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Kolmogorov's Probability Spaces for "Entangled" Data-Subsets of EPRB Experiments: No Violation of Einstein's Separation Principle

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Abstract

It is demonstrated that the use of Kolmogorov's probability theory to describe results of quantum probability for EPRB (Einstein-Podolsky-Rosen-Bohm) experiments requires extreme care when different subsets of measurement outcomes are considered. J. S. Bell and his followers have committed critical inaccuracies related to spin-gauge and probability measures of such subsets, because they use exclusively a single probability space for all data sets and sub-sets of data. It is also shown that Bell and followers use far too stringent epistemological requirements for the consequences of space-like separation. Their requirements reach way beyond Einstein's separation principle and cannot be met by the major existing physical theories including relativity and even classical mechanics. For example, the independent free will does not empower the experimenters to choose multiple independent spin-gauges in the two EPRB wings. It is demonstrated that the suggestion of instantaneous influences at a distance (supposedly "derived" from experiments with entangled quantum entities) is a consequence of said inaccuracies and takes back rank as soon as the Kolmogorov probability measures are related to a consistent global spin-gauge and permitted to be different for different data subsets: Using statistical interpretations and different probability spaces for certain subsets of outcomes instead of probability amplitudes related to single quantum entities, permits physical explanations without a violation of Einstein's separation principle.

Keywords

Bell's Theorem, Einstein's Separation Principle, EPRB Experiments

1. Introduction

The differences between classical probability and the modified probabilistic concepts used in quantum mechanics, have been the topic of many discussions related to the foundations of quantum mechanics and are in a way the root of Feynman's well-known remark that no one understands quantum mechanics. To avoid ambiguities, I define Kolmogorov's (set-theoretic) probability framework as "classical" probability and the absolute square of Feynman's probability amplitudes as the quantum probability version.

The classical-quantum distinction of probabilistic concepts has appeared in clearest relief due to the work of J. S. Bell [1] and his inequalities that involve Einstein-Podolsky-Rosen (EPR) experiments [2] and corresponding measurements of entangled pairs. Wigner [3] presented a set-theoretic version of Bell's inequalities. The actually performed experiments (see e.g. [4]) are a variation of EPR as proposed by Bohm (EPRB). The experimental results contradict the Bell-Wigner inequalities, a contradiction that has led to the common belief that instantaneous influences at a distance are at work in experiments of entangled quantum "entities" (photons, electrons etc.) and that Einstein's separation principle derived from the speed c of light in vacuum is actually violated in EPRB experiments.

Any criticism of the Bell-Wigner inequalities is currently seen by a majority of physicists as nonsensical and comparable to the attempts to build a perpetuum mobile that contradicts energy conservation. It is the purpose of this paper to show that the criticism of the Bell-Wigner approach should rather be compared to the early days of the calculus and to Berkeley's criticism of Newton's fluxions and Leibniz's infinitesimals that were put to zero after a logical procedure that regarded them as definitely not-zero. Bishop Berkeley stated that such a method of reasoning would not be allowed in Divinity. It took about a century and the work of Cauchy, Weierstrass and other notables to repair the problems convincingly.

Instantaneous influences over space-like distances do have similar logical problems as the fluxions of Newton and are indeed considered instantaneous only when the outcomes of the influences are random and when it is absolutely impossible to transmit any information instantaneously. Any instantaneous transfer of information would contradict Einstein's relativity and no sane physicist believes in such a possibility. Therefore, in a variety of descriptions, "influences" are introduced that are "instantaneous" only if the instantaneity cannot directly be proven but only statistically inferred. That statistical inference is invariably based on the violation of the Bell-Wigner type of inequalities.

It will be shown in great detail below that the commonly used logic of applying Bell-Wigner-type inequalities to actual EPRB experiments and/or the results of quantum mechanics would also not be allowed in Divinity because of a variety of reasons that uncover serious inaccuracies of epistemological, physical and mathematical nature. In contrast to the case of calculus, a repair of these problems appears unlikely.

Avoiding the mentioned inaccuracies permits us to construct a model for EPRB experiments, which is based on a statistical relation of certain subsets of measurement outcomes as opposed to interpretations regarding individual entangled pairs.

2. EPRB Experiments and Notation

I assume that the reader is reasonably familiar with EPRB experiments that deal with entangled (correlated) photon pairs. These pairs are sent into two directions or into two optical fibers. To be definite we assume that one of the entangled photons propagates perpendicular to the x, y plane that labels one face (perpendicular to the *z*-direction) of a cube-like Wollaston prism. The x-direction of this chosen coordinate system may also be called "horizontal" and the y-direction "vertical". The second photon of the entangled pair propagates perpendicular to the face of a second Wollaston prism and that face is labeled by the x', y' coordinate system, which is perpendicular to the z' direction and we may use for simplicity $z = z'$. The photons exit the Wollaston cubes along two different directions and are registered by detectors D_v^1 and D_h^1 in wing 1 of the experimental system and by detectors D_v^2 and D_h^2 in wing 2, respectively. The measurement system must also guarantee that the signals detected in the two different and spatially distant wings belong to entangled pairs. This is usually achieved by registering the measurement time and by correlating the signals by the usual space-time correlations of photons propagating with the speed of light. Furthermore, we can find an orientation of maximal correlation by fixing Wollaston prism 1 and rotating Wollaston prism 2 perpendicular to the propagation direction of the photons until we have a virtually perfect anti-correlation of the outcomes, *i.e.* when the horizontal detector clicks in wing 1, the vertical detector clicks in wing 2 or vice versa. In actual experiments this happens for about 99% of all photon pairs or even better, while in the theories that we discuss this must happen with probability 1. It is convenient to define the directions of maximal anti-correlation in wing 2 also as the x, y-directions.

As we will see later, the theoretical work of Bell deals with at least two different rotations of the Wollaston prisms for each wing and relates the corresponding detector registrations to a "horizontal" and "vertical" spin component for all the different orientations of the Wollaston prisms. Thus, we do not have a carefully defined and unique gauge for the spin measurements in Bell's work (in its most basic definition of "gauge" like that of the meter-measure in Paris) and we will see that this fact alone calls for extreme caution when formulating Bell's inequalities and using Einstein's hypothesis that elements of physical reality determine the spin outcomes and their correlations in both wings. It is certainly, in general logically inadmissible to denote different directions in a given wing by "horizontal" or "vertical". The gauge in the other wing is codetermined by the requirement of complete anti-correlation for given instrument settings, which is usually guaranteed and agreed upon through the complete experimental design of both wings. One must chose one instrument setting in one wing as gauge (like the meter measure in Paris) and relate the gauge of the other wing and the so called Bell angles θ, by the convention that for maximum anti-correlation we have $\theta = 0$. Bell and his followers emphasize the free will of the experimenters to choose the instrument settings independently, randomly and swiftly in both wings. However, the spin-gauge cannot be chosen freely and independently as will be further discussed in detail. Oaknin [5] has given careful considerations to these facts and has offered a relativistic resolution for the EPR-paradox. (See also his detailed further explanations in his recent paper [6].)

Bell's original paper does not discuss photons but instead spin $\frac{1}{2}$ quantum entities (such as entangled electrons) that propagate toward Stern-Gerlach magnets instead of Wollaston prisms (cubes). The x, y - and x', y' -coordinates are again perpendicular to the propagation direction. Unit vectors that characterize the gradient of the magnetic field are introduced and typically denoted by **a**, which points in the positive x-direction in wing 1 and **b**, which points in the x'-direction in wing 2.

3. Bell's Functions and Their Relation to Einstein's Separation Principle

Bell attempted to describe EPRB experiments by introducing in each wing functions with a domain of variables that respect Einstein's separation principle. The co-domain or range of Bell's functions describes the outcomes for the measurements of entangled pairs. The mathematical physics of these functions is required to agree with the results of quantum mechanics and/or those of actual experiments.

In order to understand Bell's task, we need to explain the precise meaning of Einstein's separation for EPRB experiments and for Bell's functions that describe them.

Consider the two wing experiments described above and the case of maximal anti-correlation of the outcomes for entangled singlet pairs and assume that the measurement equipment of the two wings is spatially separated so that light would take considerable time (say a millisecond) to propagate from the place of measurement in wing 1 to the place of measurement in wing 2. Then, according to Einstein's relativity, whatever instrument setting is used in one wing and whatever is measured within about a millisecond cannot affect the outcomes of the measurement in the other wing by any transmission of information between the two wings. The measurement stations are information-separated in this way and this fact is called Einstein's separation principle.

Why did Einstein's separation principle enter the discussions of EPRB experiments? Because of the following strange fact. At the instrument settings of maximum anti-correlation, the outcome in wing 1 (e.g. "horizontal" or "vertical") is always anti-correlated to the outcome in wing 2 (which then is "vertical" or "horizontal", respectively *i.e.* the opposite) How can that be? If we toss a coin in wing one and obtain heads, the coin-toss in wing 2 must give tails. Einstein claimed that this would only be possible if instantaneous influences are exerted from the measurement in one wing to the measurement of the other or, alternatively, there need to exist elements of physical reality (possibly "hidden" to us) that determine the measurement outcomes. Actually, one can easily agree with Einstein, when magnets are involved in the measurements. If coins with little magnets of opposite polarization are sent out from the source as entangled pairs to the measurement stations then it is fairly easy to imagine that one falls on head and the other on tail and that can also happen randomly. The hidden magnets in the coins are then Einstein's elements of physical reality.

The problem is, of course, that photons do not have little magnets inside and the elements of physical reality may be very complex and describe the dynamics of the photon equipment interaction etc. Furthermore, the experimental results need to be explained for all instrument settings and not only for maximal anti-correlation. Therefore, Bell attempted to develop a theoretical model for EPRB experiments in a more general fashion.

Bell introduced functions $A(\dots)$ for wing 1 and $B(\dots)$ for wing 2 with the following properties of the variables in the domain of the functions: one of the variables in each function is the magnet "setting", usually denoted by **a**, **b** or **d** in wing 1 and by **b** or **c** in wing 2. All other magnet settings (or equivalent settings of the Wollaston prisms) are possible but not considered in the following. The functions A and B contain each another special "variable" λ that represents the entangled pair. Bell states:

"… λ stands for any number of variables and dependences thereon of A and B are unrestricted."

In other words, λ may represent actually a whole set of variables that describe Einstein's elements of physical reality involved in the measurement of the entangled pair. Considering Einstein's theoretical thinking, such a set of variables might contain elements of the space-time continuum such as the measurement time t_m, because of possible correlations of the measurement dynamics that occurs in the two wings. Of course, such dynamic correlations may be described, in special cases, by a phase relationship between the two wings; the phase being again an element of a continuum.

The set of variables represented by λ is thus very general. In essence the set λ may contain any number of variables, but it must never contain a variable representing the instrument settings of the other wing, because these cannot influence the measurement outcomes in a given wing. In fact, to exclude instantaneous influences at a distance from the theory it is necessary and sufficient that the set λ in the domain of the function A in wing 1 must not contain any instrument-setting-variable from wing 2 and vice versa. With the given definitions, one can, mathematically speaking, use variables λ that are independent of the instrument settings of both wings. The elements of reality that λ represents may actually include local equipment interactions and, therefore, acquire some dependency on the local instrument settings. Mathematically, however, these local settings are included already in the domain of Bell's functions anyway.

(Note also that there is a slight inconsistency in Bell's notation inasmuch he uses fixed settings **a**, **b** etc. in the domain of his functions while λ is standing for a variable and only in some instances for the value of this variable (a given element of physical reality). We leave Bell's notation as it is and just make sure that this inaccuracy does not introduce a bigger mistake.)

To illustrate by an example, allowed functions in wing 1 are $A(\mathbf{a},\mu,t_m,\cdots)$ and $B(\mathbf{b},\mu,t'_m,\cdots)$ in wing 2, respectively, where μ represents some element of physical reality related to information about the photons from the source. The measurement times t_m and t'_m may appear in the function domain because of possible time-like correlations of the dynamic photon-Wollaston or electron-Stern-Gerlach interactions in the respective wings. A and B are then, in Kolmogorov's framework, the functions (random variables) of bi-variate stochastic processes.

The values of the functions *i.e.* their co-domain are the detector registrations for the given measurement times, e.g. $D_v^1(t_m)$ or (exclusive) $D_h^1(t_m)$ in wing 1 and $D_v^2(t'_m)$ or (exclusive) $D_h^2(t'_m)$ in wing 2.

4. Logical Problems with the Applicability of Bell-Type Inequalities to EPRB Experiments

4.1. Problems with Bell's Function-Domain

One of the inaccuracies related to Bell's work (that Bishop Berkeley would have criticized) is the simplistic assumption that λ represents just a finite number of elements of physical reality like coins that are sent out by the source and registered by a detector-click after passing some "evaluation" equipment. If that were the case, we could surmise that for very many measurements with given setting pairs $(\mathbf{a},\mathbf{b})$, $(\mathbf{a},\mathbf{c})$ or $(\mathbf{b},\mathbf{c})$ etc. each setting pair must encounter about the same elements of physical reality that are essentially randomly sent out from a source. In other words, the expectation value of outcomes for given instrument setting pairs would just be an average of the function outcomes for these same elements λ. This simplistic assumption together with Bell's choice of co-domain or range of the functions (see next) leads immediately to Bell's inequalities. But do these inequalities then have anything to do with actual EPRB measurements?

4.2. Problems with Bell's Function-Range

A second suspicious assumption of Bell is an oversimplification of the co-domain or range of his functions. He innocuously introduces the value of +1 for a "horizontal" result and −1 for "vertical", respectively. (Or equivalently for spin $\frac{1}{2}$ entities Bell uses +1 for an "up" deflection by the Stern-Gerlach magnets and −1 for "down".) This use of the same two integer numbers for all possible instrument settings has far reaching consequences (see beginning of next section), because it implies (without any justification) that the function-values follow the mathematical rules for the integers +1 and −1. Christian has emphasized repeatedly (most recently in [7]) that Bell oversimplified the range of his functions.

5. Bell Type Inequalities and Algebra

With the above assumptions we may deduce from the algebra of the integer numbers +1 and −1 that:

$$A(\mathbf{a},\lambda)\big(B(\mathbf{b},\lambda)-B(\mathbf{c},\lambda)\big)=A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)\big(1-B(\mathbf{b},\lambda)B(\mathbf{c},\lambda)\big) \quad (1)$$

Note that the λs of the pair with a product of two B functions correspond necessarily to different entangled pairs (if we wish to compare Equation (1) with actual experiments). Furthermore, if we define the instrument settings for maximal anti-correlation as equal, use accordingly $B=-A$ and take the absolute value, we obtain:

$$\left|A(\mathbf{a},\lambda)B(\mathbf{b},\lambda)-A(\mathbf{a},\lambda)B(\mathbf{c},\lambda)\right|=1+A(\mathbf{b},\lambda)B(\mathbf{c},\lambda), \quad (2)$$

an equation that now features the $A\cdot B$ pair instead of the $B\cdot B$, suggesting that the outcome-pair now does correspond to the measurement of an entangled pair. The triviality of Equation (2) has persuaded many to believe that Bell's inequality (which almost immediately follows from it) is a simple consequence of algebra. A moment of reflection, however, shows that the algebraic operations of Equations (1) and (2) that Bell used after his Equation (14) are physically speaking not trivial at all and require extensive justification.

Bell further assumes the existence of a single common probability density, which might be appropriate if λ just represented a finite number of elements of physical reality; such as 20 fair or not so fair coins. However, Bell did claim the generality of λ and that λ may represent a whole set of variables, including measurement times. How, then, can all of these physical variables and the instrument setting variables have the same probability density?

Only with precisely one given probability density for all function products ($A\cdot A=-A\cdot B$), do we obtain Bell's inequality: Averaging over this single probability density $\rho(\lambda)$ and noting that the absolute value of a sum is always smaller than or equal to the sum of the absolute values, one obtains the expectation value E for the function products with Bell's instrument-setting pairs $\mathbf{a},\mathbf{b}$, $\mathbf{a},\mathbf{c}$ and $\mathbf{b},\mathbf{c}$:

$$\left|E(\mathbf{a},\mathbf{b})-E(\mathbf{a},\mathbf{c})\right|\le 1+E(\mathbf{b},\mathbf{c}), \quad (3)$$

where E may, in general, be expressed by a Lebesgue integral over the product $A\cdot B$ of Bell's functions.

6. Bell Inequalities and Probability Spaces

It was soon realized [8] [9] [10] that Bell's inequality could be derived with only one necessary and sufficient powerful condition: All functions appearing in

Bell's inequality are random variables on one given probability space in the sense of Kolmogorov. This fact makes it also easy to extend the validity of the inequality to a countable infinite number of the λs. However, two major problems still exist with the application of Bell's inequality to actual experiments and the results of quantum mechanics.

First, it turns out (shown by a theorem in [9]) that the measurement time and the instrument settings cannot be random variables on one common probability space. The reason is simply that we cannot have two different instrument settings in a given wing at the same measurement time. The proof of this fact is simple and may have been known to Einstein when he enunciated: "Gott wuerfelt nicht".

I would also like to point to the recent work of Khrennikov connecting quantum probabilities and classical conditional probabilities [11]. Novel arguments about the necessity of using different probability spaces for different setting pairs have been put forward (within the framework of quantum mechanics) by Cetto, Valdes-Hernandez and Pena [12].

Second, there exists a problem with the spin-gauges. How can one logically deal with two different "horizontal" and "vertical" directions in each given wing, ($(\mathbf{a},\mathbf{b})$ in wing 1 and $(\mathbf{b},\mathbf{c})$ in wing 2), and, in addition, regard all the measurement outcomes in the above equations equal to +1 or to −1 independent of how "horizontal" or "vertical" are globally defined for a given pair measurement?

7. Wigner Inequalities and Set Theory; Selecting Global Subsets

Eugene Wigner [3] improved Bell's treatment [1] significantly by using set theory and considering certain subsets of the measurement outcomes. Instead of using outcomes +1 and −1 as Bell does, Wigner relies only on the judgement of equal **e** and not-equal **ne** for the pair measurement outcomes of a given instrument setting pair.

The actual value and physical nature of the co-domain (integer numbers or "up/down") is thus of no concern, for we need to have only a judgement of "equal" or "not-equal". Wigner also noticed that, for the purpose to derive a Bell-type inequality, it is sufficient to determine the number of (possible) outcomes that are either **e** or **ne** for a given pair of instrument settings. He, therefore, just counted the number of equal and not-equal outcomes separately for each of the 3 pairs of Bell's instrument settings $(\mathbf{a},\mathbf{b})$; $(\mathbf{a},\mathbf{c})$ and $(\mathbf{b},\mathbf{c})$ and derived his Bell-type inequality using these numbers. The spin-gauge may also be (and must be, in principle), chosen as separate and different for each such given pair of instrument settings. Note, as a preview of the more detailed discussions below, that the judgement of **e** or **ne** is a global one that determines the relative outcomes and the frequency of them in both wings. Their frequency may, in general, depend on the instrument settings of both wings, which may lead to a violation of the Bell-Wigner inequalities.

Bell [1], Wigner [3] and later d'Espagnat [13] do use, however, a single common probability space for all Bell instrument setting-pairs and the **e**-, **ne**-subsets, which enforces their inequalities. They do offer a weighty argument for this choice: If we consider a triplet-set of measurements (instead of the 3 pair measurements), we may write all possible measurement-outcomes in terms of triples:

$$A(\mathbf{a},\ldots)A(\mathbf{b},\ldots)A(\mathbf{c},\ldots), \tag{4}$$

and using $A=-B$ automatically obtain one common joint-triple probability space that lets us construct the Bell-Wigner inequalities (think of the frequency interpretation of probability).

This last step, however, presents again major problems. First, a triplet is not measured in EPRB experiments, but we deal with entangled pairs. We, therefore, run again into the problem of the assumption of all equal λs, which cannot be justified if λ encompasses variables related to a continuum. We deal then with more complex bi-variate stochastic processes. One for each of the instrument setting pairs.

This problem is easily understood from the following example. Consider the three instrument setting-pairs of Bell and let λ be the measurement time. I assume, for simplicity, the measurement times in the two wings to be equal for a given instrument setting pair and given entangled pair and thus have the following possible measurement outcomes (Bell function-pairs):

$$A(\mathbf{a},t_m)A(\mathbf{b},t_m);A(\mathbf{a},t_n)A(\mathbf{c},t_n);A(\mathbf{b},t_k)A(\mathbf{c},t_k) \tag{5}$$

here we let $m=1,2,\cdots,N$ further $n=N+1,N+2,\cdots,2N$ and $k=2N+1,2N+2,\cdots,3N$, with N indicating a large number of measurements. Then, because all the outcomes for each of the function pairs may be either equal **e** or not-equal **ne**, we have $2^{(3N)}$ possible different combinations of **e** and **ne** outcomes. Bell's inequality, however, is based on the assumption of equal λs instead of different measurement times and considers, therefore, only $2^{(2N)}$ possible combinations of **e** and **ne** outcomes. This leaves us with $2^{(3N)}-2^{(2N)}$ of these possible **e** and **ne** outcome-combinations that may contribute to violations of the Bell-Wigner inequalities. (Actually, things are not quite as drastic as suggested by these considerations, because Wigner needs for his inequality only the fraction of the number of **e** and **ne** outcomes. Considering only combinations that change that fraction, one obtains $(N+1)^3$ different combinations if we include measurement times taken out of a continuum, as compared to N^2 for a finite (countable) number of the λs [14].)

Thus, the number of combinations of equal and not-equal outcomes that may violate the Bell inequality (given general time dependent functions as they are usual in physics) is vastly larger than the number of combinations that necessarily obey Bell's inequality; which is negligibly small for large *N*. If one would bet on the odds that any experimental sequence of EPRB-type experiments (described by time dependent functions) violates the Bell inequality, one could bet with great certainty that it will. It is thus not surprising that so many different quantum experiments violate Bell-type inequalities.

As indicated above, there is also the other, even more disturbing, restriction and inconsistency in the treatment of Wigner that was later taken over by d'Espagnat [13]: they use only one common probability space, an assumption that is incorrect if one does not deal with Bell's λs, but instead with the subset of those λs that lead either to **e** or **ne** outcomes *i.e.* with $\lambda_{\mathbf{e}}$ and $\lambda_{\mathbf{ne}}$, respectively. Exactly how many of these "global" entities will lead to **e** or **ne** outcomes in the respective wings does in general depend, as I will demonstrate below, on the instrument settings of both sides without any involvement of instantaneous influences at a distance.

8. Alice and Bob to the Rescue: The Bell Game

The many objections related to infirmities of the derivation of Bell's inequality (and a large number of similar types of inequalities and equalities) made it desirable to find some way of stating Bell's findings crisply and without any possibility of objection. This was accomplished by putting the Bell inequalities themselves into the far background and just using their power of contradicting the quantum mechanical result. In this way Bell conceived a theorem for "local" theories, which is mathematically and physically always true, but only for a certain definition of "local". It was formulated by Bell in his following statement [15]:

"But if (a theory with … variables λ) is local it will not agree with quantum mechanics, and if it agrees with quantum mechanics it will not be local. This is what the theorem says."

Bell's definition of "local", however, does not only mean that the Einstein separation principle is strictly valid but adds additional, physically not necessary (actually physically impossible), conditions as explained below for the "Alice-Bob-game" or "Bell game", which was divined by Bell's followers and science writers.

Indeed, the above statement of Bell, with the addition of the "Alice-Bob-local" definition, is unassailable. As we will see, the theorem so stated is true and needs no mathematics to prove it. The problem with the theorem so stated is of epistemological nature: there exists no Alice-Bob-local physical theory of spatially distant and correlated measurements; not Einstein's relativity, not classical mechanics, not quantum mechanics, not any non-trivial theory as we will see momentarily.

What is the Alice-Bob-locality about? Alice and Bob are the experimenters in the two EPR wings and know only their own instrument setting and not that in the other wing during the measurements, because of the random switching of the instrument settings before a given measurement. It is postulated by the followers of Bell that Alice and Bob must be able to find a "local theory" that gives them the outcome for Bell's functions as soon as they obtain an element of physical reality λ that is sent to them from a source. This "local-theory-game" is, as we will see, impossible to play and thus the Bell theorem is proven. Extreme non-localities, on the other hand, make it easy to find violations of Bell's theorem: just include a setting variable from the other side in Bell's functions (and use e.g. $A(\mathbf{a},\mathbf{b},\lambda)$) and Bell's inequalities are easily violated (spooky influences yield any desired results).

To understand why the "Alice-Bob-game" (also called by many "the Bell game") cannot be played and why the corresponding definition of "local" is too narrow and physically inappropriate, we have to realize that the experimenters must use non-local or global knowledge throughout their experimental design, during the measurements and after the measurements are finished and evaluated.

Much of the knowledge that Alice and Bob need about the other wing, can be gained by the design of the global experimental set-up before any of the actual measurements are done. Alice and Bob need to know that they measure the appropriate counterpart of an entangled pair. This is commonly accomplished by careful determination of the measurement times in both wings and requires the additional assumption that the measurement time does not depend on the equipment setting (which we may accept here without consequences for the following reasoning). The global gauge for all given instrument setting pairs and the settings for maximal anti-correlations as well as the connection of the coordinate systems of Alice and Bob, may also all be fixed before the actual measurements begin.

During the measurements, Alice and Bob do not have information about the global gauge that is relevant for a particular pair-measurement, because that gauge depends on the instrument setting of both wings and these instrument settings are randomly switched. It is, therefore, impossible for them to find a local theory that describes measurement outcomes violating Bell-type inequalities at this stage of the experiment. I have pointed out in a previous publication [16] that such requirements for a local theory would not permit any theory of relativity, which necessarily works with systems as seen relative to other systems, a fact that has been discussed by Oaknin [5] in great detail. Alice and Bob certainly cannot choose, at this stage, the Wigner subsets that are the key for finding violations of Bell-type inequalities and require the knowledge of the global gauge and instrument settings. The illustration in section 9 gives further reasons why Alice and Bob cannot play the Bell game.

The information about Wigner subsets is found by Alice and Bob only after all measurements are done: they assemble the Wigner subsets from the global data to prove the experimental violations of Bell-type inequalities by counting the equal **e** vs not-equal **ne** outcomes for the Bell-pairs of instrument settings. In this way they assess outcomes as seen relative to the other wing. The fact that this relativity is introduced after the measurement run does not make a difference to the fact that the statistics of the outcomes is determined by non-local means.

I have sometimes been asked by Bell's followers: "Nature can play the Bell game and finds measurement outcomes during the experimental runs, why can you not do it?" We see from the above that nature cannot play that local game, because it involves a global experimental design, a global gauge and a global assembly of Wigner subsets with the assessment of the frequency of relative outcomes in the two wings.

The theoretician who develops a theory about the Wigner subsets for **e** and **ne** outcome pairs, needs to have, as a minimum, that same global knowledge that the experimenters have and use to produce and finally collect their data as already outlined in [16]. These latter global facts and knowledge may not only reasonably be used by the theoretician, but must be used when probabilistic theories are invoked, because the probability spaces of subsets may depend on these global facts: when we ask the question of how many **e** and **ne** pairs we have for a given instrument setting pair, we do address a global fact for which only a globally valid theory and gauge can account. In fact we ask what is the outcome relative to the outcome in the other wing and we lose the possibility to treat the two wings independent of their respective measurement outcomes. Wigner improved Bell with respect of the generality of the function range (co-domain) but had to deal, in return, with the numbers of λ that correspond to **e** and **ne** outcomes and these numbers (or their frequencies of occurrence) may depend on the instrument settings (gauge) of both wings because they depend on the relative measurement outcome of the other wing. This latter fact will be shown in more detail in section 9.

In this connection it is also important to remember that non-locality by instantaneous influences at a distance requires a specific inclusion of the instrument-settings of the other wing in the domain of Bell's functions. Such inclusion is neither present nor required at all in the following illustration.

9. Wigner-Subsets, Spin-Gauge and Probability-Spaces: The Role of Free Will and Randomly Switched Instrument Settings

The gauge for the spin ("horizontal/vertical" in a certain coordinate system) can be chosen freely in only one wing of the EPRB experiment. The gauge in the other wing is determined by the requirement of complete anti-correlation, which also determines the x, y coordinates on both sides. The outputs of the Wollaston prisms (Stern-Gerlach magnets) are then anti-correlated by definition for the same x, y coordinates, as they should be for the singlet entangled pairs that we consider.

This latter important point and its full consequences were not realized by Bell and his followers. They claim that the settings on both sides are chosen by the free will of Alice and Bob. Naturally, even the free will cannot turn instruments faster than the speed of light c, which is the reason why measurement times and instrument settings cannot be defined on one common probability space [9]. Furthermore, the gauge of the spin must be globally well-defined also in the second wing, once chosen in the first. Detailed considerations about using consistent spin-gauges were presented by David Oaknin [5]. They were also discussed independently and from a different vantage point by this author [17] and in an early stage with collaborators [18]. It is important to realize that there are certain restrictions for the choice of gauge, which have been discussed in all generality by Oaknin [5]. For our purposes here it is sufficient to adopt a definition of gauge corresponding to one given instrument setting (say in wing 1) and to connect this gauge to wing 2 as described above and below.

We may rotate the Wollaston prism in wing 2 freely, for example by an angle θ around the z-axis and replace thus the measurement directions x, y by the rotated x', y' coordinates in wing 2. Note, however, that we must now label the measurement data in wing 2 by the angle θ between x and x', if we wish to say anything about the Bell-correlations of the two wings including complete anti-correlation for the x, x' settings. Oaknin [5] states: "… only their relative orientation (referring to x, x') is a physical degree of freedom." If we do not relate the wing 2 instrument settings and wing 2 gauge to that of wing 1, we naturally cannot speak about correlations. Remember also that in actual EPRB experiments the determination of the Bell angles is usually done by the experimenters before the actual measurement-runs, while the choice of measurement sequences and setting pairs is made after the experimental runs are finished; when the Wigner subsets are collected mostly based on measurement times.

The rotation of the Wollaston in wing 2 by θ results, of course, in measurement outcomes that are different from complete anti-correlation. The corresponding correlations of the outcomes in this new situation are still determined by the physical law that governs the interactions of the photons and the Wollaston prisms and the possible measurement outcomes may, therefore, be different in wing 2 in a variety of ways.

The following illustration by an example from classical mechanics is designed to explicitly demonstrate the associated possible changes of probability spaces by rotations of the Wollaston prism and by subsequently choosing Wigner-type subsets of **e** and **ne** outcomes. We will see that such rotation changes the Kolmogorov probability spaces for the **e** and **ne** outcomes and involves θ or functions of θ. It will also become obvious that these changes of the probability spaces with θ have nothing to do with instantaneous influences at a distance in spite of the fact that θ depends on the instrument settings of both sides.

9.1. Illustration of Global Correlations between Classical EPRB-Type Measurement Pairs without Violation of Einstein's Separation Principle

This oversimplified illustration (per se) is not invalidating Bell's inequality, because it is linear in all its variables. It is demonstrating in the most elementary way, however, that a dependence of certain probabilities on θ has nothing to do with influences at a distance but arises naturally from local factors and the global experimental design and gauge.

Consider two macroscopic rods parallel and next to each other, one with a red top and blue bottom section, the other with a blue top and red bottom section. The top of one rod is always next to the bottom of the other. The center of these rods is on the z-axis of a coordinate system and the rods are always oriented perpendicular to the z-axis. Assume further that these rods are emitted from a source. The direction of the emission is random: either the first rod propagates into wing 1 and the second into wing 2 along the z-axis or vice versa, both rods exhibit the same angle ϕ, which is measured from the *x*-axis of the x, y-plane and ranges in value randomly between 0 and π.

In both wings we have instead of the Wollaston prisms simple color detectors that indicate the result "horizontal" if and only if for any ϕ between 0 and π the color is red, while they indicate "vertical" if for any such ϕ the color is blue. We can see that this simple experiment will always result in complete anti-correlation without any suspicion of instantaneous influences at a distance. There is, of course, global knowledge and fact involved in the gauge, the physical conservation law that guides the rods toward the detectors and how the rods interact with the detectors locally but in a correlated way, because of the overall design of the experiment.

Assume now that many such rod-pairs are emitted from a source. We use the same local rules for evaluating the red and blue rod-sections, but we rotate the coordinate system and color detectors in wing 2 by an angle θ that turns x into x'. We denote the angles of the rods with the x'-axis by ϕ' and determine the new "horizontal" and "vertical" outcomes in wing 2 according to the colors red or blue, respectively. Now, however, we do this for ϕ' in the x', y'-plane with values between 0 and π.

After performing many measurements, we select in wing 1 the subset of rods with the exclusive outcome "horizontal". We then ask the question, which fraction of the corresponding rods in wing 2 is "vertical" and which fraction "horizontal". The "entangled" pair of rods is determined by the angle ϕ and the fact that $\phi' = \phi - \theta$, another needed global knowledge. Because ϕ is random between 0 and π, one obtains $1 - \frac{\theta}{\pi}$ for the "vertical" fraction of rods and $\frac{\theta}{\pi}$ for the "horizontal" fraction, respectively. Thus, the probability (using the frequency interpretation) of the "horizontal" and "vertical" outcomes depends on the angle θ between the two coordinate systems.

Alice and Bob, however, could not possibly guess that simple result; they cannot know about the instrument setting in the other wing during the measurements and thus also do not know the angle θ. The probability measure $P(A = B)$ for the outcomes to be **e**, equals the probability that $0 \le \phi \le \frac{\theta}{\pi}$, which equals $\frac{\theta}{\pi}$. As is evident from the simplicity of the model, the dependence of the probability measure on the angle between the instrument settings in the two wings does not indicate any implausible non-locality and certainly not any influences at a distance but arises from the global factors of the experimental arrangement, gauge and choice of Wigner subsets.

This means that Wigner-subset probability-measures for Bell's function products and the numbers of **e** and **ne** outcomes may depend on the instrument settings of both sides, without involvement of spooky non-localities.

As a major corollary one can state that a global statistical result, obtained from many measurements at separate locations for correlated information packages and correlated measurement times, may depend on non-local variables such as θ. The global statistical result may reflect measurement arrangements of all of the separate locations, even if those arrangements do not influence each other and are unknown to anyone controlling the local measurement events. It is this corollary, which permits us to use the space-time system to exorcise spooky influences in complex situations if we choose to do so.

Note that these statistical properties of subset probability-measures do not imply that Bell's function-domain contains a variable corresponding to the equipment settings of the other wing. Nor does the random angle ϕ, which corresponds to Bell's λ (except that it is chosen out of a continuum of angles) depend on any instrument settings. It is only the probability for **e** and **ne** outcomes that depends on the instruments of both wings. Nor is the free will of Alice and Bob to choose any angle θ restricted in any way (super-determinism). They must choose, however, the spin-gauge of both wings consistent with the settings of complete anti-correlation and cannot decide that fact by their separate free will.

9.2. The Importance of Subset Selection and the Problem with Random Switching

Fast random switching of the instrument settings (and thus of θ) are declared by Bell's followers to be the vade mecum for proving Bell's theorem, because it makes it impossible to play the Alice-Bob-local game (the Bell game). Neither Alice nor Bob know θ and the gauge that is chosen in the other wing, because that gauge depends on the (rapidly switched) instrument setting. Their choice of the outcome-value for Bell's functions (of $A(\mathbf{a},\lambda)$ by Alice and $B(\mathbf{b},\lambda)$ by Bob, after they receive a value of λ) is, therefore, meaningless at the time the measurements are performed. They both do not know the global gauge at this point. This knowledge is only acquired by them when they select the Wigner subsets after the measurements are completed.

Random switching of the measurement settings on both sides does involve random changes of the spin-gauges and of the probability measures for the **e** and **ne** outcomes of selected subsets. It is, therefore, nonsensical to require that Bell's functions and the ordering of their outcome-values into subsets a la Wigner are related to only one common probability space. The probabilities $P(A=B)$ and $P(A\neq B)$ depend in our (classical mechanics) illustration on θ, which is randomly changed by randomly switching the instrument settings. The angle ϕ (chosen out of a continuum), which corresponds to Bell's λ emanating from the source, does not depend on the instrument settings as dictated by Einstein's separation principle. However, the frequencies of **e** and **ne** outcomes may depend on the instrument settings (and do depend on them in our illustration), because of the underlying physical law, global experimental arrangement and global gauge.

These facts make us appreciate Einstein's view (in his discussion with Heisenberg) that the theory codetermines what can be measured. The experimenters need to know about Wigner subsets in order to find out whether or not Wigner's inequality is obeyed by their data. The theoretician needs to help with the construction of the global design and needs to provide a consistent global gauge. The requirement that Alice and Bob should be able to find a theory at a certain point of the experimental procedure at which they have no idea of the global gauge and other factors, appears in this light as a crucial mistake.

Quantum mechanics also gives results for precisely one given instrument setting in each wing. That setting pair determines the operators that act on the designated states of a product Hilbert-space. The instrument setting thus determines the spin operators and defines the gauge through the eigenvalues and eigenvectors of these operators. The preparations of particles and their quantum states (on which the spin-operators act), determine the precise division into different subsets of the possible outcomes corresponding to **e** and **ne** values. These different subsets are, however, not necessarily defined on one common probability space but involve, in general, different probability spaces for different instrument setting pairs.

10. Explicit Statistical Interpretation and Model for EPRB

How can one model the general probability measures for the actual singlet-entangled-pair measurement outcomes, without invoking any inappropriate nonlocal influences? How can one exclusively use the data and invoke a physical law that explains the probability measures without instantaneous influences at a distance?

I specify our considerations to the case of entangled photon pairs and measurements with Wollaston prisms and also consider only Wigner's set theoretical approach, which means we need to determine only the number of **e** and **ne** outcomes for a given instrument setting pair say **a** in wing 1 and **a** as well as **b** in wing 2. This use of Wigner-type subsets, which are selected by the theoretician after all measurements are completed, is crucial.

We follow the above illustration and collect from all data, as a first step, the M "horizontal" outcomes of wing 1, as well as the corresponding measurement outcomes for the entangled photons in wing 2. For $\theta = 0$ (setting pair $\mathbf{a}, \mathbf{a}$) all the outcomes in wing 2 must be "vertical" because of maximal anti-correlation. For $\theta \neq 0$ and Wollaston prism in wing 2 set to **b** (the x axis rotated by an angle θ to form the x'-axis), one naturally expects a Malus type law at work (see e.g. The FeynmanLectures on Physics III), which results in about $M(\sin\theta)^2$ "horizontal" outcomes, because that is what is natural for a system of entangled pairs that had shown all "vertical" outcomes before the Wollaston prism was rotated by θ. The "vertical" outcomes in the rotated system are then $M - M(\sin\theta)^2 = M(\cos\theta)^2$. These results are also expected by quantum considerations as shown in the Feynman lectures.

As in our illustration above, the use of the angle θ involves no "illegal" nonlocality. We have used a Malus-type law for the probabilities of "horizontal" and "vertical" outcomes.

Symmetrically, for the set of all "vertical" outcomes in wing 1, we obtain in wing 2 about $M(\cos\theta)^2$ "horizontal" outcomes and $M(\sin\theta)^2$ "vertical" outcomes. One easily obtains then the number of equal outcomes to be $\mathbf{e} = 2M(\sin\theta)^2$, while the number of non-equal outcomes is $\mathbf{ne} = 2M(\cos\theta)^2$. Thus, we have for the difference in the outcome probabilities (frequencies):

$$E(\mathbf{a},\mathbf{b}) = -\left((\cos\theta)^2 - (\sin\theta)^2\right) = -\cos(2\theta) \tag{6}$$

Nothing in the procedure depends on the distance of the Wollaston prisms. Nor do we need to involve any instantaneous influences at a distance.

We got around the instantaneous changing of a "state" by avoiding, a la Einstein, relations of state concepts to single objects and instead using all horizontal outcomes in wing 1 as a subset and obtaining the correlated subsets in wing 2 by the probabilities as obtained from a Malus type law.

In contrast, the quantum-interpretation of Bell's followers maintains that (because the outcomes that we consider in wing 1 are all "horizontal") we are dealing in wing 1 with measurement of a single "horizontal" quantum state and only after these measurements do we know that the state in wing 2 must be a vertical quantum state and instantaneously so.

We have, with the outcome-subset treatment, not achieved any progress or improvement of the results, but have avoided instantaneous changes of single-object quantum states. In other words, we can clearly avoid any hint of instantaneous influences in this model for EPRB experiments. The major novelty is that certain subsets of data-pairs may necessarily be defined on different probability spaces. Quantum mechanics avoids dealing with this complication by only working with probability amplitudes that lead to the final subset probabilities by Born's interpretation. Andrei Khrennikov has analyzed the matter of statistical vs individual interpretations from a mathematical point of view and has contributed over many years to a large number of questions discussed above [19]

Would Einstein be satisfied with the model here presented? Not quite, because he preferred ab initio physical theories that do not invoke probability laws. However, if we do not wish to introduce a Malus type law of nature, then we must dive into the nitty gritty of the dynamic interactions of entangled pairs and measurement equipment and describe them in space-time. This presents a very complicated theoretical problem that may lead to the well-known infinite regress that arises if the equipment is treated as a many body system. This difficulty is the great barrier for ab initio Bell-type models and this barrier has contributed to many frustrations in this area. It is identical to the similar barrier of quantum theory that usually does not and cannot supply an ab initio theory of the measurement instruments.

Of course, there are numerous other quantum experiments exhibiting the same features that I have just described for EPRB: they are much easier to explain with instantaneous influences at a distance than without those influences. In the final analysis, Einstein found the use of such influences spooky and spook must be exorcised. However, spook is sometimes “entrenched” in highly valued physical concepts and strategies of explanation, simply because it solves all problems easily.

Typical examples are experiments that can be explained with interference at distant regions of space, particularly interference of probability amplitudes. However, we encounter here again a suspect logic that Bishop Berkeley would have objected to: how can the interference of “something” destruct any results at distant spatial locations, where that “something” is, ontologically speaking, not present in the first place? In other words, the concept of interference at a distance involves instantaneous influences at a distance to start with. Instead of embracing such concepts and using them to unseat other logical explanations, work must be done, as above for the model of EPRB experiments, that permits to remove instantaneous influences at a distance and exorcise spooky effects. Such work is not going to be easy, as one can see from the years of controversy with the Bell theorem. However, only such dedicated work can lead us away from spooky influences and toward more Einstein-like theories.

11. Conclusions

It has been examined whether EPRB experiments can be reasonably modeled without the use of instantaneous influences at a distance and a way was found that is numerically identical to the quantum mechanical results but interpretational different. The concept of quantum-state as a description of a single entity has been avoided by using the description of subsets of entities interacting with instruments and resulting in subsets of data. A main difference between quantum and Kolmogorov probability concepts has been pinpointed in the treatment of such subsets of the data for measurement outcomes. These subsets correspond in quantum mechanics to different quantum states as well as operators, while in Kolmogorov’s framework they correspond generally to different probability spaces.

I have also shown that Bell’s theorem does not apply to such subsets of data-pairs of distant EPRB measurement outcomes, because the definition of “local” by Bell and followers in the Alice-Bob-game is too narrow. With that definition, the Bell theorem is indeed valid, but does not apply to any nontrivial actual experiments.

I have further shown that the Bell inequalities themselves suffer from a number of logical inconsistencies and lack of generality in their dealing with probabilistic concepts, inconsistencies that also cannot even be repaired by the more general set theoretic approach of Wigner and d'Espagnat.

I believe that my findings suggest that instead of embracing instantaneous influences at a distance in physical theories and in quantum mechanics, ways must be searched for that avoid the use of such concepts by the appropriate use of different subset-probability-spaces. Quantum mechanics, of course, has accomplished that division into sub-sets through quantum states and operators. However, the interpretations that link quantum states to single ontological entities lead to temptations of suggesting instantaneous influences, which may be avoided by careful additional explanations as presented above for the special case of EPRB.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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