

Melia's $R_h = ct$ Model Is by No Means Flat

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Abstract

With the support of numerous arguments, it has been shown that Melia's claim that his cosmological $R_h = ct$ model is flat and infinite is erroneous. In contrast, the model is positively curved, closed and, therefore, finite. With respect to results of Melia's model, it is identical to our Subluminal Model.

Keywords

Globally and Locally Flat Cosmos, Melia's $R_h = ct$ Model, Subluminal Model, Curvature Parameter

1. Introduction

In numerous papers¹, Melia has proposed a cosmological model that is flat and infinite and thus contains an infinite amount of matter. Matter, space, time, and infinity were created with the Big Bang. Melia called his model the $R_h = ct$ model, where R_h represents the non-comoving radial coordinate at the cosmic horizon of the expanding model and t the cosmic time, *i.e.*, the time in the system that comoves with the expansion. Evidently, the name of Melia's model comprises two variables belonging to two different coordinate systems.

Melia has an extensive set of astrophysical data and has demonstrated in a series of articles that this data can be best adapted to the $R_h = ct$ model as compared to other Friedman-Robertson-Walker (FRW) models. Moreover, his model provides an exact solution to Einstein's field equations while most FRW models do not. Therefore, Melia's model is significantly different from the standard FRW model, where pressure is applied by hand. As Einstein's field equations cannot fully determine the FRW models, it is necessary to introduce numerous parameters, namely, the Ω s and the deceleration parameter. These quantities must be determined using astrophysical data. However, for Melia's model, only one parameter needs to be determined. This feature explains why Melia's model

¹Most papers by Melia and colleagues are listed in [1].

is favored over other models.

We proposed our Subluminal Model [1], which is positively curved and closed. The main aim of this paper is to prevent the assumption that galaxies in the universe have superluminal velocities. Surprisingly, Melia's $R_h = ct$ model and our Subluminal Model yielded the same final results. One obtains the same Friedmann equation, the same EOS, *i.e.*, $\mu_0 + 3p = 0$, and a uniform expansion of the universe.

In the second section, we oppose the two models and show that both models, although derived in different ways, are identical. We collect the results from our earlier papers and comment on them. We believe that Melia's universe is also positively curved and closed. Thus, Melia's observational results are valid for our model as well. Nevertheless, we assert that Melia's geometrical interpretation of his model is erroneous.

2. The Question of Curvature

In the following section, we will demonstrate step by step that Melia's $R_h = ct$ model is not flat at all. We have referred to our earlier papers [2] [3] where we treated the same aspect of the problem. Here, we have summarized the results.

i) We favor the view that infinite universes, be they open, flat, or negatively curved, are ruled out as a way of describing Nature. This is because infinities are hard to imagine and because we want to avoid conclusions from Hubble's law that lead to acausality and contradictions to the special theory of relativity. This is one of the reasons we have rejected the geometrical interpretation of Melia's infinite model.

ii) An infinite universe has to expand to avoid Olbers' paradox. An infinite number of stars emit an infinite amount of light. Although the intensity of light decreases with $1/r^2$, the night sky will be as bright as our sun. In the case of expansion of the universe, distant stars run away and influence the $1/r^2$ law, thereby avoiding Olbers' paradox.

iii) According to Hubble's law, $v = Hr$, where H , as the Hubble parameter, associates the recession velocity v of the galaxies with the distance r of an observer. This law emerges from astrophysical observations. Evidently, in an infinite universe, the distance r can be chosen to be arbitrarily large, and the recession velocity may reach or exceed the velocity of light. Thus, the formation of galactic islands could be possible. However, no information can be exchanged between such galaxies. The laws of special relativity are inevitably violated in an infinite universe.

iv) The boundary where the recession velocity becomes superluminal is known as the cosmic horizon. Melia introduced such a horizon rather artificially. He, building on a flat universe, created an event horizon by comparing it with the Schwarzschild theory. He referred to Weyl's cosmological principle and Birkhoff's theorem. An enclosed mass $M = M(r_h)$ of a certain volume in the universe determines the Hubble radius² $r_h = 2GM/c^2$, leading to the relation

²Melia's variables R, t correspond to our variables r, t' , which we have used in our earlier papers.

$r_h = ct' (R_h = ct)$. The Hubble radius is defined as the distance light has traveled since the Big Bang; t' represents the age of the universe and r_h the location at which the rate of expansion reaches the speed of light.

It should be mentioned that to define a cosmic horizon, the mass content of the universe is not mandatory. This shows the original version of the dS cosmos, which is empty and has a horizon.

v) In contrast, our Subluminal Model has a natural horizon. This model is based on the dS model, which can be geometrically represented by a 4-dimensional pseudo-hypersphere with a constant radius \mathcal{R} embedded into a 5-dimensional flat space. Expunging the condition $\mathcal{R} = const.$, one can obtain our Subluminal Model.

The pseudo-hypersphere of the dS cosmos is usually described with pseudo-spherical coordinates $r, \vartheta, \varphi, x^4 = it = \mathcal{R}i\psi$. Here,

$$r = \mathcal{R} \sin \eta \quad (2.1)$$

is the radial coordinate and η the polar angle of the pseudo-hypersphere. Choosing an arbitrary point on the pseudo-hypersphere as a pole, *i.e.*, the location of an observer, the associated equator surface ($\sin \eta = 1$) has

$$r_h = \mathcal{R}, \quad (2.2)$$

the maximal extension of r . This is the natural geometrical definition of the cosmological horizon and is equally valid for the Subluminal Model. It is the basic relation that connects the two models under consideration. Thus, the recession velocity is also limited via Hubble's law. As we have already shown in our paper [4], the geometric horizon can be reached by drifting galaxies only after infinite time, as experienced by the chosen observer. As observers can be fixed at any arbitrary point on the pseudo-hypersphere, each observer has an individual horizon.

vi) To examine the relationship between the two models in greater depth, let us revisit the abovementioned definition of Melia's cosmic horizon. Melia determined the Hubble radius with

$$r_h = \frac{2GM(r_h)}{c^2}.$$

Here,

$$M(r_h) = \frac{4\pi r_h^3}{3} \mu_0$$

is the mass enclosed by the sphere with radius r_h and μ_0 as the assigned mass density. Thus, with the aid of (2.2), we get

$$r_h = \sqrt{\frac{3c^4}{8\pi G \mu_0}} = \sqrt{\frac{3}{\kappa \mu_0}} = \mathcal{R}. \quad (2.3)$$

This immediately results in

$$\kappa \mu_0 = \frac{3}{\mathcal{R}^2}, \quad (2.4)$$

an expression derived in our Subluminal Model with geometrical methods.

The mass density decreases as the universe increases with the radius \mathcal{R} . This and similar relations can also be found in Einstein's universe, Friedman's universe, and in the models of the dS family. However, this relation is missing in Melia's papers. As Melia's model is assumed to be flat, a familiar relation (2.4) cannot be derived within the framework of his model.

vii) Both models the $R_h = ct$ model and the Subluminal Model describe the relation between the non-comoving radial coordinate r and the comoving coordinate r' with

$$r = \mathcal{K}(t')r', \tag{2.5}$$

where \mathcal{K} is the time-dependent scale factor. We still need to demonstrate that the $R_h = ct$ model is compatible with the features of the curvature of the pseudo-hypersphere. With

$$r = \mathcal{R} \sin \eta, \quad r' = \mathcal{R}_0 \sin \eta, \quad \mathcal{R} = \mathcal{K}\mathcal{R}_0, \quad \mathcal{R}_0 = const., \tag{2.6}$$

we can write the Hubble parameter with both the scale factor and the pseudo-hypersphere's radius of curvature as

$$H = \frac{\mathcal{R}'}{\mathcal{R}} = \frac{\mathcal{K}'}{\mathcal{K}}, \tag{2.7}$$

where \mathcal{R}_0 is the radius of the curvature of the pseudo-hypersphere if it is calculated with the aid of comoving, *i.e.*, expanding rods, and thus appears to be a constant quantity for the comoving observer.

In addition, by solving the field equations of the Subluminal Model, we can obtain the mass density, the pressure, and the EOS as follows:

$$\kappa\mu_0 = \frac{3}{\mathcal{R}^2}, \quad \kappa p = -\frac{1}{\mathcal{R}^2}, \quad \mu_0 + 3p = 0. \tag{2.8}$$

Both pressure and mass density are functions of the time-dependent radius of the universe.

viii) In the Subluminal Model, the recession velocity is *geometrically* defined by

$$v = \sin \eta = \frac{r}{\mathcal{R}}. \tag{2.9}$$

Respecting (2.2), the recession velocity at the horizon is

$$v_h = 1,$$

the velocity of light in the natural measuring system. In addition, solving Friedman's equation, we can arrive at the following simple relation:

$$\mathcal{R}' = 1. \tag{2.10}$$

With (2.7) one has $H = 1/\mathcal{R}$, and using Hubble's law, we can confirm (2.9). The dot in Equation (2.10) denotes the derivation with respect to cosmic time. Thus, we can now recover Melia's fundamental relation $R_h = ct$ using the physical measuring system.

ix) The essential difference concerning the interpretation of the models is the

question of curvature, *i.e.*, the interpretation of the curvature parameter k . The line elements of both models in comoving coordinates are the same, and we can see from them that $k = 0$.

In our Subluminal Model with the static dS metric as the seed metric has $k = 1$. The geometry of the dS cosmos is interpreted as the pseudo-hypersphere and thus as a positively curved, finite universe. It should be noted that a coordinate transformation to the comoving system cannot change the curvature of the space. Due to this, we insist that the curvature of both models is positive and the universe is finite.

It is well known to gravitational physicists that a transition to a freely falling coordinate system does not change the geometrical structure of a model. Lemaitre found such a transformation for the Schwarzschild model. Observers in a freely falling elevator tend to hover, implying that they are not exposed to gravitational forces. When writing the line element of the static Schwarzschild model in canonical form, the curvature parameter is $k = 1$; however, in a freely falling system is $k = 0$. This shows that k is not a reliable criterion for the curvature of space. In contrast, $k = 0$ denotes that an observer is in free fall. Thus, it would be more convenient to call k a form parameter for a metric. In our paper [3], we have discussed this problem in detail and extended Einstein's elevator principle to cosmology. We conclude $k = 0$ in Melia's model does not necessarily indicate the model is flat but rather indicates that the universe is in free fall.

x) In an extensively quoted paper [5], Florides discussed the relations between comoving and non-comoving coordinate systems for several cosmological models. In our papers [6] [7], we complemented the coordinate transformations of Florides with Lorentz transformations. From all these papers, it can be seen how the parameter k changes under coordinate transformations. In a table, we have provided an overview of cosmological and gravitational models in [8] and shown that k assumes rather individual values depending on the choice of coordinates for the line element. Therefore, the statement of numerous authors at the beginning of their articles that $k = (1, 0, -1)$ denotes positively curved, flat, or negatively curved spaces is definitely wrong. Florides states that the only physical acceptable member of the dS family is the de Sitter cosmos, *i.e.*, the universe with a metric that transforms $k = 1$ into $k = 0$. This is the very metric we have used as the seed metric for our Subluminal Model.

xi) To determine the structure of the universe, we cannot rely on the parameter k . Instead, Einstein's field equations need to be solved and the geometrical properties of the given quantities studied. Unfortunately, several cosmologists tend to manipulate Friedman's equation without considering the remaining components of Einstein's field equations. This way, they propose new models, trying to explain dark matter or dark energy and other possible effects in cosmology. It can be said that such solutions are not exact solutions to Einstein's field equations.

A complete treatment of the field equation can disclose the geometrical structure of the model and determine the curvatures of space. We will demonstrate

this in a somewhat pedagogical manner. Starting with a simple 2-sphere embedded into a 3-dimensional flat space, the line element on this sphere can be expressed as follows:

$$ds^2 = r^2 d\mathcal{G}^2 + r^2 \sin^2 \mathcal{G} d\varphi^2. \tag{2.11}$$

Here, r represents the radii of the greater circles and $r \sin \mathcal{G}$ the radii of the parallels. From the line element, we can read the tetrads and calculate the Ricci-rotation coefficients³

$$A_{mn}^s = B_{mn}^s + C_{mn}^s, \quad B_{mn}^s = b_m B_n^s b^s - b_m b_n B^s, \quad C_{mn}^s = c_m C_n^s c^s - c_m c_n C^s$$

$$b_m = \{0, 1, 0\}, \quad c_m = \{0, 0, 1\}, \quad m = 1, 2, 3$$

Herein, the *curvatures* are defined by

$$B_m = \left\{ \frac{1}{r}, 0, 0 \right\}, \quad C_m = \left\{ \frac{1}{r \sin \mathcal{G}} \sin \mathcal{G}, \frac{1}{r \sin \mathcal{G}} \cos \mathcal{G}, 0 \right\}. \tag{2.12}$$

We see that the Ricci-rotation coefficients contain the curvatures of the sphere $\frac{1}{r}, \frac{1}{r \sin \mathcal{G}}$. It is easy to extend this method to higher-dimensional spaces.

xi) The dS model is based on the metric

$$ds^2 = \mathcal{R}^2 d\eta^2 + \mathcal{R}^2 \sin^2 \eta d\mathcal{G}^2 + \mathcal{R}^2 \sin^2 \eta \sin^2 \mathcal{G} d\varphi^2 + \mathcal{R}^2 \cos^2 \eta d\psi^2. \tag{2.13}$$

It is the metric of a 4-dimensional pseudo-hypersphere

$$x^{a'} x^{a'} = \mathcal{R}^2, \quad a' = 0', 1', \dots, 4'$$

with a constant radius \mathcal{R} embedded into a 5-dimensional flat space, parameterized by

$$x^{3'} = \mathcal{R} \sin \eta \sin \mathcal{G} \sin \varphi$$

$$x^{2'} = \mathcal{R} \sin \eta \sin \mathcal{G} \cos \varphi$$

$$x^{1'} = \mathcal{R} \sin \eta \cos \mathcal{G}$$

$$x^{4'} = \mathcal{R} \cos \eta \sin \psi$$

$$x^{0'} = \mathcal{R} \cos \eta \cos \psi$$

Here, a' denotes the Cartesian coordinate system of the embedding space, where $x^{4'} = \mathcal{R} \cos \eta \sin \psi$ is related to an imaginary dimension of space, the “Cartesian time.” To understand the curvature problem, we can restrict ourselves to the greater circles of the spherical piece of the pseudo-hypersphere, *i.e.*, the surface $x^{\alpha'} x^{\alpha'} = r^2, \alpha' = 1', 2', 3'$, $r = \mathcal{R} \sin \eta$. In the local 5-dimensional pseudo-spherical system, the curvature quantity of these circles can be obtained as⁴

$$B_a = \left\{ \frac{1}{\mathcal{R}}, \frac{1}{\mathcal{R}} \cot \eta, 0, 0, 0 \right\}, \quad a = 0, 1, \dots, 4 \tag{2.14}$$

or with $r = \mathcal{R} \sin \eta$, the more familiar form

$$B_a = \left\{ \frac{1}{r} \sin \eta, \frac{1}{r} \cos \eta, 0, 0, 0 \right\}. \tag{2.15}$$

³The Christoffel symbols are not appropriate for this purpose.

⁴One can find a detailed calculation in [9].

The 0-dimension is the local extra dimension. Comparing the 4-dimensional part $\left\{\frac{1}{r}\cos\eta, 0, 0, 0\right\}$ of this equation with (2.12), we find that the

4-dimensional space cannot be flat. Squaring (2.15), we get $B_a B^a = 1/r^2$, r being the curvature radius of the greater circles of the spherical part of the model.

Performing a Lorentz transformation in the local [1,4]-slice, we get for (2.15)

$$B_a = \left\{ \frac{1}{r} \sin \eta, \frac{1}{r} \cos \eta \cos i\chi, 0, 0, \frac{1}{r} \cos \eta \sin i\chi \right\} \quad (2.16)$$

where $i\chi$ is the Lorentz angle. In the case of transformation to a comoving system in a universe expanding in free fall, the relative motion of a comoving observer is geometrically defined and the relation

$$\cos \eta \cos i\chi = 1 \quad (2.17)$$

is satisfied. It should be noted that the geometrical quantity $\cos \eta$ and the kinematical quantity $\cos i\chi$, the Lorentz factor of the motion, are closely related. Using this, we can easily derive

$$\sin i\chi = i \tan \eta .$$

Respecting these two relations, we can obtain from (2.16)

$$B_a = \left\{ \frac{1}{r} \sin \eta, \frac{1}{r}, 0, 0, i \frac{1}{r} \sin \eta \right\}. \quad (2.18)$$

Here, $\left\{\frac{1}{r}, 0, 0\right\}$ is the spatial part of quantity B , which seems to be flat according to (2.12). Evidently, this is a consequence of Einstein's elevator principle that we have discussed in detail in paper [3]. However, all five components of (2.18) need to be considered. Again, the square of B is $1/r^2$, with r as the radii of the greater circles. As expected, the curvature of space turns out to be an invariant property. The same holds for quantity C mentioned in (2.12).

In addition, further slices of the pseudo-hypersphere are open pseudo circles

$$x^{0^2} + x^{4^2} = \mathcal{R}^2 \cos^2 \eta$$

with radii $\mathcal{R} \cos \eta$ and curvatures $1/\mathcal{R} \cos \eta$, recalling x^{4^2} as an imaginary coordinate. This curvature is the cause of the force of acceleration in the dS cosmos. The latter is a component of the Ricci-rotation coefficients. It should be noted that for the transition of this quantity into a comoving system, the inhomogeneous transformation law of the Ricci-rotation coefficients is required. We have discussed this problem in the quoted papers.

Omitting the calculation of all the components of the Einstein tensor, we can circumvent the question of the curvature of the model. We could not find any controls in Melia's papers concerning the first three components of the Einstein tensor. They would exhibit the curvature radii of the normal and oblique slices of the pseudo-hypersphere, representing the positively curved universe.

xiii) As already mentioned in the earlier parts of this section, a transition from the dS model to the Subluminal Model is rather simple: the restriction $\mathcal{R} = \text{const.}$ needs to be expunged. Then \mathcal{R} , the radius of the pseudo-hypersphere, *i.e.*, the radius of our universe, behaves as a function of time. The evolution of our universe can be described by a series of self-similar dS universes. Evidently, the metric of the dS universe does not contain any information regarding how the universe can develop or how to calculate the change in \mathcal{R} . A second set of differential equations needs to be consulted. These are the contracted Bianchi identities $R_{[m n r s]} = 0$ that provide possible changes of the Riemann curvature tensor. To define a genuine expanding cosmological model, the following two differential equation systems are needed:

$$\begin{aligned} \text{(I)} \quad R_{mn} - \frac{1}{2} g_{mn} R &= -\kappa T_{mn} \\ \text{(II)} \quad R_{m \parallel n} - \frac{1}{2} R_{\parallel m} &= 0 \end{aligned} \quad (2.19)$$

System (II) leads to the conservation law $T_{m \parallel n} = 0$. This equation is often used in the literature to establish an outstanding relation with variables. Solving these two systems of equations, the Subluminal Model can be obtained with properties mentioned in items vii and viii.

3. Conclusion

We have demonstrated step by step that our positively curved and finite Subluminal Model is identical to Melia's $R_h = ct$ model and have extensively discussed the question of curvature. We conclude that Melia's claim that his model is flat is erroneous.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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