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A 2-Factor Model for Inclusion of Voluntary Termination Risk in Automotive Retail Loan Portfolios

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Abstract

Under the UK Consumer Act 1974, obligors of Hire Purchase and Conditional Sale contracts are allowed to perform a Voluntary Termination (VT) once certain conditions are met. Upon such an event, lenders recover the underlying assets but are potentially liable to losses upon liquidation of the assets. This poses a challenge from a risk modelling perspective, as these financial products exhibit Credit (default) risk as well as VT risk, and these two events are mutually exclusive. In this paper, we propose a modelling framework to account for Credit/Default and VT risk for Retail portfolios, designed as a 2-factor Monte Carlo simulation of loan-level termination events. The paper concludes with numerical and backtesting results from a real-life implementation of such framework in the context of an automotive loan portfolio.

Keywords

Counterparty Credit Risk, Voluntary Termination

1. Introduction

1.1. The Consumer Act 1974

The Consumer Credit Act 2006 [1] is the main UK legislation that covers consumer credit law. A large portion of the provisions in this Act are inherited from the Consumer Act 1974 [2], which introduced specific regulations on a broad range of retail financial products.

Among the products covered by the Consumer Act 1974, Conditional Sale (CS) and Hire Purchase (HP) contracts are among the most popular finance options available in the UK automotive market. In CS contracts, the customer agrees to buy specific goods (e.g. a vehicle) whereby the agreed sum, or a part of it, is payable by installments; once all repayments are made, the customer obtains ownership of the goods. In HP contracts, the customer leases the vehicle for an agreed period by paying a monthly sum; at the end of the period, the customer has the option to gain ownership by paying an additional amount (the Purchase Fee).

Section 99.1 of the Consumer Act 1974 states the conditions for a Voluntary Termination (VT) "*At any time before the final payment by the debtor under a regulated hire-purchase or regulated conditional sale agreement falls due, the debtor shall be entitled to terminate the agreement by giving notice to any person entitled or authorised to receive the sums payable under the agreement*" [2]. Section 100.1 further states that unless alternative arrangements are made, the obligor is liable to pay half of the total amount due as part of the contract, upon exercising the right to VT—this is also known as the *halves rule* in UK financial services.

1.2. Voluntary Termination as a Risk Factor

In the context of a portfolio of CS/HP loans, Voluntary Termination is effectively an additional source of financial risk for the lender. In the particular case of automotive CS/HP contracts, the lender retains ownership of the vehicle upon a VT event. In order to recover the original amount financed for the lease, the vehicle is then typically liquidated via an auction process. Commonly, this results in a net loss for the lender, as the amount received from the sale of the vehicle at the auction, net of the auction costs, is typically less than the outstanding amount owed by the obligor to the lender at the time of VT.

It is important to note that VT risk is a separate loss driver with respect to traditional Credit risk. CS and HP loans are still exposed to customer defaults and delinquencies, and the dynamics of VT and default events are fundamentally different: the VT option is a legal right (enshrined in Consumer law) to terminate a contract, whereas a default is an outright contractual breach. It also follows that, for any given loan, VT and default events are mutually exclusive: a loan terminated via the exercise of VT rights cannot default thereafter, and *vice versa*.

1.3. Benefits of VT Risk Modelling

VT risk can be a substantial contributor to financial losses. It is therefore important to accurately model VT risk in portfolio management, forecasting and balance sheet stress testing applications. The structuring and management of Significant Risk Transfer (SRT) securitization transactions also benefits from a sound modelling of VT risk Expected (EL) and Unexpected Losses (UL), as they provide validation of the risk-offsetting nature of the transaction to regulatory bodies.

1.4. Proposed Framework and Aim

Our proposal is to model portfolio losses in a holistic way, by means of jointly simulating loan-level default and VT events via a Monte Carlo scheme driven by cumulative probability of default (PD) and probability of VT curves. The result is a 2-factor model, where VT and Credit risk are sources of separate yet mutually-exclusive loan events. Monetary loss is then modelled via traditional Exposure at Default (EAD) and Loss Given Default (LGD) methods, with the introduction of specific Exposure at VT (EAVT) and Loss Given VT (LGVT) quantities for VT events. The benefits of the framework are:

1) **Adherence to real-life dynamics**: VT and default events are separate but mutually-exclusive events. It is anecdotally known that VT risk is managed as an independent model by Risk Managers—such an approach makes it difficult to calculate robust Unexpected Total (Credit + VT) Loss metrics, due to the fact that Credit and VT events are indeed not independent and the sum of extreme quantiles from independent distributions is, in general, not a good approximation for the extreme quantile of the joint distribution. This limitation is not present in our framework, as the events are jointly modelled in a single model.

2) **Output flexibility**: the Monte Carlo model computes full distributions of Credit, VT and Total (Credit + VT) losses at any time horizon;

3) **Performance**: numerical results show the feasibility of this framework as a near-real-time risk tool for portfolios of up to hundreds of thousands of loans.

Additionally, we believe the present work can be of particular interest to practitioners and academics alike, as there is no prior literature of models specifically addressing a Voluntary Termination risk factor for retail portfolios. As the key novel point of the paper is the introduction of VT risk in a Credit modelling framework, the present paper will mainly focus on the modelling/quantification of VT risk. This is also true for the backtesting results: since there are several references for analysis and benchmarking of Credit risk models (e.g. [3] [4]), the backtesting in the present paper will focus on the comparison of realized VT losses and model-predicted VT losses.

1.5. Structure

The paper is organised as follows. Section 2 describes the proposed modelling framework and methodology for the development of a 2-factor Credit and VT risk engine. Section 3 presents numerical results from a real-life implementation of the framework, calibrated against historical data and backtested on the performance of a live securitized portfolio. Section 4 provides some conclusive remarks.

2. Framework Methodology

2.1. Input Data

2.1.1. Loan-Level Inputs

The model takes several loan-level quantities as inputs. The list of inputs required for a single loan is as follows:

1) **Amortization profile**: specifies a term structure of principal cashflows to be repaid by the obligor as per its contractual obligations. Note that this term structure does not include any prepayment assumption.

2) **Cumulative Probability of Default curve**: determines the cumulative likelihood of a default event. Note that for the purpose of the present implementation, a defaulted loan is considered equivalent to an IFRS9 Stage 3 delinquent asset, but other definitions may also be used.

3) **Loss Given Default**: determines the proportion of outstanding balance (exposure) to be effectively lost due to the default event.

4) **Cumulative Probability of VT curve**: determines the cumulative likelihood of a VT event. For the purpose of the present implementation, a VT'd loan is considered as a loan for which a Voluntary Termination process has been completed and the underlying vehicle liquidated (or written off, e.g. in case of extensive damage).

5) **Loss Given VT**: determines the proportion of outstanding balance (exposure) to be effectively lost due to the VT event.

2.1.2. Simulation Parameters

In addition to general Monte Carlo parameters such as the simulation time horizon and the number of scenarios, additional macro inputs to the model are as follows:

1) **Constant Prepayment Rate (CPR)**: specifies a percentage of monthly notional paid in excess every month by all loans.

2) **Event severity probability**: specifies the likelihood of a termination event being classed as *severe*. Severe events feature a longer time lag between the termination event and their resolution, thus resulting in a higher exposure.

2.2. Modelling of Termination Events

As mentioned in the introductory section, our proposed framework revolves around a Monte Carlo simulation of a selected portfolio of loan assets, which can each be subject to default and VT termination events. This section illustrates the framework's strategy to account for the two events simultaneously for each loan. Figure 1 provides a high-level flowchart of the modelling strategy.

2.2.1. Determination of Loan Event Time

Without loss of generality, the determination of an event time is performed in a similar manner for both defaults and VT. A visual example for the default case is shown in Figure 2:

The procedure to determine the event time is as follows:

1) A uniform random variable a is drawn from the [0, 1] range;

2) The random variable is compared and intersected with the cumulative probability curve for the event. Note that the cumulative curve is provided as a vector of cumulative probabilities on a discrete (typically monthly) basis,

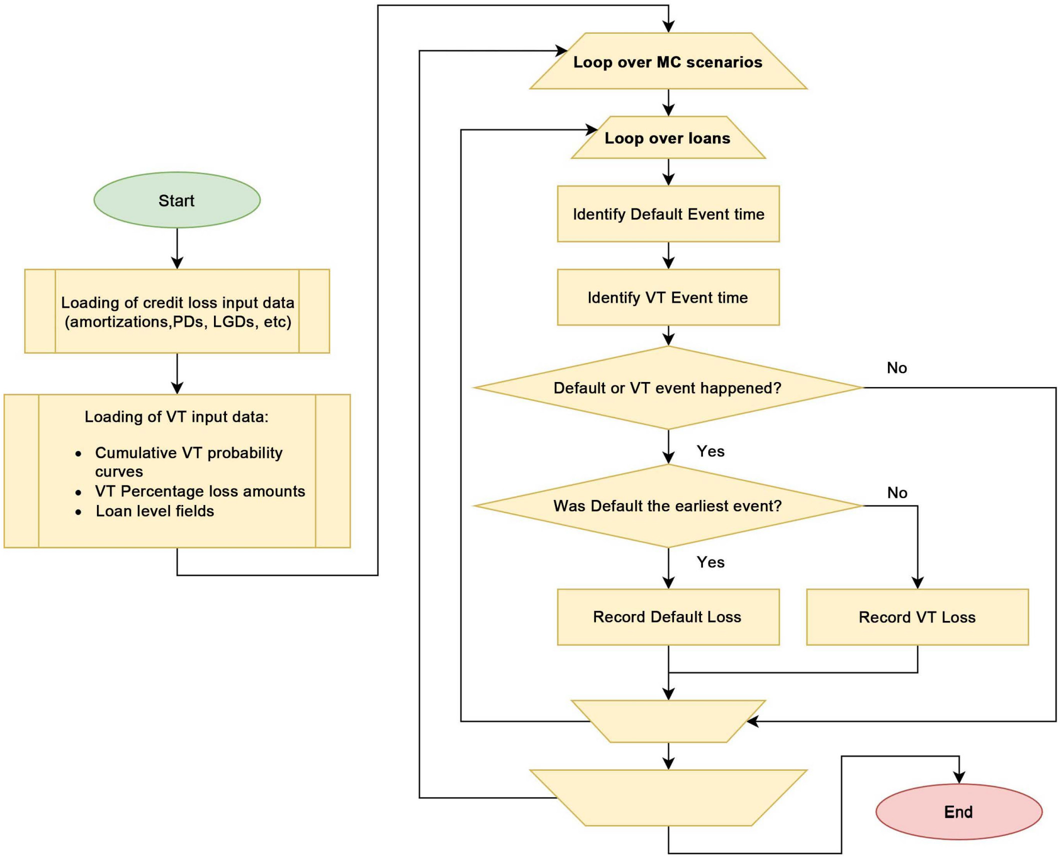

Figure 1. High-level flowchart of the 2-factor termination event modelling.

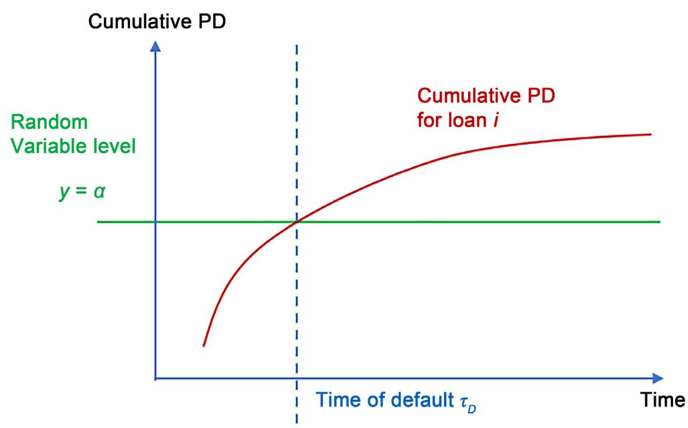

Figure 2. Example of determination of default event time.

therefore the intersection is performed in practice with a binary search of the vector with respect to α;

3) The horizontal coordinate of the intersection point determines the desired termination event time. If there is no intersection, the termination event time is conventionally assumed to be infinite, *i.e.* it does not happen within the loan's lifetime.

Note that the role of the Monte Carlo simulation is precisely that of generating the uniform random variables α, which eventually allow the explicit simulation of termination events for each loan and scenario.

The random variables can also be correlated via standard techniques, e.g. multiplying the random variable vector by the Cholesky decomposition factor of the correlation matrix.

2.2.2. Simulation of Loan Events

In every scenario of the Monte Carlo simulation, the model assesses the occurrence of a termination event on a loan-by-loan basis. A termination event is determined for each loan as follows:

1) A *potential* time of default τ_D and a *potential* time of VT τ_{VT} are calculated as per the method described in Section 0.0.3

2) A *realised* event is then recorded according to the following rules:

a) If $\tau_D \le \tau_{VT}$ and $\tau_D \ne \infty$, a default event is recorded.

b) If $\tau_D > \tau_{VT}$ and $\tau_{VT} \ne \infty$, a VT event is recorded.

c) If $\tau_D = \tau_{VT} = \infty$, no event is recorded and the loan is assumed to have naturally reached its contractual maturity.

2.2.3. Bootstrapping Method for Large Correlated Portfolios

In case the number of loans is too high and in turn correlation matrices are too large to be kept in memory and/or to be feasibly decomposed, a bootstrapping method can be used instead. The method consists of sampling a lower number of loans from the portfolio in such a way to allow the use of correlation matrices, simulate the smaller portfolio and repeat the procedure for a statistically significant number of times. The final simulation output is then taken as the average of all the smaller sub-runs.

2.3. Calibration of Cumulative VT Curves

The calibration of VT probability curves is based on a historical approach, which will be described in this section. Note that as the calibration of PD curves is a well-studied problem in the literature and not the main focus of the present paper, we direct the reader to [4] [5] [6] for treatises on industry-standard design and calibration of PD curves.

Lenders regularly monitor the performance of their loan portfolios, including VT event details such as the time of VT, the corresponding incurred loss and all details of the affected loan (e.g. LTV at origination, original loan term, etc.). Given sufficient quantity of this historical data, it is possible to infer historical curves of realized VT rates. It is reasonable to assume that these historical curves can be used to simulate the behaviour of a portfolio under analysis, provided the loans in the portfolio and those in the historical dataset are of similar type and composition.

The historical curves used as part of the present model are calibrated on the basis by a number of *bandings, i.e.* stratifications of the historical dataset used for calibration. Such bandings in general will have to be chosen in such a way to reflect the peculiarities of the financial products in the portfolio, and guaranteeing a sufficient amount of data in each banding.

As the present paper focuses on an application of the model to the automotive loan industry, the chosen bandings were:

1) Original loan term, expressed in terms of min/max brackets

2) Fuel type of the underlying vehicle

3) LTV at origination, expressed in min/max LTV brackets

4) Applicant credit score at origination, expressed as min/max credit score brackets

Prior to the start of the simulation, each loan gets mapped to the calibrated curve whose bandings match the loan's term, underlying fuel type, LTV and credit score features. The following sections provide additional details on the chosen bandings and the calibration methodology.

2.3.1. Original Loan Term

Without loss of generality, the probability of a VT event is not constant across the lifetime of a loan. It is instead usually observed to be higher closer to the point in time where the conditions to satisfy the halves rule have been met, and lower towards the start and the end of the original loan term. This behaviour is driven by:

1) **VT exercise conditions**: a customer can exercise the VT option at any point in the lifetime of the loan, but it is more economically sensible to do so when the halves' point of the loan is reached, *i.e.* when the customer has paid at least 50% of the total amount owed to the finance company[1]. This explains the spike in VT probability around the middle[2] of the overall term and the relatively low probability at the start of the term.

2) **Asset depreciation**: at the halves' point a loan can frequently be in a negative equity position due to the car' s depreciation, thus encouraging the exercise of the VT option. However, towards the end of the term, loans are more frequently found in a positive equity position, thus carrying less financial appeal towards the VT exercise and resulting in lower VT rates.

These two factors lead to typical "S"-shaped cumulative VT curves, such as the ones shown in **Figure 3**. Additionally, the precise shape of the cumulative curve is found to be highly dependent on the overall duration of the loan. For example, the halves' point (and therefore the expected point of inflexion in the cumulative curve) is directly dependent on the loan duration, and vehicles do not depreciate at constant rates across time.

2.3.2. Fuel Type

Different regulations (e.g. European-wide future bans on diesel cars) and incentives (e.g. government subsidies for the purchase of hybrid-powered vehicles) can greatly influence decisions around keeping or switching cars, influencing in

[1]Note that the halves rule is specific to the UK market. For example, Danish financial regulations also allow VT for certain products but do not include the halves rule.

[2]Note that the halves' point does not necessarily coincide with the chronological midpoint of the contractual loan term.

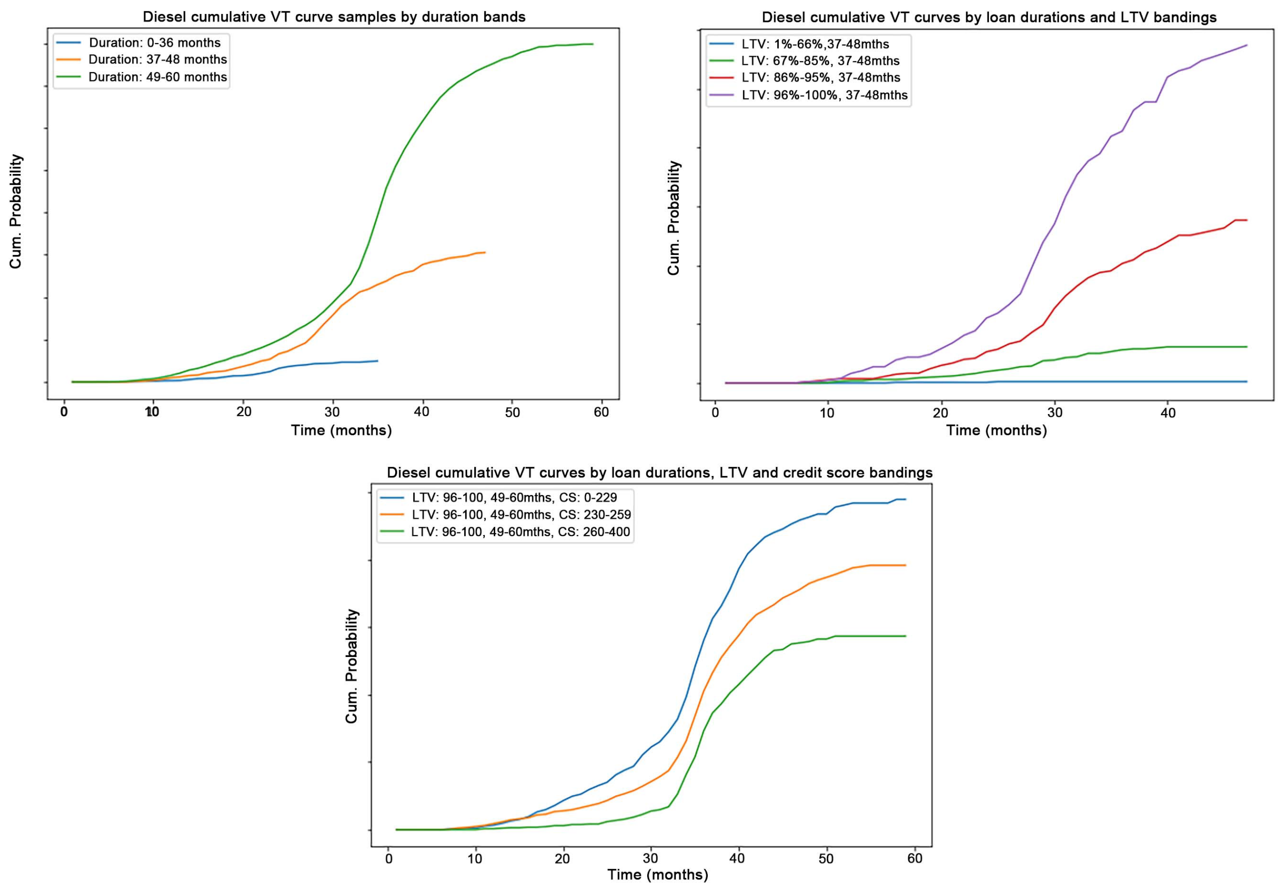

Figure 3. Sample VT curves by different banding choices. Note that the probability scales have been hidden for confidentiality reasons.

turn the overall VT rates. Note that for the specific calibration data at hand, the only Fuel Type bandings used were "Petrol" and "Diesel", as data points for electric, hybrid and alternative fuel vehicles were not sufficient.

2.3.3. LTV at Origination

The economic incentive to the customer in performing a voluntary termination is likely to be higher if, at the halves' point of the loan, the value of the underlying vehicle is significantly less than the outstanding balance. An intuitive choice for a banding is then the so-called Future Loan to Value (FLTV), defined as the ratio between the theoretical outstanding balance of the loan and the estimated future value of the underlying asset/vehicle. The estimated future value of the vehicle, however, is typically provided by 3rd-party specialist firms which may not necessarily have full data coverage for the historical dataset of interest.

As a proxy of the FLTV, the present model uses the Origination LTV as one of the bandings, defined as:

$$\text{Origin. LTV} = \frac{\text{Funded Amount}}{\text{Funded Amount} + \text{Deposit}}$$

This quantity has the added benefit of not being reliant on 3rd party data that can be missing for certain loans. Analysis of the available historical data has shown a reasonably strong differentiation between historical VT curves across LTV bands, see for example **Figure 3**.

2.3.4. Credit Score at Origination

VT events can also be the result of "near-default" situations, *i.e.* cases where customers already versing in financial difficulties end up exercising their VT option to avoid credit records' deterioration. This suggested a potential dependency between VT rates and the estimated credit quality of the applicant. Analysis of historical data confirmed this suggestion, showing reasonably strong differentiation in VT rates across credit score bands, see for example **Figure 3**. Note that the credit score utilised in the presented banding is an internal reference credit score with a value range between 0 and 400.

2.3.5. Calculation of Cumulative VT Probability

VT cumulative curves are constructed using monthly marginal historical VT rates. The marginal VT rate $pvt_b(m)$ for a given month m and portfolio banding b indicates the historical likelihood of a VT event being recorded within months m and $m + 1$, for any loan belonging to the banding. The recorded date for a VT event, in our dataset, is intended as the date of VT loss realization. This likelihood is calculated using historical data as:

$$pvt_b(m) = \frac{VT_b(m)}{N_b(m)} \tag{1}$$

where:

- $N_b(m)$ is the total number of loans in the banding b which were recorded to be active at m months from origination.
- $VT_b(m)$ is the total number of loans in the banding b which ended up in VT between m and $m + 1$ months from origination. The banding b is the combination of the loan duration, fuel type, LTV and credit score segmentations mentioned in the previous sections.

Note that both $N_b(m)$ and $VT_b(m)$ are taken across all available vintages. Therefore, the overall pool of loans that is available for analysis changes with m, due to long-term historical data not being available for the most recent vintages.

Once marginal rates are obtained for all months $m \le D_{\max}$, with $D_{\max}$ being the maximum duration in the selected duration band, cumulative VT rates $PVTC_b(m)$ are obtained as:

$$PVTC_b(m) = \sum_{n<m} pvt_b(n) \tag{2}$$

Note that the term structure given by all values of $PVTC_b(m)$ describes the cumulative VT curve from origination. Conventionally, $PVTC_b(0) = 0$, as it is assumed that any loan in the portfolio is active as of origination date $m = 0$. If a loan in the simulated portfolio is not starting from origination, *i.e.* with a non-zero time on book of t months, its corresponding VT cumulative curve for any month $m > t$ is expressed as:

$$PVTC_b(m) = PVTC_b(m+t) - PVTC_b(t) \tag{3}$$

2.4. Loss Calculation

2.4.1. Exposure Calculation

If a loan has been terminated via a default or VT event at time τ, the loan's exposure $E(\tau)$ for the event is computed as:

$$E(\tau) = \sum_{t=\max(0,\tau-\delta_t)}^{T} AM(t)$$

where:

- $AM(t)$ is the contractual cashflow at month t defined by the amortisation profile.
- T is the contractual maturity of the loan.
- τ is the termination event time (either default or VT).
- δ_t is a time lag assigned as per Section 2.4.2. As a result of the lag, the summation spans the interval $[\max(0,\tau-\delta_t),T]$, thus incorporating cashflows prior to the actual termination time to account for balance build-ups/arrears that are commonly observed in real-life portfolios.

Note that the exposure takes the name of Exposure at Default (EAD) or Exposure at VT (EAVT) for default and VT events respectively.

2.4.2. Event Severity and Time Lag Assignment

The model assigns the event's severity according to the dirty event proportion input, see Section 2.1.2. In particular, a uniform random variable α is drawn between 0 and 1, and it is compared to the dirty event proportion input d:

- If $\alpha < d$, the event is flagged as severe and the time lag δ_t is assigned as a large lag (e.g. 8 months), representative of loans that require a significant amount of time before the loss is realized (also known as dirty terminated loans).
- Otherwise, the event is flagged as non-severe and the time lag δ_t is assigned as a smaller lag (e.g. 3 months), representative of normal timeframes between an event notification and the asset's liquidation (also known as *clean* terminated loans).

Clean/dirty lags and dirty event proportions can be calibrated by analyzing the historical data in the portfolio. In the present implementation, once the termination events have been categorized as dirty or clean, the dirty event proportion has been computed as the ratio between the number of dirty events and the total number of termination events, and the clean/dirty lags have been computed as the average number of months to liquidate an asset and realize a loss after a clean/dirty event has been notified. Note that the definition of clean and dirty events is in general arbitrary, and may vary between portfolios and/or product types.

2.4.3. VT Loss

If a loan terminated via a VT event, a corresponding VT loss is recorded. The calculation of VT losses adopts an "Exposure/Loss Given Event" approach similar to what traditionally done in the Credit risk world. The rationale for this approach is that losses due to VT events are driven by a multitude of micro-drivers, such as vehicle depreciation (due to both market conditions and wear & tear), bespoke agreements between the lender and the obligor, friction costs in the liquidation process, and others. As modelling of these micro-drivers is not feasible, we propose a Loss Given VT parameter in the same spirit of the Loss Given Default (LGD) parameter used for Credit losses. The LGVT parameter can be historically calibrated, as explained in Section 2.4.4.

If a VT event is recorded for a loan as per the decision tree in Section 2.2.2, the corresponding VT loss L_{VT} is computed as:

$$L_{VT} = EAVT\left(\tau_{VT}\right) \cdot LGVT$$

where:

- τ_{VT} is the time of loss realization for the VT event.
- $EAVT\left(\tau_{VT}\right) = E\left(\tau_{VT}\right)$ is the Exposure at VT.
- $LGVT$ is the Loss Given VT.

2.4.4. Calibration of LGVT Parameters

The LGVT parameter is estimated as the average of the historical VT loss distribution, specified as a percentage of the EAVT. In the present implementation, the LGVT parameters were calibrated on the basis of the same historical dataset used to calibrate the VT probability curves.

The analysis of historical loss dataset did not highlight significant differences in LGVT values across the bandings used for VT curve calibration. However, fuel type was retained as the only banding for the LGVT. The rationale for this choice is to allow more tailored specifications of "what-if" scenarios: for example, it could be argued that future regulations will be increasingly more punitive towards Diesel and Petrol vehicles, thus reducing their appeal on the used market and increasing realized losses due to LGVT events. Therefore, having Petrol- and Diesel-specific loss parameters would allow the model user to selectively perform stress analyses.

2.4.5. Default Loss

If a loan terminated via a default event, a corresponding default loss is recorded instead, calculated using a traditional EAD/LGD approach. A default loss L_D is computed as:

$$L_D = EAD\left(\tau_D\right) \cdot LGD$$

where:

- τ_D is the time of loss realization for the default event.
- $EAD\left(\tau_D\right) = E\left(\tau_D\right)$ is the Exposure at Default.
- LGD is the Loss Given Default parameter.

2.5. Applicability to Other Product Portfolios

The introduction of Voluntary Termination as a risk factor makes the proposed model, as mentioned earlier, particularly suited for the modelling of UK automotive loans. It is worthy to conclude the description of the proposed modelling framework with a brief discussion on its applicability to portfolios of different retail finance products. In general, the proposed framework can accommodate the simulation of any other retail financial product insofar as loan-level data such as the contractual amortization profile, Probability of Default and Loss Given Defaults (and appropriate VT inputs too, if the product is subject to Voluntary Termination clauses) can be provided. If required, additional considerations for specific products can be addressed as necessary - some examples are presented in the following list:

- In the case of credit cards and other revolving credit products, particular care should be given to the amortization profile. Appropriate estimates should be made regarding the monthly exposure, which is revolving and could be subject to seasonality effects;
- In the case of asset-backed loans with hand-back options at loan maturity, this should be modelled as a separate factor as it is traditionally driven by different drivers than Voluntary Termination;
- Mortgages, especially in the US market, are known to have instead rather peculiar prepayment dynamics that should be modelled into the amortization profile. For these products, a Constant Prepayment Rate approach to be applied to the whole portfolio may not be suitable.

3. Numerical Results

3.1. Test Portfolio Description

The numerical results were produced by analyzing a real-life portfolio of approximately 95,000 UK automotive loans, of which roughly 98% are Conditional Sale contracts and 2% are Lease Purchase contracts. The loans are a mixture of newly-originated and seasoned loans, with a large prevalence of 60-months, 48-months and 36-months contractual term lengths (respectively 58%, 17% and 16% of the total). Roughly 93% of the loans feature used vehicles as the underlying asset. The portfolio was analyzed as of its inception date of 31 December 2018.

Note that for confidentiality purposes, we will not report the portfolio balance. Similarly, the numerical results in the following sections will only be reported in percentage points of balance rather than in units of currency amounts.

3.2. Simulation Setup

The portfolio was run with the setup presented in **Table 1**.

Note that CPR, Clean/Dirty lag and Dirty event probability inputs were obtained as historical averages from 5 years of Conditional Sales loan performance data. As default correlations are quite difficult to estimate in practice, we have conservatively based the default correlation input upon the BCBS recommended value of 4% asset correlation for retail credit card portfolios [7]. As a full correlation matrix is too large for the portfolio at hand, we adopted a bootstrap run as

Table 1. Simulation setup.

Simulation Parameter	Value
Reference date	31 Dec 2018
Monte Carlo Scenarios	10,000
CPR	15%
Default Correlation	5%
Clean lag	3 months
Dirty lag	8 months
Dirty event probability	40%
Timesteps	60 (monthly steps over 5 years' horizon)
Bootstrap sub-runs	100
Number of loans per bootstrap	1000

described in Section 2.2.3. For the purpose of this test, we have assumed no correlation between VT events.

The model was run as a C# implementation on a standard laptop (Intel i7 processor, 16GB RAM), taking approximately 1.5 minutes to process the portfolio simulation.

3.3. Simulation Results

Figure 4 shows the simulation results in terms of Credit and VT Expected and Unexpected Losses. Unexpected Losses are defined as 99.9%-percentiles of the corresponding loss distributions. All values are cumulative unless specifically defined as Point in Time (PiT).

Figure 5 shows cumulative loss distribution graphs for Credit, VT and Total (Credit + VT) losses, at the final time horizon for the simulation (month 60, or year 5). Note how the VT loss is bell-shaped, relatively sharp and symmetric, whereas the Credit loss distribution has a noticeably fatter tail. This is due to:

1) The impact of correlation parameters, which are zero for VT random variables and positive (5%) for default random variables;

2) The relative homogeneity of the portfolio and the bootstrap process, for which the aggregation of uncorrelated losses (such as in the case of VT) tends to converge to a binomial distribution.

3.4. Backtesting of VT Event Rate and VT Loss

In order to verify the validity of the VT Risk modelling, a backtesting exercise was performed on the period between December 2018 and June 2020, *i.e.* up to 18 months after the portfolio inception date. The rationale behind limiting the backtesting period to 18 months was to avoid periods heavily impacted by UK government interventions (e.g. *payment holidays*) in response to the COVID-19 crisis. As mentioned in the introduction, the backtesting focuses on the comparison of VT simulated risk metrics against realized values.

Year	Credit EL	Credit Loss - 99.9% percentile	Defaults per period (PiT)	EAD (PiT)	VT EL	VT- 99.9% percentile	VTs per period (PiT)	EAVT (PiT)	Total Loss (Credit +VT) - 99.9% percentile	Balance	Principal Repaid (PiT)
0										100%	
1	0.46%	2.77%	523	0.60%	0.20%	0.61%	517	0.61%	2.98%	62%	37%
2	0.92%	4.46%	599	0.59%	0.39%	0.93%	477	0.58%	4.86%	32%	29%
3	1.15%	5.19%	386	0.30%	0.52%	1.11%	327	0.38%	5.71%	13%	19%
4	1.24%	5.44%	195	0.11%	0.53%	1.13%	58	0.05%	5.98%	2%	10%
5	1.25%	5.47%	46	0.02%	0.53%	1.13%	4	0.00%	6.01%	0%	2%

Figure 4. Simulation results for the test portfolio.

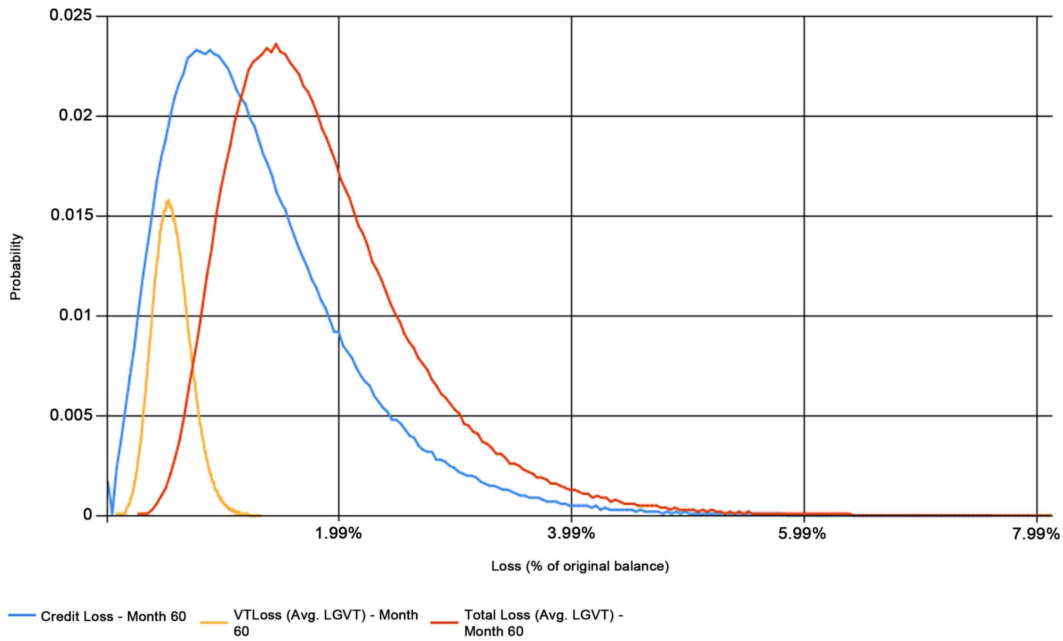

Figure 5. Credit, VT and Total (Credit + VT) Loss distributions, using 200 bins for each distribution.

Figure 6 shows a comparison of average simulated versus realized cumulative VT rates for the overall portfolio. By visual inspection, the simulated event rate matches the realized one remarkably well after approximately 7 months. Note how the realized cumulative curve shows a flattening between months 15 and 17, likely due to the onset of the COVID-19 crisis and government interventions.

Figure 7 shows a comparison of average simulated versus realized cumulative VT losses for the overall portfolio. By visual inspection, the simulated loss tracks the realized one fairly accurately. Additional analyses of the portfolio at hand revealed that the realized LGVT in this particular portfolio was higher (approximately 5% on average) than in the dataset used for calibration of the LGVT simulation parameters, thus largely explaining the larger discrepancy in the losses with respect to the VT event rates.

4. Conclusions and Future Work

We have presented a quantitative framework for the inclusion of Voluntary Termination risk in UK retail portfolios. The framework is based on a granular

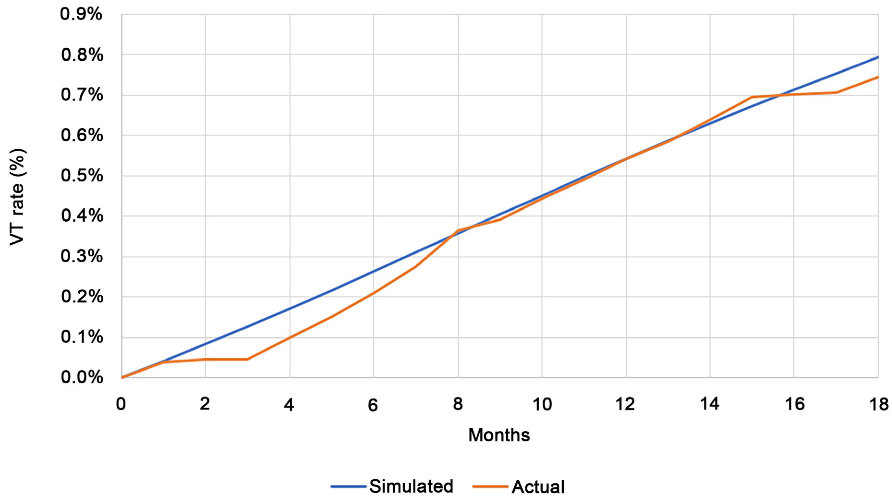

Figure 6. Simulated (average) vs. Actual cumulative VT event rate in the backtested period.

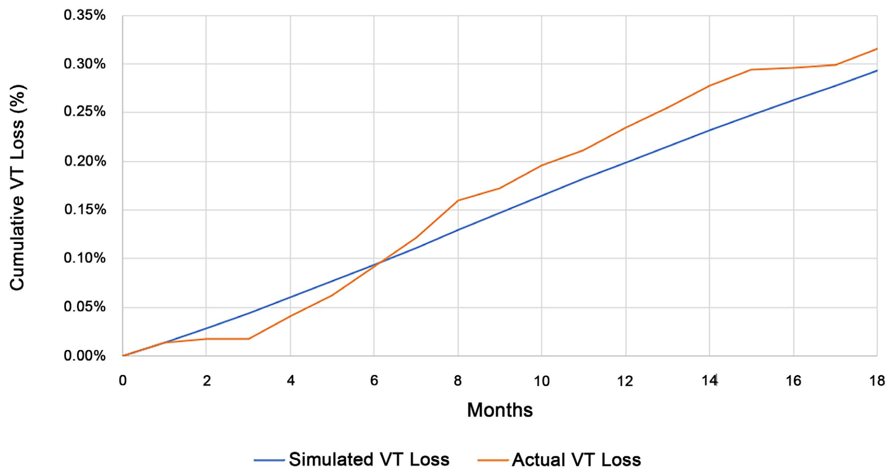

Figure 7. Simulated (average) vs. Actual cumulative VT loss in the backtested period.

loan-level simulation that accounts for mutually-exclusive default and VT events. We presented calibration strategies for the Voluntary Termination probability curves and Loss-Given-VT parameters, with special attention to the automotive business. Finally, we presented simulation and backtesting results from a real-life portfolio, showing the validity of this framework in modelling portfolio-level Voluntary Termination event rates and losses with reasonable accuracy.

Challenges in Calibration

The calibration approach described in Sections 2.3 and 2.4.4 relies, like the majority of approaches in Risk modelling for Retail Banking portfolios, on historical data. It is worthy to note that, whilst the results of the backtesting presented here showed a satisfactory degree of accuracy, the incoming years and months will likely be particularly challenging in terms of parameter recalibration. This is because:

1) The historical period that spanned the recent COVID-19 pandemic was highly influenced by government intervention activities which suppressed Credit and VT events. A calibration based on the blind application of data from this period would not tend to capture true customer behaviours;

2) At the time of writing, inflationary policies and asset prices are at peculiar historical peaks, which are unlikely to be sustained in the long period;

3) In the specific case of vehicle financing, government policies in Western countries are going more and more towards incentivizing electric vehicles. This is likely to have an asymmetrical impact on losses and the frequency of Default/VT events for different vehicle fuel types.

For these reasons, we envisage an approach based on the following high-level principles:

1) Risk managers should identify a suitable historical period to be considered as a "baseline" for the customer population under analysis;

2) Baseline parameters should be calibrated on the chosen historical period;

3) Stress factors should then be introduced to reflect Risk managers' views of future markets with respect to the chosen baseline. Note that the stress factors mentioned earlier could be calibrated as a combination of projected data (e.g. asset price index projections, as provided by specialist third party vendors) as well as internal models, and/or Subject Matter Expert (SME) inputs. We envisage a detailed analysis and discussion on parameter recalibration as a particularly stimulating basis for future work.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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