

Physical Space Is Finite

Youqi Wang

Yashentech Corporation, Shaoxing, China

Email: youqi_wang@yashentech.com

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Abstract

Metrological analysis shows that any clock in inertial motion in infinite space shall not have time dilation, due to relativity of such motion in such space. On the other hand, atomic clock in inertial motion in finite space shall exhibit time dilation, due to alteration of momentum of clock-defining particle caused by nonzero curvature of trajectory of such motion in such space. Therefore, time dilation experiment of atomic clock in inertial motion in physical space provides a direct and decisive way of determining geometry of physical space in real-time. Phenomenon of time dilation of atomic clock in inertial motion in physical space has long been observed and confirmed experimentally. Therefore, extent of physical space has to be finite, consistent with result of high precision experiment of free particle in high-speed motion conducted a decade ago.

Keywords

Geometry of Physical Space, Time Dilation, Atomic Clock, Special Relativity Theory

1. Introduction

In eras of Galileo and Newton, physical space (PHS) was perceived as having infinite extent [1]. Therefore, translation of space-time event among inertial reference frames (IRFs) followed Galilean Transformation [2]. To ease the tension between Newtonian mechanics (NM) and Maxwell electrodynamics [3], the concept of local time was conceived and motion induced time dilation was conjectured [4] [5]. In explaining null result of Michelson-Morley experiment [6], motion induced space dilation was also conjectured [7] [8]. Combination of the two conjectures formed Lorentz Transformation [9] for translating space-time event among IRFs. Meanwhile, suspicion was arisen on infiniteness of PHS and inquisition on possibility of finite space surfaced [10]. In 1905, Einstein invented

special relativity theory (SRT) that derived Lorentz transformation on basis of relativity principle of NM and constancy of speed of light in vacuo (SLV) independent of state of motion of light source [11]. Since then, motion induced time and space dilations became authentic physical phenomena known commonly as relativistic effects, although motion induced space dilation has never been observed. Wide adoption of SRT and fusion of *space* and *time* [12] also extinguished further quest for geometry of PHS. However, SRT did not alter the assumption underlying NM, *i.e.*, PHS being of infinite extent. On the other hand, general relativity theory [13] implied finite PHS that has manifested to expandable/contractible PHS [14]. Nevertheless, implication and ramification of finite PHS was and is not well understood or even aware of, let alone observation consequences thereof, due presumably to the annihilation of the notion of *space* a century ago. This study is to analyze the physical phenomenon of time dilation, with focus on that of atomic clock, to show that PHS is indeed finite therefore unfitted with the space of Newton nor that of SRT. Further evidences for finite PHS from celestial observation/measurement and some immediate impact of spacial finiteness of PHS to cosmology and astrophysics shall be presented separately.

2. Time Dilation

Observationally, time dilation refers to the phenomenon that passage of time between two temporal events as measured by one clock becomes shorter than that by another otherwise identical clock due to the difference in state of motion of the clocks and/or field environment the clocks are immersed therein. For instance, if two identical atomic clocks are displaying exactly the same readings of time while stationary in a lab on Earth and one of the clocks then takes a tour around Earth, then, when the two clocks meet with each other again in the lab, times as displayed by the two clocks shall no longer be identical and the clock exhibiting lesser time is said as having had time dilation. Similarly, if the lab is in basement and one of the clocks is then lifted to a top floor and sent back to the basement, then, when the clocks meet with each other again in the lab, times as displayed by the two clocks shall no longer be identical, and the one exhibiting lesser time is said as having had time dilation. Nevertheless, passage of the time between the events, *i.e.*, duration from the moment the clocks meeting with each other to the moment the clocks meeting with each other again, did not and does not vary, since it is and is always referring to the one and same thing, “duration from the clocks meeting with each other to the clocks meeting with each other again”, regardless of how and by what the passage of the time is measured. Therefore, if time dilation happens then it must be the unit of the time that is dilated, since passage of time = numeral \times unit of time and lesser time means lesser numeral. In other words, time dilation is synonymous with temporal dilation of unit of time. Therefore, in truth, time dilation refers to the physical phenomenon that duration of unit of time becomes longer in comparison to that in reference state chosen for time comparison.

3. Atomic Clock

In this analysis, atomic clock refers to any time-keeping device or temporal events generator that is based on quantum mechanical interaction of particle and photon. Herein, particle can be any atom, ion, nucleon, other microscopic object, macroscopic entity, etc.; photon can be of any finite positive frequency. Time defined on atomic clock is referred to hereinafter as atomic time (AT).

The quantum mechanical process defining atomic clock [15] can be expressed as

$$\text{Particle}_1 \longleftrightarrow \text{Particle}_0 + \text{Photon} . \quad (1)$$

Herein, subscript “1” denotes a particle in its relatively higher energy state (with respect to its relatively lower energy state), “0” denotes the particle in its relatively lower energy state (with respect to its relatively higher energy state), and “Photon” refers to the photon involved in transition process of the two energy states of the one and same particle. Thus, a particle in 1-state may transit to 0-state by emitting one Photon or a particle in 0-state may transit to 1-state by absorbing one Photon. The particle defining atomic clock is referred to as clock defining particle (CDP).

By the law of energy conservation (LEC), balance of energy of the transition process of Expression (1) is

$$E_p = E_1 - E_0 \equiv \Delta E . \quad (2)$$

Herein, E_1 represents total energy of a particle in 1-state, E_0 that of the particle in 0-state, ΔE difference between the total energies of the two states of the one and same particle, and E_p energy of Photon involved in the transition process.

Total energy of a particle in Equation (2) refers to aggregation of any and all forms of energy internal to the particle. In other words, the symbol E therein represents selfenergy of a particle, *i.e.*, total energy of a particle as perceived by the particle itself. Expressed explicitly,

$$\Delta E_s \equiv E_{1,s} - E_{0,s} = E_p . \quad (3)$$

Subscript s indicates self-perspective of attribute in association. In self-perspective, a particle is and is always at rest with respect to itself. Therefore, selfenergy of a particle in Equation (3) is identical in any and all aspect to any and all details to restenergy of the particle at location of the particle in any reference frame wherein the particle is at rest.

According to the law of Planck on photon energy (LPE),

$$E_p = h\nu \rightarrow \nu_{\text{AT}} = \frac{E_p}{h} = \frac{\Delta E_s}{h} . \quad (4)$$

Herein, h is Planck constant, ν_{AT} frequency of Photon involved in quantum transition process defining atomic clock.

The internationally adopted SI unit of time, *second*, is defined as “duration of 9 192 631 770 periods of the radiation corresponding to the transition between

the two hyperfine levels of the ground state of the cesium 133 atom” [16]. Therefore, SI time is defined on atomic clock hence is AT and ¹³³Cs is the CDP. By the definition,

$$\nu_{AT} [^{133}\text{Cs}_{0,\text{hfs}}] \equiv 9\,192\,631\,770 \text{ Hz, Hz} \equiv \text{s}^{-1}. \tag{5}$$

In symbolic form, such definition of unit of time is expressed as

$$\nu_{AT} \equiv \mathcal{N}_t \mathbb{U}_{AT}^{-1}, \mathcal{N}_t \equiv 9\,192\,631\,770. \tag{6}$$

ν_{AT} : Frequency of Photon involved in quantum transition process defining AT. \mathcal{N}_t : Immutable numeral assigned by definition of unit of AT. \mathbb{U}_{AT} : Unit of AT.

With Equation (4),

$$\frac{\mathcal{N}_t}{\mathbb{U}_{AT}} = \frac{\Delta E_s}{h} \rightarrow \mathbb{U}_{AT} = \frac{\mathcal{N}_t h}{\Delta E_s}. \tag{7}$$

International Bureau of Weights and Measures further specified that the SI definition of unit of *second* “refers to a cesium atom at rest at a temperature of 0 K” [17]. Therefore, by the specification, an ideal atomic clock is and is always at rest with respect to its CDP.

Assume Planck constant is state invariant (SIT), *i.e.*, an invariant regardless of state of setup for measurement of it. Then, from Equation (7), dilation of unit of AT shall occur if selfenergy difference of the CDP is altered in or by any process, regardless of nature of the process nor cause of the alteration. Conversely, dilation of unit of AT shall not occur if selfenergy difference of the CDP is not or cannot be altered in or by a process, whether such process is associated with motion of CDP, presence of field, or due to any other causes.

As adopted internationally, SLV in SI system is defined as $c \equiv 299\,792\,458 \text{ m s}^{-1}$ [17]. Therefore,

$$c \equiv \mathcal{N}_c \frac{\mathbb{U}_L}{\mathbb{U}_{AT}} \rightarrow c_x = \mathcal{N}_c \frac{\mathbb{U}_{L,x}}{\mathbb{U}_{AT,x}}. \tag{8}$$

\mathcal{N}_c : Immutable numeral assigned by definition of SLV. \mathbb{U}_L : Unit of length, meter in SI system. x Subscript indicating state of attribute in association.

Therefore, if temporal dilation occurs to unit of AT then SLV defined thereupon shall be altered accordingly, unless spacial dilation also occurs, concurrently and congruently, to unit of length. In other words, for c to be maintained as constant, dilation of space and time, if occurs, has to occur local simultaneously and to exactly the same extent as well.

4. Contradiction of Special Relativity Theory

Consider two identical clocks of any kind in inertial motion with respect to each other, referred to herein as a and b , respectively. According to relativity principle of SRT, inertial motion is relative motion. Therefore, to clock a , whether or not there exists b , direction/velocity of motion of b with respect to a , etc., shall have no shred of physical effect at all; and vice versa. Per SRT, clock in motion shall

cause time dilation of the clock. Therefore, an observer comoving with clock a shall observe b ticking slower than a whether b is approaching towards or departing from a . By relativity of the motion, observer comoving with clock b shall observe a ticking slower than b whether a is approaching towards or departing from b . Suppose a and b are approaching towards each other and, at the moment the two clocks meeting with each other, set the times as displayed by the two clocks to zero.

If the two clocks are in the motion in finite space in opposite directions along same geodesic then, without interference/intervention of any kind/party, a and b shall meet with each other again (*the event*). Then, according to SRT, a should observe the time as displayed by b at the moment of *the event* being less than that as displayed by a at the one and same moment. Likewise, b should observe the time as displayed by a at the moment of *the event* being less than that as displayed by b at the one and same moment. However, such outcome of observation is unphysical and metrologically impossible. Therefore, SRT shall fail the metrological test for inertial motion of clock in finite space.

Therefore, if inertial motion in finite space is indeed relative then the times as displayed by the clocks at the moment of *the event* must be one and same regardless of state/velocity of motion of the clocks. That is, if inertial motion in finite space is indeed relative then time should not and could not dilate for any clock in such motion in such space. However, this contradicts to that by SRT. It is then evident that SRT is not applicable to inertial motion in finite space. In other words, SRT might serve well with infinite space but definitely not to finite space.

Alternatively, it has to be construed that inertial motion in finite space is not relative, *i.e.*, absolute. As a consequence, time dilation may or may not occur to clock in such motion in such space. Nevertheless, SRT is still inapplicable in such case, because inertial motion in finite space is not relative. Therefore, even if it does occur, the occurrence of time dilation of clock in inertial motion in finite space is not, hence cannot be attributed to as, relativistic effect, since such motion in such space is not relative and SRT is not applicable therein. Either way, the role of geometry of space is pivotal in so far as to motion induced time dilation, if any.

About a half century ago, Hafele and Keating conducted time comparison experiment with on-flight atomic clocks that demonstrated the existence of time dilation of atomic clock in curved inertial motion [18]. Therefore, motion induced AT dilation is a genuine physical phenomenon, regardless of theory/interpretation. Since the phenomenon is not, hence cannot be attributed to as, relativistic effect, other cause must exist for the observed phenomenon of time dilation. In other words, time dilation of clock in motion is not exclusive of SRT. Further, it can be inferred from the Hafele-Keating experiment that atomic clock in inertial motion in finite space should exhibit time dilation, since there is no difference in essence between curved inertial motion and inertial motion in finite space.

Still further, the only distinction between curved and not-curved motion is the curvature of trajectory of motion in space, *i.e.*, the curvature being nonzero or zero. Therefore, in light of Equation (7), causal relationship must exist between curvature of trajectory of motion of atomic clock and alteration of selfenergy difference of CDP of same. That is, nonzero curvature of trajectory of motion (hence curved motion) of atomic clock shall cause alteration of selfenergy difference of the CDP associated with the clock hence time dilation of the clock in such motion. On the other hand, selfenergy, hence selfenergy difference, of any particle in inertial motion in infinite space is invariant to location, time, velocity of motion of the particle with respect to others, and curvature of trajectory of inertial motion in infinite space is zero. Therefore, by same reasoning, atomic clock in inertial motion in infinite space should not cause time dilation of such clock in such motion in such space, despite SRT.

It is therefore not unreasonable to doubt if time dilation would really occur to any clock in inertial motion in infinite space, in spite of SRT. If two identical clocks of any kind in inertial motion in infinite space in opposite directions along same straight line could meet with each other again then the truth would be found by observation and contradiction to SRT inevitable. However, clocks in inertial motion in infinite space can never meet with one another again. Therefore, the doubt cannot be cleared out by the metrological test. Thus, as the last resort, it is to reexamine some of the most basic concepts involved hereinabove, such as space, time, inertial motion, etc., in order to address the issue.

5. Infinite Space

Space can be presented as a geometric construct providing accessibility to matter or representation thereof for capturing, describing, and processing geometric information of physical world. In essence, space is but a set of geometric points having no other relationship except connectivity among the points and no other property except specifications provided.

Consider a continuous, homogeneous, isotropic, and infinite space, referred to as infinite space. Of the four specifications provided herein, the first three, *i.e.*, continuity, homogeneity, and isotropy, denoted collectively as CHI, are to ensure points of space are identical/equivalent in any and all aspect/detail hence no privileged one or group thereof is allowable hence artificial bias minimized. Based on the rationale that nowhere in physical world is more special than anywhere else in same, CHI enables establishing a plain, trivial, but coherent metrological background, against witch, nontrivial events described and physical reasoning conducted. In contrast, infinity specification on extent of space, referred to as infinite space hypothesis, is but an **assumption** aimed at mimicking spacial aspect of physical world, which also offers simplicity and convenience in conceptual operation.

Infinite space adopting Euclidean rule is known as Euclid space, denoted as E^n . In particular, distance (length of shortest path) between any pair of points of

\mathbf{E}^n is defined by Euclid norm. In labeling geometric points of \mathbf{E}^n (coordinate, coordination, etc.), any symbolic system is admissible if and only if such system conforms to the norm defined. If real numeral system is used for labeling points of \mathbf{E}^n , space thus coordinated is denoted as \mathbf{R}^n . If imaginary numeral system is used for labeling points of \mathbf{E}^n , space thus coordinated is denoted as \mathbf{I}^n . Euclid norm is invariant to such labeling systems of choosing. Therefore, without loss of generality of any kind, real numeral system shall be used for coordination, for convenience as well as in line with convention.

Any space is associated with one and only one base unit (unit not derived from other units): unit of length. In \mathbf{R}^n , length of unit of length is “1” unit by definition and understood as one length unit. Accordingly, any length between any pair of points of space is computable and computed by decoding labels associated with the points as well as that of all points contained in path of the length in accordance with norm defined for space. Thus, different paths between a pair of points may result in different lengths but all lengths are in terms of one and same unit of length of space, explicitly or implicitly. It is with respect to length unit of space that difference or indifference of lengths becomes distinguishable, concept of longer/shorter meaningful, distance between points determinable. Further, if distance between a pair of points of space is computed as one length unit then distance between points of the pair is one length unit regardless of where the pair of the points is located in space.

In relating abstract space to physical world, certain length aspect of certain physical object or phenomenon was/is assigned artificially as corresponds to one unit of length of space. Such assignment is arbitrary and may be made by pontific order, international agreement, or any other means as long as chosen length of chosen entity is not none nor infinite. Further, if assignment is made then it is to remain as such until/unless reassignment.

Any object can be projected onto space, *i.e.*, viewed/regarded/defined as occupying points of space. Such projection is said as *exist* in space. Geometric aspect of object is thus projected onto points of space occupied by/coincided with object. In particular, any length between any pair of physical points of an object is/becomes measurable via computation of corresponding length of corresponding points of space per norm defined for space. It is in such manner that geometric property of space is projected back onto physical world. It is then subject of experimentation/observation in determining if such property is in line with that of physical world. In other words, geometry of physical space cannot be assigned/assumed but only measured/determined via experiment/observation in physical space.

6. Coordination of Euclid Space

Arbitrarily assign one point of \mathbf{E}^n as origin. Arbitrarily assign one straight line in \mathbf{E}^n (Euclidean straight line, ESL) containing the origin as an axis. Arbitrarily assign another ESL containing the origin and orthogonal to the axis as another

axis, and so on and so forth, until n th axis is constructed. The set of the axes and the origin form a coordination frame (Cartesian coordination frame, CCF) capable of labeling hence identifying any and all points of \mathbf{E}^n with real numerals in unique, consistent, systematic, and coordinated manner conforming to Euclid norm.

The arbitrariness in constructing CCF is not only permitted but also demanded by CHI. Accordingly, object, phenomenon, law of physics (LOP) and associated unit, constant, parameter, etc., must be identical/one and same/invariant to specific labeling of specific CCF. In other words, size/shape of object, distance among phenomena, correlation between this and that, etc., must not depend on how a specific CCF is constructed. That is, labels of points are mere labels of points that are nonphysical and must not have any physical effect at all.

Any CCF can be mapped onto any other CCF by finite set of uniform mapping operations. In general, uniform mapping refers to establishment of one-to-one correspondence between entities of sets without altering gauge of the sets. Herein, it means translation (in linguistic sense) between labels assigned by CCFs for one and same point of space. Therefore, distance between any pair of points of \mathbf{E}^n is invariant under such translation. Therefore, such translation is nonphysical and shall have no physical effect at all.

Uniform mapping operations include shift, rotation, reflection, and combination thereof. Thus, by relabeling all points of one CCF according to finite set of rules, *i.e.*, transformation, without altering distance between any pair of the points, the CCF can become identical to any other CCF in any and all aspect/detail. Therefore, upon uniform mapping, referred to as uniform coordinate transformation (UCT), any two CCFs can become one and same. Then, arbitrariness in setup of CCF dictates that UCT must be nonphysical and have no physical effect at all. In particular, LOP and associated unit, constant, parameter must be identical/one and same/invariant under UCT.

Shift operation is also known as translation (in kinematic sense). If two CCFs can become identical by translation operation alone then the CCFs are said as being parallel to each other. Identicalness of CCFs upon translation among parallel ones is known as translation invariance of CCF or said as CCF possesses translation symmetry. Translation operation is UCT. Therefore, translation operation must be nonphysical and have no physical effect at all. In particular, LOP and associated unit, constant, parameter must be identical/one and same/invariant under translation of CCF.

Translation symmetry of CCF exists and only exists in infinite space. That is, if extent of a space is infinite then CCF in the space shall possess translation symmetry. Conversely, if CCF in a space possesses translation symmetry then extent of the space must be infinite. Therefore, it is the infinity hypothesis of a space that causes translation symmetry of CCF in the space.

In summary, the space of Newton is Euclid space, which is continuous, ho-

mogeneous, isotropic, and infinite, *i.e.*, infinite space. Infinite space possesses translation symmetry, which exists but only exists in infinite space.

7. Time in Euclid Space

By definition, all points of a CCF are and are always at rest with respect to each other and to the frame as well, *i.e.*, distance of any point of CCF to any other point of same as well as to same is and is always one and same by norm of space. From metrological point of view, if a point is labeled hence identified by a CCF then it is so labeled/identified and shall remain as such in preserving logical/metrological integrity. Since relationship among points and distance information between points are all encoded in label of point conforming to norm defined for space and decoded by same as well, distance of any point to any other point of same CCF and to the frame is and is always one and same. Therefore, by definition, a point of a frame is and is always at rest with respect to any other point of same and to same as well. Since UCT does not alter distance between any pair of points of CCF, UCT does not alter the restness of point of CCF in same.

Consider a CCF in \mathbf{E}^n . Attach identical atomic clocks to each and every point of the CCF. Such clock then defines such time at the point it is attached to, referred to as local time of the point. Thus, clocks associated with points of the CCF are and are always at rest with respect to each other and to the CCF as well. Then, by Equation (7), duration of unit of AT at any point of the CCF is identical to that at any other point of same by CHI. In general, clock of any kind can be used herein and identicalness of duration of unit of corresponding time at any and all points of the CCF is guaranteed by CHI. It is this identicalness, *i.e.*, duration of unit of local time being identical/one and same/invariant regardless of where and when a clock is located in the CCF, that makes the clocks of the CCF synchronizable. Poincaré-Einstein synchronization protocol [19] can then be used to synchronize clocks of the CCF as the follows:

- (1) from each and every point of the CCF, send a signal to origin of the frame and let it be sent back instantly upon its arrival at the origin (local simultaneous events at origin) and return to the point it was sent from;
- (2) denote passage of the time during the two events (sending and receiving same signal) as recorded by local clock at a point as $n_{t,x}\mathbb{U}_{t,x}$, $n_{t,x}$ being number of clock events during the events produced by the clock, $\mathbb{U}_{t,x}$ duration of unit of time defined by the clock, and x label (location) of the point in the CCF;
- (3) send a signal of same type from origin of the CCF in isotropic manner;
- (4) for each and every point of the CCF, at corresponding local moment of receiving the signal from the origin, set the clock associated with the point to read as one half of $n_{t,x}\mathbb{U}_{t,x}$.

In such manner and upon such operation, clocks of the CCF are said as synchronized. A CCF with synchronized clocks is referred to as synchronized CCF

(SCCF).

Any space coordinated by any labeling system and attached with any clock is referred to as a stage. Any event happens in any stage can be associated with a pointer, (x, t_x) , indicating location of happenstance of the event by location identifier x (defined by coordination system of stage) and time of happenstance of the event by time indicator t_x (defined and displayed by clock associated with location x). Such pointer points to exactly where and when of the event in stage regardless of varieties of clocks, definitions of times, durations of time units, synchronizability of clocks, correlation or lack thereof among the clocks, etc. SCCF is a stage by definition of stage. Therefore, event in SCCF can be associated with event pointer (x, t_x) .

By definition, clocks of SCCF are identical and with identical duration of unit of time for all the clocks in all the times and the clocks are synchronized among themselves. Such condition sets the metrological ground, upon which, nonlocal simultaneity of events becomes definable. Accordingly, plurality of events is said as nonlocal simultaneous if and only if times of the events (as indicated by respective clocks associated with respective locations of the events) are identical among the events regardless of locations of the events (definition of nonlocal simultaneity under SCCF). Therefore, if time of happenstance of event A (as displayed by clock at location of event A in SCCF) and that of B are identical then, by definition, A and B are nonlocal simultaneous events, regardless of locations of the events in SCCF. Nonlocal simultaneity of events thus defined is referred to as simultaneity of events (in SCCF).

Simultaneous events in a SCCF may not be simultaneous as viewed from/by other reference frames. However, any reference frame has to acknowledge the fact that the time as displayed by the clock of the SCCF at location of event A at happenstance of event A is identical to the time as displayed by the clock of the SCCF at location of event B at happenstance of event B . Therefore, by definition, A and B are simultaneous events of the SCCF, regardless of opinions of others. Further, if a signal is broadcasted in isotropic manner in a SCCF then the times are identical as displayed by the clocks of the SCCF at any and all locations of the SCCF having identical distance to the broadcaster at respective local moments the signal is received. Therefore, the events (receiving signal at location of a SCCF having identical distance to signal source in same SCCF) are, by definition, simultaneous events of the SCCF regardless of opinions of others. Likewise, if signals of identical type are sent from location A and B in a SCCF at same local time (as displayed by respective local clocks at A and B) towards a point C in same SCCF having equal distance to A and B then the signals shall arrive at C at same local moment of C hence is a local simultaneous event of C , which has to be accepted as a fact of local simultaneity by anyone regardless of existence/opinion of others.

Time of simultaneous clock events (ticking, counting, reading, displaying, etc., of clock) occurring at all points of a SCCF is known as *time* of the SCCF, which

is locally defined common time of the frame. In other words, time of SCCF refers to, is defined as, and synonymous with such an event (moment) that each and every otherwise identical clock associated with each and every point of the SCCF is displaying one and same time. It is on this ground that event pointer of SCCF, (\mathbf{x}, t_x) , can be abbreviated as (\mathbf{x}, t) , understood that the t therein refers to the time as displayed by the clock associated with \mathbf{x} which is identical to common time of the frame in any and all times of same.

It is due to the identicalness of durations of units of times of local clocks of CCF that time of SCCF, *i.e.*, display of any clock of SCCF, $t \equiv t_x$, becomes differentiable (in mathematical sense) with respect to \mathbf{x} , as such is guaranteed by continuity of local time (cf. **Appendix B**) defined on local clock in conjunction with clock synchronization and continuity of space by CHI. Therefore, among other things, it is the identicalness of duration of unit of time in CCF that is the foundation for the existence of locally defined common time of SCCF, which also provides physical and metrological meaning of the time that is metrologically consistent and legitimate.

The concept of nonlocal simultaneity (of events) is denied by/in SRT. However, *time* in SRT refers to time of IRF, which is but SCCF. Therefore, *time* referred to in SRT is but locally defined common time of SCCF, which is and has to be built on the ground of nonlocal simultaneous clock events.

8. Motion of Particle in Euclid Space

Consider a point-like object, referred to hereinbefore and hereinafter as particle, exists in SCCF of \mathbf{E}^n . By point-like, it is meant that the object has no spacial extent, *i.e.*, extent of particle is zero regardless of dimension of space it is residing in. Since particle is point-like object and geometric point of space is point-like entity, particle exists in space can be viewed as point-like object residing at or occupying/overlapping/coinciding with one and only one point of space. Therefore, existence of a particle in SCCF can be described by event pointer (\mathbf{x}, t) , which indicates where and when of existence of the particle in the stage.

Since time of SCCF is differentiable with respect to location of same, reciprocal of such differential shall also exist, *i.e.*, location of SCCF is also differentiable with respect to time of same. Accordingly, velocity of particle in SCCF is definable in differential form. Denote velocity of particle as $\mathbf{v} \equiv d(\mathbf{x}, t)/dt \equiv d\mathbf{x}/dt$. Then, according to rules of Leibniz-Newton calculus [1],

$$\mathbf{v} \equiv \lim_{t_a \rightarrow t} \frac{(\mathbf{x}_a, t_a) - (\mathbf{x}, t)}{t_a - t} = \lim_{t_a \rightarrow t} \frac{\mathbf{n}_{x_a} - \mathbf{n}_x}{n_{t_a} - n_t} \frac{\mathbb{U}_L}{\mathbb{U}_t} \equiv \frac{d\mathbf{n}_x}{dn_t} \mathbb{U}_v. \quad (9)$$

\mathbf{v} : Velocity of particle in SCCF. \mathbf{x} : Location of particle in SCCF. t : Time as defined/indicated by clock associated with \mathbf{x} at the moment particle existing at \mathbf{x} . d/dt : Differential operator with respect to t . (\mathbf{x}, t) : Event pointer pointing to where and when of existence of particle. \mathbf{x}_a : Arbitrary location of particle in vicinity of \mathbf{x} arbitrarily close to \mathbf{x} distance wise. t_a : Time as defined/indicated by clock associated with \mathbf{x}_a at the moment particle existing at \mathbf{x}_a . \mathbf{n}_x : Numeral

identifier of x . n_{x_a} : Numeral identifier of x_a . n_t : Numeral aspect of t . n_{t_a} : Numeral aspect of t_a . U_L : Unit of length of SCCF, identical at all locations of SCCF in all times of same. U_t : Unit of time of SCCF, identical at all locations of SCCF in all times of same. U_v : Unit of velocity of SCCF.

Therefore,

$$\text{If } x_a = x \text{ then } v = 0 \text{ else } v \neq 0. \quad (10)$$

That is, if a particle exists and only exists at x during infinitesimal but nonzero duration of time of SCCF then velocity of the particle is zero and particle in such state of motion is known as *at rest* at x in SCCF at moment t of same. Otherwise, velocity of the particle is nonzero and particle in such state of motion is known as *in motion* at x in SCCF at moment t of same. Further,

$$\text{If } v = \infty \text{ then } t_a = t \text{ \& } x_a \neq x \rightarrow v \neq \infty \rightarrow |v| < \infty. \quad (11)$$

That is, if velocity of a particle were infinite then the particle would exist at not-same locations nonlocal simultaneously, *i.e.*, occupying more than one point of space at same moment of SCCF time, and that would contradict with the specification of particle being point-like object. Therefore, velocity of motion of particle in SCCF cannot be infinite.

NM defines an attribute of particle known as momentum, which is defined as in proportion to velocity of particle,

$$\mathbf{P} \equiv m\mathbf{v}. \quad (12)$$

\mathbf{P} : Momentum of particle in SCCF. \mathbf{v} : Velocity of particle in SCCF. m : Proportion parameter in definition of momentum.

Parameter m in definition of momentum of particle is also known as *mass* of particle [20]. In general, m can be of form of a tensor establishing relationship between momentum vector and velocity vector. By CHI of space, m is reduced to a finite nonnegative scalar parameter,

$$0 \leq m < \infty. \quad (13)$$

m : Mass of particle in SCCF.

In NM, mass of particle is **assumed a priori** as invariant to state of motion of particle. In other words, whether a particle is at rest or in motion, mass associated with the particle is one and same in NM.

The second law of NM (LNM2) states that

$$\frac{d\mathbf{P}}{dt} = \mathbf{F}. \quad (14)$$

\mathbf{F} : Force exerted onto object.

That is, alteration of momentum of an object with respect to time is caused by and equal to *force* exerted onto the object, per LNM2. By arbitrariness of construction, SCCF has no physical effect at all to object existing therein. Under NM framework, physical effect means force. Therefore, any SCCF shall not have nor cause to have any force exerted/exerting onto any object existing therein. Therefore,

$$F_{\text{SCCF}} = \mathbf{0} \rightarrow \frac{d\mathbf{P}}{dt} = \mathbf{0} \rightarrow \mathbf{P} = \mathbf{P}_0 \subset \mathbf{x} \ \& \ t \ \text{Invariant} . \quad (15)$$

That is, momentum of free particle in SCCF is invariant to location and time of SCCF. Since mass is SIT under NM,

$$\frac{dm}{dt} = 0 \rightarrow \mathbf{v} = \mathbf{v}_0 \subset \mathbf{x} \ \& \ t \ \text{Invariant} . \quad (16)$$

Therefore, if a free particle is at rest in SCCF at a moment t_0 of SCCF, *i.e.*, $\mathbf{v}_0 = \mathbf{0}$, then the particle shall remain in such state of motion indefinitely thereafter unless/until cause causing alteration of the state; if the particle is in motion in SCCF at t_0 of SCCF, *i.e.*, $\mathbf{v}_0 \neq \mathbf{0}$, then the particle shall remain in such state of motion indefinitely thereafter unless/until interrupted for cause. Therefore, SCCF is IRF by definition of IRF, and referred to as such hereinafter. Accordingly, such motion with respect to IRF, *i.e.*, having invariant velocity with respect to location and time of IRF without involvement of any third party, is known as inertial motion, more precisely, translational free motion (TFM).

From Equation (15), if an object is in TFM with respect to an IRF then the object is in TFM with respect to any IRF. Likewise, if an IRF is in TFM with respect to another IRF then the IRF is in TFM with respect to any IRF. On the other hand, if an IRF is in not-TFM with respect to other IRF then the IRF is not an IRF. Therefore, the only motion that exists and/or can exist among IRFs is TFM. It is on this ground that TFM in IRF is said as relative. That is, TFM shall have no physical effect at all to entity in TFM. Therefore, object in TFM is and must be physically identical, equivalent, and indifferent under NM framework. Therefore, if an object is at rest in an IRF then rest property of the object shall remain intact regardless of existence of other IRFs and their velocities with respect to the IRF. Further, if an entity is in TFM with respect to another entity then such is identical/equivalent/indifferent in any and all aspect/detail to that the other entity is in TFM with respect to this entity, regardless of velocity of TFM.

In short, SCCF is IRF under NM. Inertial motion with respect to IRF is relative motion. Therefore, to any object in inertial motion, whether or not exists other object, direction/velocity of motion of that other object with respect to this object, etc., shall have no shred of physical effect at all. Note also that relativity of inertial motion, IRF, SCCF, etc., are all arisen from, caused by, and/or due to infinity hypothesis on space, exclusively. Therefore, any theory of mechanics dealing with TFM/IRF assumes/has assumed infinity of space *a priori*, such as SRT. Likewise, any transformation of space-time event among IRFs assumes/has assumed infinity of space *a priori*, such as Lorentz Transformation.

9. Clock in Translational Free Motion in Euclid Space

Consider an atomic clock, denoted as A , at rest in an IRF in \mathbf{E}^n equipped with atomic clocks of identical type, referred to herein as rest frame (RF). Then, rest properties of A such as restenergy, hence selfenergy at rest hence selfenergy dif-

ference of CDP associated with A , and Planck constant as measured at rest in RF, hence duration of unit of AT defined by A , are invariant to existence of other IRF and velocity of motion of such with respect to RF hence clock A at rest in RF.

Let δT stands for the difference between time as displayed by one clock and that by another otherwise identical clock. Suppose, at a moment of RF, δT between A and the clock of RF at location of A is observed and denoted as α . Then, at any moment of RF, δT between A and the clock of RF at location of A at that moment shall be observed as equal to α , since action of observation does not affect outcome of the observation by definition of observation. Further, at any moment of RF, δT between A and any clock of RF anywhere in RF is α , whether or not observing, since clocks of RF are synchronized among themselves.

Suppose, instead, A is in TFM with respect to RF. Then, there is no obstacle of any kind in constructing an IRF, referred to herein as moving frame (MF), so that A is a member of the clocks of MF since, by definition, SCCF in TFM with respect to IRF is an IRF. Thus, A is and is always at rest in MF in any time of MF. Therefore, rest property of A at rest in MF is invariant to existence of RF and velocity of TFM of RF with respect to MF.

Suppose, at a moment of RF, denoted as T , A is coinciding with a clock of RF and δT between A and the clock of RF is observed as α . Then, at T , δT between A and any clock of RF anywhere in RF is α whether or not observing, since clocks of RF are synchronized among themselves. Further, at T , δT between any clock of MF anywhere in MF and the clock of RF is α whether or not observing, since clocks of MF are synchronized among themselves. Therefore, at T , δT between any clock of MF and any clock of RF is α whether or not observing. Therefore, at T , δT between any clock of MF and any clock of RF coinciding therewith at T shall be observed as α regardless of velocity of TFM between the IRFs. Therefore, at any moment of any time of any IRF anywhere in \mathbf{E}^n , δT between any clock of MF and any clock of any IRF coinciding therewith shall be observed as α regardless of velocity of TFM between the IRFs.

Therefore, if an atomic clock of RF, labeled as B , observes an otherwise identical clock A in TFM with respect to B and, at the moment A and B coinciding with each other, δT between the two clocks is observed as α , then, at any moment of RF, δT between A and any clock of RF coinciding with A at that moment shall be observed as α regardless of velocity of TFM of A with respect to B . Therefore, δT between A and B is and is always α whether or not the two clocks coinciding with each other and regardless of velocity of TFM between the clocks. Therefore, atomic clock in TFM shall not slow down nor speed up regardless of velocity of the TFM. Therefore, by this metrological test, time dilation shall not occur to atomic clock in TFM. It is thus shown by the above analysis that atomic clock in inertial motion in infinite space shall not have time dila-

tion, due to relativity of such motion in such space.

By relativity of TFM under NM, rest property of clock of any kind at rest in an IRF, hence physical behavior of the clock thereat, is invariant to existence of other IRFs and state of motion of such with respect to the IRF, since TFM has no real, physical effect at all under NM. Therefore, any clock in TFM shall not slow down nor speed up regardless of velocity of the TFM. Therefore, by this metrological test, time dilation shall not occur to any clock in TFM in infinite space, due to relativity of TFM under NM. It is thus shown by the above analysis that any clock in inertial motion in infinite space shall not have time dilation, due to relativity of such motion in such space.

Suppose an observer at rest in an IRF holds a clock B that is identical in type to clocks of the IRF and has aligned B with local clock of the IRF at location of the observer. Suppose an otherwise identical clock A is in TFM with respect to the IRF hence the observer and, at the moment clock A is meeting with clock B , set time of A to equal to the time as displayed by B at the moment (hence $\delta T = 0$ as observed by the observer at the moment). Then, according to the above analysis, time of A shall be identical to that of any clock of the IRF clock A shall meet anywhere/when during its journey thereafter, regardless of velocity of the TFM, even if the velocity exceeds SLV thereat. Further, if an otherwise identical clock C is in TFM along the same line as that of A but in opposite direction and, at the moment C is meeting with A , set time of C to equal to the time as displayed by A at the moment. Then, when clock C finally meets clock B in hand of the observer, the times as displayed by the two clocks shall be the same exactly, regardless of velocities of the TFMs of A and/or C . Therefore, there shall be no “twin brother” confusion in inertial motion in infinite space.

From the above analysis, if δT between any pair of IRFs is observed as α at any one moment of an IRF, then, at any other moment of same, δT between the pair shall be observed as α regardless of TFM between the pair. This metrological fact sets the foundation for clock synchronization among all IRFs of \mathbf{E}^n . Thus, arbitrarily chose an IRF, assigned as RF, and set/reset time of any clock of any IRF at any one moment of RF to read the same as that displayed by any clock of RF coinciding with the clock at the moment of the reset. Then, regardless of state of motion of IRFs, locally defined common time of all IRFs of \mathbf{E}^n is established, which is locally defined common time of \mathbf{E}^n , also known as Newton time or absolute time, denoted as t_N , which is the common time of nonlocal simultaneous clock events occurring at any and all points of any and all IRFs of \mathbf{E}^n . In other words, time of \mathbf{E}^n , t_N , refers to and is synonymous with such event (moment) that each and every otherwise identical clock associated with each and every point of each and every IRF of \mathbf{E}^n is displaying one and same reading, t_N , regardless of state of motion of IRFs.

Thus, by relativity of TFM in infinite space under NM, duration of unit of time defined on clock of any kind at rest in any IRF in any TFM with respect to any other IRF is invariant to location, time, and velocity of TFM. Therefore,

$$\mathbb{U}_{t_N, \text{ in TFM}} = \mathbb{U}_{t_N, \text{ at rest}} \equiv \mathbb{U}_{t_N} \subset \text{SIT} . \quad (17)$$

\mathbb{U}_{t_N} : Unit of t_N .

That is, duration of unit of time of IRF is identical/one and same/invariant with respect to state of motion of IRF it is in association with, regardless of velocity of motion of such frame with respect to any IRF chosen as reference frame for time comparison.

10. Length in Translational Free Motion in Euclid Space

Metrologically, length of an object in an IRF is defined as the distance between two spacial points of the IRF coinciding with end points of the length nonlocal simultaneously at same moment of the IRF.

Consider a rod of length L in TFM with respect to an IRF, regarded as RF. Then, at any moment t_N of \mathbf{E}^n , there is one and only one pair of end points of the rod existing in \mathbf{E}^n , which is coinciding with one and only one pair of points of \mathbf{E}^n hence one and only one pair of points of any IRF regardless of state of motion of the rod nor TFM velocity of the rod with respect to any IRF. Therefore, at any moment t_N of \mathbf{E}^n , outcome of measurement of L in any IRF is single-valued regardless of state of motion of the IRF and TFM velocity of the IRF with respect to the rod. Further, at any moment of t_N of \mathbf{E}^n , there is no obstacle of any kind in constructing a comoving IRF of the rod parallel to RF, denoted as MF. Hence, upon translation operation, the MF shall be identical to RF in any and all aspect/detail at that moment. In addition, the points of MF occupied by end points of the rod shall be overlapping with the points of RF coinciding with the end points of the rod at the same moment. Therefore, distance L as measured by RF and MF is and must be identical regardless of state of motion of MF with respect to RF and TFM velocity between the IRFs. Therefore, length in TFM shall not contract nor expand regardless of velocity of the TFM, despite SRT. Therefore, by the metrological test, spacial dilation shall not occur to any object in TFM in infinite space, due to relativity of TFM under NM,

$$L_{\text{ in TFM}} = L_{\text{ at rest}} \equiv L \subset \text{SIT} . \quad (18)$$

That is, length of an object is invariant to TFM of the object. Length of unit of length is length. Therefore,

$$\mathbb{U}_{L, \text{ in TFM}} = \mathbb{U}_{L, \text{ at rest}} \equiv \mathbb{U}_L \subset \text{SIT} . \quad (19)$$

\mathbb{U}_L : Unit of length of IRF.

That is, length of unit of length of IRF is identical/one and same/invariant with respect to state of motion of IRF it is in association with regardless of velocity of motion of such frame with respect to any IRF chosen as reference frame for length comparison. It is thus shown by the above analysis that any object in inertial motion in infinite space shall not have space dilation, due to relativity of such motion in such space.

Suppose a particle p is in TFM with respect to an IRF q and q is in TFM with respect to another IRF r . Suppose, at one moment t_1 of \mathbf{E}^n , p is found at $\mathbf{x}_{p,q,1}$

of q and origin of q is found at $\mathbf{x}_{q,r,1}$ of r ; and at another moment t_2 of \mathbf{E}^n , p is found at $\mathbf{x}_{p,q,2}$ of q and origin of q is found at $\mathbf{x}_{q,r,2}$ of r . Then, by definition,

$$\mathbf{v}_{p,q} \equiv \frac{\mathbf{x}_{p,q,1} - \mathbf{x}_{p,q,2}}{t_1 - t_2}, \mathbf{v}_{q,r} \equiv \frac{\mathbf{x}_{q,r,1} - \mathbf{x}_{q,r,2}}{t_1 - t_2}. \tag{20}$$

$\mathbf{v}_{\alpha,\beta}$: TFM velocity of entity α in IRF β . $\mathbf{x}_{\alpha,\beta,\gamma}$: Location of entity α in IRF β at moment t_γ of \mathbf{E}^n .

From perspective of r , via UCT at t_1 and t_2 , respectively,

$$\mathbf{x}_{p,r,1} = \mathbf{x}_{p,q,1} + \mathbf{x}_{q,r,1}, \mathbf{x}_{p,r,2} = \mathbf{x}_{p,q,2} + \mathbf{x}_{q,r,2}. \tag{21}$$

Therefore, by definition,

$$\mathbf{v}_{p,r} \equiv \frac{\mathbf{x}_{p,r,1} - \mathbf{x}_{p,r,2}}{t_1 - t_2} = \frac{\mathbf{x}_{p,q,1} - \mathbf{x}_{p,q,2} + \mathbf{x}_{q,r,1} - \mathbf{x}_{q,r,2}}{t_1 - t_2} \equiv \mathbf{v}_{p,q} + \mathbf{v}_{q,r}. \tag{22}$$

That is, superposition of TFM of entity in \mathbf{E}^n is and must be vector additive. Therefore, translation of space-time event among IRFs has to follow Galilean Transformation instead of Lorentz Transformation. Further, there can be no speed limit imposed upon TFM of any object with respect to IRF in infinite space, due to additivity of velocity of TFM therein. Since, for any finite speed limit imposed thereupon, there can and can always exist an IRF such that velocity of TFM of object with respect to that IRF shall exceed the speed limit imposed thereupon. Further, whether or not TFM velocity of an object with respect to IRF exceeds a speed limit, there shall be no shred of physical consequence since TFM is relative motion therefore shall have no physical effect at all under NM.

As a conceptual system, NM is built on infinite space hypothesis with central scheme of force causing momentum alteration of object and mass of object being SIT. The system is self-consistent, coherent in logic, and legitimate in metrology, referred to hereinafter explicitly as NM in infinite space (NMIS). Infinite space hypothesis results in translation symmetry that, in conjunction with NM scheme, forms the basis for the concept of IRF, which can but only exist in infinite space. Under NM scheme, arbitrariness in setup of reference frames in conjunction with translation symmetry of \mathbf{E}^n result in the requirement of physical equivalence, identicalness, indifference of all IRFs in \mathbf{E}^n , which is the basis of relativity of inertial motion in \mathbf{E}^n . Accordingly, TFM of particle must be relative, velocity of TFM vector additive, and physical effect/consequence of particle in TFM must be none. In compliance, no speed limit on motion of particle in \mathbf{E}^n is permissible under NMIS.

11. Law of Mass-Energy Equivalence

The law of mass-energy equivalence of Einstein (LME) [21] is of at least two folds,

$$\frac{\delta \mathbb{E}}{\delta m} = \frac{d\mathbb{E}}{dm} = a, 0 \leq a < \infty \rightarrow \mathbb{E} = am + \mathbb{E}_{00} \\ \mathbb{E}_{00} \equiv \mathbb{E}|_{m=0} \tag{23}$$

$$c^2 = a \rightarrow \frac{dE}{dm} = c^2, E \equiv \mathbb{E} - \mathbb{E}_{00} \rightarrow E = mc^2$$

\mathbb{E} : Energy of object. m : Mass of object. a : Proportion parameter, not function of \mathbb{E} nor m . \mathbb{E}_{00} : Zeromass energy of object. E : Energy of object offset by zeromass energy of same.

That is, alteration of energy of object in any process is proportional to that of mass of same in same and the proportion parameter is real, nonnegative, and finite scalar entity independent of mass and energy of object regardless of nature of the process causing the alteration; square root of the proportion parameter is SLV; object of zeromass and nonzero energy is permissible.

Consider measuring the proportion parameter of LME in a lab at rest in an IRF via, e.g., matter-antimatter annihilation process. For instance, annihilate one Coulomb of electrons with equal amount of positrons and let energy released from the process be absorbed completely by $\frac{1}{4}$ metric ton of water and monitor temperature of the water throughout the process, which may be observed as to rise by ~ 1 Kelvin, or maybe not. Regardless of the outcome of the measurement, the proportion parameter thus determined is and must be identical and invariant no matter where in the IRF the measurement is conducted, existence or not of other IRFs, state of motion of IRFs hence opinion of such on state of motion of the lab conducting the measurement, or otherwise it would contradict CHI and/or relativity of TFM. Therefore, by Equation (23), SLV as measured at rest in any IRF is and must be invariant to state of motion of the IRF as well as that of light source, and this is the constant SLV principle of SRT. In other words, the constant SLV principle of SRT is derivable from LME in conjunction with CHI of space.

Consider a particle in motion in IRF. By definition,

$$\mathbf{P} \equiv m\mathbf{v}, \mathbf{v} \equiv \frac{d\mathbf{x}}{dt}, 0 < m < \infty. \quad (24)$$

\mathbf{P} : Momentum of particle as measured in IRF. m : Mass of particle as measured in IRF. \mathbf{v} : Velocity of particle as measured in IRF. \mathbf{x} : Location of particle in IRF. t : Locally defined common time of IRF.

Suppose force is exerted onto the particle. Then, according to LNM2,

$$\frac{d\mathbf{P}}{dt} = \mathbf{F} \rightarrow d\mathbf{w} \equiv \mathbf{F} \cdot d\mathbf{x} = \frac{d\mathbf{P}}{dt} \cdot d\mathbf{x}. \quad (25)$$

\mathbf{F} : External force exerted onto particle as perceived in IRF. $d\mathbf{w}$: Infinitesimal work done by \mathbf{F} to particle as perceived in IRF. $d\mathbf{x}$: Infinitesimal displacement of particle in IRF under \mathbf{F} as measured in IRF.

By LEC,

$$dE = d\mathbf{w} \rightarrow \frac{dE}{dt} = \frac{d\mathbf{P}}{dt} \cdot \frac{d\mathbf{x}}{dt} = v^2 \frac{dm}{dt} + \frac{m}{2} \frac{dv^2}{dt}. \quad (26)$$

dE : Infinitesimal increment of total energy of particle as measured in IRF.

By LME, Equation (23), with invariance property of the proportion parameter,

$$\frac{dE}{dt} = a \frac{dm}{dt} \rightarrow \left((a - v^2) \frac{dm}{dv^2} - \frac{m}{2} \right) \frac{dv^2}{dt} = 0. \quad (27)$$

Therefore,

$$\text{If } \frac{dv^2}{dt} \neq 0 \text{ then } \frac{dm}{m} + \frac{d(a-v^2)}{2(a-v^2)} = 0 \rightarrow d \ln \left[m\sqrt{a-v^2} \right] = 0. \quad (28)$$

That is, if external force does cause alteration of velocity of the particle in motion in the IRF then

$$m\sqrt{a-v^2} \subset t \ \& \ \mathbf{x} \ \text{Invariant} \rightarrow m = m_0 / \sqrt{1-v^2/a}, \ m_0 \equiv m|_{v=0}. \quad (29)$$

m_0 : Restmass of particle in IRF.

With Equation (23),

$$c = \sqrt{a} \rightarrow m = m_0 / \sqrt{1-(v/c)^2}. \quad (30)$$

By LME,

$$E = am_0 / \sqrt{1-v^2/a} \rightarrow E = E_0 / \sqrt{1-(v/c)^2}, \ E_0 \equiv am_0. \quad (31)$$

E_0 : Restenergy of particle in IRF.

Denote

$$\beta_u \equiv \sqrt{1-u^2}, \ \mathbf{u} \equiv v/c \rightarrow m = m_0 / \beta_u, \ E = E_0 / \beta_u, \ 0 < \beta_u \leq 1. \quad (32)$$

u : Reduced velocity of particle. β_u : Referred to as Lorentz Factor.

That is, by LME, mass of a particle is not SIT but function of velocity of the particle, which is distinctly different from that in NMIS. Further, mass of any particle is and must be real, nonnegative, and finite by definition of *mass*. Therefore, from Expression (29), finite and constant speed limit must exist for motion of particle in IRF, because if $v^2 > a$ were allowed then mass of a particle could become imaginary. In other words, LME imposes finite speed limit \sqrt{a} on motion of particle in space, despite NMIS.

If external force does not or cannot cause alteration of velocity of particle then, from Equation (26),

$$\frac{dv^2}{dt} = 0 \rightarrow (a-v^2) \frac{dm}{dt} = 0. \quad (33)$$

Then,

$$\text{If } v^2 \neq a \text{ then } \frac{dm}{dt} = 0 \rightarrow \frac{d\mathbf{P}}{dt} = \frac{dm}{dt} \mathbf{v} + m \frac{d\mathbf{v}}{dt} = \mathbf{0} \neq \mathbf{F}. \quad (34)$$

That is, such would cause LNM2 violation. Therefore, if external force does not or cannot cause alteration of velocity of a particle then velocity of the particle must be $v = \sqrt{a}$. However,

$$\text{If } v^2 = a \text{ then } \frac{dE}{dt} = a \frac{dm}{dt} = \mathbf{F} \cdot \mathbf{v} \rightarrow \frac{dm}{dt} = \frac{\mathbf{v}}{a} \cdot \mathbf{F}. \quad (35)$$

Therefore, by exerting force in opposite direction of motion of the particle, mass of the particle could become negative hence contradicting with definition of *mass*. Therefore, if a particle is in motion at the speed limit then no force of any kind shall be experienced by the particle, even though reference frame may not

concur. In other words, if particle in motion with respect to IRF were to maintain invariant velocity under external force then that velocity must be and always be identical to the speed limit, \sqrt{a} . Further, if a particle is in motion at the speed limit with respect to an IRF then no force of any kind shall be experienced by the particle, regardless of opinion of the IRF.

Therefore, confliction between NMIS and LME is the sharpest on velocity of motion, which is not limitable in NMIS but mandated by LME to be constrained by finite speed limit. In SRT, the constraint took precedence over relativity of TFM, that resulted in the need for spacial and temporal dilations of TFM. As analyzed in **Section 9** and **10**, however, such dilations do not and cannot occur to entity in TFM due to relativity of TFM. Therefore, the confliction is irreconcilable via the dilations.

On the other hand, TFM, relativity of TFM, IRF, etc., are all due to/based on/originated from, among other things, translation symmetry of space that is uniquely possessed by and only by infinite space while LME, hence finite speed limit, is supported by experiment conducted in physical world. Therefore, in essence, the confliction is but manifestation of the confrontation between infinity hypothesis of space and LME of physical world. That is, if PHS is infinite then $E \neq mc^2$. Conversely, if $E = mc^2$ then PHS cannot be infinite.

LME is LOP confirmed/verified by plurality of fantastic experiments conducted in physical world to beyond reasonable doubt. Infinity hypothesis of space is but an assumption on PHS aimed at emulating physical world. Therefore, in binding with the law while resolving the confliction, infinite space hypothesis has to be abandoned. Therefore, by LME, PHS is not and cannot be infinite, *i.e.*, extent of PHS must be finite.

12. Finite Space

Finite space refers to any CHI space of finite extent. If finite space were to have boundary of some sort, continuity, homogeneity, or isotropy of the space would be compromised. Therefore, by CHI, finite space has no boundary.

Finite space can be presented as surface of n -ball in \mathbf{E}^n , which is but a set of all points of \mathbf{E}^n having identical Euclidean distance, referred to as external radius of the space, to an arbitrarily chosen point of \mathbf{E}^n , referred to as external origin of the space. Such set of geometric points is known as m -sphere, denoted as \mathbf{S}^m if real numeral system is used for labeling and coordinating the points. Such collection of spacial points is also known as a Riemann space, in fact, the simplest one, in which, all points are of identical, finite, and nonzero curvature radius equaling external radius of the space.

Finiteness of finite space results in fundamental difference/distinction between Riemann and Euclid space. For instance, there is no ESL in finite space hence translation symmetry of \mathbf{S}^m . Instead, there is Riemann straight line (RSL) in \mathbf{S}^m , *i.e.*, shortest path (length of path defined by Euclid norm) between any pair of points of \mathbf{S}^m . More precisely, RSL is largest circle in \mathbf{S}^m , *i.e.*, having long-

est circumference by Euclid norm. That is, RSL is circle in \mathbf{S}^m having external radius equal to that defining \mathbf{S}^m . Instead of translation symmetry of \mathbf{E}^n , \mathbf{S}^m is of rotation symmetry, *i.e.*, being identical, invariant, indifferent, indistinguishable under any rotation with respect to its external origin.

RSL is commonly known as geodesic of Riemann space and denoted as \mathbf{S}^1 , since it is 1-sphere by definition and a subset of \mathbf{S}^m . One unique property of geodesics is that any geodesic of \mathbf{S}^m must intersect with any other geodesic of same at two and only two points in same.

Consider coordinating finite space. Since \mathbf{S}^m can be viewed as subset of \mathbf{E}^n or said as embedded in \mathbf{E}^n , points of \mathbf{S}^m can be coordinated in same manner as that for \mathbf{E}^n , e.g., by CCF, but with constraint. Such coordination is simple or convenient but not transparent. An alternative approach is to coordinate points of finite space from within. Internal coordination is transparent but may not be as simple or convenient. For instance, consider \mathbf{S}^3 having defining radius R , *i.e.*, external radius of the space, which is inaccessible from within \mathbf{S}^3 . Let points of \mathbf{S}^3 be identified as

$$\mathbf{x} \equiv R(x_1, x_2, x_3, x_4) + \mathbf{x}_{00}, \begin{cases} x_1 = \cos \varphi & 0 \leq \varphi \leq \pi \\ x_2 = \sin \varphi \cos \vartheta & 0 \leq \vartheta \leq \pi \\ x_3 = \sin \varphi \sin \vartheta \cos \theta & 0 \leq \theta < 2\pi \\ x_4 = \sin \varphi \sin \vartheta \sin \theta \end{cases} \quad (36)$$

\mathbf{x} : Coordinate of point of \mathbf{S}^3 with CCF label of \mathbf{E}^4 . R : External radius of \mathbf{S}^3 in unit of length of \mathbf{E}^n .

\mathbf{x}_{00} : Coordinate of external origin of \mathbf{S}^3 with CCF label of \mathbf{E}^4 , assigned as $\mathbf{0}$.

Such approach, known as spherical coordination, is capable of identifying each and every point of \mathbf{S}^3 , and such point only, in manner that is coherent, systematic, and conditionally unique. Therefore, spherical coordination is transparent, *i.e.*, geometric relationship among points of finite space being explicit, but not as simple or convenient as that of CCF. Further, identicalness of all points of \mathbf{S}^m could be compromised due to artificial bias introduced in spherical coordination system towards certain points in such coordination. By arbitrariness of construction of coordination frame, physical phenomenon presented/described with any coordination is required to be identical/invariant/indifferent/indistinguishable under any and all uniform spherical transformation of coordinates.

At each and every point of \mathbf{S}^3 , with spherical coordinate, four local directions can be established as the follows

$$\mathbf{e}_R \equiv \frac{\partial \mathbf{x}}{\partial R}, \mathbf{e}_\varphi \equiv \frac{1}{R} \frac{\partial \mathbf{x}}{\partial \varphi}, \mathbf{e}_\vartheta \equiv \frac{1}{R \sin \varphi} \frac{\partial \mathbf{x}}{\partial \vartheta}, \mathbf{e}_\theta \equiv \frac{1}{R \sin \varphi \sin \vartheta} \frac{\partial \mathbf{x}}{\partial \theta}. \quad (37)$$

The 4-vector \mathbf{e}_R is pointing towards outside of \mathbf{S}^3 hence the direction is imperceptible from within \mathbf{S}^3 ; and the rest of the directions are internal to \mathbf{S}^3 , *i.e.*, accessible from within. The set of the vectors also satisfies

$$\begin{aligned} \mathbf{e}_\varphi \times \mathbf{e}_\vartheta \times \mathbf{e}_R &= \mathbf{e}_\theta \\ \mathbf{e}_i \cdot \mathbf{e}_j &= \delta_{i,j}, i, j \in \{R, \varphi, \vartheta, \theta\}; \mathbf{e}_\vartheta \times \mathbf{e}_\theta \times \mathbf{e}_R = \mathbf{e}_\varphi \\ \mathbf{e}_\theta \times \mathbf{e}_\varphi \times \mathbf{e}_R &= \mathbf{e}_\vartheta \end{aligned} \quad (38)$$

$\delta_{i,j}$: Kronecker delta function.

That is, the four 4-vectors form a set of orthogonal unit vectors. Therefore, at each and every point of \mathbf{S}^3 , there are three and only three orthogonal directions within the space. Therefore, any point and its vicinity in \mathbf{S}^3 shall resemble zone of \mathbf{E}^3 , but only approximately. In general, any region of \mathbf{S}^m can be approximated as zone of \mathbf{E}^m if maximal distance between points of the region is small in comparison with external radius of \mathbf{S}^m . However, such region is not flat nor approaching flat regardless of size of the region, because curvature of any region of finite space is constant and nonzero regardless of how small a region may be, even if only containing one single point. In other words, finite space is not flat, no matter how large it may be, nor region of finite space, no matter how small it may be.

Two orthogonal line elements internal to \mathbf{S}^3 and perpendicular to e_φ can be constructed as

$$ds_\varrho = e_\varrho \cdot \frac{\partial \mathbf{x}}{\partial \varrho} d\varrho = R \sin \varphi d\varrho, \quad ds_\theta = e_\theta \cdot \frac{\partial \mathbf{x}}{\partial \theta} d\theta = R \sin \varphi \sin \varrho d\theta. \quad (39)$$

ds_ϱ : Line element of variable ϱ internal to \mathbf{S}^3 and perpendicular to e_φ . ds_θ : Line element of variable θ internal to \mathbf{S}^3 and perpendicular to e_φ .

Therefore,

$$dS_\varphi = ds_\varrho ds_\theta = R^2 \sin^2 \varphi \sin \varrho d\varrho d\theta \rightarrow S_\varphi = 4\pi R^2 \sin^2 \varphi, \quad \varphi = s/R. \quad (40)$$

S_φ : Surface area of a three-dimensional ball in \mathbf{S}^3 centered at $\mathbf{x}_0 \equiv R(1,0,0,0)$, \mathbf{x}_0 is referred to as internal origin of \mathbf{S}^3 . s : Internal radius of the 3-ball.

Therefore,

$$V_\varphi = \int_0^\varphi S_\varphi ds = 2\pi R^3 (\varphi - \sin \varphi \cos \varphi) \rightarrow V_{S_R^3} = 2\pi^2 R^3. \quad (41)$$

V_φ : Volume of 3-ball of internal radius s centered at \mathbf{x}_0 . $V_{S_R^3}$: Volume of \mathbf{S}^3 of external radius R .

RSL is largest circle in finite space. Therefore, longest internal distance between points of \mathbf{S}^m is $\frac{1}{2}$ circumference of such circle, referred to as internal diameter of \mathbf{S}^m , and half of that internal radius of same,

$$D_i = \pi R, \quad R_i = \frac{\pi}{2} R \rightarrow V_{S_R^3} = \frac{2}{\pi} D_i^3 = \frac{16}{\pi} R_i^3. \quad (42)$$

D_i : Internal diameter of finite space. R_i : Internal radius of finite space.

13. Motion in Finite Space

Consider motion of free particle in \mathbf{S}^m . \mathbf{S}^m can be viewed as a subset of \mathbf{E}^n and identified as surface of a ball centered at origin of a SCCF of \mathbf{E}^n . SCCF is IRF of \mathbf{E}^n hence its motion with respect to any other IRF of \mathbf{E}^n shall have no physical effect at all. Therefore, SCCF embedding \mathbf{S}^m can be regarded as a rest frame of \mathbf{E}^n (RFE) without prejudice of any kind. Accordingly, \mathbf{S}^m is and is always at rest with respect to RFE.

There is no translation symmetry in finite space hence TFM of particle in \mathbf{S}^m . If a particle is not at rest with respect to RFE then it shall experience centrifugal force caused by curvature of \mathbf{S}^m in conjunction with motion of the particle with respect to RFE. Centrifugal force experienced by the particle shall force the particle to move towards along a RSL unless the particle is already in motion along RSL. From perspective of RFE, such motion of particle is geodesic rotation of particle in \mathbf{S}^m . Therefore, free particle in \mathbf{S}^m must be in motion in geodesic of \mathbf{S}^m if not at rest with respect to RFE.

Free particle does not have to move along shortest path between points of finite space because any geodesic has two and only two directions. In comparison to any other path in same space, if a path between two points of a geodesic is the shortest along one direction of the geodesic then the path between the same points in same geodesic along the other direction is the longest in terms of RSL distance between the points. No matter which direction of geodesic is taken by a particle, centrifugal force associated with motion of the particle in geodesic is pointing to direction outward of \mathbf{S}^m hence projection of such force in \mathbf{S}^m is none therefore shall not be experienced by the particle in motion in geodesic. Therefore, it is the centrifugal force caused by motion of particle in finite space that causes motion of free particle in geodesic, wherein, projection of such force is none regardless of velocity and direction of motion of particle.

Centrifugal force is a borrowed term for narrative convenience. For particle in consideration, there is no involvement of any real, physical force by others but only curvature of space that is finite and nonzero for \mathbf{S}^m . From perspective of \mathbf{E}^n , such curvature shall cause alteration of momentum of particle if particle is in motion. According to LNM2, alteration of momentum of particle is caused by force, hence the term centrifugal force.

Restness of particle in \mathbf{S}^m is measurable from within \mathbf{S}^m without referencing to RFE. For such purpose, consider a group of at least three free particles, referred to herein as Cluster, which is dispersed in \mathbf{S}^m in nondegenerate configuration (NDC). By free particle, it is meant that there is no interaction of any kind between particle and any other entity, nor constraint of any kind imposed upon or experienced by particle except finiteness of space. NDC refers to configuration of Cluster that at least two of the geodesics containing plurality of members of Cluster are not one and same. Therefore, at least one member of Cluster shall be in motion (along a geodesic) unless none is in motion. Accordingly, internal distance between at least one pair of particles of Cluster shall be varying with time (of any reference clock) unless none is in motion. Therefore, if all members of a Cluster of NDC shall remain at rest indefinitely with respect to each other, *i.e.*, internal distance between any pair of particles of the Cluster does not vary with any time, then such state of motion of such group of such particles is referred to as Rest State (RS) and reference frame comoving with such Cluster Rest Frame (RF).

If a free particle is at rest with respect to a Cluster in RS (CRS), *i.e.*, distance of

the particle to each and every member of CRS shall remain invariant indefinitely then, by definition, the particle is includible as a member of the CRS hence the particle is in RS. If a group of free particles is at rest with respect to CRS, *i.e.*, distance between each particle of the group and every member of CRS shall remain invariant indefinitely, then particles of the group are qualified as members of the CRS hence the particles are all in RS. Further, distance between any pair of the particles of the group shall also remain invariant indefinitely. Therefore, by definition, the group is also a CRS if it has at least three members and in NDC. Therefore, if a Cluster in one RS is at rest with respect to another Cluster in another RS then the Cluster is qualified as member of the other and vice versa. Therefore, RS of the Cluster is one and same as that of the other.

If a free particle is not at rest with respect to RF then it must be in motion along geodesic trajectory according to LNM and centrifugal force caused by motion of the particle in \mathbf{S}^m . If a Cluster is not in RS, then at least one member of the Cluster is in motion along its own geodesic trajectory. Therefore, distance between at least one pair of members of the Cluster shall vary with time, since any two nondegenerate geodesics shall intersect at some point in finite space hence distance between particles in motion along such geodesics shall vary with time, even if velocity of motion of the particles were identical and direction of motion parallel. Therefore, the Cluster can be determined as not in RS by measurement/observation. Therefore, plurality of Clusters cannot all be in RS unless they are all at rest with respect to each other and at least one of them is in RS.

Therefore, RS is a unique state of motion of particle in finite space, *i.e.*, there is one and only one RS in \mathbf{S}^m , which is not determined by referencing to other entity, e.g., with respect to reference state or frame, but only on its own measurable. Therefore, motion of particle in \mathbf{S}^m is said as absolute, *i.e.*, with respect to RS, a unique and self-determined state of motion. Further, it is only with respect to RS, hence RF, that motion of free particle in finite space shall follow geodesic trajectory.

RS is a rest state of motion in \mathbf{E}^n and RF a rest frame of \mathbf{E}^n , in fact, an IRF of \mathbf{E}^n . Therefore, local clocks associated with RF are synchronizable among themselves and locally defined common time of RF can exist, referred to as Rest Time (RT). Therefore, nonlocal simultaneity of events in RF is definable hence the concept meaningful.

Geodesic motion of particle in \mathbf{S}^m is unique in that, although centrifugal force caused by motion of particle is maximal in magnitude along external radius of \mathbf{S}^m , projection of such force in \mathbf{S}^m is none hence the force is imperceptible to particle in such motion in such space. Therefore, free particle in motion in geodesic shall remain in such state of motion indefinitely. This is known as inertial motion in finite space and referred to as uniform rotation of free particle in \mathbf{S}^1 of \mathbf{S}^m .

14. Length and Time in Geodesic Motion

Consider a set of identical clocks dispersed arbitrarily at plurality of locations in

\mathbf{S}^1 . Suppose each and every clock of the set is aligned hence in sync with corresponding local clock of RF. Then, by specification, clocks of the set are also in sync among themselves. Suppose each and every clock of the set is equipped with an otherwise identical propeller, which shall be turned on at one preset moment, off at another preset moment, all according to the clock it is in association with, and direction of propulsion of propeller is along \mathbf{S}^1 in consistent manner.

By specification, the set of the clocks is confined in \mathbf{S}^1 . Therefore, at any moment of RT, clocks of the set shall be found locating at somewhere in \mathbf{S}^1 regardless of state/operation history of propeller. Further, clocks of the set are, by specification, identical in every aspect/detail except their locations in \mathbf{S}^1 and have had, are having, and will have otherwise identical experience in \mathbf{S}^1 according to their own times, and \mathbf{S}^1 is of CHI. Therefore, guaranteed by CHI, clocks of the set shall display identical reading of time anywhere they are found in \mathbf{S}^1 at any moment of RT regardless of where/when they are found in \mathbf{S}^1 and state/operation history of propeller, although the time as displayed by the set of the clocks may not be identical to RT. Therefore, clocks of the set are and are always in sync among themselves regardless of state of motion of the set, such as at rest with respect to RF, in rotational acceleration, in uniform rotation, etc., as well as state/operation history of propeller. Therefore, clocks of the set can have locally defined common time among themselves regardless of state of motion of the set and state and operation history of propeller.

By specification, clocks of the set are at rest with respect to RF before onset of propeller. Therefore, internal distance (arc length in \mathbf{S}^1 by Euclid norm) between any pair of clocks of the set is well defined before onset of propeller. Since clocks of the set are identical in every aspect/detail except their initial locations in \mathbf{S}^1 and have had, are having, and will have otherwise identical experience in \mathbf{S}^1 according to their own times, and \mathbf{S}^1 is of CHI, therefore, distance between any pair of clocks of the set is guaranteed by CHI to be invariant with respect to state/operation history of propeller. Therefore, clocks of the set are and are always having invariant distances among themselves regardless of state of motion of the set, such as at rest with respect to RF, in rotational acceleration, in uniform rotation, etc., as well as state/operation history of propeller. Therefore, any clock of the set is and is always at rest with respect to any other clock of same regardless of state of motion of the set and state and operation history of propeller.

Therefore, reference frame of \mathbf{S}^1 comoving with the set of the clocks can be set up, referred to as co-rotation frame (CRF). Thus, arbitrarily assign location of one of the clocks of the set as internal origin of CRF; arbitrarily assign one direction in \mathbf{S}^1 as positive direction; label location of each and every clock of the set with a real numeral (sign of numeral being consistent with direction chosen) according to its internal distance to the chosen origin; expand the set of the clocks so that each and every point of CRF is in association with a clock of the

set, which defines local time of the point. Then, according to the analysis above, locally defined common time of CRF can exist hence nonlocal simultaneity of events in CRF is definable hence the concept meaningful. Further, any point of CRF is and is always at rest with respect to any other point of same regardless of state of motion of the frame with respect to RF. Therefore,

$$s_{r, \text{ in uniform rotation}} = s_{r, \text{ in acceleration}} = s_{r, \text{ at rest}} = s_i \cdot \quad (43)$$

s_r : Internal distance between pair of points of CRF. s_i : \mathbf{S}^1 distance between corresponding pair of points of RF.

Assign internal distance of one pair of points of CRF as unit of length of the frame. Then,

$$\mathbb{U}_{L,r, \text{ in uniform rotation}} = \mathbb{U}_{L,r, \text{ in acceleration}} = \mathbb{U}_{L,r, \text{ at rest}} = \mathbb{U}_{L,i} \cdot \quad (44)$$

$\mathbb{U}_{L,r}$: Unit of length of CRF. $\mathbb{U}_{L,i}$: Unit of length of RF.

That is, length of unit of length of CRF is invariant to state of motion of CRF with respect to RF. Therefore, length of unit of length of CRF shall not expand nor contract with respect to any other rotation frame of \mathbf{S}^1 , RF, and any IRF of \mathbf{E}^n , regardless of state of motion of CRF, including acceleration. It is thus shown by the above analysis that any object in motion in finite space shall not have space dilation.

15. Speed of Light in Vacuo in Geodesic Motion

In general, rotation frame is not inertial rotation frame. If a rotation frame is in uniform rotation with respect to RF then motion of free particle in such reference frame shall remain in its state of motion indefinitely. Such rotation frame is referred to as geodesic inertial frame (GIF), which is, by definition, a one-dimensional inertial frame in \mathbf{S}^1 .

Consider a particle in motion in GIF. By definition,

$$\mathbf{P}_r \equiv m_r \mathbf{v}_r, \mathbf{v}_r \equiv \frac{d\mathbf{s}}{dt_r} = \boldsymbol{\omega}_r \times \mathbf{r}_r, \boldsymbol{\omega}_r \equiv \frac{d\theta_r}{dt_r} \mathbf{k} \cdot \quad (45)$$

\mathbf{P}_r : Momentum of particle as measured at rest in GIF. m_r : Mass of particle as measured at rest in GIF. \mathbf{v}_r : Velocity of particle as measured at rest in GIF. \mathbf{s} : Location of particle in GIF. t_r : Locally defined common time of GIF. $\boldsymbol{\omega}_r$: Angular velocity of particle with respect to RF. \mathbf{r}_r : Location vector of particle, a vector connecting external origin of \mathbf{S}^1 and particle location in GIF. θ_r : Angle between \mathbf{r}_r and reference vector connecting external origin of \mathbf{S}^1 and internal origin of GIF. \mathbf{k} : Unit vector along rotation axis of GIF, a vector at external origin of \mathbf{S}^1 and perpendicular to \mathbf{S}^1 .

If external force is exerted onto the particle then the particle shall alter its state of motion in response to force experienced by the particle. LEC requires

$$\frac{dE_r}{dt_r} = \mathbf{F}_r \cdot \mathbf{v}_r = \frac{d\mathbf{P}_r}{dt_r} \cdot \mathbf{v}_r = v_r^2 \frac{dm_r}{dt_r} + \frac{m_r}{2} \frac{dv_r^2}{dt_r} \cdot \quad (46)$$

E_r : Total energy of particle as measured at rest in GIF. \mathbf{F}_r : External force exerted onto particle as perceived in GIF. v_r : Magnitude of \mathbf{v}_r , $v_r \equiv \|\mathbf{v}_r\|$.

From Expression (45),

$$v_r^2 = r_r^2 \omega_r^2, r_r \equiv \|r_r\| \rightarrow \frac{dE_r}{dt_r} = r_r^2 \omega_r^2 \frac{dm_r}{dt_r} + \frac{m_r r_r^2}{2} \frac{d\omega_r^2}{dt_r}. \quad (47)$$

By LME,

$$E_r = m_r c_r^2 \rightarrow c_r^2 \frac{dm_r}{dt_r} + 2m_r c_r \frac{dc_r}{dt_r} = r_r^2 \omega_r^2 \frac{dm_r}{dt_r} + \frac{m_r r_r^2}{2} \frac{d\omega_r^2}{dt_r}. \quad (48)$$

c_r : SLV as measured/defined in GIF.

By definition of SLV, Expression (8),

$$c_r \equiv \mathcal{N}_c \frac{\mathbb{U}_{L,r}}{\mathbb{U}_{t,r}} \rightarrow \frac{dc_r}{dt_r} = \frac{\mathcal{N}_c}{\mathbb{U}_{t,r}} \frac{d\mathbb{U}_{L,r}}{dt_r} - \frac{\mathcal{N}_c \mathbb{U}_{L,r}}{\mathbb{U}_{t,r}^2} \frac{d\mathbb{U}_{t,r}}{dt_r}. \quad (49)$$

$\mathbb{U}_{L,r}$: Unit of length of GIF. $\mathbb{U}_{t,r}$: Unit of time of GIF.

From Expression (44),

$$\frac{d\mathbb{U}_{L,r}}{dt_r} = \frac{d\mathbb{U}_{L,i}}{dt_r} = 0 \rightarrow \frac{dc_r}{dt_r} = -\frac{\mathcal{N}_c \mathbb{U}_{L,r}}{\mathbb{U}_{t,r}^2} \frac{d\mathbb{U}_{t,r}}{dt_r}. \quad (50)$$

$\mathbb{U}_{L,i}$: Unit of length of RF.

By the rule of numeration (cf. **Appendix A**),

$$\frac{d\mathbb{U}_{t,r}}{dt_r} = 0 \rightarrow \frac{dc_r}{dt_r} = 0. \quad (51)$$

Equation (48) becomes

$$(1-u^2) \frac{dm_r}{dt_r} = \frac{m_r}{2} \frac{du^2}{dt_r}, u \equiv \frac{r_r \omega_r}{c_r} = \frac{v_r}{c_r}. \quad (52)$$

Therefore,

$$\text{If } \frac{du^2}{dt_r} \neq 0 \text{ then } \frac{d}{dt_r} \ln[\beta_u m_r] = 0, \beta_u \equiv \sqrt{1-u^2}, 0 < \beta_u \leq 1. \quad (53)$$

That is, if external force does cause alteration of velocity of the particle then

$$\beta_u m_r \subset t_r \text{ \& } r_r \text{ Invariant.} \quad (54)$$

Accordingly,

$$m_r = m_{r,0} / \beta_u, m_{r,0} \equiv m_r|_{u=0} \rightarrow E_r = E_{r,0} / \beta_u, E_{r,0} \equiv m_{r,0} c_r^2. \quad (55)$$

$m_{r,0}$: Restmass of particle as measured at rest in GIF. $E_{r,0}$: Restenergy of particle as measured at rest in GIF.

That is, if external force alters velocity of a particle, then both mass and energy of the particle shall be altered that are function of velocity of the particle. Further, mass of any particle is and must be real, nonnegative, and finite by definition of *mass*. Therefore, under LME, there must exist speed limit c_r for motion of particle in GIF, because if $v_r > c_r$ were allowed then mass of particle could become imaginary.

If external force does not or cannot alter velocity of the particle then

$$\frac{du^2}{dt_r} = 0 \rightarrow (1-u^2) \frac{dm_r}{dt_r} = 0. \quad (56)$$

Therefore,

$$\text{If } u^2 \neq 1 \text{ then } \frac{dm_r}{dt_r} = 0 \rightarrow \frac{d\mathbf{P}_r}{dt_r} = \frac{dm_r}{dt_r} \mathbf{v}_r + m_r \frac{d\mathbf{v}_r}{dt_r} = \mathbf{0} \neq \mathbf{F}_r. \quad (57)$$

That is, such would cause LNM2 violation. Therefore, if external force does not or cannot cause alteration of velocity of a particle then velocity of the particle must be $v_r = c_r$. However,

$$\text{If } u^2 = 1 \text{ then } \frac{dE_r}{dt_r} = c_r^2 \frac{dm_r}{dt_r} = \mathbf{F}_r \cdot \mathbf{c}_r \rightarrow \frac{dm_r}{dt_r} = \frac{\mathbf{c}_r}{c_r^2} \cdot \mathbf{F}_r. \quad (58)$$

Therefore, by exerting force in opposite direction of motion of the particle, mass of the particle could become negative hence contradicting with definition of *mass*. Therefore, if a particle is in motion at the speed limit then no force of any kind shall be experienced by the particle, even though reference frame may not concur. In other words, if particle in motion with respect to GIF were to maintain invariant velocity under external force then that velocity must be and always be equal to c_r . Further, if particle is in motion at c_r with respect to GIF then no force of any kind shall be experienced by the particle regardless of opinion of GIF.

16. Atomic Clock in Geodesic Motion

Consider a particle in motion in \mathbf{S}^1 of external radius r . According to Expression (55),

$$E_{i,u} = E_{i,0} / \sqrt{1 - (v_i/c_i)^2}, \quad v_i = r\omega_i. \quad (59)$$

$E_{i,u}$: Total energy of particle in motion in \mathbf{S}^1 as measured in RF. $E_{i,0}$: Restenergy of particle as measured in RF. v_i : Velocity of particle in motion in \mathbf{S}^1 as measured in RF. c_i : SLV as measured/defined in RF. r : External radius of \mathbf{S}^1 . ω_i : Angular velocity of particle in motion in \mathbf{S}^1 as measured in RF.

By specification, motion of the particle is confined in \mathbf{S}^1 , which is a subset of \mathbf{E}^n . If the spacial confinement were revoked at any moment of RT then, unless the particle was at rest with respect to RF, the particle would have tendency to move spontaneously towards outward of \mathbf{S}^1 along tangent direction of \mathbf{S}^1 at location of the particle at the moment the confinement was revoked, and the particle would have the total energy and TFM velocity as measured at rest in \mathbf{E}^n as expressed in Expression (59).

The aforementioned tendency/potential of the outward motion of the particle is caused by centrifugal force in association with motion of the particle in \mathbf{S}^1 while no real, physical force of any kind was/is exerted onto the particle but only spacial confinement, which is unique in that centrifugal force caused by motion under such confinement is undetectable from within the confined space hence

imperceptible to the particle existing in the space confined, including internals of any such object to any level of details, if projection of such force in such space is none, as is the case for motion of particle in \mathbf{S}^1 .

While centrifugal force due to absolute motion of a particle in \mathbf{S}^1 is imperceptible to the particle in motion therein, the particle would have tendency/potential to move along direction of external radius of \mathbf{S}^1 if such displacement were allowed. If allowed, on the other hand, such motion would be spontaneous and self-driven hence energy required for such displacement, if any, would have to come from the particle itself. Therefore, by LEC,

$$\partial_r E_{s,r} = -\mathbf{F}_c \cdot \partial \mathbf{r} . \tag{60}$$

∂_r : Partial differential operator with respect to external radius of \mathbf{S}^1 . $E_{s,r}$: Selfenergy of particle in motion in \mathbf{S}^1 . \mathbf{F}_c : Centrifugal force caused by absolute motion of particle in \mathbf{S}^1 . \mathbf{r} : Location vector of particle in RF, connecting external origin of \mathbf{S}^1 to particle location in \mathbf{S}^1 .

From perspective of RF, with Expression (55),

$$\mathbf{F}_c = m_{i,u} \omega_i^2 \mathbf{r} = \frac{m_{i,0}}{\beta_u} \omega_i^2 \mathbf{r}, \beta_u \equiv \sqrt{1-u^2}, u \equiv \frac{r\omega_i}{c_i} \rightarrow \frac{dE_{s,r}}{d\beta_u} = E_{i,0} . \tag{61}$$

$m_{i,u}$: Total mass of particle in motion in \mathbf{S}^1 as measured in RF. $m_{i,0}$: Restmass of particle as measured in RF. $E_{s,i}$: Selfenergy of particle as measured in RF.

Therefore,

$$E_{s,r} = \beta_u E_{i,0} = \beta_u E_{s,i} . \tag{62}$$

That is, selfenergy of a particle in motion in finite space is not SIT but function of velocity of absolute motion of the particle. Therefore, it is indeed the nonzero curvature of trajectory of motion, inertial motion included, that causes the alteration of selfenergy of particle in such motion.

If the particle in consideration is CDP of an atomic clock in motion in \mathbf{S}^1 then

$$E_{1,s,r} = \beta_u E_{1,s,i}, E_{0,s,r} = \beta_u E_{0,s,i} \rightarrow \Delta E_{s,r} = \beta_u \Delta E_{s,i} . \tag{63}$$

$E_{1,s,r}$: Selfenergy of CDP in 1-state in motion in \mathbf{S}^1 . $E_{0,s,r}$: Selfenergy of CDP in 0-state in motion in \mathbf{S}^1 . $E_{1,s,i}$: Selfenergy of CDP in 1-state at rest in RF. $E_{0,s,i}$: Selfenergy of CDP in 0-state at rest in RF. $\Delta E_{s,r}$: Selfenergy difference between 1-state and 0-state of CDP in motion in \mathbf{S}^1 . $\Delta E_{s,i}$: Selfenergy difference between 1-state and 0-state of CDP at rest in RF.

By definition of atomic clock, with Equation (7),

$$\mathbb{U}_{AT,r} \equiv \frac{\mathcal{N}_i h_r}{\Delta E_{s,r}} = \frac{\mathcal{N}_i h_r}{\beta_u \Delta E_{s,i}} = \frac{\beta_h \mathcal{N}_i h_i}{\beta_u \Delta E_{s,i}} \equiv \frac{\beta_h}{\beta_u} \mathbb{U}_{AT,i}, \beta_h \equiv \frac{h_r}{h_i} . \tag{64}$$

$\mathbb{U}_{AT,r}$: Unit of AT at particle location in CRF of particle in motion in \mathbf{S}^1 . $\mathbb{U}_{AT,i}$: Unit of RT defined on AT (RAT). h_r : Planck constant as measured at particle location in CRF of particle in motion in \mathbf{S}^1 . h_i : Planck constant as measured in RF.

Assumption 1: Planck constant is SIT,

$$h_r = h_i \equiv h \subset \text{SIT} \rightarrow \beta_h = 1 . \tag{65}$$

That is, it is **assumed** that Planck constant is one and same whether h is meas-

ured by setup comoving/corotating with particle (hence at rest with respect to particle) or at rest with respect to IRF of \mathbf{E}^n or reference frame of any kind.

Under the condition of **Assumption 1** (conditioned AT, CAT),

$$\mathbb{U}_{AT,r} = \mathbb{U}_{AT,i} / \beta_u \rightarrow \mathbb{U}_{AT,r} > \mathbb{U}_{AT,i}, \quad 0 < \beta_u < 1. \tag{66}$$

That is, time dilation shall occur to atomic clock in geodesic motion and, in general, any motion anywhere having nonzero curvature of trajectory. It is thus shown that, under CAT, atomic clock in inertial motion in finite space shall exhibit time dilation, due to alteration of momentum of CDP of the clock caused by nonzero curvature of trajectory of such motion in such space.

Therefore, if two identical atomic clocks, in inertial motion along same geodesic but in opposite directions, set their times to read the same when they are meeting with each other, then, unless magnitude of the velocities of the clocks were identical, the times as displayed by the clocks, when meeting with each other again, shall be different, and the clock moving faster shall display lesser time than the one moving slower. If the clocks, in inertial motion along different geodesics in same space in same velocity, set their times to read the same if and when they meet with each other, then, when they meet with each other again, the times as displayed by the clocks shall be exactly the same.

Therefore, in finite space under **Assumption 1**, duration of unit of AT is not SIT but function of state of motion of CDP. Note that reduced velocity u in Equation (66) is of absolute meaning, *i.e.*, referring to velocity of absolute motion of particle with respect to RF, hence is different from that in SRT, which is relative, even though expression for motion induced time dilation appears identical to that of SRT.

Thus, particle clock, hence selftime (ST) of particle defined on AT (SAT), in geodesic motion shall have time dilation, caused by centrifugal force associated with such motion in finite space if **Assumption 1** is valid thereat. In other words, centrifugal force shall alter selfenergy of particle in motion in finite space even though such force is imperceptible to the particle.

By definition of SLV, Expression (8), with Equation (44),

$$\begin{aligned} c_r &\equiv \mathcal{N}_c \mathbb{U}_{L,r} \mathbb{U}_{t,r}^{-1} \\ c_i &\equiv \mathcal{N}_c \mathbb{U}_{L,i} \mathbb{U}_{t,i}^{-1} \end{aligned} \rightarrow \frac{c_r}{c_i} = \frac{\mathbb{U}_{L,r}}{\mathbb{U}_{L,i}} \frac{\mathbb{U}_{t,i}}{\mathbb{U}_{t,r}} = \frac{\mathbb{U}_{t,i}}{\mathbb{U}_{t,r}}. \tag{67}$$

c_r : SLV as measured at particle location in CRF of particle in motion in \mathbf{S}^1 . $\mathbb{U}_{L,r}$: Unit of length of CRF of particle in motion in \mathbf{S}^1 . $\mathbb{U}_{L,i}$: Unit of length of RF.

Therefore,

$$c_r = \frac{\mathbb{U}_{AT,i}}{\mathbb{U}_{AT,r}} c_i = \beta_u c_i \rightarrow \begin{matrix} c_r < c_i \\ 0 < \beta_u < 1 \end{matrix}. \tag{68}$$

That is, under CAT, c_r shall be slower than c_i . Therefore, if time is defined on atomic clock and SLV is defined on AT then, in finite space under **Assumption 1**, SLV is not SIT but function of state of motion of lab whereat SLV is measured/defined. Therefore, measured/defined on AT at location of particle in mo-

tion in finite space, hence from perspective of particle, SLV is not constant but function of state of motion of particle if **Assumption 1** is valid thereat.

Under CAT,

$$\frac{c_r}{\Delta E_{s,r}} = \frac{\mathcal{N}_c \mathbb{U}_{L,r}}{\mathcal{N}_i h_r} = \frac{\mathcal{N}_c \mathbb{U}_{L,i}}{\mathcal{N}_i h_i} = \frac{c_i}{\Delta E_{s,i}} \rightarrow \frac{c_r}{c_i} = \frac{\Delta E_{s,r}}{\Delta E_{s,i}}. \quad (69)$$

Therefore, it is the state dependency of selfenergy of CDP that causes the state dependency of SLV defined on AT under **Assumption 1**.

By the rule of numeration, with Equation (68),

$$\begin{aligned} u_r &\equiv \frac{v_r}{c_r} \equiv \frac{1}{c_r} \frac{ds}{dt_r} = \frac{1}{c_r} \frac{ds}{\mathbb{U}_{t,r} dn_t} = \frac{c_i \mathbb{U}_{t,i}}{c_r \mathbb{U}_{t,r} c_i} \frac{1}{\mathbb{U}_{t,i}} \frac{ds}{dn_t} \\ &= \frac{1}{c_i} \frac{ds}{\mathbb{U}_{t,i} dn_t} \equiv \frac{1}{c_i} \frac{ds}{dt_i} \equiv \frac{v_i}{c_i} \equiv u_i \equiv u \end{aligned} \quad (70)$$

ds : Line element of trajectory of motion of particle in space.

Therefore, if velocity of motion of particle is expressed in unit of SLV measured/defined at rest at exactly the same location, reference frame, and perspective, referred to as reduced velocity, then reduced velocity of motion of particle is independent of and invariant to definition of time, location, reference frame, and perspective.

17. Additivity of Velocity of Motion in Finite Space

Consider a particle in uniform motion in GIF of \mathbf{S}^1 . Suppose, at one moment of RT, the particle is coinciding with internal origin of GIF at point of \mathbf{S}^1 labeled as a , and at another moment of RT, origin of GIF is found at b and particle at c of \mathbf{S}^1 , referred to as *the events*. Then, by definition of velocity of the motion,

$$v_{i,f} \equiv \frac{\widehat{ab}}{\delta t_i}, v_{f,i} \equiv \frac{\widehat{ba}}{\delta t_f}, v_{s,i} \equiv \frac{\widehat{ca}}{\delta t_s}, v_{i,s} \equiv \frac{\widehat{ac}}{\delta t_i}, v_{f,s} \equiv \frac{\widehat{bc}}{\delta t_f}, v_{s,f} \equiv \frac{\widehat{cb}}{\delta t_s}. \quad (71)$$

$v_{x,y}$: Velocity of entity y as measured in x -reference frame in x -time. x, y : $i = \text{RF}$, $f = \text{GIF}$, $s = \text{Particle}$. $\widehat{\alpha, \beta}$: Directional internal distance between points of \mathbf{S}^1 , from α to β . δt_x : Duration in x -time between *the events* of finding entities in \mathbf{S}^1 .

Then,

$$v_{f,i} = -v_{i,f} \frac{\delta t_i}{\delta t_f}, v_{s,i} = -v_{i,s} \frac{\delta t_i}{\delta t_s}, v_{f,s} = -v_{s,f} \frac{\delta t_s}{\delta t_f}, v_{s,f} = v_{s,i} + v_{i,f} \frac{\delta t_i}{\delta t_s}. \quad (72)$$

Denote reduced velocity of entity as

$$u_{x,y} \equiv v_{x,y} / c_x, x, y \in \{i, f, s\} \rightarrow |u_{x,y}| < 1. \quad (73)$$

$u_{x,y}$: Reduced velocity of entity y measured in x -reference frame with x -time and in x -SLV. c_x : SLV measured/defined in x -reference frame in x -time.

That is, reduced velocity of entity shall be less than one by speed limit of motion of entity in space as imposed by LME.

The δt in Expression (71) refers to “duration between *the events*”. There-

fore, $L = c\delta t$ is the distance for photon travel in \mathbf{S}^1 during *the* events. Therefore, L must be identical during one and same δt regardless of perspective, measurement, definition of time, and that of SLV,

$$L = c_x \delta t_x = c_y \delta t_y \quad x, y \in \{i, f, s\} \rightarrow \frac{\delta t_x}{\delta t_y} = \frac{c_y}{c_x} \rightarrow \begin{array}{l} v_{f,i} c_i = -v_{i,f} c_f \\ v_{s,i} c_i = -v_{i,s} c_s \\ v_{f,s} c_s = -v_{s,f} c_f \end{array} \rightarrow \begin{array}{l} u_{f,i} = -u_{i,f} \\ u_{s,i} = -u_{i,s} \\ u_{f,s} = -u_{s,f} \end{array} . \quad (74)$$

That is, reduced velocities of motion of entities are of reciprocal symmetry, *i.e.*, equal in magnitude and opposite in sign. The expression is expandable to any motion of particle anywhere in \mathbf{S}^m , since velocity is an attribute definable at single point of geodesic of space. Further, with Equation (72),

$$\begin{array}{l} u_{s,i} + u_{i,f} + u_{f,s} = 0 \\ u_{f,s} + u_{s,i} + u_{i,f} = 0 \\ u_{i,f} + u_{f,s} + u_{s,i} = 0 \end{array} \rightarrow \begin{array}{l} u_{s,i} + u_{i,f} = u_{s,f} \\ u_{f,s} + u_{s,i} = u_{f,i} \\ u_{i,f} + u_{f,s} = u_{i,s} \end{array} \rightarrow \begin{array}{l} |u_{s,i} + u_{i,f}| < 1 \\ |u_{f,s} + u_{s,i}| < 1 \\ |u_{i,f} + u_{f,s}| < 1 \end{array} . \quad (75)$$

The inequation is also expandable to include any plurality of superposition of motions of particle since specialty of RF herein is inconsequential due to reciprocal symmetry of motion of entity and reference frame if and when velocity is expressed in reduced form. Therefore, in and only in reduced form, superposition of velocities of motions in finite space follows the same rule of vector addition as that in infinite space. However, unlike that in infinite space, superposition of reduced velocities of motions of particle in \mathbf{S}^m complies with finite speed limit imposed upon by LME.

Therefore, in sufficiently small region of \mathbf{S}^m , kinematic behavior of particle shall appear to conform to NMIS, LME, and SRT (without space dilation) simultaneously while still preserving logical and metrological integrity of the system of concepts without causing confliction nor resorting to blending of *space* and *time*. Such system of concepts is referred to as NM in finite space (NMFS).

18. Extended Object

If an object is not a particle, *i.e.*, if the object is of nonzero spacial extent hence an extended object (EO), then an EO in motion in finite space shall experience real, physical force caused by absolute motion of the EO in \mathbf{S}^m without intervention of others, unless the entire EO is coinciding with a section of RSL in motion in same. Therefore, in general, if an EO is in geodesic motion, *i.e.*, an assigned point of the EO (chosen to represent the EO) is in motion along a geodesic, then centrifugal force outside the geodetic is perceptible to the EO and shall act as transverse pressure exerted onto each and every point of the EO except those in the geodesic. Further, duration of unit of SAT associated with point of EO is, in general, not identical among points of EO due to differences in state of motion of points of EO with respect to RF even if EO is a rigid body. Therefore, in general, SATs of points of EO are not synchronizable among themselves hence locally defined common AT of EO does not exist hence concept of nonlocal si-

multaneity of EO events on AT meaningless metrologically. Therefore, in general, there is no such thing as SAT of EO in \mathbf{S}^m unless EO is in RS or arc-like object coinciding with RSL (RL object, RL) in motion in same. In contrast, SAT of particle is and is always definable, e.g., via particle clock at rest at location of particle in comotion frame of particle (CMF) and meaningful regardless of state of motion of particle.

For same reason as that for EO, locally defined common AT does not exist for CMF in motion in \mathbf{S}^m with respect to RF unless CMF is in RS or RL entity in motion in same. Consequently, nonlocal simultaneity of events on AT is indefinable in CMF hence the concept meaningless, unless CMF is in RS, in such case, time of CMF is identical/equivalent to RAT; or CMF is RL in motion in same, in such case, CMF is equivalent/identical to geodesic reference frame (GRF) in \mathbf{S}^1 . As analyzed in **Section 14**, ticking rates of identical clocks associated with points of GRF are identical. Therefore, locally defined common time of GRF can exist therefore nonlocal simultaneity of events of GRF is definable hence meaningful.

For same reason, RL in motion in same can have ST, *i.e.*, RL object time, which is locally (point of the EO) defined common time of the RL that is equivalent/identical to locally defined common time of comoving GRF. However, GRFs of different velocity/acceleration in same \mathbf{S}^1 do not have common AT among them even though they are in one and same subspace. On the other hand, GIFs in different \mathbf{S}^1 of same \mathbf{S}^m with identical magnitude of rotation velocities with respect to RF are synchronizable therefore can have common time among them even though they are not in same subspace. In general, under CAT, translation of space-time event among GIFs is impossible metrologically. Therefore, such transformation is meaningless in metrology, e.g., Lorentz transformation or alike.

Any EO in \mathbf{S}^m shall have or cause to have spin, *i.e.*, rotation of object with respect to itself. For instance, RL in motion along its own geodesic shall have spin, which is in sync with rotation of the entity in \mathbf{S}^m . If an EO has intrinsic spin, *i.e.*, at least one point of the EO is at rest in comoving frame of the EO while at least one point of the EO is in motion with respect to the same frame, then such spin shall cause Coriolis force [22] ($\propto \mathbf{P}_\omega \times \mathbf{v} \times \mathbf{e}_R$, \mathbf{P}_ω spin momentum of EO in comoving frame, \mathbf{v} velocity of the frame with respect to RF) that shall be experienced by the EO in motion in \mathbf{S}^m . Such force shall affect selfenergy state of EO hence time dilation of particle clocks associated with EO. Further, there are two and only two ways of spinning for a rigid body. Therefore, effect of intrinsic spin to inertial motion of rigid EO in finite space is of two levels.

19. Doppler Effect in Finite Space

Consider a signal generator, referred to herein as source, in uniform motion in \mathbf{S}^1 with velocity $0 < |v_i| < c_i$ towards a signal receiver at rest in \mathbf{S}^1 . Suppose, at \mathbf{S}^1 moment $t_i = 0$, internal distance between source and receiver is L and source starts sending signals (temporal events) towards receiver and stops sending sig-

nal at the moment source coinciding with receiver. Suppose signal travels in SLV. Then, from perspective of S^1 ,

$$L = c_i \tau_i = v_i T_i \rightarrow v_{r,0,i} \equiv \frac{n_r - 1}{T_i - \tau_i} = \frac{n_r - 1}{T_i(1 - u)}, u \equiv \frac{v_i}{c_i}, 0 < u < 1. \quad (76)$$

L : Internal distance between source and receiver at S^1 moment $t_i = 0$. c_i : SLV as measured/defined in S^1 . τ_i : Arrival time of first signal (sent by source in motion) at receiver according to S^1 clock. v_i : Velocity of source as measured in S^1 . T_i : Arrival time of source at receiver according to S^1 clock. $v_{r,0,i}$: Frequency of the signals as measured by receiver at rest in S^1 in S^1 time. n_r : Number of the signals received by receiver.

By definition of *frequency*, frequency of temporal events is reciprocal of duration between pair of temporal events, or number of temporal events during duration of the events, or number of temporal events occurs during duration of unit of time.

All signals sent by source are received by receiver. Therefore, from perspective of source at rest in CRF of source,

$$n_s = n_r \rightarrow v_{s,u,r} \equiv \frac{n_s - 1}{T_r} = \frac{n_r - 1}{T_i} \frac{T_i}{T_r} = (1 - u) \frac{T_i}{T_r} v_{r,0,i}. \quad (77)$$

n_s : Number of the signals sent by source. $v_{s,u,r}$: Frequency of the signals as measured by source at rest in CRF in CRF time. T_r : Time taken for source to travel from L to receiver according to CRF clock.

Therefore,

$$L = v_r T_r = v_i T_i \rightarrow v_{s,u,r} = (1 - u) \frac{v_r}{v_i} v_{r,0,i} = (1 - u) \frac{c_r}{c_i} v_{r,0,i}. \quad (78)$$

v_r : Velocity of motion of source with respect to S^1 in CRF time. c_r : SLV as measured/defined in CRF in CRF time.

By definition of SLV, with Expression (44),

$$\frac{c_r}{c_i} = \frac{\mathbb{U}_{t,i}}{\mathbb{U}_{t,r}} \rightarrow v_{s,u,r} = (1 - u) v_{r,0,i} \frac{\mathbb{U}_{t,i}}{\mathbb{U}_{t,r}} \rightarrow \begin{matrix} v_{s,u,i} = (1 - u) v_{r,0,i} \\ v_{s,u,r} = (1 - u) v_{r,0,r} \end{matrix}. \quad (79)$$

$\mathbb{U}_{t,i}$: Unit of S^1 time. $\mathbb{U}_{t,r}$: Unit of CRF time. $v_{s,u,i}$: Frequency of the signals as measured by source at rest in CRF in S^1 time. $v_{r,0,r}$: Frequency of the signals as measured by receiver at rest in S^1 in CRF time.

Suppose instead, at S^1 moment $t_i = 0$, source is coinciding with receiver and starts sending signals towards receiver along S^1 and stops sending signal at the moment source coinciding with milestone L (from receiver) in S^1 . Then,

$$\begin{aligned} L = v_i T_i & \rightarrow v_{r,0,i} \equiv \frac{n_r - 1}{T_i + \tau_i} = \frac{n_r - 1}{T_i} \frac{1}{1 + u} \\ = c_i \tau_i & \rightarrow \\ = v_r T_r & \rightarrow v_{s,u,r} \equiv \frac{n_s - 1}{T_r} = (1 + u) v_{r,0,i} \frac{\mathbb{U}_{t,i}}{\mathbb{U}_{t,r}} \end{aligned} \rightarrow \begin{matrix} v_{s,u,i} = (1 + u) v_{r,0,i} \\ v_{s,u,r} = (1 + u) v_{r,0,r} \end{matrix}. \quad (80)$$

T_i : Arrival time of source at milestone L according to S^1 clock. τ_i : Travel time of last signal sent

from source at L to arrive at receiver according to S^1 clock. T_i : Time taken for source to travel from receiver to milestone L according to source clock.

Suppose receiver is in uniform motion in S^1 with velocity $0 < |v_i| < c_i$ towards source at rest in S^1 . Suppose, at S^1 moment $t_i = 0$, internal distance between receiver and source is L and source starts sending signals towards receiver and stops sending signal at the moment receiver coinciding with source. Then, from perspective of S^1 ,

$$L = c_i \tau_i + v_i \tau_i = v_i T_i, \begin{matrix} v_{s,0,i} \equiv (n_s - 1)/T_i \\ v_{r,u,i} \equiv (n_r - 1)/(T_i - \tau_i) \end{matrix} \rightarrow \begin{matrix} v_{r,u,i} = (1 + u)v_{s,0,i} \\ v_{r,u,r} = (1 + u)v_{s,0,r} \end{matrix}. \quad (81)$$

τ_i : Arrival time of first signal (sent by source at rest) at receiver per S^1 clock. T_i : Arrival time of receiver at source according to S^1 clock. $v_{s,0,i}$: Frequency of signals as measured by source at rest in S^1 in S^1 time. $v_{r,u,i}$: Frequency of signals as measured by receiver in CRF in S^1 time. $v_{r,u,r}$: Frequency of signals as measured by receiver in CRF in CRF time. $v_{s,0,r}$: Frequency of signals as measured by source at rest in S^1 in CRF time.

Suppose instead, at S^1 moment $t_i = 0$, receiver is coinciding with source and source starts sending signals towards receiver along S^1 and stops sending signal at the moment receiver coinciding with milestone L (from source) in S^1 . From perspective of S^1 ,

$$L = c_i \tau_i - v_i \tau_i = v_i T_i, \begin{matrix} v_{s,0,i} \equiv (n_s - 1)/T_i \\ v_{r,u,i} \equiv (n_r - 1)/(T_i + \tau_i) \end{matrix} \rightarrow \begin{matrix} v_{r,u,i} = (1 - u)v_{s,0,i} \\ v_{r,u,r} = (1 - u)v_{s,0,r} \end{matrix}. \quad (82)$$

τ_i : Time taken for last signal sent by source at rest to arrive at receiver per S^1 clock. T_i : Arrival time of receiver at milestone L per S^1 clock.

In summary,

$$\begin{matrix} S_u \ \& \ R_0 \\ R_u \ \& \ S_0 \end{matrix} \cdot \begin{matrix} v_{s,u} = (1 \mp u)v_{r,0} \\ v_{r,u} = (1 \pm u)v_{s,0} \end{matrix}, \begin{matrix} - : S \rightarrow R \quad + : S \leftrightarrow R \\ + : R \rightarrow S \quad - : R \leftrightarrow S \end{matrix}. \quad (83)$$

S: Source. R: Receiver. u : Subscript indicating associated entity in motion. 0: Subscript indicating associated entity at rest.

That is, for geodesic motion of particle in S^m , if units involved in equation are consistent and velocity of motion expressed in reduced form, then Doppler Effect is of identical expression as that for translation motion of particle in E^n , regardless of perspective, setup for measurement, and definition of time and SLV.

20. Test of Atomic Clock in Geodesic Motion

As shown, inertial motion of atomic clock in finite space (along RSL of S^m) is absolute and shall cause time dilation of the clock while inertial motion of any clock in infinite space (along ESL of E^n) is relative and shall not cause time dilation of the clock. Therefore, test of atomic clock in straight-line motion (SLM) in PHS shall reveal geometric nature of PHS.

Currently, the most precise test of particle clock in SLM is optical-optical double resonance (OODR) spectroscopic experiment [23]. In such test, particle is accelerated to velocity comparable to local SLV. While particle is in high speed

motion along SL (approximately horizontal, ignoring gravitation effect of Earth and Sun to particle in motion) with respect to lab frame (approximated as RF), two laser beams are directed towards same section of the SL (approximately), one in same direction as that of motion of particle and the other in opposite direction. A tristate of particle is selected for such test, in which, transition of the particle between one relatively higher energy state and two relatively lower but energetically different states of same particle is allowed by rule of quantum mechanics. Accordingly, by tuning frequencies of the stationary lasers to meet the double resonance condition, OODR phenomenon of the particle is observed.

From Equation (63),

$$\Delta E_{1,s,u} = \Delta E_{1,s,0} \beta_u, \Delta E_{2,s,u} = \Delta E_{2,s,0} \beta_u, \beta_u \equiv \sqrt{1-u^2}, u \equiv \|\mathbf{u}\|, \mathbf{u} = \mathbf{v}_0/c_0. \quad (84)$$

$\Delta E_{1,s,u}$: Selfenergy difference between higher energy state and one lower energy state of particle in motion. $\Delta E_{1,s,0}$: Selfenergy difference between higher energy state and one lower energy state of particle at rest in lab. $\Delta E_{2,s,u}$: Selfenergy difference between higher energy state and the other lower energy state of particle in motion. $\Delta E_{2,s,0}$: Selfenergy difference between higher energy state and the other lower energy state of particle at rest in lab. v_0 : Velocity of particle along SL as measured in lab frame. c_0 : SLV as measured/defined at rest in lab.

From Equation (83),

$$v_r = (1 \pm u)v_s \rightarrow h_r v_r = (1 \pm u)(h_r/h_s)h_s v_s. \quad (85)$$

v_r : Frequency of photon as perceived by particle in motion with respect to laser at rest in lab. v_s : Frequency of photon as perceived by laser at rest in lab. h_r : Planck constant as measured at rest at particle location in CMF of particle. h_s : Planck constant as measured at rest in lab.

With LPE, under **Assumption 1**,

$$E_{p,r} = h_r v_r = (1 \pm u)(h_r/h_s)E_{p,s} = (1 \pm u)E_{p,s}. \quad (86)$$

$E_{p,r}$: Energy of photon as absorbed by particle in motion with respect to laser at rest in lab. $E_{p,s}$: Energy of photon as measured by laser at rest in lab.

Therefore, condition for double resonance absorption is

$$\begin{aligned} E_{p,r,a} = \Delta E_{1,s,u} &\rightarrow (1+u)E_{p,s,a} = \sqrt{1-u^2} \Delta E_{1,s,0} \\ E_{p,r,b} = \Delta E_{2,s,u} &\rightarrow (1-u)E_{p,s,b} = \sqrt{1-u^2} \Delta E_{2,s,0} \end{aligned} \quad (87)$$

$E_{p,r,a}$: Photon energy as measured by particle in motion towards photon. $E_{p,r,b}$: Photon energy as measured by particle in motion away from photon. $E_{p,s,a}$: Photon energy as measured in Lab for laser beam in opposite direction of particle motion. $E_{p,s,b}$: Photon energy as measured in Lab for laser beam in same direction of particle motion.

Therefore,

$$\frac{\Delta E_{1,s,0} \Delta E_{2,s,0}}{E_{p,s,a} E_{p,s,b}} = \frac{v_1 v_2}{v_a v_b} = 1. \quad (88)$$

v_1 : Frequency of photon of transition of $\Delta E_{1,s,0}$ of particle at rest in lab. v_2 : Frequency of photon of transition of $\Delta E_{2,s,0}$ of particle at rest in lab. v_a : Resonance frequency of laser "a" as measured

at rest in lab. ν_b : Resonance frequency of laser “b” as measured at rest in lab.

The OODR experiment [23] reported that, for lithium ion with selected trisate, at $u \approx 0.338$,

$$\begin{aligned} \nu_1 = 546\,455\,143.0 \text{ MHz} \quad \nu_a = 384\,225\,534.98 \text{ MHz} \quad \sqrt{\nu_a \nu_b / \nu_1 / \nu_2} - 1 \\ \nu_2 = 546\,474\,960.7 \text{ MHz} \quad \nu_b = 777\,210\,326.98 \text{ MHz} \quad = (1.5 \pm 2.3) \times 10^{-9}. \end{aligned} \quad (89)$$

OODR test of such level of precision demonstrated that the SL is not ESL but RSL, *i.e.*, curvature of section of the SL as measured in the lab on Earth is non-zero. Therefore, by CHI, extent of PHS must be finite.

In contrast, if PHS were of infinite extent then trajectory of free particle (approximated in the test by the ion at onset of interaction with the photons) must be a section of an ESL (ignoring gravity). Therefore, CMF of the particle at onset of particle-photon interaction must be an IRF. Therefore, selfenergy of the particle, hence selfenergy difference of same, and Planck constant as measured at rest at location of particle in the IRF must be identical/one and same/invariant to that as measured at rest in rest frame (approximated by lab frame). Therefore, the test and the setup would be identical to, indifferent from, indistinguishable with that for measurement of Doppler Effect of free particle in TFM in infinite space. Accordingly, double resonance absorption condition would become

$$\begin{aligned} (1+u)E_{p,s,a} = \Delta E_{1,s,0} \\ (1-u)E_{p,s,b} = \Delta E_{2,s,0} \end{aligned} \rightarrow \frac{\Delta E_{1,s,0} \Delta E_{2,s,0}}{E_{p,s,a} E_{p,s,b}} = \frac{\nu_1 \nu_2}{\nu_a \nu_b} = 1 - u^2 \approx 0.886 \neq 1. \quad (90)$$

Therefore, the SL as measured in the OODR test was not ESL but RSL. Therefore, PHS must be finite.

From Equations (86) and (88), and Expression (89),

$$\frac{h_r}{h_s} = \sqrt{\frac{\Delta E_{1,s,0} \Delta E_{2,s,0}}{E_{p,s,a} E_{p,s,b}}} = \sqrt{\frac{\nu_1 \nu_2}{\nu_a \nu_b}} \rightarrow \frac{h_r}{h_s} = 1 + (1.5 \pm 2.3) \times 10^{-9}. \quad (91)$$

That is, up to the precision of the test, the OODR experiment also validated **Assumption 1** on invariance of Planck constant to state of motion of setup in finite space for measurement of h . That is, Planck constant is indeed SIT regardless of state of motion of setup for measurement of such, up to precision of the test.

The frequencies measured in the test, as expressed in Expression (89), were determined with frequency standard, *i.e.*, atomic clock. However, duration of unit of AT, hence frequency of frequency standard, is not SIT but function of state of the clock defining the AT. Therefore, if the frequencies were not measured all at same location in same time then unit of the AT associated with the frequencies may not be cancelled out from Expression (89) since, if not generated at same place in same time, *second* and *second*, hence MHz and MHz ($10^6 \text{ s}_{\text{lab}}^{-1}$), may not have identical duration. Therefore, systematic errors may be introduced as result of frequency measurements at different location/time due to differences in state of frequency standard used thereat/in. For instance, relative variation of frequency of atomic frequency standard due to annual variation of Earth-Sun distance is $\pm 6.6 \times 10^{-10}$, that due to daily variation of lab velocity

caused by spin of Earth is $\pm 6 \times 10^{-11}$ maximal (pending on latitude of lab on Earth), etc. Such is suspected to be partial cause for the 0.81 MHz bias underlying the frequencies reported. In other words, further improvement of precision of the OODR test by another order of magnitude is possible with even more meticulous frequency measurement and rest particle experiment.

21. Atomic Clock in Circular Motion

Particle motion in non-geodesic circle in \mathbf{S}^m can be visualized as motion of particle in \mathbf{S}^2 , which is 2-sphere, *i.e.*, surface of 3-ball of external radius identical to that defining \mathbf{S}^m . All points of \mathbf{S}^2 can be coordinated in terms of latitude and longitude. Accordingly, circle in consideration can be viewed as a set of all points of \mathbf{S}^2 having equal latitude.

Centrifugal force associated with motion of particle in circle is along direction of external radius of the circle. Therefore, projection of such force in \mathbf{S}^2 is non-zero since, by specification, circle in consideration is not geodesic. Therefore, such force is perceptible from within \mathbf{S}^2 . If not counterbalanced, such force shall drive particle towards suitable geodesic on spot (infinite number of geodesics are available at each and every point of \mathbf{S}^2). Therefore, in \mathbf{S}^2 , and \mathbf{S}^m in general, particle tends to move along geodesic. Accordingly, real and physical constraint or confinement is necessary for particle to maintain in circular motion. Aside from said real, physical confinement, motion of particle in circle is identical/indifferent to that in geodesic. Therefore, from Equation (55), (62), and (68), by LME, under CAT,

$$\begin{aligned} E_{i,u} &= \beta_u^{-1} E_{i,0}, & E_{s,u} &= \beta_u^{+1} E_{i,0}, & c_{s,AT} &= \beta_u^{+1} c_{i,AT} \\ m_{i,u} &= \beta_u^{-1} m_{i,0}, & m_{s,AT} &= \beta_u^{-1} m_{i,0}, & \mathbb{U}_{AT,s} &= \beta_u^{-1} \mathbb{U}_{AT,i} \end{aligned} \quad (92)$$

$E_{i,u}$: Total energy of particle in motion in \mathbf{S}^2 as measured at rest in RF. $E_{i,0}$: Restenergy of particle as measured at rest in RF. β_u : Lorentz Factor for particle in motion in \mathbf{S}^2 . $m_{i,u}$: Total mass of particle in motion in \mathbf{S}^2 as measured at rest in RF. $m_{i,0}$: Restmass of particle as measured at rest in RF. $E_{s,u}$: Selfenergy of particle in motion in \mathbf{S}^2 as measured at rest at particle location in CMF of particle. $m_{s,AT}$: Selfmass of particle in motion in \mathbf{S}^2 as measured at rest at particle location in CMF of particle under CAT. $c_{s,AT}$: SLV defined on AT as measured at rest at particle location in CMF of particle. $c_{i,AT}$: SLV defined on AT as measured at rest in RF. $\mathbb{U}_{AT,s}$: Unit of SAT of particle in motion in \mathbf{S}^2 . $\mathbb{U}_{AT,i}$: Unit of RAT.

22. Test of Atomic Clock in Circular Motion

Consider a pair of identical clocks, labeled as A and B respectively. Suppose A is at rest at location \mathbf{x} of RF and B is in uniform circular motion (β_u of B being invariant to location and RT) with respect to a point of RF. Suppose B shall meet with A at \mathbf{x} during its journey, regarded as an event. Then, B shall meet with A at \mathbf{x} again upon completion of each rotation (if starts from \mathbf{x}), regarded as another event. Denote T as duration between any two consecutive events at \mathbf{x} , *i.e.*, A and B meeting with each other to A and B meeting with each other again.

Such pair of the events is referred to herein as *the* events. Then,

$$T_A = n_A \mathbb{U}_{t,A}, T_B = n_B \mathbb{U}_{t,B}. \quad (93)$$

T_A : Time passage of *the* events as measured by A . T_B : Time passage of *the* events as measured by B .
 n_A : Counts of ticks during *the* events as read from A . n_B : Counts of ticks during *the* events as read from B . $\mathbb{U}_{t,A}$: Unit of time of Clock A . $\mathbb{U}_{t,B}$: Unit of time of Clock B .

Since the T herein stands for “duration between *the* events” then T is and is always referring to the one and same thing, *i.e.*, “duration between *the* events”, regardless of how and by what the T is measured. Therefore, T_A and T_B must refer to the one and same entity, T ,

$$T_A = T = T_B \rightarrow n_A \mathbb{U}_{t,A} = n_B \mathbb{U}_{t,B} \rightarrow n_B = n_A \mathbb{U}_{t,A} / \mathbb{U}_{t,B}. \quad (94)$$

Therefore, if A and B are atomic clock of same type then, from Equation (92),

$$n_B = \beta_u n_A, 0 < \beta_u < 1 \rightarrow n_B < n_A. \quad (95)$$

That is, counts (number of ticks) as read from clock B (in motion) shall be lesser than that from clock A (at rest) for the one and same T , *i.e.*, passage of the time between *the* events. Such phenomenon is commonly known as motion induced time dilation and described as, *e.g.*, clock in motion runs slower, time slows down in motion, etc. However, the passage of the time, T , *i.e.*, “duration between *the* events”, is neither longer nor shorter but exactly “duration between *the* events”. Therefore, in preserving logical integrity, the T is not and cannot be altered by measurement of the T . On the other hand, tool for measurement may be affected by state of motion of tool in measurement. From Equation (94), it is the unit of the time defined by the clock that is altered, due in this case to state of motion of the clock. Therefore, “time dilation” is synonymous with “temporal dilation of unit of time”, *i.e.*, duration of unit of time being/becoming longer in comparison to that chosen as reference for time comparison.

Phenomenon of motion induced time dilation was predicted by SRT, as necessity in maintaining constancy of SLV in motion. In NMFS, such is due to alteration of selfenergy of particle caused by absolute motion of particle in curved trajectory (with respect to ESL) hence alteration of duration of unit of SAT of CDP under **Assumption 1**. Despite the difference in physical origin and/or interpretation of the phenomenon, Lorentz Factor from SRT and NMFS are identical in form,

$$\beta_{\text{SRT}} \equiv \sqrt{1 - (v/c)^2} = \sqrt{1 - (r\omega/c)^2} \equiv \beta_u. \quad (96)$$

A test of conceptual simplicity hence clarity for rotation induced time dilation of AT is the Mössbauer rotor experiment [24]. In such test, a gamma ray source is placed at center of a rotor, a matching absorber on rim of the rotor, and gamma ray detector behind absorber. Reversal of the configuration is equivalent in effect, *i.e.*, source can be placed on rim of rotor and absorber/detector at the center. Due to Mössbauer effect [25], *i.e.*, suppression of recoil of particle by lattice interaction, high-energy photon such as gamma ray can be emitted/absorbed with narrow line width. Thus, if rotor is not rotating, less number of photons

shall penetrate through absorber due to resonance absorption of photons by matching absorber. If rotor is spinning, selfenergy difference of states of nucleon involved in transition process of photon emission/absorption of entity in rotation shall be reduced by a factor of β_u in comparison to that of entity at center of rotor (approximate rest state). Therefore, if absorber is rotating with respect to source at rest, photon emitted by the source shall be more energetic than corresponding selfenergy difference of absorber; if source is rotating with respect to absorber at rest, photon emitted by the source shall be less energetic than corresponding selfenergy difference of absorber at rest. In quantitative measurement, mechanical drive is installed with either the source or the absorber, causing relative translation motion between source and absorber. If Doppler Effect induced by the mechanical modulation compensates for the alteration of selfenergy difference of entity caused by the rotation, photon signal at detector is minimized. Resonance condition of such test is, if source is at center of rotor with mechanical modulation,

$$E_{p,r} = \Delta E_{s,r} \rightarrow \frac{\Delta E_{s,i}}{1 + u_M} = \sqrt{1 - u_\omega^2} \Delta E_{s,i} \rightarrow u_M = \frac{1}{\sqrt{1 - u_\omega^2}} - 1. \quad (97)$$

$E_{p,r}$: Photon energy as measured by absorber in rotation. $\Delta E_{s,r}$: Selfenergy difference of the transition of absorber in rotation. $\Delta E_{s,i}$: Selfenergy difference of the transition of source/absorber at rest. u_M : Magnitude of reduced resonance modulation velocity of source at center. u_ω : Magnitude of reduced rotation velocity of absorber.

Given the test parameters of

$$s = 9.3 \text{ cm}, \omega = 35000 \text{ rpm} \rightarrow v_M \approx 1.94 \times 10^{-8} \text{ m s}^{-1}. \quad (98)$$

s : Distance between center of rotation and absorber on rim. v_M : Resonance modulation velocity of source at center of rotation.

Reported value for the resonance modulation velocity was $1.95 \times 10^{-8} \text{ m s}^{-1}$ [24].

The Kündig experiment demonstrated that, up to precision of the test, curved motion of object does cause alteration of selfenergy of the object. Therefore, time dilation shall occur to unit of SAT of particle in curved motion, under **Assumption 1**. Nevertheless, the test did not validate nor falsify **Assumption 1** since Planck constant is not involved in such test.

23. Summary and Discussion

NMIS assumed *a priori* that PHS is of infinite extent. One unique property of infinite space is the translation symmetry that, in conjunction with force-central scheme and state-invariant mass, leads to relativity of inertial motion of entity therein. Accordingly, superposition of velocity of motion in infinite space is and must be vector additive. As a consequence, no speed limit is permissible on motion of object in infinite space. On the other hand, in conjunction with CHI of space, LME imposes finite speed limit on motion of object in PHS. SRT inherited the relativity principle of NM [26] without altering the underlying assumption of NM but attempted resolving the conflict by enforcing space and time dilations

on inertial motion of object in infinite space. As analyzed herein, however, such dilations do not and cannot occur to object in inertial motion in infinite space. Therefore, the conflict was not really resolved then and is irresolvable without abandoning the infinity hypothesis on PHS. Therefore, SRT is internally incoherent and self-contradicting even though LME was leapt forwarded therefrom.

LME causes fundamental alterations to NM that mass of object is no longer SIT. Under LME, in conjunction with LEC, LPE, etc., it is shown that (1) PHS is not infinite; (2) motion in finite space is not relative; (3) SLV in motion in PHS is not SIT under CAT; (4) time dilation of atomic clock in inertial motion is caused by finiteness of PHS.

The phenomenon of time dilation of AT reveals a scientific fact, which was unnoticed or otherwise unnoticeable, that duration of unit of time may not be genuine constant but function of state of clock defining the unit. In general, unit of physical attribute may be conditioned on state of the unit instead of being genuine SIT. In contrary to this fact, however, constancy of unit of physical attribute, *i.e.*, invariance of unit of attribute on state of the unit, time included, has always been **assumed a priori** in all disciplines of natural science, metrology included, since the beginning of quantitative science, physics included. As shown herein, physical effect of unit dilation/contraction may be minuscule but impact of such on metrological ground of foundation of physics may not be as minute, and results of this analysis are but tips of the iceberg.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Appendix A. Rule of Numeration

Assertion A: Any numerable attribute can be expressed as

$$a \equiv n_a \mathbb{U}_a. \quad (\text{A.1})$$

a : Numerable attribute. n_a : Numeral aspect of a . \mathbb{U}_a : Unit of a .

Examples of numerable attributes in physics include, but not limited to, *length, time, velocity, mass, energy, momentum*, etc. Conversely, if an entity can be expressed in form of Expression (A.1) then the entity is referred to as numerable attribute or attribute in short.

Rule A: Unit of attribute cannot vary with that attribute (or consistency of numeration tarnished),

$$\frac{d\mathbb{U}_a}{da} = 0. \quad (\text{A.2})$$

From Expression (A.1),

$$da = \mathbb{U}_a dn_a + n_a d\mathbb{U}_a \rightarrow 1 = \frac{da}{da} = \mathbb{U}_a \frac{dn_a}{da} + n_a \frac{d\mathbb{U}_a}{da}. \quad (\text{A.3})$$

By Equation (A.2),

$$1 = \mathbb{U}_a \frac{dn_a}{da} \rightarrow da = \mathbb{U}_a dn_a \equiv dn_a \mathbb{U}_a. \quad (\text{A.4})$$

Numeral is also an attribute and unit of the attribute is one.

Appendix B. Time

To understand *time*, consider observing a physical point in physical world. Thus, if no event happens at the point then nothing happens there. If one event happens there then there happens one event regardless of what the event may be. If more than one event happens there then all the events can be ascribed to two and only two types: simultaneous and not-simultaneous. If event A and B are simultaneous events, then A and B happen simultaneously (at same point), and vice versa. Further, if event B and C are also simultaneous events, then A and C are also simultaneous events, and so on. It is self evident that any event is simultaneous with respect to itself.

If event A and B are not-simultaneous events then A and B do not happen simultaneously but do happen at same point. Therefore, event A (or B) must happen before/after B (or A). The descriptor before/after is a single term of relative character, *i.e.*, pending on perspective/context. That is, if event A is ascribed as before B then it is of same meaning as ascribing B as after A ; if A is said as after B then it is of same meaning as saying B is before A . Therefore, if event A (or B) happens not-before/after B (or A), *i.e.*, neither before nor after, then A and B are simultaneous events.

The pair of the descriptors, before/after and not-before/after (synonymous with simultaneous), is sufficient in capturing and describing one aspect of events in physical world known as temporal order of events. In other words, events are

of temporal order, which is either before/after or not-before/after. With such pair of descriptors alone, it suffices to describe temporal order of any plurality of any events regardless of what events that may be, as long as all events are local, *i.e.*, happen at one and same point. Although cumbersome and inefficient, such manner of describing temporal order of events is nevertheless true and accurate.

Among all possible events, natural or artificial, real or imaginary, there is a particular category of events known as recurring events, referring to set of identical events, or perceived, regarded, defined, or approximated as such, that happens and/or can happen not-simultaneously. Recurring events can be any recurring phenomenon, or perceived, regarded, defined, or approximated as such, occurring in nature or generated by artificial device, mechanism, or setup. Recurring events are and/or can be used as temporal marker in facilitating description of temporal order of events. In such use, any event is ascribed either as before/after or not-before/after with respect to recurring event (event mark). For such use, recurring phenomenon or events generator is known as clock and corresponding event clock event, wherein, clock is understood as local clock and event local event. Although not quantitative, such manner of describing temporal order of events, *i.e.*, in line with event mark/mark event, is less cumbersome and more efficient in comparison with that mentioned above while retaining same truthfulness and accuracy of the description.

On the other hand, whether occurring naturally or setup artificially, clock does enable quantitative description of temporal order of events via quantification of temporal order of events. For such purpose, temporal order of any pair of events is referred to as temporal duration of the pair or duration (of the events) in short, and duration of any consecutive pair of clock events or assigned plurality thereof is defined/assigned as unit of duration. Accordingly, duration of any pair of events becomes numerable entity in terms of unit of duration hence a physical attribute by definition of attribute (cf. **Appendix A**) known commonly as *time*. In such manner, duration of any pair of events becomes measurable and expressible in quantity with respect to unit of duration. For instance, simultaneous events can be expressed as events of zero unit of duration among themselves (while happen at same point); before/after of events are quantified as plurality/fraction/combination thereof of unit of duration defined hence comparable in quantitative manner; when, moment, long/short, fast/slow, etc., can all be expressed and described economically. While retaining the same truthfulness and accuracy, such manner of describing temporal order of events is most compact and efficient in comparison with other alternatives mentioned above. Another advantage of such approach is that recognition of pattern among events, if any, may become easier.

Therefore, *time* is a metrological construct providing background and reference events (event marks, mark events) for purpose of capturing and describing hence processing information on temporal order of events of physical world. In essence, *time* is but sequence of recurring events having no other relationship among themselves except before/after and no other property except locality of

clock and definition/assignment of unit of *time* as duration between any consecutive clock events of same or aggregation thereof.

Any recurring event can be regarded as clock event to define corresponding time. For instance, moment of minimum length of shadow of sunlight of an object located somewhere on Earth surface may be taken as clock event to define unit of such time there, *i.e.*, duration between one such moment and next recurring moment before/after may be defined as one unit of such time, named as, e.g., sun-day. Or, moment of full Moon showing at horizon of somewhere may be viewed as clock event to define unit of such time there, *i.e.*, duration between one such moment and next recurring moment before/after may be assigned as one unit of such time, tagged as, e.g., moon-month. Or, moment of some star pattern seen at local zenith may be chosen as clock event to set unit of such time there, *i.e.*, duration between one such moment and next recurring moment before/after may be set as one unit of such time, labeled as, e.g., earth-year. Or, moment of voltage across a LC circuitry measured as zero volt while polarity of the voltage is changing from negative to positive may be assigned as clock event in defining unit of such time, *i.e.*, duration between one such moment and next recurring moment before/after may be agreed upon as one unit of such time, called oscillator-time, and assigned aggregation of such duration in consecutive order may be called as, e.g., oscillator-second; and so on.

Although it is arbitrary in appointing clock and assigning unit of corresponding time, basic rule of metrology dictates that once clock is chosen and unit of time defined then it is so chosen/defined and must be retained as such in preserving logical and metrological integrity. That is, no other clock and/or unit of time can be defined/assigned unless entities involved are one and same. For instance, if time is defined on basis of LC oscillator then sun-day can no longer be appointed as unit of time and duration of sun-day cannot be assigned but only measured, by oscillator-time, and there is no guarantee for identicalness of duration of this sun-day and that sun-day in terms of oscillator-second, and vice versa. Likewise, if time is defined on basis of sundial then moon-month can no longer be appointed as unit of time and duration of moon-month cannot be assigned but only measured, by sun-day, and there is no guarantee for identicalness of duration of this moon-month and that moon-month in terms of sun-day, and vice versa; and so on.

On the other hand, for any time chosen/appointed/agreed upon, duration of unit of the time is guaranteed being identical from the time to the time and for all the times, not by measurement, computation, LOP, or anything else but definition of the unit of the time. Such aspect of such subject may appear counterintuitive but is a consequence of logical and metrological consistency that is the basic prerequisite at most fundamental level for any quantification of any entity, as reflected in the basic rule of numeration (cf. **Appendix A**). Therefore, identicalness by definition is an inevitable price paying for quantification, *time* included, and economical benefits such brings but may be significant, if not the

biggest, approximation in mathematical handling of physical affairs.

While arbitrary in choosing time and unit thereof, there is property of *time* that is common to any and all times: unidirection and continuity. If event *A* happens after event *B* then event *A* happens after event *B* regardless of nature of the event, even if event *A* may be identical in any and all other aspect to any and all other details to event happens before or not-after *B*. Therefore, at any moment of any time, there are only two potential outlooks at the most: event happens or not. If event does happen then it must happen after the moment regardless of nature of the event, including recurring event. If no event happens nor recurring event after the moment then state of the affair is identical in any and all aspect to any and all details to the moment hence is the moment by definition of the moment. Therefore, in logical reality, there is one and only one outlook at any moment of any time. In other words, no event shall happen after any moment of any time until happenstance of an event, regardless of nature of the event, recurring or otherwise. This aspect of *time* is commonly known as unidirection of *time*. Further, no event shall happen after any moment of any time before happenstance of an event. In other words, there can be no event happens/happening inbetween any consecutive events or otherwise logical integrity would be compromised. This aspect of *time* is referred to as continuity of *time*. Therefore, in addition to locality, *time* is unidirectional and continuous. However, such aspects are not due to property of nature or LOP or anything else but mere logical consistency requirement imposed thereupon.

Due to arbitrariness in defining *time*, quality of *time* cannot be judged by its properness, correctness, truthfulness, or alike but only by goodness. While unquantifiable, good *times* are those that might make recognition of pattern among phenomena easier and/or relationship between entities simpler and/or LOP subtler. Judged by such criteria and comparing to other alternatives available, AT, *i.e.*, time defined on atomic clock, or in general, particle clock, *i.e.*, those that are based on principle of quantum transition of particle between selfenergy states involving photon, may be a good choice, for at least such would make energy-time relationship more transparent and Planck constant less mystical. However, state dependency of unit of AT, if any, shall cause LOP to become less simple.