

# Mathematical and Numerical Modelling of Copper Tube Extrusion after Optimizing Geometrical and Operating Parameters

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## Abstract

This study aimed to optimize cross-sectional area of the extruder and the flow rate of the material through the extruder. Extrusion process was first modeled using the continuum mechanics method before being simulated with the commercial software LS-DYNA using the Johnson-Cook model based on the finite element method after optimization of the factors with simplex algorithm. The study was carried out on a connecting rod: diameter of 145 mm, cross-section of 1.65 m<sup>2</sup>, length of 0.25 m, volume of 0.41 m<sup>3</sup> and a mass of 37 kg. Optimum parameters obtained were temperature of 850°C, flow rate of 0.237 m/s, die diameter of 19.2 mm, pin diameter of 14 mm, extrusion strength of 565.6 kN and press pressure of 245 GPa. From these factors, a blank was obtained with external and internal diameters of 19.2 and 14 mm respectively over a length of 28 m with a thickness of 2.6 mm. Simulation using LS-DYNA software resulted in a difference in values of 7.4% between optimization and simulation when the difference of 2.4% was found between modelling and simulation. These values make the process optimal for industrial application to improve the extrusion requirements of copper tubes.

## Keywords

Copper Extrusion, Mathematical Modelling, Numerical Simulation, Simplex Algorithm, LS-DYNA Software

## 1. Introduction

The manufacturing of copper tubes is a process that requires several steps in a manufacturing chain leading to a final product that meets the customer's requirements. Extrusion, drawing and thermal treatment are the most important of these [1]. To be able to satisfy customers, the extrusion process should achieve the industrial standard of high quality and efficiency. These criteria are impacted by the extrusion process selected, the shape of the die and the manufacturing tolerances.

Extrusion can be used to produce bars, solid or shaped sections, tubes, wires, and rods. The extruded product quality depends on different factors such as press, materials, equipment, and metal temperature; that lead to a careful choice of the extrusion technique to be applied, which could be direct extrusion, indirect extrusion, hydrostatic extrusion, conforming process, impact extrusion or through extrusion [1]-[6].

Metal extrusion depends on several factors: materials, flow, temperature, press [1] [5] [7] [8] [9]. In addition, the choice of die shape to be used will depend on the profiles to be manufactured: mandrel and core die or finned die (tubes), screw die (tubes, cables, films), jacket-type die (tubes, cables) [2] [6] [7] [8] [9].

Most significant extrusion defects during tube manufacture are oxides (due to the passage of the billet through the press, the surface of which is coated with oxides, which become locked into the structure of the section. This generates areas where the material becomes more delicate and consequently has a negative impact on the mechanical properties of the material), cracks (which occur when the extrusion speed is too high and the temperature is too high, or when the friction involved is too high), peeling defects (caused by the accumulation of waste on the internal surface of the die) and central defects (caused by extrusion with a non-uniform flow of the S or A type) [2] [3] [6] [10] [11] [12].

The control of the previously mentioned factors is very delicate at the industrial scale, which is the reason why the Gécamines Laminoirs et Câbleries (LC/GCM) has launched this research, which allows the improvement of the efficiency of the copper tube extrusion process, whose main aim is to propose a schema of homogenisation of the flow speed distribution of the materials at the exit of the dies by the control and optimization of the operative and geometrical parameters. For this purpose, the optimization using the non-deterministic method by the simplex algorithm, the modelling using the continuum mechanics model and the numerical simulation using the commercial software LS-DYNA based on the finite element method using the Johnson-Cook model have been done [2] [11]-[21].

## 2. Material and Methods

### 2.1. Raw Materials

The raw material in this study is a copper war bar of 99.86 wt% purity from Usines Chimiques de Shituru/Gécaminesgroupe centre (US/GCM) and manu-

factured into a rod at LC/GCM. Chemical analysis of the material was carried out by atomic absorption spectrometry in the chemical analysis EMT Gecamines laboratory.

## 2.2. Rods Preparation

Rods are prepared from a 120 kg war bar, which has been separated into three parts of 40 kg each. The resulting portions are then pre-heated in a diesel furnace at 850 °C for two hours and then pressed into a 150 mm diameter container to give rod round shape. The rod is subsequently processed to remove the oxide layer and reduce its diameter to 5 mm and its mass to 3 kg. The equipment used for this purpose was: a saw, a diesel furnace, a 750-ton press, a receiver, a pointing device, a die, a die holder, studs, a counter-die, a pin, an extrusion disc or pad, a spanner, a pair of pliers, a scale, a calliper, a tape measure.

## 2.3. Extrusion Tests

Extrusion tests of the billets were carried out in the laboratory of LC/Gecamines Groupe Ouest. They have consisted of billet pre-heating in a diesel furnace up to the temperature of 850 °C, loading the billet in the receiver to evaluate the flow speed of the material according to the formula (1).

$$v = (2p/\rho)^{1/2} \quad (1)$$

where  $\rho$  is the copper density and  $v$  is the flow rate.

At the end of the test, the quality control of the resulting blank is performed by:

- Sizing the billet (diameter:  $145 \pm 2$  mm, cross-section:  $1.65 \pm 0.01$  m<sup>2</sup>, length:  $0.250 \pm 0.001$  m, volume:  $0.41 \pm 0.01$  m<sup>3</sup> and mass of the billet: 37 kg).
- Calculation of the ratio between sections  $R = \frac{A_0}{A_1}$  (2) where  $A_0$  and  $A_1$  are the sections of the billet and the blank respectively.
- Calculation of the extrusion force  $F = \frac{p \times A_0}{\ln R}$  (3);  $p$  is the pressure applied by the press [22].

## 2.4. Extrusion Optimization

The extrusion die and pin will determine the external and internal diameter of the blank. Thus, three factors influence their choice: the billet pre-heating temperature of 850 °C (taken as constant in this study), the pressure supplied by the press of 245 GPa (constant in this study) and the material flow speed.

The optimization method chosen is the non-deterministic method. It is a stochastic method of the simplex algorithm with the objective of optimizing the die section and the material flow rate [2] [23] [24] [25] [26].

At each iteration, a set of points is generated, by eliminating the point corresponding to the highest value of  $z$  and determining a new point using the reflection operation. The position of this point is then modified by contraction or ex-

pansion depending on the value of  $z$  at that point. When the optimal value is achieved and no previous operation is satisfactory, the points are contracted in all directions, reducing the search area [27] [28] [29] (Table 1).

**Table 1.** Input parameters of the algorithm.

Mass ( $m$ )	34
Die cross-section ( $X$ )	$\geq 73.005$
Section C	$(m/X) = 250$
State function ( $Z = \text{pressure}$ )	$\lim Z = X/m = 250$

### 2.5. Mathematical Modelling of Extrusion

To carry out the mathematical modelling of the extrusion of copper tubes, the approach adopted was as follows:

- 1) The further statements [2] [30] [31]:
  - The material (copper) used is perfectly bonded, any chemical or physical transformation of the material is ignored. The internal energy is dependent on the absolute temperature  $T$  ( $e = e(T)$ ) via the specific heat capacity  $c$  of the material.
  - The volume rate term of the received heat ( $r \cong 0$ ) is negligible. The energy creation is defined by the mechanical work rate.
  - The deformation of the billet is uniaxial. Since the material under consideration is isotropic, the deformation tensor is equal to the value of the deformation.
  - The material flow is in a permanent flux.
  - Copper heated to  $850^\circ\text{C}$  is viscous, it acts like an incompressible fluid.

$$\left\{ \begin{array}{l} \nabla \cdot (2\eta(\dot{\gamma}) \dot{\epsilon}(v)) - \nabla p = 0 \\ \nabla \cdot \bar{v} = 0 \end{array} \right. \quad (4)$$

$$\left\{ \begin{array}{l} \rho \hat{c} \frac{dT}{dt} = -\nabla \cdot q + \sigma : \dot{\epsilon}(v) \end{array} \right. \quad (6)$$

$$\dot{\gamma} = \sqrt{2 \sum_{i,j=1}^3 \dot{\epsilon}_{ij}^2} \quad (7)$$

With:

$\hat{c}$  : expansion factor

$\eta$  : the medium viscosity

$\dot{\epsilon}$  : the tensor of strain rates

$\dot{\gamma}$  : is the generalized shear rate

$T$ : the temperature in celcius

$v$ : the flow of the material

$p$ : the pressure

$q$ : the heat flux

The heat flux can be determined by Fourier's law:

$$q = k \frac{A(T_1 - T_2)}{L} \quad (8)$$

With:

$k$ : thermal conductivity. For copper,  $k = 388$  W/mK

$A$ : transferred surface (m<sup>2</sup>)

$L$ : length.

## 2) The Law of Material Performance

It is experimentally established that molten metals do not have Newtonian behaviour: their viscosity is not constant but changes with the shear rate. The molten copper is taken as a plastic ideal fluid whose coefficient will be determined by the following formula [31]:

$$\tau = \tau_0 + \eta \left( \frac{\Delta V}{\Delta h} \right) \quad (9)$$

With:

$\tau$ : shearing stress

$\Delta V$ : increase in particle velocity

$\Delta h$ : fluid film thickness

## 2.6. Numerical Modelling of Extrusion

To carry out the numerical modelling of the extrusion of copper tubes, the main considerations are as follows [32] [33] [34]:

### 1) Assumptions

- The billets used for extrusion have a round shape.
- The modeling is performed on the LS DYNA finite element software.
- The material is viscoplastic, *i.e.* a thermomechanically coupled law of behaviour.

### 2) Geometry

The copper billet is round with the dimensions of  $145 \pm 2$  mm in diameter,  $1.65 \pm 0.01$  m<sup>2</sup> in cross-section,  $0.25 \pm 0.001$  m in length and  $0.41 \pm 0.01$  m<sup>3</sup> in volume. It had a mass of 37 kg and was then heated to a temperature of 850 °C.

### 3) The following material behaviour law:

The Johnson-Cook model was used to define the performance of the material during the extrusion of the billet, it is frequently used in hot forming, particularly in compression and forging, and offers an excellent correlation with the experimental data [12] [13]-[31]:

With:

$A$ : the average flow stress (MPa)

$B$ : the work-hardening modulus (MPa)

$n$ : the work hardening coefficient

$C$ : the coefficient of strain rate sensitivity

$\dot{\epsilon}_0^p$ : the reference strain rate (s-1)

$\epsilon^p$ : the plastic strains

$\dot{\epsilon}^p$ : the strain rate (s-1)

$T$ : the temperature of the deformed billet (°C)

$T_{\text{limit}}$ : the ambient temperature of the material (°C)

$T_f$  : material melting temperature.

4) Limit conditions:

The limit conditions are such that:

- The die is recessed.
- The die penetrates the billet prior to the extrusion operation for the inner diameter of the blank.
- The extrusion of the billet is carried out at temperature 850°C and the operation is carried out in an adiabatic environment.

5) Finite element model:

The tube starts its stroke outside the dies. A parallel displacement to the die axis is specified at the nodes at the end of the tube. The displacement is a function of the percentage reduction and the simulation time. However, for all simulations, the time is one second in order to simplify the calculations [31] [32].

### 3. Results and Discussion

#### 3.1. Chemical Composition

The chemical composition of the war bar is shown in **Table 2**.

#### 3.2. Extrusion Tests

The aim of the billet extrusion was to manufacture a tube blank that could be used to produce a tube during cold drawing with a dimension of 17/14 mm [22]. The results of the extrusion tests are given in **Table 3**.

As the press force is a function of die diameter and billet temperature, the smaller die diameter and billet temperature, the greater the force. After the extrusion process, the blank obtained is of  $D_{\text{ext}} = 54 \pm 2$  mm,  $d_{\text{int}} = 48.4 \pm 2$  mm and thickness of  $2.8 \pm 0.1$  mm when applying an extrusion force of 493.52 kN.

The extrusion temperature of 850°C resulted in easier billet flow and deformation. However, the maximum extrusion rate is reduced because of the high

**Table 2.** Chemical composition of the billet.

Elements	% Cu	ppm Co	ppm Fe	ppm Pb	ppm Ni	% S	% Si
Mass contents	99.86	15	42	457	<10	<0.01	<0.04

**Table 3.** Extrusion test results.

Billet mass (kg)	34
$D_{\text{external}}$ (mm)	54
$d_{\text{interior}}$ (mm)	48.4
Thickness (mm)	2.8
Section (m <sup>2</sup> )	450.15
Length (m)	8.51
$\varepsilon$	8.51
Extrusion force (kN)	493.515

temperature peaks which could initiate melting of the billet.

The extrusion force is dependent on the billet temperature and the section ratio. The latter gave us a value of 493.5145 kN for a ratio of 3700.

### 3.3. Extrusion Optimization

Several factors were optimized while keeping the same operating mode [23]:

- The diameter of the die which was obtained as a function of the pressure.
- The flow rate of the material was optimized by considering a permanent flow.

The results of this optimization are reported in **Table 4**.

The tooling optimization, using the non-deterministic method with the simplex algorithm, was carried out keeping the same material characteristics as at the industrial scale. The press force resulted in a blank with a dimension of 19.2/14 mm external and internal diameter, a thickness of 2.6 mm and a total blank length of 28 m with an extrusion force of 565.6 kN.

### 3.4. Mathematical Modelling

#### 3.4.1. Extrusion Properties of Copper

Copper is assumed to be elasto-viscoplastic at all temperatures. Its viscoplastic part will be modelled by Johnson-Cook's law. Its properties are abstracted in **Table 5**.

The copper properties presented in **Table 5** are within the range of rheological conditions for an optimal copper extrusion during its heat treatment from 850°C.

#### 3.4.2. Modelling at Extrusion

Mathematical modelling of the billet extrusion was carried out using the continuum mechanics approach, considering the material properties presented previously [30] [31]. The following results were obtained:

- Stress: 366.07 MPa
- Extrusion force: 593 kN
- Flow rate: 0.237 m/s

**Table 4.** Optimization results for copper tube extrusion parameters.

Temperature (°C)	850
Pressure (kg/cm <sup>2</sup> )	250
Flow rate (m/s)	0.237
Die diameter (mm)	19.2
Pin diameter (mm)	14
Force (kN)	565.6

**Table 5.** Extruded copper properties.

$\dot{\epsilon}^p$ (s <sup>-1</sup> )	$\rho$ (kg/m <sup>-3</sup> )	$T$ [K]	$q$ [W·m <sup>-2</sup> ·K <sup>-1</sup> ]	$c$
1	8900	1123	2,357,100	274.7

### 3.4.3. Comparing the Results of the Optimization and the Modelling Approach

To evaluate the efficiency of the modelling approach, the significant difference between the results of the optimization and the modelling approach was calculated; the results are shown in **Table 6**.

The results of the optimization and modelling provide a significant deviation of 4.6%, which is rather small.

## 3.5. Numerical Modelling

### 3.5.1. Properties of Copper at 850°C

The copper properties at 850°C are shown in **Table 7**.

The results shown in **Table 7** after stochastic optimisation using the simplex algorithm were used as operating parameters for the simulation and modelling of optimal copper extrusion.

### 3.5.2. Comparing the Results of Optimization, Modelling and Simulation

To assess the efficiency of the simulation carried out, the significant difference between the results of the optimization, modelling and simulation was calculated. The results are presented in **Table 8**.

The deviation of the results of the optimization from the simulation is 7.4% and that of the results of the modelling from the simulation is 2.4%, proving that the factors optimized during the billet extrusion can be applied at the industrial scale. These results allow for time and cost savings in tube manufacturing as the simulated forces are more or less like those measured.

**Table 6.** Optimization results compared to modelling results.

Extrusion force (kN)	Optimization	565.6
	Modelling	593
Deviation (%)		4.6

**Table 7.** Copper properties at 850°C.

$A$ (MPa)	$B$ (MPa)	$C$	$N$	$M$	$T$ (°C)	$T_f$ (°C)	$T_{limit}$ (°C)	$\varepsilon_p$
2	40.19	0.01	0.1	5.01	850	1083	20	9.41

**Table 8.** Comparison of optimization, modeling and simulation results.

Extrusion force (kN)	Optimization	565.6
	Modeling	593
	Simulation	610.9
Deviation 1 (%)		7.4
Deviation 2 (%)		2.4



The result of the simulation of the copper flow rate during extrusion is given in **Figure 1** below:

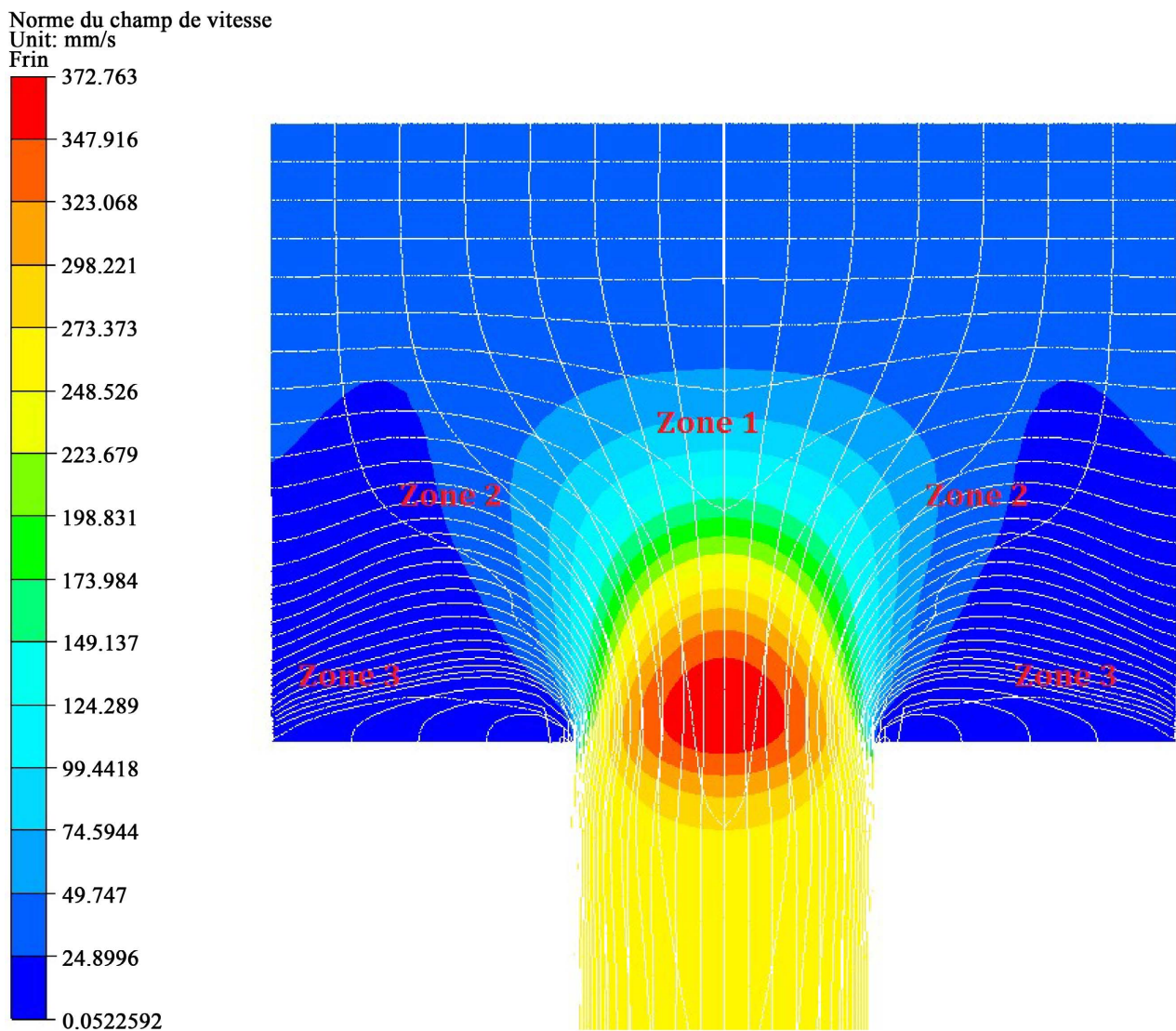
The factors that control the hot extrusion process are:

- The flow rate of the material.
- The coefficient of friction.
- The initial temperature of the tools.

Thus, after simulation of the flow rate, three zones were investigated, namely

- Zone 1: deformation zone located in the central part of the billet.
- Zone 2: intermediate zone which separates the two other zones.
- Zone 3: dead material zone at the contact of the billet with the die and the container.

It is established that a high flow rate is obtained in the deformation zone in the central part of the billet with a rate around 0.233 m/s, giving the same result as in the experiment.



**Figure 1.** Simulated copper flow rate results from extrusion.

According to Palengat M. *et al.* 2008, the deviation of the modelling results from the experimental results should not be more than 25%. This deviation may be related to the numerical stabilization used in the calculations or to the coefficient of friction with tungsten carbide. The models adopted in this approach are suitable and can be used at industrial scale as the deviation of the modelling results from the experimental results does not increase beyond 25%.

#### 4. Conclusions

The approach applied in this research was to perform an industrial extrusion of a copper tube from a billet. To achieve the objectives, an optimization of the die cross-section and the material flow rate using the non-deterministic method of the simplex algorithm was performed. The experimental results led to a mathematical modelling and simulation using the LS-DYNA software.

For a billet with a diameter of 145 mm, a cross-section of 1.65 m<sup>2</sup>, a length of 0.25 m, a volume of 0.41 m<sup>3</sup> and a mass of 37 kg, the main results obtained are following:

- Optimization of the factors resulted in a temperature of 850°C, a flow rate of 0.237 m/s, a die diameter of 19.2 ± 0.02 mm, a punch diameter of 14 ± 0.02 mm, an extrusion force of 565.6 kN and a pressure of 245 GPa. These factors have resulted in a blank with dimensions of 19.2 ± 0.02 mm and 14 ± 0.02 mm outer and inner diameters, 28 ± 0.1 m in length and 2.6 ± 0.01 mm in thickness.
- Modelling of the factors led to a temperature of 850°C, a flow rate of 0.237 m/s, an extrusion force of 593 kN and a pressure of 245 GPa. The values of these factors have resulted in a blank of similar dimensions to those obtained after the optimization.
- Simulating of the factors led to the same values as above with an extrusion force of 610.9 kN and the same blank.

After simulation using LS-DYNA software, deviations of 7.4% between the optimization and the simulation and 2.4% between the modelling and the simulation were obtained. These values are within sustainable limits and confirm the industrial application of the approach to improve the tube extrusion process. A study to optimize and simulate the heat treatment of copper and to determine the defects resulting from the heat treatment could be carried out for a more refined extrusion.

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#### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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