Journal of Mathematical Finance, 2023, 13, 380-393
https://www.scirp.org/journal/jmf
ISSN Online: 2162-2442
ISSN Print: 2162-2434

Spread-Based Direct Alpha (SBDA) as a Performance Measure for PE Funds

Koichi Miyazaki*, Kazuhiro Shimada

Department of Investment Strategy, Government Pension Investment Fund, Tokyo, Japan
Email: *f-research@gpif.go.jp

How to cite this paper: Miyazaki, K. and Shimada, K. (2023) Spread-Based Direct Alpha (SBDA) as a Performance Measure for PE Funds. *Journal of Mathematical Finance*, **13**, 380-393.
https://doi.org/10.4236/jmf.2023.133024

Received: July 19, 2023
Accepted: August 22, 2023
Published: August 25, 2023

Abstract

In this study, we explore the Public Market Equivalent (PME) as a measure of Private Equity (PE) fund performance relative to the listed market. While various PME methods exist, the "direct alpha method" has been identified as the superior approach. However, concerns arise regarding its suitability from a financial theory perspective, given its derivation process and the questions surrounding its role as a performance measurement method for PE funds relative to listed markets. To address these issues, we propose a novel and more accurate measurement method, the Spread Based Direct Alpha (SBDA), along with a technique for deriving the alpha amount based on SBDA, enabling a more precise comparison of PE fund performance against traditional assets.

Keywords

PE Funds, Public Market Equivalent (PME), Direct Alpha, Spread Based Direct Alpha (SBDA), Performance Measurement

1. Introduction

Although the title of this study states "a performance measure for PE funds", the approach taken here is a performance measure that is applicable not only to PE funds but also to other alternative assets such as infrastructure and real estate. While the performance of traditional assets such as stocks and bonds is often measured by time-weighted rates of return, the performance of alternative assets is generally measured by the internal rate of return (IRR) since inception. The reason for this is explained in "(Column) Method of Measuring the Rate of Return on Alternative Assets" on page 55 of the GPIF annual report (FY2021 version) [1] as follows: "In contrast to the fund manager for traditional assets, in the management of alternative assets, the fund manager decides the timing of investing funds and selling out assets, asks investors to contribute funds each time point of investing funds (capital call), and distributes investors the funds (distribution) each time point of selling out assets. Therefore, the internal rate of return (IRR) is used with the notion that determining the timing and size of cash flows is part of the fund manager's capabilities".

In measuring the performance of PE funds, according to BVCA's Limited Partner Committee and Investor Relation Advisory Group [2], the abovementioned internal rate of return ("IRR") and investment multiple, which measure the absolute value of the investment, have been observed. While these are excellent for the purpose of understanding the absolute return of each PE fund, they are not suitable for comparing the performance of PE funds with that of traditional assets. In contrast, the PME methodology assumes that, at the time point of a capital call, the same amount in question was invested in the benchmark and the performance is compared with that of the real PE fund. The major existing PMEs include the PME of Long and Nickels [3], the PME+ of Rouvinez [4], the mPME of Cambridge Associate [5], and the Direct Alpha Method of Gredil *et al.* [6]. Papers that explain these evaluation methods in an easy-to-understand manner using specific examples include Gredil *et al.* [6] and Shiraki and Miyata [7]. In this study, we will refer to Shiraki and Miyata [7]. With regard to the existing major PME methods, while the paper recognizes the advantage that the above PME methods are able to compare the IRR of the fund and that of the benchmark, it insists the limitations of the PME methods except the direct alpha that they are unable to separate the alpha (excess return) from the beta (benchmark return) such as $r(t) = \alpha + \beta(t)$, where $r(t)$, α, $\beta(t)$ are the total return of the PE fund, the excess return of the PE fund and the benchmark return corresponding to the cash flows of the PE fund, in order. The paper provides an explanation of the valuation method for each PME method, and then states that "For the valuation of excess return against benchmarks, among the major PME methods, the direct alpha method, which has no mathematical defects and does not require any artificial corrections, is considered to be the best method for measuring PE fund performance at present".

Shiraki and Miyata [7] evaluated that the direct alpha method "seems to be the most reasonable measurement method currently considered", but there may be some questions about the conception of the direct alpha method and thus there is the need for a more robust measurement approach. In this study, we propose the spread based direct alpha method (Spread Based Direct Alpha, hereinafter referred to as "SBDA".) as a measurement method that the performance of the PE fund can be compared fairly accurately with those of traditional assets by splitting the performance of PE funds into the beta part, which is the performance of the benchmark and the alpha part, which represents the pure performance of PE funds. We will also clarify questions regarding the conception of the direct alpha method using the tools used to introduce SBDA. Furthermore, based on numerical examples, the mechanisms and properties of SBDA and alpha amounts will be understood.

The structure of this study is as follows: Section 2 clarifies the questions in the conception on the direct alpha method, relying on specific examples by Shiraki and Miyata [7]. In Section 3, we discuss the conception and definition of SBDA, the method for deriving the alpha amount (the amount of excess return due to the pure skill of the PE fund) from SBDA, and clarify the questions in the conception regarding the direct alpha method. In Section 4, the mechanisms and properties of SBDA and alpha amount will be confirmed by numerical examples. In the final section, a summary and future issues will be discussed.

2. Questions Regarding the Direct Alpha Method

Questions in the conception of the direct alpha method are explained using Figure 4 (reproduced here as **Table 1**), which was used by Shiraki and Miyata [7] in their explanation of the direct alpha method. In the cash flow column of the PE fund in **Table 1**, capital call is the amount of the contribution of investors, the distribution is the amount of fund that the PE fund distributes to investors each time point of selling out assets, and NAV is the residual market value. The sum of these three amounts at each time point (from Year 0 to Year 4, year by year) is shown in the Net column. The IRR is nothing other than the rate of return obtained by relying on the cash flows listed in the net column.

Next, in the Benchmark column, the level of the benchmark against which the PE returns are compared is noted for each year. Depending on the level of these benchmarks, each of the contributions, distributions, remaining market value, and net amounts in the PE fund's cash flow column are all converted to present value (value in Year 0) and listed in the Investment in Benchmark column. As an example, the amount "−60" in the cash flow column of the PE fund that will be the fund contribution in 2 years is converted to the present value "−50" as in "−60 × 100 ÷ 120 = −50" because the level of the benchmark in 2 years is "120". The amount "−60" is converted to the present value "−50" and is listed in the fund contribution after 2 years (present value) in the "Investment in Benchmark" column.

The question in the direct alpha method is whether it is appropriate, from the perspective of finance theory, to obtain the IRR by converting the cash flows generated at each year to present value at the benchmark return and then considering these as having occurred at the year in question. This question is clarified by the mathematical expression of the direct alpha method using the tools we prepared to define our SBDA in Section 3 and the implications of the direct alpha method in terms of finance theory are not always clear.

3. SBDA

3.1. Conception and Definition of SBDA

The inspiration for the SBDA comes from the concept of credit spreads in the bond market, more specifically, the credit spread over the spot rate of bonds with the relevant maturity. In the spot rate curve, the current time point is 0, and the future time point is taken on the horizontal axis. In the SBDA, a "benchmark

Table 1. Direct alpha method.

	PE fund's cash flow				Investment in Benchmark				
Year	Contribution	Distribution	NAV	Net	Benchmark	Contribution (PV)	Distribution (PV)	Market Value (PV)	Net Cash Flow (PV)
0	−1000	0		−1000	100	−1000	0	0	−1000
1	0	560		560	112	0	500	0	500
2	−60	0		−60	120	−50	0	0	−50
3	0	280		280	140	0	200	0	200
4	0	240	300	540	120	0	200	250	450
	−1060	1080	300			−1050	900	250	
	0	1	2	3	4	IRR			
Fund	−1000	560	−60	280	540	11.75%			
Benchmark	−1000	500	−50	200	450	3.87%			
					diff	7.88%			

spot rate curve" is defined as the equivalent of the spot rate curve. Since the SBDA is a measure of the performance of alternative assets, it is presented as a value obtained at the time point of the valuation. The benchmark spot rate curve is the performance of the benchmark from the time point of investment commitment to the time point of valuation (see Figure 1 with numerical examples in Table 1 for the benchmark dynamics). Thus, in the benchmark spot rate curve, Year 0 on the horizontal axis is the investment commitment time point and the final time point on the horizontal axis is the valuation time point. The "benchmark spot rate" in the benchmark spot rate curve is the annualized cumulative rate of return of the benchmark from the time point of investment commitment to each time point up to the valuation time point (see Figure 2 with numerical examples from Table 1).

Normally, in bond investments, the investor invests in the bond at present and receives the coupon payment and face value at a pre-determined future time point. In contrast, with alternative assets, the investment is not made immediately at the time point of commitment, but rather, the commitment begins, sometime later when contributions of funds are made at first time in response to a capital call, which continues until the cumulative amount of contributions reaches the commitment limit (in Figure 3, which uses the numerical example in Table 1, there are capital calls at Year 0 and Year 2). The amount of the capital call at each time point is invested in alternative assets and the investor receives a certain amount as distribution at a certain time point (in Figure 3 using the numerical example in Table 1, there are distributions at Year 1, Year 3, and Year 4). At the time point of valuation (Year 4 in Figure 3, which uses the numerical example in Table 1), not all the funds invested in response to the capital call are collected as a distribution, in which case the value of the alternative

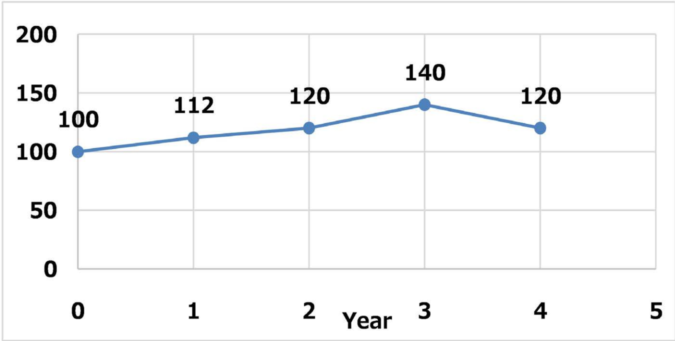

Figure 1. Benchmark dynamics.

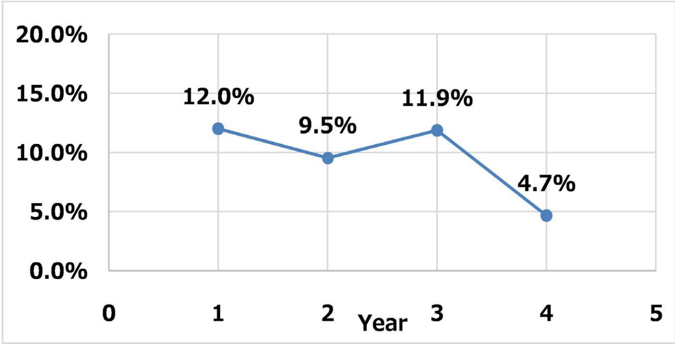

Figure 2. Benchmark spot rate curve (annualized).

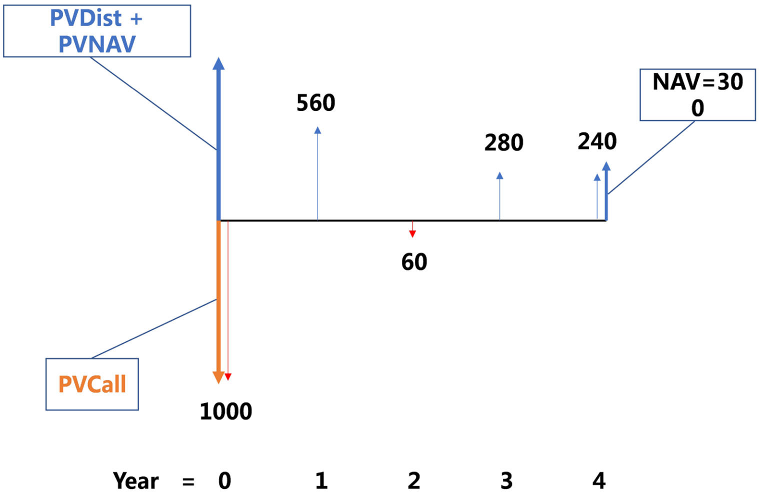

Figure 3. Diagram of cashflows in Table 1.

assets under management is recognized as NAV. The SBDA represents the degree to which, on average, the PE fund has exceeded the benchmark return over the investment period in the process of obtaining distribution.

The valuation method for SBDA relies on the valuation method for interest rate swaps with credit risk. Theoretical underpinning of SBDA is the principle that the present value of capital calls discounted back to the commitment time by funding rate in funding side should be equal to the present value of the distributions discounted back to the commitment time by funding rate plus SBDA in investing side. In other words, the present value of capital calls discounted back to the commitment time is invested in the PE at the commitment time and the PE generates in the future time the distributions, which are the grown-up capital calls compounded by funding rate plus SBDA. Thus, the theoretical underpinning of SBDA is crystal clear and is expected to be more accurate than previous PME methods. First, we consider the funding side. As an alternative asset, we will use a foreign PE as an example. This is an alternative asset that falls under the category of foreign (Non-Japanese) equities in the GPIF's policy benchmark portfolio, and the GPIF's policy benchmark for foreign equities is the MSCI ACWI ex Japan. Therefore, the benchmark spot rate at a given time point is the annualized cumulative rate of return of the MSCI ACWI ex Japan from the time point of commitment to that time point. For the funding side, the total investment is considered to be the sum of the funds invested in each capital call, discounted by the benchmark spot rate to the time point of commitment. The total investment is then considered to be the sum of the distributions (including NAV) available at each time point on the investment side, discounted to the time point of commitment at the benchmark rate plus the SBDA at that time point. Relying on the swap valuation method, the SBDA is determined so that the present value of the funding side equals the present value of the investment side at the time point of commitment.

We now define SBDA by expressing the above ideas in a mathematical formula.

(Definition) SBDA

SBDA is defined as s satisfying the following equation:

$$\sum_{j=1}^{m}\frac{Call(j)}{(1+r_j)^j}=\sum_{i=1}^{n}\frac{Dist(i)}{(1+r_i+s)^i}+\frac{NAV}{(1+r_n+s)^n} \tag{1}$$

Here, $Call(j)$ and r_j are, respectively, the amount of funds invested in response to the capital call and the benchmark rate at the time point *j*, $Dist(i)$ and r_i are the amount of distribution and the benchmark rate at the time point *i*, respectively, NAV and r_n represent the amount of alternative assets under management and the benchmark rate at the time point *n* of valuation. The above equation shows that there were *m* capital calls and *n* distributions.

(Remark)

The usual derivation of IRR does not separate the funding side cash flows from the investment side cash flows, and also assumes that distributions made during the period are reinvested in IRR. Therefore, the IRR can be derived either based on the present value of the cash flows (value at the time point of commitment) or based on the future value of the cash flows (value at the time point of valuation), and the IRRs from both derivation methods are consistent. In this study, the objective is to compare the performance of the PE fund with that of traditional assets as accurately as possible after separating the alpha portion of the PE fund, which is the *s* in Equation (1) from the beta portion of it, which is the r_i and r_j in Equation (1). After the distribution $Dist(i)$ at the time point i is distributed to investors, the corresponding present value of capital call $\frac{Dist(i)}{(1+r_i+s)^i}$ is no longer invested in the PE fund, and therefore the concept of reinvestment after the time point i does not exist. In fact, once the distribution is made, GPIF will invest it again in the MSCI ACWI ex Japan, so there is no room for an alpha portion that expresses the pure management skill of the PE fund, since the PE fund has not managed it. Therefore, unlike the IRR derivation method, SBDA can only be derived based on the present value of cash flows (value at the time point of commitment).

3.2. Conversion from SBDA to Alpha Amount

The alpha, which is the excess return from the benchmark in the management of traditional assets, corresponds to the SBDA obtained in a way that satisfies Equation (1) in the case of PE funds. In this section, we describe a method for deriving the alpha amount, which is the amount of excess return from the benchmark, using the SBDA. There are three possible ways to capture the alpha amount assumed to have been generated by the PE fund. Before presenting the formula for deriving the alpha amount, we will explain the background conception using **Figure 4**.

The amount of distribution at time point 2 (Year 2) is shown as the area of the largest square in **Figure 4**. We decompose this and consider three different alpha amounts in terms of which part can be regarded as the alpha amount generated by the PE fund. The area of the largest square in **Figure 4** is $(1+r_2+s)^2$ and this is the amount of distribution at time point 2 (Year 2). This can be decomposed into nine parts (①, two ②, ③, two A, two C, and B in **Figure 4**) since it is the square of the trinomial. The area of the square in ① represents the present value at the time point of the commitment (the investment amount at time point 0 that generates the distribution at time point 2 (Year 2)). The area of 2 × ② + ③ should be called beta because it is the amount of return obtained by investing the area of ① in the MSCI ACWI ex Japan at the time point of commitment and managing it until the time point 2 (Year 2). The area of 2 × A + B can be obviously regarded as the amount of PE's alpha in relation to the investment amount of area ①, and the alpha amount based on this conception is called the alpha amount (1). The area of 2 × C is the alpha amount obtained with respect to the part where the capital call is grown with beta, and can be viewed as the alpha amount mixed with beta. If we consider this portion as the alpha amount generated entirely by the PE, and we call 2 × A + B + 2 × C alpha amount (2). The alpha amount 2 × A + B + C that we consider half of the portion to have been generated by the PE is called the alpha amount (3). Note that the amount of distribution at time point 3 (Year 3) is the volume of a cube with one side $1 + r_3 + s$. The volume of the cube $(1 + r_3 + s)^3$ can be decomposed into 27 parts by the third power of 3 and considered in the same way as the amount of distribution at

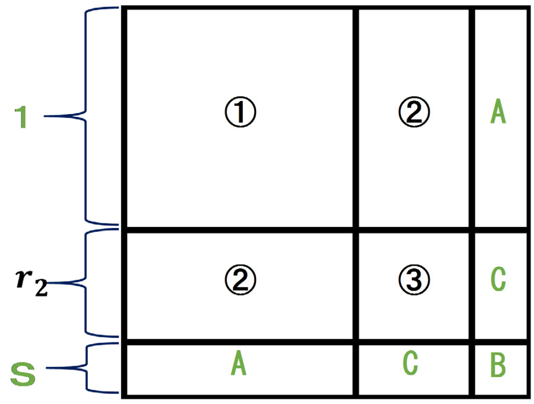

Figure 4. Illustration of the alpha amount (case at time point 2).

time point 2 (Year 2). In the same manner, the amount of distribution at time point n can be assumed to be the volume of a n-dimensional cube.

In line with the above approach, it can be easily confirmed that alpha amounts (1) through (3) can be expressed as in Equations (2) through (4) using the capital call amount, distribution amount, NAV, benchmark spot rate, and SBDA that appear in Equation (1).

Alpha Amount (1)

$$\sum_{i=1}^{n} \frac{Dist(i)}{(1+r_i+s)^i}\left((1+s)^i-1\right)+\frac{NAV}{(1+r_n+s)^n}\left((1+s)^n-1\right) \quad (2)$$

Alpha Amount (2)

$$\sum_{i=1}^{n} \frac{Dist(i)}{(1+r_i+s)^i}\left((1+r_i+s)^i-(1+r_i)^i\right)+\frac{NAV}{(1+r_n+s)^n}\left((1+r_n+s)^n-(1+r_n)^n\right) \quad (3)$$

Alpha Amount (3)

$$\text{Alpha Amount (1)}+\frac{1}{2}\left(\text{Alpha Amount}(2)-\text{Alpha Amount}(1)\right) \quad (4)$$

3.3. Clarification of Questions in the Direct Alpha Method

We define the direct alpha using the tools prepared in Section 3.1 in line with the method of deriving direct alpha presented with concrete examples in Section 2. The present value of the invested funds in response to the capital call at time point j is $\frac{Call(j)}{(1+r_j)^j}$ and the present value of the distribution at time point i and the present value of NAV at time point n are $\frac{Dist(i)}{(1+r_i)^i}$ and $\frac{NAV}{(1+r_n)^n}$, respectively.

Since direct alpha is the internal rate of return obtained by considering each of these amounts converted to present value as if they occurred at the same time point, it can be defined as follows:

(Definition) Direct Alpha

Direct alpha is α that satisfies the following equation;

$$\sum_{j=1}^{m} \frac{Call(j)}{(1+r_j)^j} \cdot \frac{1}{(1+\alpha)^j} = \sum_{i=1}^{n} \frac{Dist(i)}{(1+r_i)^i} \cdot \frac{1}{(1+\alpha)^i} + \frac{NAV}{(1+r_n)^n} \cdot \frac{1}{(1+\alpha)^n} \tag{5}$$

Here, all notations except for α are the same as those used in defining SBDA. Although the original equation to define direct alpha is the one that the left-hand side of Equation (5) is transposed to the right hand side, it is defined in the form of Equation (5) for ease of comparison with the Equation (1) to define SBDA.

The denominators of both sides of Equation (5) are organized as follows:

$$\sum_{j=1}^{m} \frac{Call(j)}{(1+r_j+\alpha+r_j \cdot \alpha)^j} = \sum_{i=1}^{n} \frac{Dist(i)}{(1+r_i+\alpha+r_i \cdot \alpha)^i} + \frac{NAV}{(1+r_n+\alpha+r_n \cdot \alpha)^n} \tag{5'}$$

It follows that the financial implications are not certain for $r_j \cdot \alpha$ and $r_n \cdot \alpha$ in the denominator of Equation (5').

Additionally, since $r_i \cdot \alpha$ and $r_n \cdot \alpha$ considered negligible second-order terms, an approximate equation $(1+r_j)(1+\alpha) \cong 1+r_j+\alpha$ is attained and rewriting both sides of Equation (5) using it:

$$\sum_{j=1}^{m} \frac{Call(j)}{(1+r_j+\alpha)^j} \cong \sum_{i=1}^{n} \frac{Dist(i)}{(1+r_i+\alpha)^i} + \frac{NAV}{(1+r_n+\alpha)^n} \tag{6}$$

is obtained. Comparing the approximate Equation (6) with Equation (1), which is the defining equation of the SBDA, α in the investment side of the PE fund implied by the right-hand side of approximate Equation (6) is identical to s in Equation (1). However, in the funding side of the PE fund implied by the left-hand side of the approximate Equation (6), capital call $Call(j)$ at time point j is discounted by $1+r_j+\alpha$ instead of $1+r_j$ in Equation (1). In the funding side of the PE fund, the fund raised from the funds invested in MSCI ACWI ex Japan, which is the benchmark, so when discounting capital call to present value, the return of MSCI ACWI ex Japan should be used and capital call should be discounted by $1+r_j$. But instead, the return attained by adding direct alpha to this return is used and capital call is discounted by $1+r_j+\alpha$. Therefore, in the usual case where capital call occurs after the time point of commitment and α is positive, the left side of Equation (1) defining SBDA will be larger than the left side of Equation (5) defining direct alpha, and SBDA will be smaller than direct alpha due to this effect. In other words, using direct alpha in evaluating the performance of a PE fund would result in an overestimation of the fund's performance. Although it is a special case, we should also check the case where a capital call occurs only at the time point of commitment. In this case, the left-hand side of Equation (1) defining SBDA and the left-hand side of Equation (5') defining direct alpha have the same value, whereas the denominator of the right-hand side of Equation (5') has $r_j \cdot \alpha$, which does not exist in the denominator of the right-hand side of Equation (1). Under the influence of such factors, as r_i and α are positive, direct alpha is slightly smaller than SBDA in the usual case.

4. Numerical Examples

In this section, based on numerical examples, we first compare SBDA and direct alpha ("DA" in Section 4 below), which is the only PME that is able to separate the alpha (excess return) from the beta (benchmark return) such as r(t) = α + β(t) and is able to be compared with SBDA on an equal basis and see to what extent using DA in evaluating the performance of a PE fund will lead to an overestimation of the fund's performance in many cases. In addition, we will reconfirm the pricing mechanism of SBDA and DA through numerical examples. Furthermore, we will obtain three different alpha amounts and review the magnitude of the alpha amounts for SBDA and capital calls, as well as the differences in magnitude among the three alpha amounts.

As numerical examples, we consider five different PE funds from Case 1 to Case 5, *i.e.*, the 5 kinds of sets of capital calls and distributions (hereinafter referred to as the "set"). Case 1 is the set shown in **Table 1**; Case 2 and Case 3 are the sets of Case 1 in which 500, half of the invested funds in response to the capital call at Year 0, is moved to Year 1 and Year 2, respectively. Case 4 and Case 5 are the sets of Case 1 in which the distribution of 280 at Year 3 is moved to Year 4 and Year 2, respectively. The benchmark spot rates to each year were obtained from and the numerical examples in **Table 1** and shown in **Figure 2**. **Table 2** shows the DA, SBDA, and 3 kinds of alpha amounts for these 5 PE funds.

4.1. SBDA and DA

First, we compare Case 1 with Case 2 and Case 3. The present value (hereinafter referred to as PV) is evaluated on the time point of commitment to the PE (at Year 0). The PV of the invested funds in response to the capital call becomes smaller from Case 1 to Case 2 and Case 3. On the other hand, since the distribution Case 1 to Case 3 are all the same, the excess rate of return is expected to increase from Case 1 to Case 2 and Case 3. In fact, **Table 2** shows that SBDA and DA both increase from Case 1 to Case 3. It is important to note that while there is only a slight difference between SBDA and DA in Case 1, DA increases significantly more than SBDA in Case 2 and Case 3. This is because, as pointed out in Section 3.3, the funds invested in response to the capital call are raised from funds invested in the benchmark MSCI ACWI ex Japan, so when discounting to present value, the return of the MSCI ACWI ex Japan should be used and discounted with $1+r_j$. In the DA method, however, they are discounted by $1+r_j+\alpha+r_i\cdot\alpha$. As expected, it is confirmed that the DA method considerably overestimates excess returns.

Next, compare Case 1 with Case 4 and Case 5. This time, the PV of the invested funds in response to the capital call is the same in all cases, but a part of the distribution is moved to a later time point (Case 4) and a part of the distribution is moved to an earlier time point (Case 5), respectively. The point of interest is the comparison between Case 1 and Case 4, since the distribution (280) that was at Year 3 in Case 1 is moved to a later time point (Year 4) in Case

Table 2. Numerical example.

		Year 0	Year 1	Year 2	Year 3	Year 4	NAV	DA	SBDA	Alpha amount		
										(1)	(2)	(3)
Case 1	Capital Call	1000	0	60	0	0	300	3.87%	4.00%	107.64	122.62	115.13
	Distribution	0	560	0	280	240						
Case 2	Capital Call	500	500	60	0	0	300	7.77%	6.43%	165.61	188.29	176.95
	Distribution	0	560	0	280	240						
Case 3	Capital Call	500	0	560	0	0	300	12.24%	7.89%	197.92	224.76	211.34
	Distribution	0	560	0	280	240						
Case 4	Capital Call	1000	0	60	0	0	300	4.72%	4.82%	141.44	158.35	149.90
	Distribution	0	560	0	0	520						
Case 5	Capital Call	1000	0	60	0	0	300	5.59%	5.73%	142.84	158.05	150.45
	Distribution	0	560	280	0	240						

4, it is usually assumed that the PV of the distribution in Case 4 is smaller than that in Case 1. However, **Figure 2** shows that the benchmark spot rates up to Year 3 and Year 4 are 11.9% and 4.7%, respectively, and the values (discount function values) to be multiplied when discounting the PV from Year 3 and Year 4 are 0.714 and 0.832, respectively, indicating that the PV discounted from Year 4 is larger than the PV discounted from Year 3. From this effect, both SBDA and DA are larger in Case 4 than in Case 1. In a similar fashion, both SBDA and DA are larger in Case 5 than in Case 4. A closer look at Case 1, Case 4, and Case 5 shows that DA is slightly smaller than SBDA in all cases. When considering the reason for this, it is important to first confirm that in these cases, the PV of invested funds according to the capital call used in the definition of SBDA and that used in the definition of DA are almost the same. This is because the majority (1000) of the funds invested in response to the capital call (1060 in total) occurred at Year 0, so the difference in discount rates (*i.e.*, the difference between $1+r_j$ and $1+r_j+\alpha+r_j\cdot\alpha$) hardly appears. Thus, the reason is related to distribution; in the definition of DA, the discount rate in the denominator of the right-hand side of Equation (5') has terms such as $r_i\cdot\alpha$ and $r_n\cdot\alpha$ that are not present in SBDA and have no concrete financial implications. Because these terms were positive, DA was smaller than SBDA.

4.2. Three Kinds of Alpha Amounts

The size of the SBDA is the most important factor because alpha amount (1) through alpha amount (3) are, in turn, derived based on Equations (2) through (4) using the SBDA. In Section 4.1, we observed that the SBDA increases in order from Case 1 to Case 2 and Case 3. **Table 2** shows that the alpha amounts (1) through (3) corresponding to these cases also increase from Case 1 to Case 2 and Case 3. In other words, the order of magnitude of SBDA, which was introduced as an indicator of (excess return) rate, is maintained when converted to an indicator of (excess return) amount, *i.e.*, the alpha amount, if the SBDA is compared among PE funds with the same distribution.

Next, we consider the use of SBDA to calculate the alpha amount in a comparison between PE funds with the same invested fund in response to a capital call and different distributions. In this section, we consider the alpha amount based on the alpha amount (1), which is representative of the alpha amount. It is interesting to note that Case 4 has an increase of 33.8 in alpha amount (1) even though SBDA is only 0.82% larger than Case 1, while Case 5 has an increase of only 1.4 in alpha amount (1) even though SBDA is 0.91% larger than Case 4. The reason for this can be largely explained by the difference in the time point at which distributions occur; distribution 280 at Year 3 in Case 1 is moved to Year 4 in Case 4 and to Year 2 in Case 5. In Case 4, the portion of the alpha amount in distribution 280 that is the increased amount in PV of the capital call compounded over four years (20.72%) at the rate of SBDA (4.82%), while in Case 5, the portion of the alpha amount in distribution 280 that is the increased amount in PV of the capital call compounded over only two years (11.79%) at the rate of SBDA (5.73%). This effect is so strong that even though SBDA is about 0.91% larger in Case 5 than in Case 4, the alpha amount (1) remains at the same level. Thus, if SBDA is positive and the same among PE funds, the alpha amount (1) tends to be larger for PE fund with distribution at a later time point due to the compounding effect of SBDA over a longer period of time.

Finally, we examine the relative comparison of alpha amounts (1) through (3) in each case. **Table 2** shows that in all cases, alpha amount (1) is the smallest, alpha amount (2) is the largest, and alpha amount (3) is between the two. The reason for this is the benchmark dynamics (see **Figure 1**), which is another factor that determines the SBDA. **Figure 1** shows that in this numerical example, the benchmark dynamics are generally upward trending, *i.e.*, as shown in **Figure 2**, the benchmark spot rate is positive at all time points, so the PV of invested funds in response to a capital call at Year 0 (① in **Figure 4**) will be inflated at the benchmark spot rate (two ② and ③ in **Figure 4** will be added). This generates an alpha amount (C in **Figure 4**) for the portion of the PV inflated by the benchmark spot rate. Thus, in this numerical example, the alpha amount (1) was the smallest and the alpha amount (2) was the largest in all cases. Contrary to the benchmark dynamics in this example, if the benchmark dynamics are generally downward trending, *i.e.*, the benchmark spot rate is negative at all time points, the PV of the capital call at Year 0 will tend to shrink with the benchmark spot rate, and the alpha amount associated with this portion (the two C) will be recognized as a negative value. Therefore, in this case, alpha amount (1) is the maximum and alpha amount (2) is the minimum. If the benchmark dynamics are horizontal, *i.e.*, the benchmark spot rate is zero at all times, the PV of the invested funds in response to the capital call at Year 0 is not affected by the benchmark spot rate, and therefore the alpha amount related to this portion (the two C) is not generated. The PV of funds invested in response to a capital call at

Year 0 at all times is not affected by the benchmark spot rate. Thus, in this case, the three kinds of alpha amounts will have the same value.

5. Summary and Future Issues

In this study, we proposed a measurement method that can compare the performance of PE funds quite accurately with that of traditional assets, splitting the performance of PE funds into a beta portion, which is the market performance, and an alpha portion, which expresses the pure investment skill of PE funds, by way of the spread based direct Alpha (SBDA) and the alpha amount based on SBDA. One of the valuable practical implications of using SBDA is to provide the way deriving the alpha amount of the PE, which is the return amount purely attained only from managers' investment skill. Once the alpha amount of the PE is derived, referring to the amount, investors are able to decide the fee amount fairly paid to the PE fund managers. The tools prepared in introducing these concepts also clarified the question of the conception of the direct alpha method, which is regarded as the best among the existing PMEs. Furthermore, based on numerical examples, the mechanism and nature of the SBDA and alpha amounts were ascertained in terms of the time point of occurrence of capital calls and distributions and the benchmark spot rate.

The SBDA and the alpha amount based on the SBDA were devised to measure the performance of PE funds in order to satisfy the double mandate: 1) the alpha portion, which expresses the pure skill of the PE fund, should be extractable, and 2) the performance relative to the MSCI ACWI ex Japan, the GPIF's policy benchmark for foreign equities, should be measurable. Whether or not the double mandate is actually met in practice will need to be examined from various perspectives in the future. Especially, the empirical research applying SBDA to real-world data and providing a thorough interpretation of the findings is left for an important future research topic. In the process, it will also be essential to improve the SBDA and the corresponding alpha amounts.

We hope this paper will be of some help when considering "better PE fund performance measurement methods".

Disclaimer

This paper is a compilation of research results by GPIF staff, and the contents and opinions expressed in the text do not represent the official views of the GPIF.

Acknowledgements

We heartily thank anonymous reviewers for their helpful comments and suggestions to improve initial version of this paper.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] Government Pension Investment Fund (2022) Annual Report Fiscal Year 2021.

[2] BVCA's Limited Partner Committee and Investor Relation Advisory Group (2015) Private Equity Performance Measurement (BVCA Perspectives Series).

[3] Long, A.M. and Nickeles, C.J. (1996) A Private Investment Benchmark. Working Paper.

[4] Rouvinez, C. (2003) Private Equity Benchmarking with PME+. *Venture Capital Journal*, 34-38.

[5] Cambridge Associates (2013) Private Equity and Venture Capital Benchmark—An Introduction for Readers of Quarterly Commentaries.

[6] Gredi, O.R., Griffiths, B. and Stucke, R. (2014) Benchmarking Private Equity: The Direct Alpha Method. *Journal of Corporate Finance*, **81**, Article ID: 102360. https://doi.org/10.2139/ssrn.2403521

[7] Shiraki, S. and Miyata, S. (2018) Comparison of Performance Measurements of Private Equity Fund Investments. *Securities Analysts Journal*, **10**, 42-52. (In Japanese)