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An Introspection of Luxury Auto Sales Using Revealed Preference Theory

Derek J. Streidl

Department of Mathematics, Indiana University, Richmond, IN, USA
Email: dstreidl@iu.edu

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Abstract

Revealed preference is a powerful tool within the realm of consumer theory and behavioral economics used to determine consumer proclivity towards goods and services which maximize their individual utilities in scenarios where the alternatives would have only suppressed utility, first coined by American economist Paul Samuelson in 1938. The consumer then prefers the good or service from which this maximation stems. Using a cross-sectional dataset from Carvana Co. on used automobile sales from January to September 2022, this study uses the Weak Axiom of Revealed Preference (WARP) and binary relation to show consumer preference amongst three bundles (brands and prices). All examined brands were compared in the quantity of units moved across sixty identical price points, so price is considered a controlled variable for the terms of this research. We then represent our finding graphically and in terms of WARP as well as construct an ordinal utility function to prove maximization is achieved amongst the examined brands.

Keywords

Consumer, Demand, Assumption, Behavior, Axiom, Carvana, Used Cars, Auto, Logic, Revealed Preference, Samuelson, Consumer Theory

1. Introduction

Revealed Preference is a theory of consumer consumption that contends that each consumer holds individually or as some group, which dictates their unique buying patterns and was first coined by economist Paul Samuelson in 1938; the theory generally dictates what consumers will purchase under different circumstances, most notably incomes and price levels [1]. With applied preference theory, one can theoretically determine the rational purchasing behavior of groups of customers and apply more direct marketing, which can help lead to increases in

a firm's revenue. This method within consumption theory shows consumers consider goods over prices as a more important factor in purchasing decisions, holding the price of items constant, as consumers are more concerned with the producers or brands than the actual cost of the item when utility level from the purchased item(s) is already providing maximum utility to the consumer and the consumer's income level is capable of affording all alternatives within the set.

There are three generally accepted axioms within revealed preference, each with a varying degree of feasibility in truth. The Weak Axiom of Revealed Preference (WARP) states that given constant incomes and price levels when one product or service is purchased over another, consumers will always make that same choice and never buy a different product or service from a different brand unless that brand offers a higher convenience factor or quality and that our buying decisions will always remain constant. In our particular study, we use three different brands of automobile to show that consumers choice in brands will generally always follow this axiomatic assumption. In the Strong Axiom of Revealed Preference (SARP), a market only sells two goods from which consumers must choose and in this two-dimensional market, both the strong and weak purchasing decisions are equivalent. The third axiom, Generalized Axiom of Revealed Preference (GARP), states for a given income and/or price level, consumers gain the same benefit from more than one consumption choice, called bundles; GARP essentially covers the missing space within preference theory for when no unique bundle which maximizes consumer utility exists within the market.

In 2021, the U.S. market size of the used car industry was valued at about $195.84 billion, with an expected annual growth of 7.51% and projected to hit around $302.47 billion by the year 2027, according to Mordor Intelligence, an India-based macro & micro think tank firm specializing in research analytics [2]. Carvana is a used car retailer which initially burst onto the US used automobile scene with its unique approach to buying cars in late 2013. The company offers consumers a completely "touch-free" experience, allowing their used vehicles to be purchased through their website and mobile application, then either delivered to their front door via Carvana's delivery system or offering consumers the chance to pick up their purchase from a "car vending machine". The vending machine model made Carvana a household name in the U.S. throughout the late 2010's, with the company owning and operating 32 fully automated, multistory glass encased structures that have the ability to retrieve and produce cars in their inventories, on-demand, for a consumer much like you would get a can of coke from the machine in your office break room [3]. Though the company has had some financial setups during the last year, including a tumbling stock price and a gnarly corporate restructuring, The Motley Fool still reports the company's current market capitalization to be around $1.2 billion, or 0.61% share of the country's used automobile market [4]. In the second quarter of 2023 however, Carvana was able to command a new market capitalization of $4.58 billion, or 2.34% of the market share reported for 2021 as the stock once again soared 50% from

its previously deflated price for the first time in 12 months [5]. With an expected growth of 7.51% annually, the used car market is undoubtedly becoming larger within the U.S. as the years pass and encompassing a larger share of the country's GDP. Amidst a fragmented market of many start-ups attempting to do the same thing, Carvana and its uniquely technological, *no touch* approach make the company a prime source for sales data, including consumer preference trends.

In this empirical paper, we sought to explore the theory of revealed preference and its existence within the luxury automobile market utilizing Binary Relation and the Weak Axiom of Revealed Preference (WARP) to establish consumer proclivity, before describing the relation formally in an ordinal utility function. In order to do this, we approached the experiment in a unique scope from others who have studied Revealed Preference, holding the income constant and instead focusing on how many units of some good (in this case, luxury automobiles) are bought by a customer at some particular price point. The forthcoming research initially appears to reveal Tesla is preferred over competitor brands BMW and Audi respectively, however, once the data is analyzed using WARP, and the use of binary relation to establish preference, these findings shift to show BMW is revealed to Tesla and eventually, to a third bundle representative of the Audi brand. Furthermore, we also establish an original choice function for the members of the alternative set to BMW, showing that all three examined brands do in fact belong to the same nonempty set of luxury automobiles from which consumers can choose to purchase. We have held income constant throughout this entire study, focusing solely on the quantity of each brand sold at the same sixty price levels, in a concentrated effort to ensure we were focusing on the true preference of brand and not simply the price of each vehicle.

2. Data

The data used in this study represents used automobile sales from Carvana Co., compiled for analysis by Saturn Data and distributed to users via Amazon Web Services Data Marketplace, an open-source management system that allows users to upload data sets as well as download them, in both free and paid subscriptions and provides a record of all used automobiles sold by Carvana in the United States from January to September 2022. It should be noted that these are not sales for new automobiles, nor was this data provided by the manufacturers of the automobiles, but rather by a third-party re-seller. The data was then refined and cleaned to include only sixty equal price points, with the minimal being $35,990 to a maximal of $91,000. Setting this price constraint allowed us to examine the actual units being moved for each examined brand at each set price point, which eliminates units moved above or below the price constraint and set a more concise path for quantifying our preferences through WARP across 753 observations. The price constraint allows us to effectively ignore externalities caused by price and income in determining consumer preference, with the notion that all consumers in the market for luxury automobiles have a suitable income to

afford any of the studied brands, choosing one over the other purely from a preferred point-of-view.

This cross-sectional dataset records sales for 44 different brands of auto manufacturers across 44,366 total observations. In order to study our chosen "luxury" brands or brands whose average price of a vehicle is greater $25,000, we first sorted all vehicles sold by brand and sold price, eliminating any whose average sold price per vehicle was less than our $25,000 benchmark and any brands who lacked enough total observations to be equally compared across our chosen price points. Of the original 44,366 vehicles observed, the Audi, BMW and Tesla brands comprised 3431 of these observations and were chosen as the luxury brands. Of the observations of these three brands was then again sorted by the sold price variable, where we set the minimal sold price at $35,990 and the maximal sold price of $91,000, drastically reducing our total observations to a total of 753 and allowing us to focus on the actual quantity of vehicles moved at the same price points across brands.

We begin by performing a direct consistency test amongst the Tesla and BMW brands, a test which has been applied to Revealed Preference since at least the 1960s and proven to be effective "for two purchases and the general consistency test for three or more purchases [and] if we should find that a complete set of consistency conditions is satisfied for all possible market situations, we can deduce transitivity of his choice and can, therefore, construct an ordinal utility function for him" [6]. Koo, in his landmark paper published in *Econometrica* Volume 31, goes on to discuss the importance too, of holding maximal subsets of observations in order to determine that the consumers' scale of preference remains unchanged. We achieve this by using a capped observational set, with subsets being the brands we examine being sold at varying price points and utilizing a non-stationary dataset for experimentation. In our case, prices do not vary over time, but rather over automobile models, and we hold all income constant, as we are assuming all consumers who can afford a Tesla, can also afford a comparative brand such as Audi or BMW. As Revealed Preference is a theory resting on the axiom of assumption, we are safe to include relevant assumptions within our testing.

Our initial hypothesis was that Tesla (X^a) would be directly preferred to BMW (X^b), given the Tesla brand's global explosion in popularity over the last few years, indicated by positively increasing annual production numbers. The same sixty price points for each vehicle brand were set, and then a quantity of units moved for each brand, at each respective price was taken and can be represented via matrices (**Table 1(a)** & **Table 1(b)** below) for easier clarification. **Table 1(c)** is a matrix representation of the sixty equal price points attached to the respective quantities.

Please note, the last four values in 1 c are null, as the matrices are constructed in 8 × 8 form for more transparent matrix arithmetic, but there are four null values in each vector representing quantity as well for compensation.

Table 1. (a) The Matrix of BMW brand Quantity Sold at Sixty Equal Price Points; (b) The Matrix of Tesla brand Quantity Sold at Sixty Equal Price Points; (c) The Matrix of the Sixty Equal Price Points used for Quantity Comparison of all observed brands.

(a)

BMW							
20	15	17	19	26	8	15	6
4	3	5	8	4	4	4	4
2	1	4	9	4	3	1	3
4	4	3	4	5	4	4	3
1	2	1	2	3	2	1	1
1	2	1	2	1	1	1	1
2	1	1	2	1	1	1	1
1	1	1	1	0	0	0	0

(b)

Tesla							
1	1	1	5	2	1	3	1
4	1	3	4	3	1	5	6
1	8	5	6	2	9	3	9
1	9	2	8	2	4	5	4
2	2	6	0	6	5	6	3
3	2	3	5	3	3	2	6
9	2	3	4	4	3	1	3
1	1	2	1	0	0	0	0

(c)

Prices							
35,990	36,990	37,590	37,990	38,990	39,590	39,990	40,990
41,990	42,590	42,990	43,990	44,590	44,990	45,990	46,990
47,590	47,990	48,590	48,990	49,590	49,990	50,590	50,990
51,590	51,990	52,590	52,990	53,590	53,990	54,590	54,990
55,590	55,990	56,990	57,590	58,000	59,990	60,990	61,990
62,990	63,990	64,590	64,990	65,590	66,990	67,590	68,990
69,990	71,990	72,590	72,990	75,990	78,990	80,990	82,590
82,990	84,990	89,990	91,000	0	0	0	0

3. Methodology & Results

Before beginning our formal proof, we should lay out some basic definitions to give the reader a more concise introduction to preference theory. Using WARP, as we do in this study, can be defined formally; if (X_B, Y_B) is directly preferred to (X_A, Y_A), and both bundles are unique, then it cannot be inferred that

(X_A, Y_A) is directly preferred to (X_B, Y_B) as preference is always demonstrated by a consumers choice; we can say a choice function satisfies the weak axiom of revealed preference if whenever $x \succsim^c y$, but not when the case is such $y \succ^c x^1$ as the larger the collection Σ, the more restrictive the weak axiom becomes. Particularly, if the Σ includes all sets where the cardinality is three at the most, then the weak axiom is equivalent to rationalizability by a preference relation [7]. We apply WARP to this dataset through binary relation, completing a rationalizing relation to show that the weak axiom is indeed, fulfilled. Functions in revealed preference theory are also always assumed to be convex. The prices for each bundle presented are the exact same, so the comparison rests heavier on the quantity purchased by consumers of each brand at each price point from the same observational pool, showing that consumers are maximizing their individual utilities according to their brand preference.

To establish formality between the two bundles (Tesla & BMW), we must establish binary relation. Let us begin with some basic yet essential definitions. The quantities purchased by consumers, as has been stated, are represented in simple matrices across sixty identical and stated price points, to focus on quantity moved where price is not a factor. As such, our set of alternatives are all positive integers which can be represented by $\mathbb{R}_+^2$, where:

X = Tesla Brand, X^P = equal price points of Tesla Brand

Y = BMW Brand, Y^P = equal price points of BMW Brand

We can then define our null hypothesis in the form of a binary relation as:

$$\Leftrightarrow xRy : \left(X, X^P\right) \gtrsim \left(Y, Y^P\right) \text{ iff } \left(X, X^P\right) \geq \left(Y, Y^P\right)$$

where our revealed preference bundle is denoted by $\gtrsim$. We must now prove this is a binary preference relation, by proving reflexivity, completeness, and transitivity to securely define that this is true choice behavior.

The reflexivity of X and Y

We know that $(X,Y) \in \mathbb{R}_+^2 \geq (X,Y) \Leftrightarrow (X) \approx (Y)$, so we can confidentially report reflexivity.

Completeness: In our data where the price level is not a factor in relation to units consumed, quantity consumed becomes our vital indicator for showing preference. Thus, we must show:

$\left(X, X^P\right), \left(Y, Y^P\right) \in \mathbb{Z}_+$ that either: $\left(X, X^P\right) \geq \left(Y, Y^P\right)$ or $\left(Y, Y^P\right) \geq \left(X, X^P\right)$

We then want to understand that the quantity consumed in either bundle (X, Y) is greater or equal to the opposing bundle, as price is not a concerning factor. The BMW brand moved 257 units between the minimal and maximal price constraints, whereas Tesla moved 211.

As such:

$$\left(257, Y^P\right) \geq \left(211, X^P\right) \in \mathbb{R}_+^2$$

Transitivity; In order to properly show these bundles are transitive, we must add in an additional bundle of goods and prove this bundle to not be preferred to our original. From the same dataset, we chose the Audi brand to represent our

Z bundle along the same sixty price points used for comparison in our original binary relation. The Audi brand shows a movement of 162 units along those same sixty price points and within our aforementioned 753 observations.

Here,

$$(Y=257),(X=211)\,\&\,(Z=162)$$

$$\Leftrightarrow \text{iff } \left(X,P^X\right),\left(Y,P^Y\right),\left(Z,P^Z\right)\in\mathbb{R}^2_+ \text{ s.t.}$$

$$\left(Y,P^Y\right)\geq\left(X,P^X\right)\&\left(Y,P^Y\right)\geq\left(Z,P^Z\right)$$

So,

$$\left(257,Y^P\right)\geq\left(211,X^P\right)\&\left(257,Y^P\right)\geq\left(211,X^P\right)$$

As such, $yRx;zRx$ and so Y (BMW) is our preferred bundle to the alternatives within our predetermined set $\mathbb{R}^2_+$ in rejection of the null hypothesis.

Preference relation can also be represented graphically, the below **Figure 1(a)** shows the preference between our original binary relation (Tesla vs. BMW).

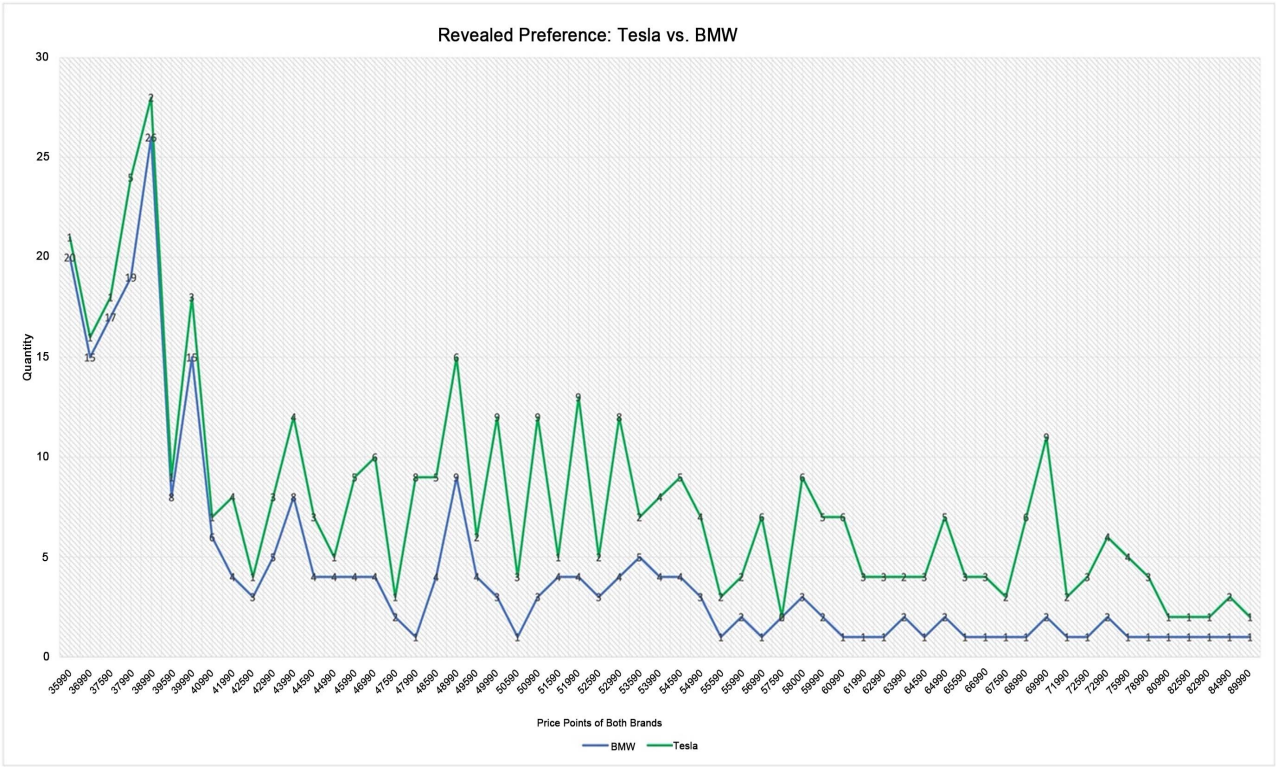
As the price points increase, the preference of the two brands is almost matching in volume of units moved, but as one can see from this figure, Tesla sells better than BMW does at higher price points. While this does not negate the findings of our binary relation, it does point to the sensitivity of price felt primarily by BMW consumers. Why Tesla brand sells better at higher prices is of course conjecture, but the reality is that regardless of the brands' ability to pull in higher values even for used automobiles, BMW still wins in a pure "units sold" comparison. It is possible that longevity in brand awareness for BMW contributes heavily to this, as Tesla is a much newer competitor in the oligopolist automobile industry.

When an additional bundle is added into the comparison from a third and separate manufacturer (Audi brand), we see almost the same phenomenon occurring; the lower price points on our scale are being dominated by the Audi and Tesla brands both, only to lose out on overall units moved to the BMW brand when the summation of all three is completed. Audi competes well in the low region of our price scale, from $35,990 to $46,990 but unlike Tesla, loses this advantage as the average price increases, as shown in **Figure 1(b)**, the below preference relation still shows the BMW bundle moving more units than the alternate bundles.

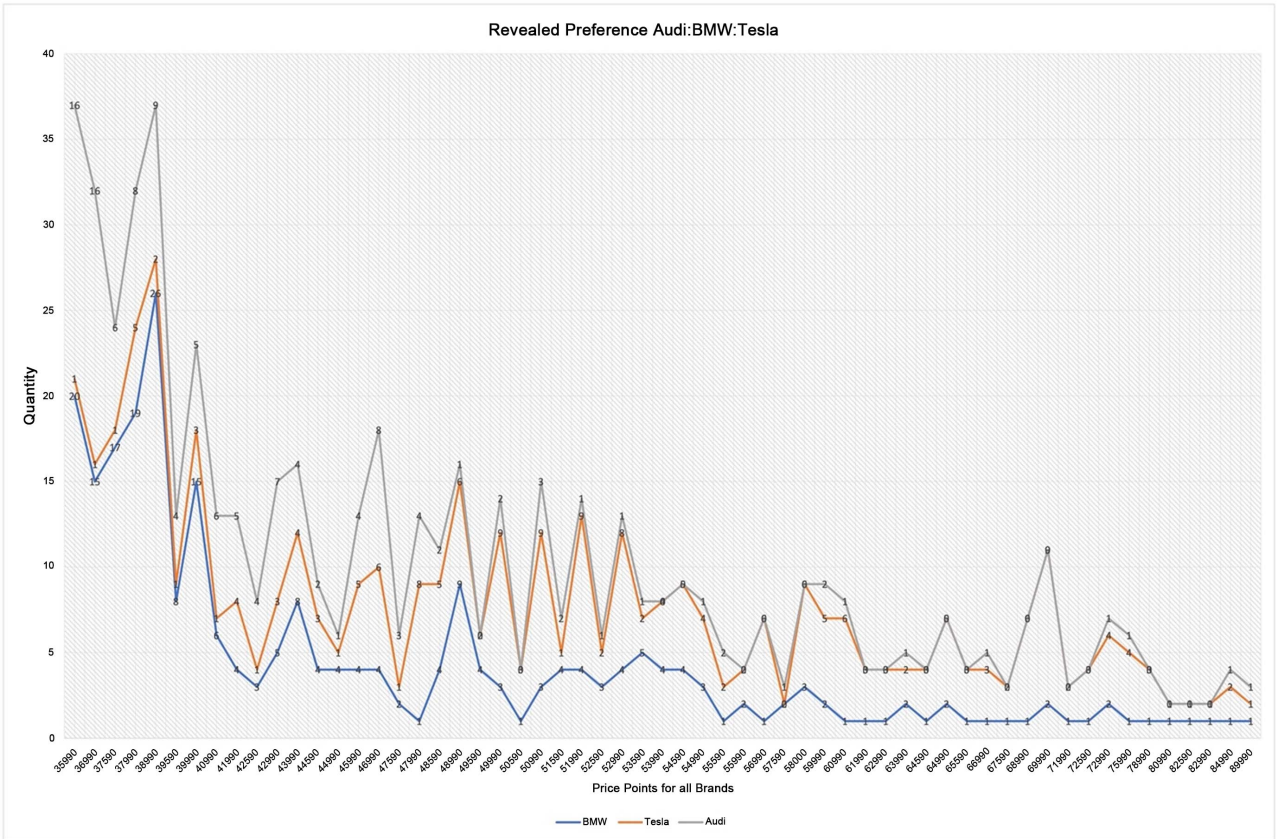
The Weak Axiom of Revealed Preference (WARP) states that if good X is chosen while good Y is also affordable (which is true in this case), then a consumer has suffered a loss in utility by not choosing good X, though both were initially affordable and as such, the consumer has *revealed* X is *preferred* to Y, as demonstrated for our bundles below in **Figure 1(c)**.

Here: $Y\left(p,q\right)\in\mathbb{Z}_+\Rightarrow Y\left(p',q'\right)\notin\mathbb{Z}_+$ and so, $Y\succsim X$

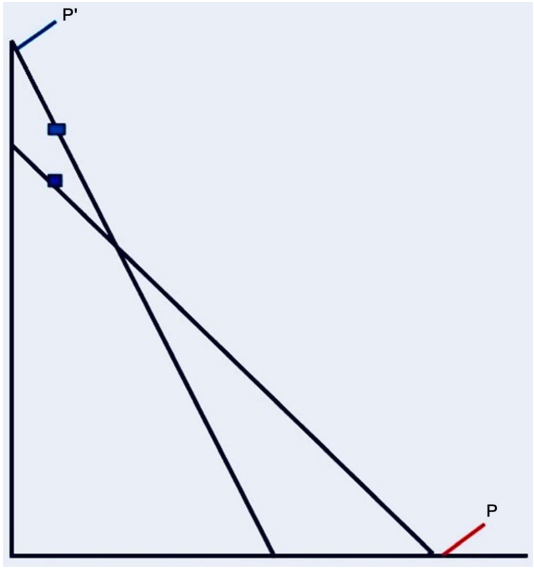
Formally, preference relations can also be described via a rational utility function $u:X\to\mathbf{R}$ by $x\succsim y$ if $u(x)\geq u(y)$ where u is represented by $\succsim$ and are said to possess said utility function if there exists a countable set $Z\subseteq X$ for all $x,y\in X$ for which $x\succ y$, there exists $z\in Z$ for which, $x\succsim z\succsim y$.[1]


Revealed Preference: Tesla vs. BMW
Quantity
Price Points of Both Brands
BMW
Tesla

(a)


Revealed Preference Audi:BMW:Tesla
Quantity
Price Points for all Brands
BMW
Tesla
Audi

(b)



(c)

Figure 1. (a) The Preference Relation of Tesla brand to BMW brand at Sixty Equal Price Points; (b) The Preference Relation of all three Observed Brands at Sixty Equal Price Points; (c) The WARP Representation of bundles X to Y.

Within our dataset, our countable set Z is the quantities of brands moved $\left(Q_x, Q_y, Q_z\right)$ within the complete set of examined alternatives X in our domain $\mathbb{R}_+^2$ and as such, makes $Z \subseteq X$ for all units $\to Z$ belong to the complete set of examined alternatives X. Furthermore, we have already shown $y \gtrsim$ to both $\left(x, z\right): X$ and that $z \in X : \left(y \gtrsim z, x\right)$. We can confidently show through quantities moved that $y \gtrsim x \gtrsim z$ where BMW brand is our Q_y.

We can also say that our binary relation $\gtrsim$ on X is rational if its choice function c strongly rationalizes c if for all $Z \in \Sigma$, $c\left(Z\right) = \left\{y \in Z : \forall x \in Z, y \gtrsim x\right\}$.[1] According to the classical abstract choice theory, a utility function is weakly rational if the choice function of the binary relation is defined by $y \gtrsim x \Leftrightarrow u\left(y\right) \gtrsim u\left(x\right)$ weakly rationalizes the choice function itself. Because "weak rationalizability allows for the existence of feasible alternatives that are equally as good as the chosen ones, but [not the observable choices]" we can see that the choice function is weakly rationalized as we only know our chosen quantities moved, but we did not personally observe the choices being made [7]. Through binary relation, we are able to define a maximizable utility function based on the sets of quantities and equal price points and show it to be a weakly rationalizable choice function, but one is that is both rational and not in violation of WARP.

This dataset does of course have limitations, mostly in the fact that these sales comprise only one year of used automobile sales and were compiled from one single source, Carvana Co. The actual market for used automobiles in the U.S. is massive as we have stated previously, around $195.84 billion while the total sales of our observed brands in this dataset calculate to $1.12 billion. Bias of course may also be present here due to this same reason, as we did not have available data from larger sources of used car sales such as independent lots, rival corporate dealers and others who share in the culmination of this massive resale mar-

ket. In a future study, we would want to gather multiple datasets from a plethora of used auto dealers and perform the same consistency testing using much larger observational sets, using binary relation to prove preference once more. Carvana is indeed one of the most successful corporate dealers in the U.S. but limiting our collection to only this single dataset may slightly impact the outcome of similar tests performed on larger datasets, a notion we will consider in future research on revealed preference.

4. Conclusions

Revealed preference can be a powerful tool in the ever-growing field of consumer theory, using data to help both econometricians and businesses alike understand the behavioral aspect of consumer purchases. It should be implicitly stated though, as others have, that "the modern versions of [revealed preference] are not entirely sufficient to determine whether or not a consumer's preferences can be described by a utility function due to the issue of integrability, in all cases other than the unrealistic scenario of only two commodities" [8]. The insight gained from studies utilizing revealed preference allows us to better understand market structures, the effects of price discrimination, anticipate growing and shrinking industries with better accuracy, and more.

In this study, we used a particular dataset on the sales of used luxury automobile brands to gain a better understanding of consumer preference amongst three of the market's top competitors: BMW, Tesla, and Audi. Initially, it was our assumption that Tesla would be the brand revealed preferred, but BMW quantitatively still reigned supreme to the electric vehicle manufacturer. As Samuelson himself has routinely stated, "all combinations of goods on or within the budget line could have been bought in preference to what was actually bought, but they weren't; hence they were all 'revealed' inferior to A [and] no other line of reasoning is needed" [9]. Because we were effectively able to hold the budget line of our experiment constant by testing quantity moved at equal price points, we could then simply show inferiority relevance purely through units moved at the same prices, in an experiment where the observations were too, constrained to the same equality.

Though it is outside the scope of this paper, there are possibly numerous obstacles the Tesla brand still faces to driving the majority of consumer decisions towards themselves, some within the firm's control and some outside of it. Revealed preference and preference relation theory have come a long way since first proposed by Samuelson in 1938 and we still have a way to continue its progress as economists, academics, and the public's unseen problem solvers in hopes we may gain a wider array of knowledge in consumer theory as a whole.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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