] has been rounded to $z = 1.7$. Also, since it was found in [4] [5] [6] that there was no dispersion over the range of wavelengths that were observed, it is assumed, as was done earlier, that the index of refraction is without dispersion over all wavelengths, as it is for the vacuum, both in special relativity and general relativity. In this respect, the index of refraction in this model differs significantly from its behavior in standard electromagnetic theory.

The above considerations lead to the conclusion that the revised model should introduce a variable index of refraction $n(z)$ for $0 \le z \le 1.7$, and a constant index $n(z)=1$, for $z \geq 1.7$. The behavior of the function $n(z)$ is determined by the conditions: $n(1.7)=1$, and $n(0.5)$ should have the value required to fit the ΛCDM model at the redshift $z=0.5$, for the present density values $\Omega_m = 0.315$, $\Omega_\Lambda = 0.685$. To arrive at a possible expression for $n(z)$, it is convenient to review the original argument in [7] that led to the expression for the increase in apparent magnitude given by $\delta m = 5\log(1+(n-1)\ln(1+z))$, and the related increase in logarithm of distance given by $d = \log(1+(n-1)\ln(1+z))$, for a constant index of refraction. There, one derived the increase of the apparent magnitude, due to the reduction in the speed of light, by first obtaining the infinitesimal extra amount of time δt it takes light to travel an infinitesmal distance $\mathrm{d}\sigma$ through the dark energy with a dispersionless index of refraction *n*, compared with that through the vacuum with its dispersionless index of refraction of unity. One has that through the dark energy, the infinitesimal time $\mathrm{d}t'$ for light to travel an infinitesimal distance $\mathrm{d}\sigma$ is $\mathrm{d}t' = n\mathrm{d}\sigma/c$, while that through the vacuum is $\mathrm{d}t = \mathrm{d}\sigma/c$, hence $\delta t = \mathrm{d}t' - \mathrm{d}t = (n-1)\mathrm{d}\sigma/c = (n-1)\mathrm{d}t$. During that infinitesimal extra amount of time the universe will have expanded by the infinitesimal extra amount $\dot{a}\delta t = (n-1)\dot{a}\mathrm{d}t$, and therefore the infinitesimal fractional amount of extra expansion is

$$(n-1)\dot{a}\mathrm{d}t/a = (n-1)\mathrm{d}\ln a. \tag{8}$$

This expression is to be integrated from the time of emission of the light to the present, in order to obtain the total fractional amount of extra expansion. However since it is redshift rather than time that is measured, it is appropriate to convert this relation to one that is a function or redshift using $a = a_0/(1+z)$, which holds for arbitrary FLRW expanding universes. Under this assumption, and the assumption that *n* is now a function of redshift, $n = n(z)$, the integral of (8) takes the form

$$\int_0^z (n(z')-1)\mathrm{d}\ln(1+z'). \tag{9}$$

Now in [7], since *n* was assumed to be a constant, the factor (*n* − 1) was removed from under the integral sign to obtain the value of the integral as $(n-1)\ln(1+z)$, but here it will be assumed, for simplicity, that

$$n(z) = n_0 - bz, \quad 0 \leq z \leq 1.7, \quad n(z) = 1, \quad z \geq 1.7. \tag{10}$$

Under this assumption, (9) takes the form

$$\int_0^z (n_0 - bz' - 1)\mathrm{d}\ln(1+z'). \tag{11}$$

The value of this integral is

$$(n_0 - 1 + b)\ln(1+z) - bz, \tag{12}$$

so that the original luminosity distance D_L is increased by the factor $(1+(n_0-1+b)\ln(1+z)-bz)$, and hence the resulting luminosity distance D_L' is given by

$$D_L' = (1+(n_0-1+b)\ln(1+z)-bz)D_L. \tag{13}$$

Then, following the argument on page 82 of [7], for the apparent magnitude δm, and for the increase in the logarithm of distance *d*, one has

$$\delta m = 5\log\left(1+\left(n_0-1+b\right)\ln\left(1+z\right)-bz\right), \tag{14}$$

$$d = \log\left(1+\left(n_0-1+b\right)\ln\left(1+z\right)-bz\right). \tag{15}$$

Note that in the limit $b \to 0$, the above expressions reduce to the previous ones for a constant index of refraction, upon setting $n_0 = n$, and note also that $\delta m = 5d$. The parameters n_0 and b are next determined from the requirements described above. Thus from (10), one has

$$n_0 - b1.7 = 1, \tag{16}$$

and at $z = 0.5$, one has

$$\log\left(\mathrm{X}_\Lambda(0.5)/\mathrm{X}_m(0.5)\right) = d(0.5) = \log\left(1+\left(n_0-1+b\right)\ln(1.5)-b(0.5)\right), \tag{17}$$

where $\mathrm{X}_\Lambda, \mathrm{X}_m$ are defined and worked out in Equations (17), (18) in [7], except that there it was for the case that $\Omega_m = 0.3, \Omega_\Lambda = 0.7$, so that $\Omega_\Lambda/\Omega_m = 2.3333$, whereas here, using the more recent rounded values $\Omega_m = 0.315, \Omega_\Lambda = 0.685$, and hence $\Omega_\Lambda/\Omega_m = 2.1746$. For these new values, for Equations (17), (18) in [7], one finds the following replacements

$$\mathrm{X}_\Lambda = (0.315)^{-1/2}\int_0^{0.5}\left((1+z)^3+2.1746\right)^{-1/2}\mathrm{d}z = 0.4387, \tag{18}$$

where the integral has been evaluated numerically, while by direct integration one has

$$\mathrm{X}_m = \int_0^{0.5}(1+z)^{-3/2}\mathrm{d}z = 0.3670, \tag{19}$$

hence $\log\left(\mathrm{X}_\Lambda(0.5)/\mathrm{X}_m(0.5)\right) = 0.0775 = d(0.5)$. Since from (16) $n_0 - 1 = b1.7$, upon substitution in (17), the following relation for b results

$$\log\left(1+2.7b\ln(1.5)-0.5b\right) = 0.0775, \tag{20}$$

from which one obtains, with the aid of (16), the following values for the two parameters

$$b = 0.3285,\quad n_0 = 1.558. \tag{21}$$

Obviously, the above values are only approximate because of the uncertainties in Ω_m and the value of redshift for which $n = 1$. It will be noted that for $z = 0.5$, from (10) and (21) one has that $n(0.5) = 1.394$. This is significantly lower than the values for constant *n* that was found earlier, depending on the values of Ω_m, to be in the range 1.47 - 1.5. It is interesting to compare the values of $\log\left(\mathrm{X}_\Lambda/\mathrm{X}_m\right)$ with the values of *d*, as was done in earlier papers, to see how well this model fits the ΛCDM model. This is done in **Table 1** that covers a greater range of redshifts than was done in these earlier papers, so as to include much of the range in [13], and also to include some beyond $z = 1$, where future astronomical observations of SNe Ia will provide more data, and opportunity for comparison. It will be noted that in **Table 1** *d*(1.7), *d*(1.9), and *d*(2.1) are all equal, this is because for $z \ge 1.7$, $n(z) = 1$, and hence the contribution to the

Table 1. In columns 2 - 5, comparison of $\log\left(X_\Lambda/X_m\right)$ with $d \equiv \log\left(1+\left(n_0-1+b\right)\ln\left(1+z\right)-bz\right)$ for $n_0=1.558$, $b=0.3285$, $\Omega_m=0.315$, $\Omega_\Lambda/\Omega_m=2.1746$, $\Delta \equiv d-\log\left(X_\Lambda/X_m\right)$, and $R_\Lambda \equiv \log\left(X_\Lambda/X_m\right)$.

z	$\log\left(X_\Lambda/X_m\right)$	d	Δ	Δ/R_Λ %
0.005	0.00110	0.00121	0.00011	10.0
0.02	0.00433	0.00470	0.00037	8.5
0.04	0.00864	0.00930	0.00066	7.7
0.06	0.01273	0.01367	0.00094	7.4
0.08	0.01675	0.01786	0.00111	6.6
0.10	0.02061	0.02189	0.00128	6.2
0.30	0.05330	0.05467	0.00137	2.6
0.50	0.07752	0.07752	0	0
0.70	0.09580	0.09367	−0.00213	-2.2
0.90	0.10989	0.10505	−0.00484	-4.4
1.10	0.12095	0.11284	−0.00811	-6.7
1.30	0.12979	0.11783	−0.01196	-9.2
1.50	0.13698	0.12056	−0.01642	-12.0
1.70	0.14292	0.12140	−0.02152	-15.1
1.90	0.14790	0.12140	−0.02650	-17.9
2.10	0.15212	0.12140	−0.03072	-20.2

integral (11) from $z=1.7$ to $z=2.1$ vanishes.

In concluding this section, it should be emphasized that the above linear model for the index of refraction is only a preliminary attempt to go beyond the constant index of refraction of the previous publications. It is possible a deeper understanding of dark matter and its proposed phase transition to dark energy will emerge from the comparison of the above predictions with that of the ΛCDM model, as well as with other accelerating models, that will lead to an improved model for the behavior of the index of refraction.

5. Age of the Universe

The age of the universe, *i.e.* the time back to the Big Bang where the expansion parameter $a(t)$ was arbitrarily small, was first taken up in [8], where it was shown that although the EdS universe leads to an age T_0 given by $T_0=\left(2/3\right)H_0^{-1}$, which would rule out the model, the reduction in the speed of light, occasioned by the dark energy that has a constant index of refraction n, leads to an age $T_0=\left(2/3\right)nH_0^{-1}$, and since in the original model $n \approx 3/2$, this well-known objection to the EdS universe was removed. However, before examining what the revised model with a variable index of refraction has to say about this issue, it will be helpful to recapitulate and comment on how the current disagreement about the Hubble constant impacts on the age of the universe, as well as the latter, reciprocally, on the Hubble constant.

The recent Planck CMB rounded value of the Hubble constant is $H_0 = 67.4 \pm 0.5\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$ [20]. Likewise, Planck gives the age of the universe as 13.797 ± 0.023 Gyr, which will be replaced by $T_0 = H_0^{-1}F$, where $F = 0.95$, as will be derived below, and since $H_0^{-1} = 14.5 \pm 0.1\ \text{Gyr}$, the rounded age for Planck CMB is $T_0 = 13.8 \pm 0.1\ \text{Gyr}$. There will be further comment on the meaning of this result below. To derive the above value of *F*, one makes use of Einstein's field equation for the energy density

$$G_0^0 = -\kappa T_0^0 - \Lambda, \tag{22}$$

where $\kappa \equiv 8\pi G/c^4$. For $T_0^0 = \rho c^2$, where ρ is the matter density consisting of the dark matter mass density and the baryonic mass density, while $\kappa^{-1}\Lambda$ is the density of dark energy in the ΛCDM model. This is the simplified two component energy density source model for ΛCDM that has been used throughout this work. Upon introducing the FLRW line element for a flat universe (analysis for a slightly closed universe is in [9]), one has

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - a(t)^2\,\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j,\ i, j = 1,2,3. \tag{23}$$

After the metric is substituted in G_0^0, and suitably rewritten,(22) takes the form

$$\frac{\dot{a}^2}{2} - \frac{4\pi G\rho a^2}{3} - \frac{\Lambda a^2c^2}{6} = 0. \tag{24}$$

Since the matter source tensor T_ν^μ is that for cold dark matter $T_\nu^\mu = diag\left(T_0^0, 0, 0, 0\right)$, there is no pressure. It follows from the covariant conservation law $T_{\nu\ ;\mu}^{\mu} = 0$ that ρa^3 is a constant of the motion. Hence, upon introducing the mass $M \equiv 4\pi\rho a^3/3$, (24) may also be written as

$$\frac{\dot{a}^2}{2} - \frac{GM}{a} - \frac{\Lambda a^2c^2}{6} = 0. \tag{25}$$

In this form, one recognizes (25) as the Newtonian equation for a test particle moving radially outside a fixed spherical body of mass *M* with kinetic energy per unit mass $T = \dot{a}^2/2$, and potential energy per unit mass $V = -(GM/a) - \Lambda a^2c^2/6$, so that the total energy per unit mass is $T + V = 0$. The fact that the mass of the test body cancels out completely from the equation can be seen as a manifestation of the Newtonian principle of equivalence, inertial mass equals gravitational mass, which is not only obeyed by the standard gravitational potential energy term, in accordance with Newtonian mechanics, but by the cosmological potential energy per unit mass term as well. One sometimes refers to the cancelled mass that would multiply *M* as the "passive" gravitational mass, while *M*, since it comes from the source tensor T_ν^μ, is described as the "active" gravitational mass. However, since the source energy-momentum tensor exists in special relativity where there is no gravitation, *M* is necessarily inertial mass. But since it also has been shown that inertial mass is passive gravitational mass, it follows that all three masses are the same in general relativity, as they are in Newtonian mechanics. Thus, at this level, general relativity has a deep connection with Newtonian mechanics, so that if general relativity were to break down, as has been occasionally proposed as an alternative explanation for the diminished brightness of the SNe Ia, Newtonian mechanics would also have to break down as well, at its most basic level, something that is frequently not discussed in such proposals.

Returning now to the derivation of F which, although standard, is given here for completeness. From (25), after solving for $\dot{a}$, and taking the positive root, one has

$$\dot{a}/a = \left(\left(8\pi G\rho/3\right) + \left(\Lambda c^2/3\right)\right)^{1/2}. \tag{26}$$

The negative root, not indicated above, corresponds to the descending branch of the ΛCDM universe, since, as discussed in conjunction with the EdS universe in [9], to be consistent with the time-reversal invariance of the field equations, ΛCDM should be thought of as a cyclic universe with infinite period, and infinite amplitude, with a cusp at $t = 0$ where the collapsing part of the cycle meets the expanding part; although this feature of the ΛCDM universe is rarely discussed. However, for the purpose of this work, one is only concerned with the positive root, corresponding to the expanding portion of the cycle. Upon introducing the parameter $H \equiv \dot{a}/a$, the Hubble constant $H_0 \equiv \dot{a}_0/a_0$, and using the relation $a = a_0/(1+z)$, so that $\rho = \rho_0(1+z)^3$, and with the following definitions, $\rho_c \equiv 3H_0^2/8\pi G$, $\rho_\Lambda \equiv \Lambda c^2/8\pi G$, $\Omega_m = \rho/\rho_c$, and $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$, (26) takes the form

$$H/H_0 = \Omega_m^{1/2}\left(\left(1+z\right)^3 + \left(\Omega_\Lambda/\Omega_m\right)\right)^{1/2}. \tag{27}$$

Since $\mathrm{d}t = H^{-1}\mathrm{d}a/a$, it follows, after integrating, that the age of the universe is given by

$$T_0 \equiv \int_0^{T_0}\mathrm{d}t = H_0^{-1}\Omega_m^{-1/2}\int_0^{a_0}\left(\left(1+z\right)^3 + \left(\Omega_\Lambda/\Omega_m\right)\right)^{-1/2} a^{-1}\mathrm{d}a. \tag{28}$$

Then, upon using $a^{-1}\mathrm{d}a = -a_0\mathrm{d}z/(1+z)$, and $T_0 = H_0^{-1}F$, one has

$$F = \Omega_m^{-1/2}\int_0^{\infty}\left(\left(1+z\right)^3 + \left(\Omega_\Lambda/\Omega_m\right)\right)^{-1/2}\left(1+z\right)^{-1}\mathrm{d}z. \tag{29}$$

Since $\Omega_\Lambda/\Omega_m = 2.1746$, as noted preceding (18), and $\Omega_m^{-1/2} = 1.7817$, (29) yields $F = 0.951$. Since, more accurately, one has $\Omega_m = 0.315 \pm 0.017$ [20], (29) yields $F(\Omega_m = 0.308) = 0.957$, and $F(\Omega_m = 0.322) = 0.945$, so that

$$F = 0.951 \pm 0.006. \tag{30}$$

However, in view of the uncertainty in H_0, and the simplification of a two component flat ΛCDM model, it is clearly appropriate to set $F = 0.95$ in what follows.

If one now determines the age of the universe for the SHoES LDL value of H_0 given by Riess *et al.* [20] as $H_0 = 74.03 \pm 1.42\ \mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$, that will be rounded here to $H_0 = 74.0 \pm 1.4\ \mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$, so that $H_0^{-1} = 13.2 \pm 0.2\ \mathrm{Gyr}$, one has for the age of the flat ΛCDM universe

$$T_0 = H_0^{-1}F = 12.6 \pm 0.2\ \text{Gyr}. \tag{31}$$

This age is in possible conflict with the ages of the globular clusters in the MW, for which the cut-off as to lower age was given as 12.6 Gyr by Krauss and Chaboyer [42]. While quite recently, Gellart *et al.* [43], using color-magnitude diagrams, and the new distances to stars in the MW found by Gaia [44], obtained a peak age for stars of ~13.4 Gyr. Here the disagreement of the stellar ages with the SHoES LDL age in (31) is even greater than it is for the globular clusters. Nevertheless, this short age in (31) would seem to have found support by the above mentioned findings of HOLiCOW XIII [25] that, in contrast with the LDL method based on Cepheids, used a time delay system based on gravitationally lensed quasars, as given in Wong *et al.* [25]. Since HOLiCOW XIII found $H_0 = 73.3^{+1.7}_{-1.8}\ \text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, this results in a Hubble time $H_0^{-1} = 13.3 \pm 0.4\ \text{Gyr}$, and under the assumption that this value, as with the SHoES LDL value, is for a flat ΛCDM universe, so that with $F = 0.95$, one has for HOLiCOW's age of the universe

$$T_0 = 12.7 \pm 0.4\ \text{Gyr}. \tag{32}$$

It is in good agreement with (31), and again in possible disagreement with the above stellar ages. However, there is another issue arising with HOLiCOW's determination of H_0. Since their determination involves a *distance determination of* H_0, and according to the arguments presented earlier in Section 2, it should agree with the CMB and BAO determinations to within the uncertainties, which it obviously does not, and hence, *either the decelerating model proposed here is wrong, or there is possibly some problem with HOLiCOW's determination of* H_0. To examine this, it will be noticed that their value for H_0 is based on six independent determinations that can be divided into two groups, A and B. Group A has four members (units, $\text{kms}^{-1}\cdot\text{Mpc}^{-1}$, in what follows are omitted for brevity): $68.9^{+5.4}_{-5.1}, 71.0^{+2.9}_{-3.3}, 71.6^{+3.8}_{-3.9}, 71.7^{+4.8}_{-4.5}$, which have been simplified to those given in column A below, since more accuracy is not needed for the analysis. The second group B has two members: $78.2^{+3.4}_{-3.4}, 81.1^{+8.0}_{-7.1}$, which again have been simplified in column B, and so one has

$$\begin{array}{cc} A & B \\ 69.8 \pm 5.3 & 78.2 \pm 3.4 \\ 71.0 \pm 3.1 & 81.1 \pm 7.6 \\ 71.6 \pm 3.9 & \\ 71.7 \pm 4.7 & \end{array} \tag{33}$$

Before proceeding further, it is desirable to re-evaluate H_0 using the data in the above columns, and the weighted average method used earlier, to see how it compares with their more precise determination of H_0. It was found that $\langle H_0 \rangle = 73.3 \pm 2.0$, which is in perfect agreement with the value of H_0 obtained by HOLiCOW, given by $73.3^{+1.7}_{-1.8}$ and is therefore in more than sufficient agreement with their value to justify the analysis that follows. It was found that the weighted average of the values in column A yielded $\langle H_0(\mathrm{A})\rangle = 71.1 \pm 2.2$, while that in column B yielded $\langle H_0(\mathrm{B})\rangle = 78.7 \pm 4.2$. One sees that the difference, ignoring the uncertainties in the mean values, $\langle H_0(\mathrm{B})\rangle - \langle H_0(\mathrm{A})\rangle = 7.6$ is significantly greater than the difference $\langle H_0(\mathrm{A})\rangle - H_0(\mathrm{CMB}) = 3.7$, so that one may question combining the data in B with that in A. A further objection is the age of the universe implied by $\langle H_0(\mathrm{B})\rangle$, since one has $\left(\langle H_0(\mathrm{B})\rangle\right)^{-1} = 12.5 \pm 0.7\ \mathrm{Gyr}$, and, if this is to be associated with the flat ΛCDM universe, then upon multiplication by F = 0.95, one has $T_0(\mathrm{B}) = 11.9 \pm 0.7\ \mathrm{Gyr}$, an age range that is hardly acceptable for the globular clusters, let alone the oldest stars in the MW. This suggests that in determining the value of the Hubble constant by HOLiCOW III, one should reject the data from column B, and base the determination solely on that from column A, and clearly the small value for the difference $\langle H_0(\mathrm{A})\rangle - H_0(\mathrm{CMB})$ at 1.6*σ*, does not disprove the proposed decelerating model.

It is now appropriate to examine the age of the decelerating universe predicted by the revised model, in which the index of refraction is no longer a constant *n*, but a variable $n(z)$. It will be assumed that the age takes the form

$$T_0 = (2/3)\langle n\rangle H_0^{-1}, \tag{34}$$

where $\langle n\rangle$ is the average of the index of refraction over a suitable redshift distance ζ that is determined from the integral

$$\langle n\rangle = \zeta^{-1}\int_0^{\zeta} n(z)\,\mathrm{d}z. \tag{35}$$

To obtain $\langle n\rangle$, it will be assumed (to be discussed further below) that just as the CMB determines a Hubble constant for a decelerating universe, the age that it predicts is also for the decelerating universe, hence

$$T_0 = (2/3)\langle n\rangle H_0^{-1} = 0.95 H_0^{-1}, \tag{36}$$

so that $\langle n\rangle = 1.43$. In evaluating the above integral, it is convenient to break it up into two parts: The first part extends from the terrestrial observer to the edge of the local group, since $n(z) = 1$ over this range. For simplicity, this redshift distance will be taken to be the redshift ℓ determined by the diameter of the local group, since it is going to prove to be negligible. To be sure, since the local group is a tight cluster, it is not expanding, but one can imagine a galaxy, with negligible peculiar velocity, located just outside the local group, that is experiencing the Hubble expansion at the redshift ℓ. To obtain a value for ℓ it is convenient to use the expression for the first order Doppler effect $cz/\langle n\rangle = H_0 D_L$, since the desired value of redshift is so small, higher order terms may be neglected. Now, with D_L taken to be the diameter of the local group, that is estimated to be ~10^7 lyr [45], and with the value, $H_0 = 67.4\ \mathrm{km\cdot s^{-1}\cdot Mpc^{-1}}$ from (20), one obtains an upper bound for ℓ given by $\ell = 10^{-4}$. Upon performing the integration in (34) with $n(z) = 1, 0 \le z \le \ell$, and with $n(z) = n_0 - bz, \ell \le z \le \zeta$, (34) becomes $\langle n\rangle = \ell\zeta^{-1} + n_0 - n_0\ell\zeta^{-1} - (b/2)\zeta + (b/2)\ell^2\zeta^{-1}$. Upon multiplication of both sides equation by ζ, and rewriting it, the following quadratic equation for ζ results

$$\zeta^2 - 2b^{-1}\left(n_0 - \langle n\rangle\right)\zeta + 2b^{-1}\ell\left(n_0 - 1 + (b/2)\ell\right) = 0. \quad (37)$$

After inserting the numerical values, $n_0 = 1.56$, $\langle n\rangle = 1.43$, $b = 0.3285$, to sufficient accuracy, (36) becomes

$$\zeta^2 - 0.79\zeta + 3.75\times10^{-4} = 0. \quad (38)$$

The larger root $\zeta = 0.79$ is clearly the root of physical interest. It is significant that ζ exceeds the redshift $z = 0.5$, where the model and ΛCDM are in perfect agreement (by selection) with the diminished brightness of the SNe Ia. On the other hand, as noted in the previous section, the function $n(z)$ should emerge from a theory that goes beyond the present model. It is possible in such a development ζ will also emerge, and hence one will be able to use (34) to obtain $\langle n\rangle$, and compare it with the value $\langle n\rangle = 1.43$ found above. Also, a matter for further investigation is the assumption that the age of the decelerating universe is $0.95H_0^{-1}$, where H_0 is the CMB Hubble constant that is claimed here to be that for a decelerating universe, since, as was shown above, $F = 0.95$ was obtained for the flat ΛCDM accelerating universe. Thus, rigorously, the age 13.8 ± 0.1 Gyr is actually the age of an accelerating universe that presumably has the same age as that of the decelerating universe proposed here. However, under the assumption the distances used in obtaining the Hubble constant found in [13] are correct, in contrast with the SHoES value, the age of the currently proposed ΛCDM accelerating universe is

$$0.95H_0^{-1} = 0.95\left(69.8 \pm 2.0\ \text{km/s/Mpc}\right)^{-1} = 13.2 \pm 0.2\ \text{Gyr}. \quad (39)$$

As was shown in previous discussions, the age of the decelerating universe is greater than the age of the accelerating universe, consequently, while the age 13.8 ± 0.1 Gyr used here for the decelerating universe is *qualitatively* correct, since it is greater than the above age of the accelerating universe, it will require further theoretical study to determine whether the use of $F = 0.95$ in (36) is justified in obtaining the age of the decelerating universe.

Meanwhile, just as this work was nearing completion, it was reported that the Atacama Cosmology Telescope (ACT) group [46], by studying the polarization in the CMB, were led to values of the Hubble constant and the age of the universe sufficiently close to those found by the Planck collaboration that no changes in the above considerations are needed.

6. Conclusions

Undoubtedly, the major support for the model, so far, is the finding in Section 2 that when $q_0 = -0.53$ for an accelerating universe, that was used by Freedman *et al.* [13] in obtaining their value of H_0, is replaced by $q_0 = 0.50$ for a decelerating universe, and the distance they determined is assumed to be the same as that required for the model for the redshift $z = 0.07$, then their value of H_0 comes down to excellent agreement (0.1%) with the CMB value, and likewise to within 0.3% of the weighted average of the BAO values for H_0, despite the uncertainties of a few to several percent in these different measured values of H_0. This result partially validates the claim underlying this work that the disagreement about the Hubble constant stems mainly from the proposal that the CMB and BAO values for H_0 are actually for a decelerating universe, rather than for an accelerating universe, as is currently believed. However, to fully validate the claim, it remains to be seen what value Freedman *et al.* [13] obtain for H_0 when its re-evaluation has been made for the full range of redshifts that were used in obtaining their present result.

Additional support for the model stems from the fact, as discussed in Section 3, that there are tight clusters, such as the local group, of which the Milky Way and the LMC are members, in which the gravitational binding of the dark matter content is so strong that these clusters do not undergo the Hubble expansion, and, as a consequence, in accordance with the model, their dark matter does not undergo a phase change into dark energy, so that the speed of light within these tight clusters remains c, as it does within galaxies themselves. This leads to a simple explanation as to why the light from SN1987a in the LMC was able to arrive 3 hours after the neutrinos that were detected. On the other hand, the reduction in the speed of light that is traveling through the dark energy of the space that is experiencing the Hubble expansion leads to the prediction that there should not be any correlation between neutrinos and gamma ray bursts (GRBs). As reported in [1] and more fully in [39], this turned out to be well satisfied in thousands of cases for low MeV neutrinos, and about a hundred cases of TeV neutrinos, and a few cases for PeV neutrinos. However, there is a possible exception in the TeV range that was not referenced in [1]. This is the case of a neutrino that at the 3σ level seems to have been emitted from the blazar TXS-0506+56 that was flaring at the time. Also a study of past records indicates there appears to have been other possible neutrino emissions from the same blazar. It is suggested in the text that these cases are possibly accidental correlations associated with a more distant neutrino source that happens to lie within 0.1 degrees, angle-wise, near the blazar. But if this should prove not to be the case, this would be a strong indication that the model is wrong, despite the support described in Section 2.

In Section 4, an attempt was made to improve on the original assumption of a constant index of refraction by introducing a variable index of refraction $n(z)$ that was taken to be linear for simplicity. However, this complicated the determination of the age of the universe, as discussed in Section 5, and clearly more study of this issue is needed. On the other hand, the analysis showed that the higher values of the Hubble constant found by different groups is problematic, since their values lead to ages of the universe in possible conflict with the ages of the oldest stars in the Milky Way.

In conclusion, as noted earlier in [39], a fundamental challenge to the model arises from the fact that GWs from a binary neutron star merger [24] [25] traveled to the earth with the same speed (to within 1.7 s) as the electromagnetic waves (EMWs) [47] [48]. This agreement implies, according to the model, that GWs have the same dispersionless index of refraction as EMWs when traveling through dark energy. Such a relation, as well as the proposed reduction in speed itself, falls well outside present general relativistic and electromagnetic theories, and supports the need for finding a unified theory of gravitation and electromagnetism, despite well-known failures in the past. Such a theory would also lead to an improved understanding of dark matter and dark energy.

Acknowledgements

I would like to thank Prof. George Fuller for his long-term interest in the work, and helpful suggestions. I am further indebted to Prof. Thomas Murphy for critical comments.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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