

The Hubble Constant Problem and the Solution by Gravitation in Flat Space-Time

Walter Petry

Mathematical Institute of the University Düsseldorf, Düsseldorf, Germany

Email: wpetry@meduse.de

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Abstract

General Relativity implies an expanding Universe from a singularity, the so-called Big Bang. The rate of expansion is the Hubble constant. There are two major ways of measuring the expansion of the Universe: through the cosmic distance ladder and through looking at the signals originated from the beginning of the Universe. These two methods give quite different results for the Hubble constant. Hence, the Universe doesn't expand. The solution to this problem is the theory of gravitation in flat space-time where space isn't expanding. All the results of gravitation for weak fields of this theory agree with those of General Relativity to measurable accuracy whereas at the beginning of the Universe the results of both theories are quite different, *i.e.* no singularity by gravitation in flat space-time and non-expanding universe, and a Big Bang (singularity) by General Relativity.

Keywords

Gravitation in Flat Space-Time, Cosmological Models, Hubble Constant, No Big Bang, No Singularity, Non-Expanding Universe

1. Introduction

General Relativity (GR) implies an expanding universe where the expansion rate is the Hubble constant. There are two different methods to measure the Hubble constant. The results of these two methods are two different values for the Hubble constant (see e.g. [1] [2]). Hence, the assumption that the universe expands is not correct and the universe doesn't expand (see e.g. [2]). The expansion is a generally accepted assumption supported by GR. We can say that GR isn't a correct description of gravitation. There are authors who ask for new physics (see [1]). Therefore, we will use the theory of gravitation in flat space-time (GFST) instead of GR which is studied by the author in the book and in several

articles (see e.g. the articles [3] [4] [5] [6]). GFST gives non-expanding space for the universe. The metric is the pseudo-Euclidean geometry and the proper time is formally similar to the metric of GR. The source of the gravitation field is the total energy-momentum tensor including that of gravitation. This is in full agreement with Einstein who stated that matter is equal to energy and reverse. GR doesn't satisfy this condition and in addition the energy-momentum of gravitation by GR is not a tensor. It is worth to mention that GFST was already studied in article [7] with application to non-singular cosmological models in [8]. Surface data show evidence for a non-expanding universe [9]. The possibility of non-expanding, cosmological models is already given in the article [10] by the use of GFST. Non-singular universes by GFST with matter creation and entropy production are also studied in [11].

2. GFST

The theory of GFST is shortly summarized. The metric is flat space-time given by

$$(ds)^2 = -\eta_{ij} dx^i dx^j \quad (1)$$

where (η_{ij}) is a symmetric tensor. Especially, pseudo-Euclidean geometry has the form

$$(\eta_{ij}) = (1, 1, 1, -1). \quad (2)$$

Here, $(x^i) = (x^1, x^2, x^3)$ are the Cartesian coordinates and $x^4 = ct$. Let

$$\eta = \det(\eta_{ij}). \quad (3)$$

The gravitational field is described by a symmetric tensor (g_{ij}) . Let (g^{ij}) be defined by

$$g_{ik} g^{kj} = \delta_i^j \quad (4)$$

and put similar to (3)

$$G = \det(g_{ij}). \quad (5)$$

The proper time τ is defined by

$$(cd\tau)^2 = -g_{ij} dx^i dx^j. \quad (6)$$

The Lagrangian of the gravitational field is given by

$$L(G) = -\left(\frac{-G}{-\eta}\right)^{1/2} g_{ij} g_{kl} g^{mn} \left(g_{/m}^{ik} g_{/n}^{jl} - \frac{1}{2} g_{/m}^{ij} g_{/n}^{kl} \right) \quad (7)$$

where the bar “/” denotes the covariant derivative relative to the flat space-time metric (1). The Lagrangian of dark energy (given by the cosmological constant Λ) has the form

$$L(\Lambda) = -8\Lambda \left(\frac{-G}{-\eta}\right)^{1/2}. \quad (8)$$

Let

$$\kappa = 4\pi k/c^4 \tag{9}$$

where k is the gravitational constant. Then, the mixed energy-momentum tensor of gravitation, of dark energy and of matter of a perfect fluid is

$$T(G)_j^i = \frac{1}{8\kappa} \left[\left(\frac{-G}{-\eta} \right)^{1/2} g_{kl} g_{mn} g^{ir} \left(g_{/j}^{km} g^{ln} - \frac{1}{2} g_{/j}^{kl} g^{mn} \right) + \frac{1}{2} \delta_j^i L(G) \right] \tag{10a}$$

$$T(\Lambda)_j^i = \frac{1}{16\kappa} \delta_j^i L(\Lambda) \tag{10b}$$

$$T(M)_j^i = (\rho + p) g_{jk} u^k u^i + \delta_j^i p c^2. \tag{10c}$$

Here, ρ , p and u^i denote density, pressure and four-velocity of matter. it holds by (6)

$$c^2 = -g_{ij} u^i u^j. \tag{11}$$

Define the covariant differential operator

$$D_j^i = \left[\left(\frac{-G}{-\eta} \right)^{1/2} g^{kl} g_{jm} g^{mi} \right]_{/k} \tag{12}$$

of order two. Then, the field equations for the gravitational potentials (g_{ij}) have the form

$$D_j^i - \frac{1}{2} \delta_j^i D_k^k = 4\kappa T_j^i \tag{13}$$

where

$$T_j^i = T(G)_j^i + T(M)_j^i + T(\Lambda)_j^i. \tag{14}$$

Define the energy-momentum tensor

$$T(M)^{ij} = g^{ik} T(M)_k^j. \tag{15}$$

Then, the equations of motion in covariant form are

$$T(M)_{i/k}^k = \frac{1}{2} g_{kl/i} T(M)^{kl}. \tag{16}$$

In addition to the field Equation (13) and the equations of motion (16) the conservation law of the total energy-momentum holds, *i.e.*

$$T_{i/k}^k = 0. \tag{17}$$

The results of this chapter may be found in the book [12] and in the subsequently appeared articles [3] [4] [6]. In article [5] the gravitation theories of GFST and GR and their results are compared with one another. Furthermore, the redshift formula for GFST is derived.

4. GFST and the Universe

GFST is defined in flat space-time metric, e.g. in the pseudo-Euclidean geometry which is used in the following to study homogeneous, isotropic, cosmological

models. The matter tensor is given by a perfect fluid with velocity equal to zero. The total matter is given by the sum of density of matter ρ_m and of radiation ρ_r with the corresponding pressure density of matter $p_m = 0$ and of radiation $p_r = \frac{1}{3}\rho_r$. It holds for homogeneous, isotropic, cosmological models

$$\begin{aligned} g_{ij} &= a(t)^2 \quad (i = j = 1, 2, 3) \\ g_{ij} &= -1/h(t) \quad (i = j = 4) \\ g_{ij} &= 0 \quad (i \neq j). \end{aligned}$$

The initial conditions at present time $t_0 = 0$ are

$$a(0) = h(0) = 1, \dot{a}(0) = H_0, \dot{h}(0) = \dot{h}_0, \rho_m(0) = \rho_{m0}, \rho_r(0) = \rho_{r0}$$

where H_0 is the Hubble constant and \dot{h}_0 is an additional constant not appearing in GR. Relation (16) for $i = 4$ implies under the assumption that matter and radiation do not interact

$$\rho_m = \rho_{m0}/h^{1/2}, \quad \rho_r = \rho_{r0}/(ah^{1/2}) \tag{18}$$

It follows by the use of the field Equation (13)

$$\frac{d}{dt} \left(a^3 \sqrt{h} \frac{\dot{a}}{a} \right) = 2\kappa c^4 \left(\frac{1}{2} \rho_m + \frac{1}{3} \rho_r + \frac{\Lambda}{2\kappa c^4} \frac{a^3}{\sqrt{h}} \right) \tag{19a}$$

$$\frac{d}{dt} \left(a^3 \sqrt{h} \frac{\dot{h}}{h} \right) = 4\kappa c^4 \left(\frac{1}{2} \rho_m + \rho_r + \frac{1}{8\kappa c^4} L(G) - \frac{\Lambda}{2\kappa c^2} \frac{a^3}{\sqrt{h}} \right) \tag{19b}$$

where

$$L(G) = \frac{1}{c^2} a^3 \sqrt{h} \left(-6 \left(\frac{\dot{a}}{a} \right)^2 + 6 \frac{\dot{a}}{a} \frac{\dot{h}}{h} + \frac{1}{2} \left(\frac{\dot{h}}{h} \right)^2 \right)$$

The expression $\frac{1}{16\kappa} L(G)$ is the density of gravitation field. The conservation law of the total energy is

$$(\rho_m + \rho_r)c^2 + \frac{1}{16\kappa} L(G) + \frac{\Lambda}{2\kappa} \frac{a^3}{\sqrt{h}} = \lambda c^2 \tag{20}$$

where λ is a constant of integration. Define the quantity

$$\varphi_0 = 3H_0 \left(1 + \frac{1}{6} \frac{\dot{h}}{H_0} \right).$$

The field Equation (19) imply by the use of the conservation law (20) and the initial conditions the relation

$$a^3 \sqrt{h} = 2\kappa c^4 \lambda t^2 + \varphi_0 t + 1. \tag{21}$$

It follows from (20) with the present time $t_0 = 0$ by the use of the initial conditions and the standard definitions of the density parameters of matter, radiation and of the energy given by the cosmological constant with the abbreviation

$$\kappa_0 \Omega_m = \Omega_r + \Omega_m + \Omega_\Lambda - 1 \tag{22}$$

the differential equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{H_0^2}{(2\kappa c^4 \lambda t^2 + \varphi_0 t + 1)^2} \left(-\Omega_m \kappa_0 + \Omega_r a^2 + \Omega_m a^3 + \Omega_\Lambda a^6\right). \tag{23a}$$

Here, Ω_r , Ω_m and Ω_Λ are the density parameters of radiation, matter and the energy given by the cosmological constant. The initial condition for the differential Equation (23a) is

$$a(0) = 1. \tag{23b}$$

Relation (20) with $t = t_0 = 0$ gives by elementary calculations

$$\frac{8\kappa c^4 \lambda}{(H_0)^2} - \left(\frac{\varphi_0}{H_0}\right)^2 = 12\Omega_m \kappa_0 \tag{24}$$

The assumption

$$0 < \kappa_0 \tag{25}$$

implies that the solution of (23) is non-singular for all $t \in \mathbb{R}$. It exists $t_1 < 0 = t_0$ with $\dot{a}(t_1) = 0$, that is

$$a(t) > a(t_1) = a_1 > 0 \text{ for all } t \neq t_1. \tag{26}$$

It follows from (23a)

$$\Omega_r a_1^2 + \Omega_m a_1^3 + \Omega_\Lambda a_1^6 = \Omega_m \kappa_0.$$

The time t_1 must be long time before the present time $t_0 = 0$ implying $0 < a_1 \ll 1$, *i.e.*

$$\kappa_0 \ll 1. \tag{27}$$

Therefore, $a(t)$ starts at a positive value at time equal to minus infinity, decreases to a_1 at $t = t_1$ and then increases for all t . The function $h(t)$ can then be calculated from relation (21). Let us introduce the proper time $\tilde{\tau}$ instead of the time t by

$$\tilde{\tau}(t) = \int_{-\infty}^t 1/\sqrt{h(t)} dt \tag{28}$$

The differential Equation (23a) can by the use of (21) be rewritten

$$\left(\frac{1}{a} \frac{da}{d\tilde{\tau}}\right)^2 = H_0^2 \left(-\frac{\Omega_m \kappa_0}{a^6} + \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda\right). \tag{29}$$

This differential equation is for not too small functions $a(t)$ nearly identical with that of GR for a flat homogeneous, isotropic universe by virtue of (25) and (27).

Then, the conditions (25) and (27) give

$$0 < a_1 \ll 1, \tag{30}$$

i.e. t_1 corresponds to the time of the big bang of GR with value a_1 very small but not zero. This result is received by GFST without any additional assumption

or change of the theory.

5. Conclusions

There are two methods of measuring the Hubble constant of the universe: the cosmic distance ladder and looking at the signals originated from the beginning of the universe. Two different results for the Hubble constant are received. Therefore, the universe doesn't expand because the methods use the expansion of the universe. It is worth to mention that GR implies expansion because the universe starts from a point singularity and the observed universe is very big. Furthermore, the universe must be inflationary expanding because the observed universe is flat. Summarizing, it follows that GR doesn't correctly describe gravitation if two Hubble constants are measured.

A theory of gravitation in pseudo-Euclidean geometry has been given in article [12]. Later on, it is studied more generally in flat space-time. The applications of this theory to homogeneous, isotropic, cosmological models are given in article [8] where non-singular solutions are received, *i.e.* big bang did not exist. It was proved that for weak gravitational fields the results of GFST and GR agree to measurable accuracy. The theory and the applications of GFST is studied in several articles and summarized in the book [12]. Differences of the results of GFST and GR arise for cosmological models in the beginning of the universe. The metric of GFST is the pseudo-Euclidean geometry, *i.e.* space is not expanding. It is worth to mention that by virtue of the covariance of GFST an expansion of the universe would also be possible by a suitable transformation. But this is not realistic. A non-expanding universe is important because expansion of the universe implies two different Hubble constants. For cosmological models of GFST the source is the total energy-momentum tensor inclusive that of the gravitational field (as it should be by Einstein: matter is equal to energy and reverse) whereas the source is only the matter tensor and no gravitational energy-momentum for cosmological models of GR which is no tensor for GR. The redshift of distant objects follows by the energy of time-dependent gravitational fields which is converted to matter where the total energy is conserved and it doesn't follow from velocities (expanding space).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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