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Relativistic Reduction of the Electron-Nucleus Force in Bohr's Hydrogen Atom and the Time of Electron Transition between the Neighbouring Quantum Energy Levels

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Abstract

The aim of the paper is to get an insight into the time interval of electron emission done between two neighbouring energy levels of the hydrogen atom. To this purpose, in the first step, the formulae of the special relativity are applied to demonstrate the conditions which can annihilate the electrostatic force acting between the nucleus and electron in the atom. This result is obtained when a suitable electron speed entering the Lorentz transformation is combined with the strength of the magnetic field acting normally to the electron orbit in the atom. In the next step, the Maxwell equation characterizing the electromotive force is applied to calculate the time interval connected with the change of the magnetic field necessary to produce the force. It is shown that the time interval obtained from the Maxwell equation, multiplied by the energy change of two neighbouring energy levels considered in the atom, does satisfy the Joule-Lenz formula associated with the quantum electron energy emission rate between the levels.

Keywords

Hydrogen Atom, The Bohr Model, Lorentz Transformation Done with the Aid of the Electron Orbital Speed, Maxwell Equation Applied to Calculate the Time Interval of Electron Transitions between Two Quantum Energy Levels, Comparison with the Joule-Lenz Law for Energy Emission

1. Introduction

In an application of the Bohr model to the energy spectrum of the atomic hydrogen, the electric field—acting between the electron and nucleus—plays a dominant role; see e.g. [1]. Nevertheless the magnetic field—due to circulation of the electron along its orbit—though it does not enter the spectral calculations, can be important for the Lorentz transformations of different kind applied to the vector fields active in the atom. For, in some previous paper [2], we noticed that quantum properties of the magnetic field

$$B_n = \frac{e^3 m^2 c}{\hbar^3 n^3} \tag{1}$$

can be also applied in calculating the energy quanta of the electron in the hydrogen atom. These quanta of energy can be obtained when the magnetic moments associated with the circulating electron particle are interacting with corresponding quanta of the magnetic field. In effect of that interaction the spectrum of the energy levels identical to that known from the Bohr atom can be calculated.

Another application of B_n in (1)—taken into account together with the quanta E_n of the electric field acting between the nucleus and electron—concerns the calculation of the drift velocity of electron possessed in the hydrogen atom; see [2] [3].

Evidently E_n—when multiplied by the electron charge $-e$—gives the attractive force between the nucleus of charge e and the electron having charge $-e$:

$$-eE_n = -\frac{mv_n^2}{r_n}; \tag{2}$$

see [4]. The right-hand side of (2) is the centripetal force which provides us with an equilibrium of the electron motion along a circular orbit having the radius [4]

$$r_n = \frac{n^2\hbar^2}{me^2} \tag{3}$$

and the speed [4]

$$v_n = \frac{e^2}{n\hbar}. \tag{4}$$

In the result of (2)-(4) we obtain the electric field

$$E_n = \frac{mv_n^2}{er_n} = m\frac{e^3}{n^2\hbar^2}\frac{me^2}{n^2\hbar^2} = \frac{m^2e^5}{n^4\hbar^4}. \tag{5}$$

In [5] we have shown that a classical Lorentz force—which combines both E_n and B_n at the same time—can attain zero on condition both electric and magnetic fields are represented by parameters characteristic for some electron orbit n. The first aim of the present paper—being a supplement of [5]—is to point out that E_n can be reduced to zero due to a suitable Lorentz transformation done in the presence of B_n. In effect we find with the use of one of the Maxwell equations that the time interval seeked for the electron transition can be obtained in terms of both the electric and magnetic field intensities characteristic for some quantum state in the atom.

2. Lorentz Transformation of the Electric and Magnetic Field Present in the Hydrogen Atom

If the velocity v_L entering the Lorentz transformation is relatively small in comparison with the speed of light c [6], so

$$v_L \ll c, \tag{6}$$

the components of the transformation matrix which are [6]

$$(F_{ik}) = \begin{pmatrix} 0 & B_z & -B_y & -iE_x \\ -B_z & 0 & B_x & -iE_y \\ B_y & -B_x & 0 & -iE_z \\ iE_x & iE_y & iE_z & 0 \end{pmatrix} \tag{7}$$

can be simplified into the approximate expressions

$$\begin{gathered} E_x = E'_x, \quad E_y = E'_y + \frac{v_L}{c} B'_z, \quad E_z = E'_z - \frac{v_L}{c} B'_y, \\ B_x = B'_x, \quad B_y = B'_y - \frac{v_L}{c} E'_z, \quad B_z = B'_z + \frac{v_L}{c} E'_y, \end{gathered} \tag{8}$$

The primes indicate the components of the transformed field.

The formulae entering (8) and (9) can be combined into the vector relations

$$\boldsymbol{E} = \boldsymbol{E}' + \frac{1}{c}[\boldsymbol{B}' \times \boldsymbol{v}_L], \tag{9}$$

$$\boldsymbol{B} = \boldsymbol{B}' - \frac{1}{c}[\boldsymbol{E}' \times \boldsymbol{v}_L]. \tag{10}$$

Formally there exist two kinds of the transformation possibilities which reduce the external fields to zero [6]:

$$(\text{a}) \qquad \boldsymbol{B}' = 0 \tag{11a}$$

which implies

$$\boldsymbol{B}^{(\text{a})} = \frac{1}{c}[\boldsymbol{v}_L \times \boldsymbol{E}], \tag{11b}$$

or

$$(\text{b}) \qquad \boldsymbol{E}' = 0 \tag{12a}$$

which implies

$$\boldsymbol{E}^{(\text{b})} = -\frac{1}{c}[\boldsymbol{v}_L \times \boldsymbol{B}]. \tag{12b}$$

From the calculations done in Sec. 3 it becomes evident that

$$|\boldsymbol{B}| \gg |\boldsymbol{E}|, \tag{13}$$

so we examine solely the case of (b) in (12a).

As the size of the transformation speed v_L we assume

$$|v_L| = |v_n| = \frac{e^2}{n\hbar} \tag{14}$$

given in (4). Because the direction of $\boldsymbol{v}_n$ is normal to that of B_n, we obtain in this case from (12b) the result

$$E_n^{(\mathrm{b})} = -\frac{1}{c} v_n B_n = -\frac{1}{c} \frac{e^2}{n\hbar} \frac{e^3 m^2 c}{\hbar^3 n^3} = -\frac{e^5 m^2}{n^4 \hbar^4} \tag{15a}$$

which is the size of the electric field acting on the electron located on the orbit n, but having an opposite sign than E_n in (5). In effect the field in (5) added to the correcting term in (15a) give the Lorentz force equal to zero:

$$e\boldsymbol{E}_n - \frac{e}{c}\left[\boldsymbol{B}_n \times \boldsymbol{v}_n\right] = 0. \tag{15b}$$

3. Equation for the Field Invariants and Its Solution

The matrix (6) can be applied in solving the equation

$$\left|F_{ik} - \lambda \delta_{ik}\right| = 0 \tag{16}$$

the solution of which gives the invariants λ; see [6]. A substitution of the components F_{ik} entering (7) into (16) provides us with the equation

$$\lambda^4 + \lambda^2\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right) - \left(\boldsymbol{B} \cdot \boldsymbol{E}\right)^2 = \lambda^4 + \lambda^2\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right) = 0. \tag{17}$$

The last step on the left of (17) holds because the field $\boldsymbol{B}$ is normal to $\boldsymbol{E}$.

Evidently a substitution of

$$\boldsymbol{B}^2 = B_n^2 = \frac{e^6 m^4 c^2}{n^6 \hbar^6}, \tag{18}$$

and of $E_n = E_n^{(\mathrm{b})}$ from (15a), so

$$\boldsymbol{E}^2 = E_n^2 = \frac{e^{10} m^4}{n^8 \hbar^8}, \tag{19}$$

gives

$$\boldsymbol{B}^2 - \boldsymbol{E}^2 = B_n^2 - E_n^2 = \frac{e^6 m^4 c^2}{n^6 \hbar^6} - \frac{e^{10} m^4}{n^8 \hbar^8} = \frac{e^6 m^4 c^2}{n^6 \hbar^6}\left(1 - \frac{e^4}{n^2 c^2 \hbar^2}\right). \tag{20}$$

Since the atomic constant is equal to

$$\frac{e^2}{c\hbar} \cong \frac{1}{137.04}, \tag{21}$$

we obtain in (20) the relation

$$B_n^2 \gg E_n^2. \tag{22}$$

For example for $n = 1$ the first term entering the bracket expression in (20) is more than 10^4 times larger than the second term.

This makes (17) transformed into the expression

$$\lambda^2 = -\left(B_n^2 - E_n^2\right) < 0 \tag{23}$$

equal to a negative number which implies an imaginary λ.

4. Reduction of the Lorentz Force and Its Effect on the Electromotive Force Acting on the Electron in the Hydrogen Atom

A basic content of the Maxwell equation which provides us with a combination of the electric field, magnetic field and an interval of time is [7]

$$\oint \boldsymbol{E}_c \mathrm{d}\boldsymbol{l} = -\frac{\partial}{c\partial t}\int \boldsymbol{B}\mathrm{d}\boldsymbol{f}. \tag{24}$$

On the left-hand side of (24) there is presented the integral of a non-vanishing electric field $\boldsymbol{E}_c$ performed along a closed contour line which—in the present case—is the cross-section line of the electron orbit normal to the orbit axis; on the right-hand side we have a time derivative of the integral concerning the magnetic field vector $\boldsymbol{B}$ enclosed by the contour mentioned on the left-hand side of (24); t is time and c is a speed of light.

We assume that field $\boldsymbol{B}$ for some quantum state n is a constant equal to (1). On the other hand, $\boldsymbol{E}_c$ is a non-vanishing electric field along the contour; the field size

$$|\boldsymbol{E}_c| = \left|\boldsymbol{E}^{(b)}\right| = E_{cn} = \frac{e^5 m^2}{n^4 \hbar^4} \tag{25}$$

is given in (15a). The contour, being a cross-section line of the electron orbit, can be identified with a circumference of the electron microparticle; see below (27).

The only simplification we apply in (24) is the replacement

$$\frac{\partial}{\partial t} \to \frac{1}{\Delta t}, \tag{26}$$

so Δt becomes the time interval dividing the integral given on the right-hand side of (24).

The electron particle is moving about the atomic nucleus along a circular orbit n, but for any n we have the same length of the cross-section line of the orbit namely

$$\oint \mathrm{d}\boldsymbol{l} = 2\pi r_{el} \cong 2\pi \frac{e^2}{mc^2} \tag{27}$$

where r_{el} is the radius of the electron particle given in [7]:

$$r_{el} \cong \frac{e^2}{mc^2}. \tag{28}$$

In effect the area enclosing the electron orbit becomes

$$\int \mathrm{d}\boldsymbol{f} = 2\pi r_{el} 2\pi r_n. \tag{29}$$

The expression on the right of (29) is approximately equal to a surface of a cylinder having its length equal to the orbit length equal to

$$2\pi r_n \tag{30}$$

and the circumference of the cylinder is assumed to be equal to (27).

In effect, by neglecting the negative sign in (24), we obtain from the Maxwell equation the formula

$$E_{cn} 2\pi r_{el} \cong \frac{1}{c\Delta t} B_n 2\pi r_{el} 2\pi r_n, \tag{31}$$

or

$$E_{cn} \cong \frac{1}{c\Delta t} B_n 2\pi r_n. \tag{32a}$$

This relation, because of (1) and (25), is equivalent to

$$\frac{e^5 m^2}{n^4 \hbar^4} \cong \frac{1}{c\Delta t} \frac{e^3 m^2 c}{\hbar^3 n^3} 2\pi \frac{n^2 \hbar^2}{me^2}. \tag{32b}$$

Therefore

$$\Delta t \cong \frac{e^3}{\hbar^3 n^3} \frac{n^4 \hbar^4}{e^5} 2\pi \frac{n^2 \hbar^2}{me^2} = 2\pi \frac{n^3 \hbar^3}{me^4}. \tag{33}$$

This result is identical to the time interval associated with the electron transition between neighbouring quantum levels obtained earlier (see [8]) on the basis of an examination of the dynamical properties characteristic for the electron particle moving in the Bohr's hydrogen atom.

5. Comparison with the Joule-Lenz Law Coupling the Time and Energy of the Quantum Transitions

The energy ΔE emitted between two quantum levels $n+1$ and n is coupled with the emission time Δt (see [9] [10] [11]) by the formula

$$\Delta E \Delta t = h \tag{34}$$

derived with the aid of the quantum reasoning applied to the Joule-Lenz law of the energy emission in classical thermodynamics [12].

The size of the time emission interval Δt in case of the hydrogen atom is calculated in (33). In the next step, the energy interval between two neighbouring quantum levels becomes

$$\begin{aligned} \Delta E = E_{n+1} - E_n &= -\frac{me^4}{2\hbar^2}\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] \\ &\cong \frac{me^4}{2\hbar^2} \frac{(n+1)^2 - n^2}{n^2 (n+1)^2} \cong \frac{me^4}{2\hbar^2} \frac{2n}{n^4} = \frac{me^4}{\hbar^2 n^3}. \end{aligned} \tag{35}$$

Therefore we obtain for the product entering (34) the result

$$\Delta E \Delta t = \frac{me^4}{\hbar^2 n^3} 2\pi \frac{\hbar^3 n^3}{e^4 m} = 2\pi\hbar = h \tag{36}$$

which is identical to that given in (34). Evidently the energy symbols E_{n+1} and E_n in (35) should not be confused with the electric field strength in (5), (15a) and (25).

6. Summary

The aim of the paper was to get an insight into the time interval connected with the electron transition between two neighbouring quantum energy levels in the hydrogen atom. To this purpose a non-probabilistic approach to the quantum atomic levels characteristic for the Bohr semiclacssical theory became useful.

This is so because such approach allowed us to apply the classical Maxwell electrodynamics in describing the electron behaviour on the orbits present in the atom.

In result, in the first step, the paper demonstrates that when the size of the velocity v_L entering the Lorentz transformation amounts the size v_n of the electron speed along some orbit n in the hydrogen atom, the effect of transformation reduces the electric field E_n by the term $-E_n$ making the electric field between the electron and nucleus equal to zero. The presence of a corresponding magnetic field in the atom normal to the orbit n [see (1)] is essential for the effect.

In the second step of the paper the Maxwell equation for the electromotive force joining the electric and magnetic field with the time interval of the electron transition is taken into account. In this case the presence of a non-zero electric field active along the cross-section line of the electron orbit is assumed. A characteristic point is that the size of this field, obtained with the aid of the Lorentz transformation making the electrostatic interaction between the electron and atomic nucleus equal to zero, is equal to the size of E_n; see (5) and (15a). The new electric field active along the cross-section of the orbit is much smaller than the magnetic field normal to the orbit plane.

When a substitution of the both—electric and magnetic—fields to the Maxwell equation is done, the time interval for the electron transition due to the electromotive force is found. This interval is very close to the time intervals obtained earlier from an analysis of the dynamical properties of the electron in the hydrogen atom (see [8]), or on the basis of an examination of the Ehrenfest adiabatic invariants; see [11]. In a final step it is shown that the product of intervals of transition time and that of energy between two neighbouring states in the hydrogen atom does satisfy the quantum formula for the Joule-Lenz energy emission in that atom; see (34) and (36).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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