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Optimal Water Allocation Model of Inter-Basin Water Transfer Based on Option Contracts under Uncertainty

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Abstract

Inter-basin water transfer is a large-scale artificial method to transfer water from water-surplus areas to water-deficient areas, so as to promote the economic development of water-deficient areas. In this paper, water call options are introduced to improve the management of inter-basin water transfer. As the seller of water call options, the water diversion area benefits from water call options, as well as bears the risk of a water shortage caused by the exercises of water call options. On the one hand, the economic benefit of the system can be maximized by choosing the maximum water availability and the exercise prices of water call options. On the other hand, by using water call options, the water diversion area obtains certain economic compensation and the water receiving area gains additional water to ensure water security in dry seasons. By considering the uncertainties in the process of water resource management, an interval two-stage stochastic multi-objective mixed integer programming (ITSMMIP) model is developed for supporting decisions of water resource allocation when water call options are applied in inter-basin water transfer. The results prove the effectiveness of the model.

Keywords

Inter-Basin Water Transfer, Inexact Optimisation, Water Call Option, Water Allocation

1. Introduction

Due to the rapid development of economy and society and the uneven temporal and spatial distribution of water resources, the water resources in many economically developed areas in China have been unable to meet the water demand. In order to solve the contradiction between the temporal and spatial distribution of water resources and the imbalance of regional economic development, China has successively constructed dozens of inter-basin water transfer projects, such as a water diversion project from Luan River to Tianjin City, a water diversion project from Biliu River to Dalian City, Dongshen water supply project and south to North Water Transfer Project, which solved the problem of the uneven temporal and spatial distribution of water resources from the infrastructure level [1].

Inter-basin water transfer has been an important measure for water resource allocation, which has greatly alleviated the water shortage in municipal, industrial and agricultural, and ensured the growing water demand for economic and social development in water-deficient areas. However, there are still many management problems to be solved in the field of inter-basin water transfer. The first is the lack of effective management of water resources. Generally, there is no relevant water transfer agreement between the water diversion area and the water receiving area, and the responsibilities and obligations are not clear. Especially in the dry season, disputes between the two parties are easy to occur, and the water demand in the water receiving area can not be guaranteed, which increases the difficulty of water resource management. Second, the optimization of water resource allocation in each area is imperfect. Adjustment of the water consumption structure and balance of the priority of water resources utilization in the water diversion area and the water receiving area need to be considered in the water transfer process according to the economic and social development of each area. Third, the water resources trading system has not been established, and the water market is still a quasi-market. The establishment of water resources trading mechanism under the macro-control of government is more conducive to the optimal allocation of inter-basin water transfer, and can improve water use efficiency and provide more reasonable economic compensation for water diversion areas [2].

There are many uncertainties in the process of inter-basin water transfer, which increases water supply risk in both areas. For example, available water is the most uncertain factor affecting water transfer. In the high-water season, because the available water is sufficient in both areas, the water transfer willingness of the water receiving area is not strong even if the water price is decreased; in the low-water season, the available water is insufficient in the water diversion area, but the water receiving area has a strong willingness to transfer water even if the water price increases. In addition, complex relationships and uncertain information, such as economic benefit, water transfer cost and water shortage loss, have great impacts on the optimal allocation of inter-basin water transfer [3].

In view of the above problems, water options can be introduced as risk management tools in the optimal water allocation decisions of inter-basin water transfer [4] [5] [6] [7]. A water option is a sales contract for a certain amount of water between the water diversion area and the water receiving area, with clear responsibilities and obligations for both parties [6]. Taking call options as an example, the water transfer area, as the seller of the call option, can obtain the option fee as compensation for the promise to sell a certain amount of water at a certain price.

At the same time, it has the obligation to sell a certain amount of water according to the contract. On the other hand, by paying the option fee, the water receiving area has the right to buy a certain amount of water at the agreed price. By executing the water option, the water demand of water in the receiving area (especially in low-water season) is guaranteed to a certain extent [8]. In this way, water options can facilitate water trade and benefit both water diversion areas and water receiving areas [9].

To deal with the complexities and uncertainties, many uncertain optimization methods are developed. Such as Jafarzadegan *et al.* [10] developed an integrated stochastic dynamic programming model to provide monthly policies for water allocation to users in water donors and receiving basins. Luo *et al.* [11] proposed an inexact two-stage stochastic nonlinear programming model for water resources management with water trading. Gu *et al.* [12] developed an interval parameter multistage joint-probability programming model to deal with water resource allocation under joint probability and interval uncertainties. María *et al.* [13] proposed a global optimisation model involving multiple supply sources and multiple users by considering the water distribution efficiency and the physical connections between water supply sources and water users. Han *et al.* [14] developed a multi-objective linear programming model with interval parameters to improve the allocation of multi-source water resources to multiple users.

Therefore, an interval two-stage stochastic multi-objective mixed integer programming (ITSMMIP) model is established and applied to the optimal water allocation of inter-basin water transfer based on options contracts. The model takes the economic benefit of the water diversion area and the water receiving area as the target, and can reflect the decision-making process of water transfer including water call options. The rest of the paper is arranged as follows. In Section 2, the ITSMMIP model is presented. In Section 3, a case study is provided to illustrate the efficiency of the method. In Section 4, a conclusion is drawn.

2. Methodology

2.1. Problem Description and Hypothesis

Inter-basin water transfer is a redistribution process of water resources among different areas, which has a great impact on the society, economy and environment of water diversion and receiving areas. The water manager should consider both the water shortage situation in the water receiving area and the transferable water volume in the water diversion area. The optimal water transfer volume should be determined on the premise that the water demand in the water diversion area is basically met, and that the reasonable water transfer compensation mechanism is established. Therefore, it is necessary to establish a scientific and reasonable water allocation scheme taking into account the interests of various parties in the process of inter-basin water transfer, so as to maximize the utilization of water resources [15].

It is assumed that the water receiving area is economically developed with serious shortage of water resources. The water diversion area is economically underdeveloped with abundant water resources. Water users in both areas can be divided into various types (such as municipal, industrial and agricultural water users). In order to meet the water demand of the water receiving area, an inter-basin water transfer project has been built. Through the project, water transfer can be carried out and an inter-basin water market formed. It is assumed that the water receiving area, water diversion area and inter-basin water transfer project are uniformly managed by water manager. In order to maximize the system economic benefit, avoid the risk of water shortage, and pay compensation to water diversion area, water manager decides to adopt water call options as a management tool of inter-basin water transfer.

The optimal water allocation of inter-basin water transfer based on water call options can be divided into two stages. In the first stage, before the beginning of a water transfer period, water manager sets the option premium, exercise price, maximum exercisable amount and determines the maximum amount of options to be sold by the water diversion area according to the relevant hydrological information. According to the relevant information of water call option, the water receiving area determines the purchase amount of water call options and pays the option fee. At the same time, water managers need to determine the water allocation targets for the water users in both areas. At this stage, water resources management is faced with uncertainties such as climate and economic benefit. If the amount of water call options sold by water diversion area is too large, once the water receiving area exercises the options, the cost of water transfer will be increased, and the water demand in the water diversion area may not be met, resulting in water shortage loss. As the goal of water manager is to maximize the system economic benefit, it is needed to reduce the risk of water shortage in the water receiving area and provide appropriate economic compensation to the water diversion area by reasonably setting the maximum exercisable amount and exercise price of the water call options.

In the second stage, the available water as a random event has been determined. If the execution conditions of the water call options are met, the water receiving area shall determine the amount of water gained by exercising water call options and cover the expense. According to the available water and the principle of maximum economic benefit, water manager revises the decision-making in the first stage and allocate water resources to water users in each area.

Through the above analysis, in addition to the relevant parameters of water call option, the model should also include water allocation targets, water supply benefit, water shortage, water shortage loss, water transfer volume, water transfer cost and other parameters. The decision-making objective of the model is to maximize the economic net benefit of both areas. The parameters and variables used in the mathematical model established in this chapter are defined as follows:

f_A: System net benefit of the water receiving area ($);

f_B: System net benefit of the water diversion area ($);

u_A: Number of water users in the water receiving area;

u_B: Number of water users in the water diversion area;

NB_{Aj}: Net benefit per unit water of water users j in the water receiving area ($/m^3);

NB_{Bj}: Net benefit per unit water of water users j in the water diversion area ($/m^3);

T_{Aj}: Water allocation target promised to users j in the water receiving area (m^3);

T_{Bj}: Water allocation target promised to users in the water diversion area (m^3);

$E[\cdot]$: Expectation of random variables;

Q_A: Random variable, the available water quantity of water receiving area (m^3);

Q_B: Random variable, the available water quantity of water diversion area (m^3);

Q_P: Maximum water transfer capacity of the water diversion project (m^3);

C_{Aj}: Loss to user j per unit of water caused by water shortage in the water receiving area ($/m^3);

C_{Bj}: Loss to user j per unit of water caused by water shortage in the water diversion area ($/m^3);

D_{AjQ}: Shortage volume of user j by which the water allocation target T_{Aj} is not met when present flow is Q (Q_A, Q_B) (m^3);

D_{BjQ}: Shortage volume of user j by which the water allocation target T_{Bj} is not met when present flow is Q (Q_A, Q_B) (m^3);

$T_{Aj\max}$: Maximum water demand of water user j in the water receiving area (m^3);

$T_{Bj\max}$: Maximum water demand of water user j in the water diversion area (m^3);

$T_{Aj\min}$: Minimum amount that should be allocated to user j to ensure the basic needs in the water receiving area (m^3);

$T_{Bj\min}$: Minimum amount that should be allocated to user j to ensure the basic needs in the water diversion area (m^3);

R_i: Binary variable, 0 means not to buy the water call option i, 1 otherwise ($i = 1, 2, \cdots, n$);

NO_i: The maximum exercisable amount of the water call option i ($i = 1, 2, \cdots, n$);

$NO_{i\max}$: The upper bound of the maximum exercisable amount of water call options i ($i = 1, 2, \cdots, n$);

$NO_{i\min}$: The lower bound of the maximum exercisable amount of water call options i ($i = 1, 2, \cdots, n$);

M_i: Preestablished value, the water call option can be exercised if the inflow Q_B is larger than M_i ($i = 1, 2, \cdots, n$);

WO_i: Number of calls executed ($i = 1, 2, \cdots, n$);
PO_i: The strike price of the water call option i (\$/m^3, $i = 1, 2, \cdots, n$);
OP_i: Premium of the water call option i (\$/m^3);
PC: Water transfer cost (\$/m^3).

2.2. Model Establishment and Solution

The decision-making objective is to maximize the economic benefit in each area, that is

$$f_A = \sum_{j=1}^{u_A} NB_{Aj} \times T_{Aj} - \sum_{i=1}^{n} OP_i \times R_i - E\left[\sum_{i=1}^{n} WO_i \times PO_i\right] - E\left[\sum_{j=1}^{u_A} C_{Aj} \times D_{Aj}\right]. \quad (2.1)$$

$$\begin{aligned} f_B = &\sum_{j=1}^{u_B} NB_{Bj} \times T_{Bj} + \sum_{i=1}^{n} OP_i \times R_i + E\left[\sum_{i=1}^{n} WO_i \times PO_i\right] \\ &- E\left[\sum_{i=1}^{n} WO_i \times PC\right] - E\left[\sum_{j=1}^{u_B} C_{Bj} \times D_{Bj}\right]. \end{aligned} \quad (2.2)$$

The constraints of the model include

1) Water availability:

$$\sum_{j=1}^{u_A} \left(T_{Aj} - D_{Aj}\right) \le Q_A + \sum_{i=1}^{n} WO_i, \quad (2.3)$$

$$\sum_{j=1}^{u_B} \left(T_{Bj} - D_{Bj}\right) \le Q_B - \sum_{i=1}^{n} WO_i. \quad (2.4)$$

2) Water allocation targets:

$$T_{Aj\max} \ge T_{Aj} \ge D_{Aj} \ge 0, \forall j, \quad (2.5)$$

$$T_{Aj} - D_{Aj} \ge T_{Aj\min}, \forall j. \quad (2.6)$$

$$T_{Bj\max} \ge T_{Bj} \ge D_{Bj} \ge 0, \forall j, \quad (2.7)$$

$$T_{Bj} - D_{Bj} \ge T_{Bj\min}, \forall j. \quad (2.8)$$

3) Water call options:

Since the holder of the water call options has the right to exercise the options, it is necessary to set up exercise conditions for the water call options to avoid the impact on the basic water needs in the water diversion area because of excessive exercise of the options. In order to meet the water transfer demand of the water receiving area under different available conditions, the water diversion area can sell several options with different parameters, but the total exercisable amount cannot exceed the maximum water transfer capacity of the water diversion project. It can be expressed as

$$\begin{cases} WO_i \le NO_i \times R_i, & Q_B \ge M_i, \\ WO_i = 0, & Q_B < M_i, \end{cases} \quad (2.9)$$

$$\sum_{i=1}^{n} WO_i \le Q_P. \quad (2.10)$$

To sum up, the optimal allocation model of inter-basin water transfer based on the call water options can be expressed as follows:

$$\max\begin{cases} f_A=\sum_{j=1}^{u_A}NB_{Aj}\times T_{Aj}-\sum_{i=1}^{n}OP_i\times R_i-E\left[\sum_{i=1}^{n}WO_i\times PO_i\right]-E\left[\sum_{j=1}^{u_A}C_{Aj}\times D_{Aj}\right],\\ f_B=\sum_{j=1}^{u_B}NB_{Bj}\times T_{Bj}+\sum_{i=1}^{n}OP_i\times R_i+E\left[\sum_{i=1}^{n}WO_i\times PO_i\right]\\ \quad -E\left[PC\times\sum_{i=1}^{n}WO_i\right]-E\left[\sum_{j=1}^{u_B}C_{Bj}\times D_{Bj}\right].\end{cases} \tag{2.11a}$$

Subject to:

Water receiving area:

$$\sum_{j=1}^{u_A}\left(T_{Aj}-D_{Aj}\right)\le Q_A+\sum_{i=1}^{n}WO_i, \tag{2.11b}$$

$$T_{Aj\max}\ge T_{Aj}\ge D_{Aj}\ge 0,\forall j, \tag{2.11c}$$

$$T_{Aj}-D_{Aj}\ge T_{Aj\min},\forall j. \tag{2.11d}$$

Water diversion area:

$$\sum_{j=1}^{u_B}\left(T_{Bj}-D_{Bj}\right)\le Q_B-\sum_{i=1}^{n}WO_i, \tag{2.11e}$$

$$T_{Bj\max}\ge T_{Bj}\ge D_{Bj}\ge 0,\forall j, \tag{2.11f}$$

$$T_{Bj}-D_{Bj}\ge T_{Bj\min},\forall j. \tag{2.11g}$$

Water call options:

$$\begin{cases} WO_i\le NO_i\times R_i,\ Q_B\ge M_i,\\ WO_i=0,\qquad Q_B<M_i,\end{cases} \tag{2.11h}$$

$$\sum_{i=1}^{n}WO_i\le Q_P. \tag{2.11i}$$

Non-negative constraint:

$$T_{Aj},T_{Bj},WO_i,D_{Aj},D_{Bj}\ge 0,\ R=0,1,\ \forall i,j.. \tag{2.11j}$$

To solve this model, suppose Q_A and Q_B take discrete values q_{Ak} and q_{Bk} with probabilities p_{kl} ($k=1,2,\cdots,n_A$, $l=1,2,\cdots,n_B$, $\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}=1$), we have

$$\max\begin{cases} f_A=\sum_{j=1}^{u_A}NB_{Aj}\times T_{Aj}-\sum_{i=1}^{n}OP_i\times R_i-\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{i=1}^{n}WO_{ikl}\times PO_i\right]\\ \quad -\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{j=1}^{u_A}C_{Aj}\times D_{Ajkl}\right],\\ f_B=\sum_{j=1}^{u_B}NB_{Bj}\times T_{Bj}+\sum_{i=1}^{n}OP_i\times R_i+\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{i=1}^{n}WO_{ikl}\times PO_i\right]\\ \quad -\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[PC\times\sum_{i=1}^{n}WO_{ikl}\right]-\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{j=1}^{u_B}C_{Bj}\times D_{Bjkl}\right].\end{cases} \tag{2.12a}$$

Subject to:

Water receiving area:

$$\sum_{j=1}^{u_A}\left(T_{Aj}-D_{Ajkl}\right)\le q_{Ak}+\sum_{i=1}^{n}WO_{ikl},\forall k,l, \tag{2.12b}$$

$$T_{Aj\max}\ge T_{Aj}\ge D_{Ajkl}\ge 0,\forall j,k,l, \tag{2.12c}$$

$$T_{Aj}-D_{Ajkl}\ge T_{Aj\min},\forall j,k,l. \tag{2.12d}$$

Water diversion area:

$$\sum_{j=1}^{u_B}\left(T_{Bj}-D_{Bjkl}\right)\le q_{Bl}-\sum_{i=1}^{n}WO_{ikl},\forall k,l, \tag{2.12e}$$

$$T_{Bj\max}\ge T_{Bj}\ge D_{Bjkl}\ge 0,\forall j,k,l, \tag{2.12f}$$

$$T_{Bj}-D_{Bjkl}\ge T_{Bj\min},\forall j,k,l. \tag{2.12g}$$

Water call options:

$$\begin{cases}WO_{ikl}\le NO_i\times R_i, q_{Bl}^{\pm}\ge M_i,\\ WO_{ikl}=0, \qquad q_{Bl}^{\pm}<M_i,\end{cases} \tag{2.12h}$$

$$\sum_{i=1}^{n}WO_{ikl}\le Q_P. \tag{2.12i}$$

Non-negative constraint:

$$T_{Aj},T_{Bj},WO_{ikl},D_{Ajkl},D_{Bjkl}\ge 0, R=0,1,\forall i,j,k,l. \tag{2.12j}$$

The above model can effectively reflect stochastic uncertainties, but the water manager may find it difficult to make deterministic water allocation targets ($T_{\bullet j}$) and the economic parameters ($C_{\bullet j}$ and $NB_{\bullet j}$) may not be described by deterministic values or random variable but intervals. To reflect such uncertainties, interval parameters can be introduced into the model [16]. This results in an interval two-stage stochastic multi-objective mixed integer programming (ITSMMIP) model:

$$\max\begin{cases}f_A^{\pm}=\sum_{j=1}^{u_A}NB_{Aj}^{\pm}\times T_{Aj}^{\pm}-\sum_{i=1}^{n}OP_i\times R_i-\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{i=1}^{n}WO_{ikl}^{\pm}\times PO_i\right]\\ \quad -\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{j=1}^{u_A}C_{Aj}^{\pm}\times D_{Ajkl}^{\pm}\right]\\ f_B^{\pm}=\sum_{j=1}^{u_B}NB_{Bj}^{\pm}\times T_{Bj}^{\pm}+\sum_{i=1}^{n}OP_i\times R_i+\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{i=1}^{n}WO_{ikl}^{\pm}\times PO_i\right]\\ \quad -\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[PC\times\sum_{i=1}^{n}WO_{ikl}^{\pm}\right]-\sum_{k=1}^{n_A}\sum_{l=1}^{n_B}p_{kl}\times\left[\sum_{j=1}^{u_B}C_{Bj}^{\pm}\times D_{Bjkl}^{\pm}\right]\end{cases} \tag{2.13a}$$

Subject to:

Water receiving area:

$$\sum_{j=1}^{u_A}\left(T_{Aj}^{\pm}-D_{Ajkl}^{\pm}\right)\le q_{Ak}^{\pm}+\sum_{i=1}^{n}WO_{ikl}^{\pm},\forall k,l, \tag{2.13b}$$

$$T_{Aj\max}\ge T_{Aj}^{\pm}\ge D_{Ajkl}^{\pm}\ge 0,\forall j,k,l, \tag{2.13c}$$

$$T_{Aj}^{\pm} - D_{Ajkl}^{\pm} \geq T_{Aj\min}, \forall j,k,l. \tag{2.13d}$$

Water diversion area:

$$\sum_{j=1}^{u_B} \left(T_{Bj}^{\pm} - D_{Bjkl}^{\pm} \right) \leq q_{Bl}^{\pm} - \sum_{i=1}^{n} WO_{ikl}^{\pm}, \forall k,l, \tag{2.13e}$$

$$T_{Bj\max} \geq T_{Bj}^{\pm} \geq D_{Bjkl}^{\pm} \geq 0, \forall j,k,l, \tag{2.13f}$$

$$T_{Bj}^{\pm} - D_{Bjkl}^{\pm} \geq T_{Bj\min}, \forall j,k,l. \tag{2.13g}$$

Water call options:

$$\begin{cases} WO_{ikl}^{\pm} \leq NO_i \times R_i, q_{Bl}^{\pm} \geq M_i, \\ WO_{ikl}^{\pm} = 0, \qquad q_{Bl}^{\pm} \geq M_i, \end{cases} \tag{2.13h}$$

$$\sum_{i=1}^{n} WO_{ikl}^{\pm} \leq Q_P. \tag{2.13i}$$

Non-negative constraint:

$$T_{Aj}^{\pm}, T_{Bj}^{\pm}, WO_{ikl}, D_{Ajkl}^{\pm}, D_{Bjkl}^{\pm} \geq 0, R = 0,1, \forall i,j,k,l. \tag{2.13j}$$

There are two different methods to solve the stochastic multi-objevtive programming prolem [17] [18]. One is called multi-objective method by Ben Abdelaziz [19] that transforms the stochastic multi-objective problem into its equivalent multi-objective deterministic problem. This method transforms each stochastic objective into its equivalent deterministic separately by selecting a criterion, but not take the stochastic correlation between stochastic objectives into considration. The other method is called stochastic method that transform the stochastic multi-objective problem into a stochastic single-objective problem, and then into a deterministic single objective optimization problem. For practical problems with correlation, this method is more appropriate [20]. Therefore, for the interval two-stage stochastic mixed integer programming model, we will transform it into a single objective interval two-stage stochastic mixed integer programming problem to solve. Because the water receiving area is the holder of the call option and has the right to exercise the option, it has a higher priority in the multi-objective problem, our model can be solved by using the hierarchical ordering method. In this method, the objective functons are ranked in order of importance, and then a series of single-objective problems are solved to obtain efficient solutions. The specific steps are as follows:

Step 1: According to the water availability, water demand and related economic data, the water manager determines the upper and lower bounds of the maximum exercisable amount $NO_{i\min}, NO_{i\max}$ and the strike price $PO_{i\min}, PO_{i\max}$ of the water call option *i*. Let $PO_i \in \left[PO_{i\min}, PO_{i\max}\right]$, $NO_i \in \left[NO_{i\min}, NO_{i\max}\right]$.

Step 2: Solve the optimization problem of the water receiving area.

1) Since the objective function of the water receiving area is the system net benefit, the upper bound of the objective function f_A^+ can be solved first. Let $T_{Aj}^{\pm} = T_{Aj}^{-} + y_{Aj}\left(T_{Aj}^{+} - T_{Aj}^{-}\right)$, $y_{Aj} \in [0,1]$, then the sub model of f_A^+ can be expressed as follows:

$$\max f_A^+ = \sum_{j=1}^{u_A} NB_{Aj}^+ \times \left(T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right)\right) - \sum_{i=1}^{n} OP_i \times R_i$$
$$-\sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{i=1}^{n} WO_{ikl}^- \times PO_i\right] - \sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{j=1}^{u_A} C_{Aj}^- \times D_{Ajkl}^-\right]. \quad (2.14a)$$

Subject to:

$$\sum_{j=1}^{u_A}\left(T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right) - D_{Ajkl}^-\right) \le q_{Ak}^+ + \sum_{i=1}^{n} WO_{ikl}^-, \forall k, l, \quad (2.14b)$$

$$T_{Aj\max} \ge T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right) \ge D_{Ajkl}^- \ge 0, \forall j, k, l, \quad (2.14c)$$

$$T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right) - D_{Ajkl}^- \ge T_{Aj\min}, \forall j, k, l, \quad (2.14d)$$

$$\sum_{j=1}^{u_B}\left(T_{Bj}^- + y_{Bj}\left(T_{Bj}^+ - T_{Bj}^-\right) - D_{Bjkl}^-\right) \le q_{Bk}^+ + \sum_{i=1}^{n} WO_{ikl}^-, \forall k, l, \quad (2.14e)$$

$$T_{Bj\max} \ge T_{Bj}^- + y_{Bj}\left(T_{Bj}^+ - T_{Bj}^-\right) \ge D_{Bjkl}^- \ge 0, \forall j, k, l, \quad (2.14f)$$

$$T_{Bj}^- + y_{Bj}\left(T_{Bj}^+ - T_{Bj}^-\right) - D_{Bjkl}^- \ge T_{Bj\min}, \forall j, k, l, \quad (2.14g)$$

$$\begin{cases} WO_{ikl}^- \le NO_i \times R_i, q_{Bl}^\pm \ge M_i, \\ WO_{ikl}^- = 0, \qquad q_{Bl}^\pm < M_i, \end{cases} \quad (2.14h)$$

$$\sum_{i=1}^{n} WO_{ikl}^- \le Q_P, \quad (2.14i)$$

$$WO_{ikl}^-, D_{Ajkl}^-, D_{Bjkl}^- \ge 0, R_i = 0, 1, 0 \le y_{Aj} \le 1, 0 \le y_{Bj} \le 1, \forall i, j, k, l. \quad (2.14j)$$

2) Let f_{Aopt}^+, R_{iopt}, WO_{iklopt}^-, $D_{Ajklopt}^-$, y_{Ajopt} be the solution of the model in 1), then the optimal decision to buy the water call options is R_{iopt}, and the optimal water allocation targets are

$$T_{Ajopt}^\pm = T_{Aj}^- + y_{Ajopt}\left(T_{Aj}^+ - T_{Aj}^-\right). \quad (2.15)$$

3) Based on above results, the optimization model of the lower bound of the objective function f_A^- is established as follows:

$$\max f_A^- = \sum_{j=1}^{u_A} NB_{Aj}^- \times \left(T_{Aj}^- + y_{Ajopt}\left(T_{Aj}^+ - T_{Aj}^-\right)\right) - \sum_{i=1}^{n} OP_i \times R_{iopt}$$
$$-\sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{i=1}^{n} WO_{ikl}^+ \times PO_i\right] - \sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{j=1}^{u_A} C_{Aj}^+ \times D_{Ajkl}^+\right]. \quad (2.16a)$$

Subject to:

$$\sum_{j=1}^{u_A}\left(T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right) - D_{Ajkl}^+\right) \le q_{Ak}^- + \sum_{i=1}^{n} WO_{ikl}^+, \forall k, l, \quad (2.16b)$$

$$T_{Aj\max} \ge T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right) \ge D_{Ajkl}^+ \ge 0, \forall j, k, l, \quad (2.16c)$$

$$T_{Aj}^- + y_{Aj}\left(T_{Aj}^+ - T_{Aj}^-\right) - D_{Ajkl}^+ \ge T_{Aj\min}, \forall j, k, l, \quad (2.16d)$$

$$\sum_{j=1}^{u_B}\left(T_{Bj}^- + y_{Bj}\left(T_{Bj}^+ - T_{Bj}^-\right) - D_{Bjkl}^+\right) \le q_{Bk}^- + \sum_{i=1}^{n} WO_{ikl}^+, \forall k, l, \quad (2.16e)$$

$$T_{Bj\max} \geq T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) \geq D_{Bjkl}^{+} \geq 0, \forall j,k,l, \tag{2.16f}$$

$$T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) - D_{Bjkl}^{+} \geq T_{Bj\min}, \forall j,k,l, \tag{2.16g}$$

$$\begin{cases} WO_{ikl}^{+} \leq NO_i \times R_{iopt}, q_{Bl}^{\pm} \geq M_i, \\ WO_{ikl}^{+} = 0, \qquad\qquad q_{Bl}^{\pm} < M_i, \end{cases} \tag{2.16h}$$

$$\sum_{i=1}^{n} WO_{ikl}^{+} \leq Q_P, \tag{2.16i}$$

$$WO_{ikl}^{+}, D_{Ajkl}^{+}, D_{Bjkl}^{+} \geq 0, \forall i,j,k,l. \tag{2.16j}$$

4) Let f_{Aopt}^{-}, WO_{iklopt}^{+}, $D_{Ajklopt}^{+}$ be the solution of the model in 3), then we have

$$f_{Aopt}^{\pm} = \left[f_{Aopt}^{-}, f_{Aopt}^{+}\right], \tag{2.17a}$$

$$D_{Ajklopt}^{\pm} = \left[D_{Ajklopt}^{-}, D_{Ajklopt}^{+}\right], \ \forall j,k,l, \tag{2.17b}$$

$$W_{iklopt}^{\pm} = \left[W_{iklopt}^{-}, W_{iklopt}^{+}\right], \ \forall\, k,l. \tag{2.17c}$$

Step 3: Solve the optimization problem of the water diversion area.

1) Since the objective function is the system net benefit, the upper bound of the objective function can be solved first. Let $T_{Bj}^{\pm} = T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right)$, $y_{Bj} \in [0,1]$, and put the optimal solution obtained in step1 into the sub model of f_B^{+}, we have

$$\begin{aligned} \max f_B^{+} = & \sum_{j=1}^{u_B} NB_{Bj}^{+} \times \left(T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right)\right) - \sum_{i=1}^{n} OP_i \times NO_{iopt} \\ & - \sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{i=1}^{n} WO_{iklopt}^{-} \times PO_i\right] - \sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{j=1}^{u_B} C_{Bj}^{-} \times D_{Bjkl}^{-}\right]. \end{aligned} \tag{2.18a}$$

Subject to:

$$\sum_{j=1}^{u_A}\left(T_{Aj}^{-} + y_{Aj}\left(T_{Aj}^{+} - T_{Aj}^{-}\right) - D_{Ajkl}^{-}\right) \leq q_{Ak}^{+} + \sum_{i=1}^{n} WO_{iklopt}^{-}, \forall k,l, \tag{2.18b}$$

$$T_{Aj\max} \geq T_{Aj}^{-} + y_{Aj}\left(T_{Aj}^{+} - T_{Aj}^{-}\right) \geq D_{Ajkl}^{-} \geq 0, \forall j,k,l, \tag{2.18c}$$

$$T_{Aj}^{-} + y_{Aj}\left(T_{Aj}^{+} - T_{Aj}^{-}\right) - D_{Ajkl}^{-} \geq T_{Aj\min}, \forall j,k,l, \tag{2.18d}$$

$$\sum_{j=1}^{u_B}\left(T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) - D_{Bjkl}^{-}\right) \leq q_{Bk}^{+} + \sum_{i=1}^{n} WO_{iklopt}^{-}, \forall k,l, \tag{2.18e}$$

$$T_{Bj\max} \geq T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) \geq D_{Bjkl}^{-} \geq 0, \forall j,k,l, \tag{2.18f}$$

$$T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) - D_{Bjkl}^{-} \geq T_{Bj\min}, \forall j,k,l, \tag{2.18g}$$

$$\begin{cases} WO_{iklopt}^{-} \leq NO_i \times R_{iopt}, \quad q_{Bl}^{\pm} \geq M_i, \\ WO_{iklopt}^{-} = 0, \qquad\qquad q_{Bl}^{\pm} < M_i, \end{cases} \tag{2.18h}$$

$$\sum_{i=1}^{n} WO_{iklopt}^{-} \leq Q_P, \tag{2.18i}$$

$$D_{Bjkl}^{-} \geq 0, 0 \leq y_{Bj} \leq 1, \forall j,k,l. \tag{2.18j}$$

2) Let f_{Bopt}^{+}, $D_{Bjklopt}^{-}$, y_{Bjopt} be the solution of the model in 1), then the optimal water allocation targets in the water diversion area is as follows:

$$T_{Bjopt}^{\pm} = T_{Bj}^{-} + y_{Bjopt}\left(T_{Bj}^{+} - T_{Bj}^{-}\right). \tag{2.19}$$

3) Based on the above results, the optimization model of the lower bound of the objective function f_{B}^{-} is established as follows:

$$\begin{aligned} \max f_{B}^{-} = & \sum_{j=1}^{u_B} NB_{Bj}^{-} \times \left(T_{Bj}^{-} + y_{Bjopt}\left(T_{Bj}^{+} - T_{Bj}^{-}\right)\right) - \sum_{i=1}^{n} OP_i \times NO_{iopt} \\ & - \sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{i=1}^{n} WO_{iklopt}^{+} \times PO_i\right] - \sum_{k=1}^{n_A}\sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{j=1}^{u_B} C_{Bj}^{+} \times D_{Bjkl}^{+}\right]. \end{aligned} \tag{2.20a}$$

Subject to:

$$\sum_{j=1}^{u_A}\left(T_{Aj}^{-} + y_{Ajopt}\left(T_{Aj}^{+} - T_{Aj}^{-}\right) - D_{Ajkl}^{+}\right) \le q_{Ak}^{-} + \sum_{i=1}^{n} WO_{iklopt}^{+}, \forall k, l, \tag{2.20b}$$

$$T_{Aj\max} \ge T_{Aj}^{-} + y_{Aj}\left(T_{Aj}^{+} - T_{Aj}^{-}\right) \ge D_{Ajkl}^{+} \ge 0, \forall j, k, l, \tag{2.20c}$$

$$T_{Aj}^{-} + y_{Aj}\left(T_{Aj}^{+} - T_{Aj}^{-}\right) - D_{Ajkl}^{+} \ge T_{Aj\min}, \forall j, k, l, \tag{2.20d}$$

$$\sum_{j=1}^{u_B}\left(T_{Bj}^{-} + y_{Bj}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) - D_{Bjkl}^{+}\right) \le q_{Bk}^{-} + \sum_{i=1}^{n} WO_{iklopt}^{+}, \forall k, l, \tag{2.20e}$$

$$T_{Bj\max} \ge T_{Bj}^{-} + y_{Bjopt}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) \ge D_{Bjkl}^{+} \ge 0, \forall j, k, l, \tag{2.20f}$$

$$T_{Bj}^{-} + y_{Bjopt}\left(T_{Bj}^{+} - T_{Bj}^{-}\right) - D_{Bjkl}^{+} \ge T_{Bj\min}, \forall j, k, l, \tag{2.20g}$$

$$\begin{cases} WO_{ikl}^{+} \le NO_i \times R_{iopt}, q_{Bl}^{\pm} \ge M_i, \\ WO_{ikl}^{+} = 0, \qquad q_{Bl}^{\pm} < M_i, \end{cases} \tag{2.20h}$$

$$\sum_{i=1}^{n} WO_{ikl}^{+} \le Q_P, \tag{2.20i}$$

$$D_{Bjkl}^{+} \ge 0, \forall j, k, l. \tag{2.20j}$$

4) Let f_{Bopt}^{-}, WO_{iklopt}^{+}, $D_{Bjklopt}^{+}$ be the solution in 3), then we have:

$$f_{Bopt}^{\pm} = \left[f_{Bopt}^{-}, f_{Bopt}^{+}\right], \tag{2.21a}$$

$$D_{Bjklopt}^{\pm} = \left[D_{Bjklopt}^{-}, D_{Bjklopt}^{+}\right], \ \forall j, k, l. \tag{2.21b}$$

Step 4: Repeat steps 1 to 3 for $PO_i \in \left(PO_{i\min}, PO_{i\max}\right)$, $NO_i \in \left(NO_{i\min}, NO_{i\max}\right)$ and the solution with the largest net benefit of the whole system ($f_A^{\pm} + f_B^{\pm}$) is the optimal water allocation strategy.

In order to illustrate the advantages of water call options in inter-basin water resource management, we will compare our model with other inter-basin water resource management models.

Model 1: An interval two-stage stochastic programming model for inter-basin water resources management under uncertainty without water diversion

In this model, the water receiving area and the water diversion area allocate the water resources to water users (such as municipal, industrial and agricultural) within their areas respectively, and there is no water transfer between areas. The model can be expressed as follows:

$$\max f^{\pm} = \sum_{j=1}^{u_A} NB_{Aj}^{\pm} \times T_{Aj}^{\pm} + \sum_{j=1}^{u_B} NB_{Bj}^{\pm} \times T_{Bj}^{\pm} - \sum_{k=1}^{n_A} p_{Ak} \times \left[\sum_{j=1}^{u_A} C_{Aj}^{\pm} \times D_{Ajk}^{\pm} \right] - \sum_{l=1}^{n_B} p_{Bl} \times \left[\sum_{j=1}^{u_B} C_{Bj}^{\pm} \times D_{Bjl}^{\pm} \right]. \tag{2.22a}$$

Subject to:

Water availability:

$$\sum_{j=1}^{u_A} \left(T_{Aj}^{\pm} - D_{Ajk}^{\pm} \right) \le q_{Ak}^{\pm}, \forall k, \tag{2.22b}$$

$$\sum_{j=1}^{u_B} \left(T_{Bj}^{\pm} - D_{Bjl}^{\pm} \right) \le q_{Bl}^{\pm}, \forall l. \tag{2.22c}$$

Water allocation targets:

$$T_{Aj\max} \ge T_{Aj}^{\pm} \ge D_{Ajk}^{\pm} \ge 0, \forall j, k, \tag{2.22d}$$

$$T_{Aj}^{\pm} - D_{Ajk}^{\pm} \ge T_{Aj\min}, \forall j, k, \tag{2.22e}$$

$$T_{Bj\max} \ge T_{Bj}^{\pm} \ge D_{Bjl}^{\pm} \ge 0, \forall j, l, \tag{2.22f}$$

$$T_{Bj}^{\pm} - D_{Bjl}^{\pm} \ge T_{Bj\min}, \forall j, k, l. \tag{2.22g}$$

Non-negative constraint:

$$T_{Aj}^{\pm}, T_{Bj}^{\pm}, D_{Ajkl}^{\pm}, D_{Bjkl}^{\pm} \ge 0, \forall j, k, l. \tag{2.22h}$$

Model 2: An interval two-stage stochastic programming model for inter-basin water resources management under uncertainty by treating the two areas as a whole

In this model, the water resources in the water receiving area and the water diversion area are uniformly allocated by the water manager. The water manager can transfer water between areas with the objective of maximizing the overall economic benefit. Let $L_{kl}^{\pm}$ be the water quantity transfered from the water diversion area to the water receiving area. Then the model can be expressed as follows:

$$\max f^{\pm} = \sum_{j=1}^{u_A} NB_{Aj}^{\pm} \times T_{Aj}^{\pm} + \sum_{j=1}^{u_B} NB_{Bj}^{\pm} \times T_{Bj}^{\pm} - \sum_{k=1}^{n_A} \sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{j=1}^{u_A} C_{Aj}^{\pm} \times D_{Ajkl}^{\pm} \right] - \sum_{k=1}^{n_A} \sum_{l=1}^{n_B} p_{kl} \times \left[\sum_{j=1}^{u_B} C_{Bj}^{\pm} \times D_{Bjkl}^{\pm} \right] - \sum_{k=1}^{n_A} \sum_{l=1}^{n_B} p_{kl} \times \left[PC \times L_{kl} \right]. \tag{2.23a}$$

Subject to:

Water availability:

$$\sum_{j=1}^{u_A} \left(T_{Aj}^{\pm} - D_{Ajkl}^{\pm} \right) \le q_{Ak}^{\pm} + L_{kl}^{\pm}, \forall k, l, \tag{2.23b}$$

$$\sum_{j=1}^{u_B}\left(T_{Bj}^{\pm}-D_{Bjkl}^{\pm}\right)\leq q_{Bl}^{\pm}-L_{kl}^{\pm},\forall k,l. \tag{2.23c}$$

Water allocation targets:

$$T_{Aj\max}\geq T_{Aj}^{\pm}\geq D_{Ajk}^{\pm}\geq 0,\forall j,k,l, \tag{2.23d}$$

$$T_{Aj}^{\pm}-D_{Ajk}^{\pm}\geq T_{Aj\min},\forall j,k,l, \tag{2.23e}$$

$$T_{Bj\max}\geq T_{Bj}^{\pm}\geq D_{Bjkl}^{\pm}\geq 0,\forall j,k,l, \tag{2.23f}$$

$$T_{Bj}^{\pm}-D_{Bjl}^{\pm}\geq T_{Bj\min},\forall j,k,l. \tag{2.23g}$$

Non-negative constraint:

$$T_{Aj}^{\pm},T_{Bj}^{\pm},D_{Ajkl}^{\pm},D_{Bjkl}^{\pm},L_{kl}^{\pm}\geq 0,\forall j,k,l. \tag{2.23h}$$

The above models are interval two-stage stochastic programming models, and the solving process can be divided into two steps. The first step is to solve the sub model for the upper bound of the objective function f^{+} to obtain the optimal water allocation targets T_{Aj},T_{Bj}; the second step is to solve the sub model for the lower bound of the objective function f^{-} based on the results of the first step. The optimal solutions are obtained in the following form:

$$f_{opt}^{\pm}=\left[f_{opt}^{-},f_{opt}^{+}\right], \tag{2.24a}$$

$$D_{Aijklopt}^{\pm}=\left[D_{Ajklopt}^{-},D_{Ajklopt}^{+}\right],\ \forall j,k,l, \tag{2.24b}$$

$$D_{Bijklopt}^{\pm}=\left[D_{Bjklopt}^{-},D_{Bjklopt}^{+}\right],\forall j,k,l, \tag{2.24c}$$

$$L_{klopt}^{\pm}=\left[L_{klopt}^{-},L_{klopt}^{+}\right],\ \forall k,l. \tag{2.24d}$$

3. Case Study

Suppose a water manager is responsible for managing water resource in two areas: the water receiving area and the water diversion area. There are three users in each area: a municipality, an industrial concern and an agricultural sector. The available water in the water receiving area is scarce, but the water diversion area has abundant water resources. Table 1 shows the water availability data, which is divided into three levels with relative appearance probability. Table 2 provides the water allocation targets of each area, and Table 3 presents the economic data of each area. The case data in this section partly refers to the literature [21]. The water manager must make a effective plan to minimize the risk of water shortage and maximize the expectation of economic benefit in the two areas.

Table 1. Available water distribution.

	Probability	Water availability (10^6 m^3)	
		Water receiving area	Water diversion area
Low (L)	0.2	[5, 7]	[12, 14]
Medium (M)	0.6	[7, 10]	[14, 17]
High (H)	0.2	[10, 12]	[17, 20]

Table 2. Water allocation targets.

Water allocation targets $T_{\cdot j}^{\pm}$ (10^6 m^3)	Users		
	Municipal ($j=1$)	Industrial ($j=2$)	Agricaltural ($j=3$)
Water receiving area	[3.5, 5.0]	[5.5, 7.0]	[2.0, 3.0]
Water diversion area	[1.5, 2.5]	[2.0, 3.0]	[3.0, 4.5]
Maximum allowable allocation of the water receiving area $T_{Aj\max}$ (10^6 m^3)	5.5	7.5	3.5
Minimum allowable allocation of the Water diversion area $T_{Bj\min}$ (10^6 m^3)	1.0	1.5	2.5

Table 3. Economic data.

		Users		
		Municipal ($j=1$)	Industrial ($j=2$)	Agricaltural ($j=3$)
Net benefit when water demand is satisfied $NB_{\cdot j}^{\pm}$ ($/m^3)	Area A	[100, 120]	[70, 90]	[45, 60]
	Area B	[80, 95]	[55, 70]	[35, 55]
Penalty when water is not delivered $C_{\cdot j}^{\pm}$ ($/m^3)	Area A	[200, 240]	[130, 160]	[80, 100]
	Area B	[160, 200]	[110, 150]	[70, 95]
Cost for water diversion CP ($/m^3)		[20, 25]		

Assumes that there is only one kind of water call option that can be purchased by the water receiving area. The upper and lower bound of the maximum exercisable amount of the water call option are 7.0 × 10^6 m^3 and 3.0 × 10^6 m^3, the variation range of the strike price is $40/m^3 - $80/m^3, and the variation range of the option premium is $4.0/m^3 - $8.0/m^3. The specific analysis is as follows.

1) The optimal maximum exercisable amount and strike price of the water call option

Figure 1 presents the relationship between the system net benefit and the maximum exercisable amount when the strike price is $40/m^3. When the maximum exercisable amount increases 3.0 × 10^6 m^3 from to 5.0 × 10^6 m^3, the system net benefit increases from $ [953.5, 1857] × 10^6 to $ [1093.5, 1951] × 10^6, while the maximum exercisable amount is 7.0 × 10^6 m^3, the system net benefit is $ [1015, 1939] × 10^6. It can be seen that the system net benefit first increases with the increase of the maximum exercisable amount, and then turns to decrease. This is because the larger maximum exercisable amount reduces the available water in the water diversion area, resulting in larger water shortage loss and water transfer cost.

Figure 2 shows the relationship between the system net benefit and the strike price when the maximum exercisable amount is 5.0 × 10^6 m^3. When the strike price is $40/m^3, the system net benefit is $ [1093.5, 1951] × 10^6, and the net benefit of the water diversion area is $ [492.5, 793] × 10^6, the net benefit of the water receiving area is $ [601, 1158] × 10^6. When the strike price moves up to $80/m^3, the net benefit is $ [712.5, 877] × 10^6 for the water diversion area and $ [381, 974] × 10^6 for the water receiving area. It can be seen that the system net benefit decreases with the increase of the strike price, the net benefit of the water diversion area increases with the increase of the strike price, and the net benefit of the water receiving area decreases with the increase of the strike price. This is because with the increase of the strike price, the water diversion area can obtain more income from selling water resources, but the water demand of the water receiving area is restrained by the high water price, which leads to the decrease of the overall net benefit of the system.

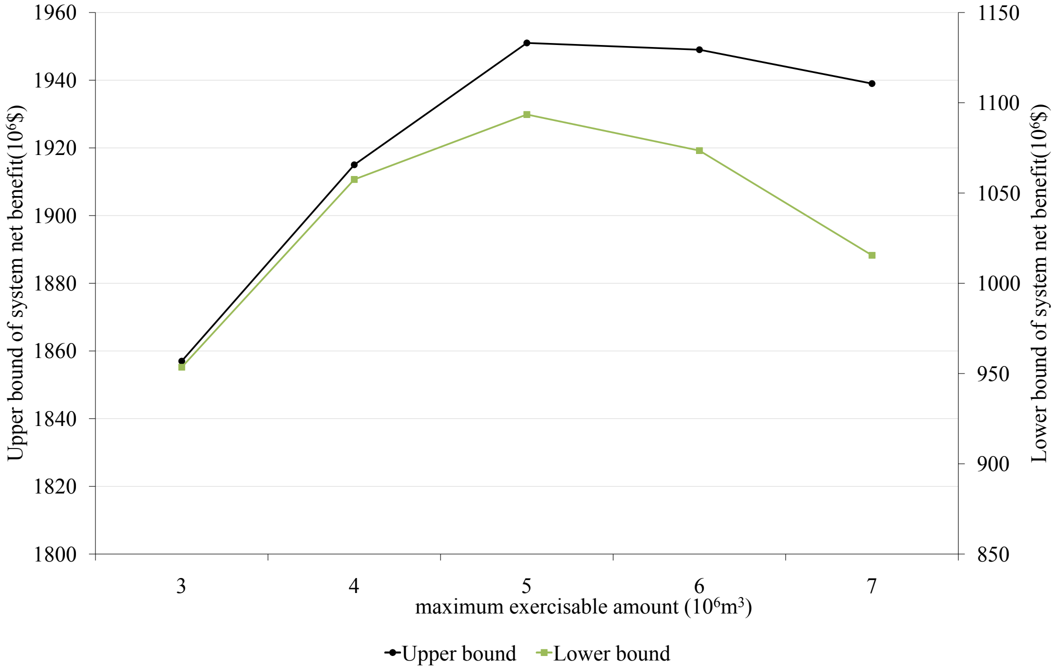

Figure 1. Relationship between system net benefit and maximum exercisable amount.

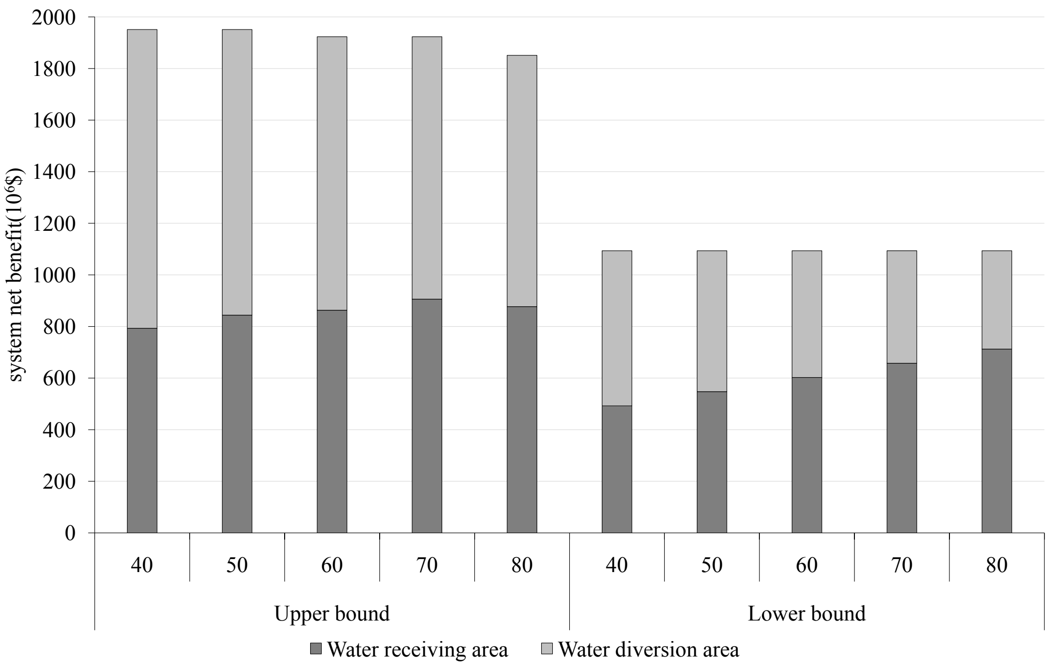

Figure 2. Relationship between system net benefit and strike price.

Through the above analysis, when the strike price is $40/m^3, the maximum exercisable amount is 5.0×10^6 m^3, the overall system net benefit is the maximum, and the corresponding optimal water allocation strategy is optimal.

2) System net benefit under different inter-basin water allocation models

Figure 3 shows the system net benefit under different inter-basin water allocation models. When there is no water diversion between the two areas (Model 1), the system net benefit is $ [799.5, 1644] $\times 10^6$, in which the net benefit of the water diversion area is $ [522.5, 695] $\times 10^6$, the net benefit of the water receiving area is $ [277, 949] $\times 10^6$. When the water resources were allocated by treating the two areas as a whole (Model 2), the system net benefit is $ [1096.5, 1951] $\times 10^6$, which is [18.7%, 37.1%] higher than the model of no water diversion. The net benefit of the water diversion area is $ [427.5, 681] $\times 10^6$, [18.2%, 2.0%] lower; the net benefit of the water receiving area is $ [669, 1270] $\times 10^6$, [33.8%, 141.5%] higher. It can be seen that by treating the two areas as a whole the system net benefit increase significantly. However, due to the overall water shortage, the lower bound of the net benefit of the water diversion area is reduced significantly.

As shown in **Figure 3**, the maximum exercisable amount of the water call option is 5.0×10^6 m^3, the strike price is $40/m^3, and the option premium is $4/m^3. By using this option, the system net benefit is $ [1093.5, 1951] $\times 10^6$, in which the upper bound is equal to the system net benefit of Model 2 , and the lower bound is 0.55% less than Model 2. The net benefit of the water receiving area is $ [601, 1158] $\times 10^6$, [22%, 117%] higher than that of Model 1, and [8.8%, 10.2%] lower than that of Model 2. The net benefit of the water diversion area is $ [492.5, 793] $\times 10^6$, which is increased [−5.7%, 10.8%] by compared with Model 1, and is increased [15.2%, 16.4%] compared with Model 2. It can be seen that compared with the no water diversion model, the use of water call option can significantly increase the system net benefit, and the upper bounds of the net benefit of both areas have a greater growth. Compared with the model treating the two areas as a whole, the system net benefit is almost equal, and especially, the water diversion area obtains economic compensation through the water call option.

3) Optimal water allocation under different inter-basin water allocation models

Suppose that the maximum exercisable quantity of the water call option is 5.0×10^6 m^3, the exercise price is $40/m^3, and the option premium is $4/m^3. **Figure 4** shows the water allocation targets under different inter-basin water allocation models. Without water diversion, the water allocation targets of industrial, agricultural and municipal water users in the receiving area are 4.5×10^6 m^3, 5.5×10^6 m^3 and 2.0×10^6 m^3 respectively. The water allocation targets by using water call option are 5.0×10^6 m^3, 7.0×10^6 m^3 and 3.0×10^6 m^3, which is the same as that by treating the two areas as a whole and increased by 11.1%, 27.3% and 50.0%. The water allocation targets of the water diversion area remain unchanged under different inter-basin water allocation models, and the use of water call option doesn't affect the targets of this area.

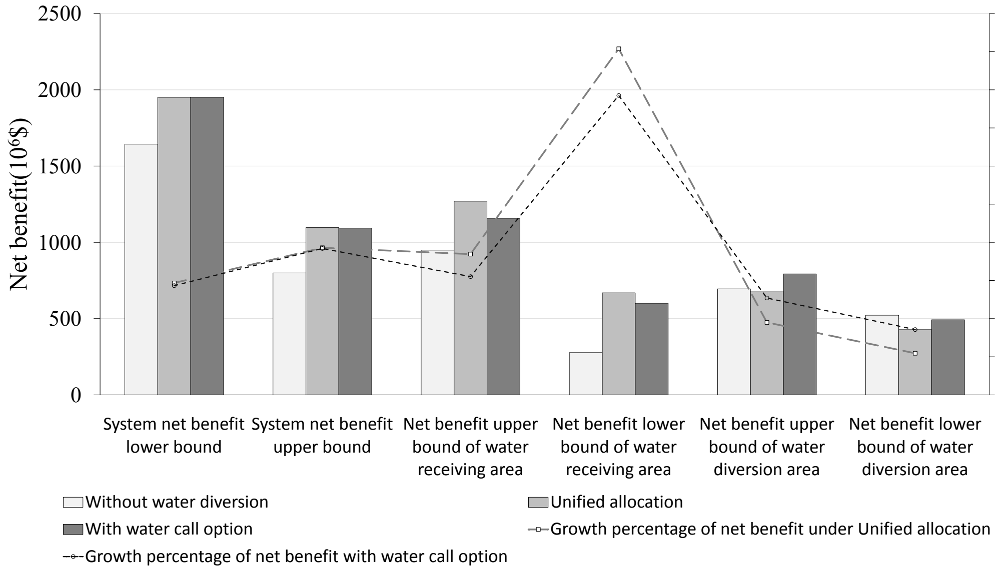

Figure 3. Net benefit under different inter-basin water allocation models.

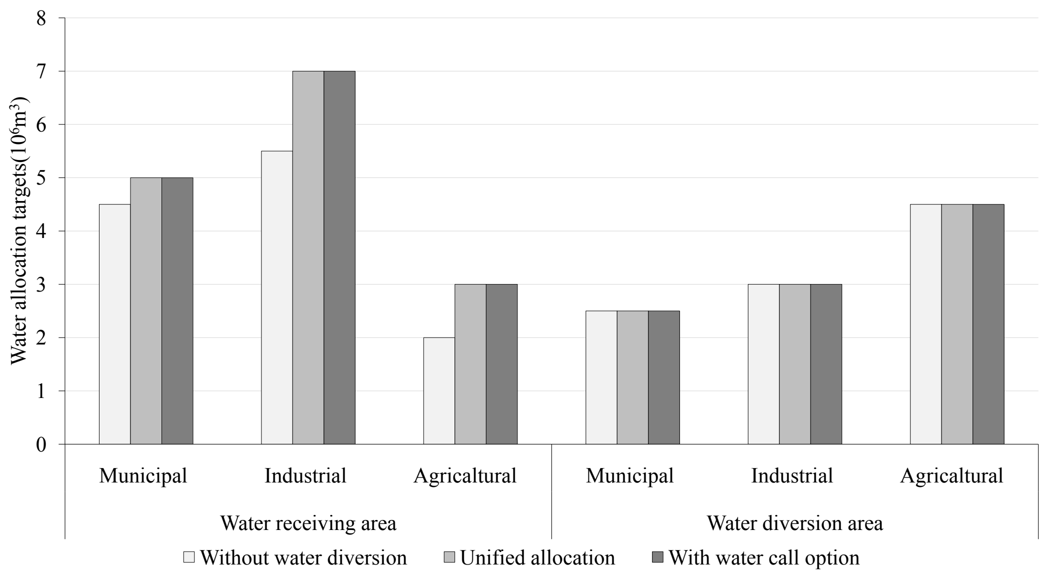

Figure 4. Water allocation targets under different inter-basin water allocation models.

Figure 5 and Figure 6 show the optimal water allocation results under different inter-basin water allocation models. Compared with no water diversion, the water allocation in the water receiving area has been greatly improved by using the water call option. For example, in the dry season, the optimal water allocation of industrial water users increases from $[0.5, 2.5] \times 10^6$ m^3 to $[5.0, 7.0] \times 10^6$ m^3. Different from the water receiving area, the water allocation of agricultural water users and industrial water users in the water diversion area slightly decreased. For example, in the dry season, the optimal water allocation of agricultural water users decreased from 4.5×10^6 m^3 to $[2.5, 3.5] \times 10^6$ m^3. It can be seen that through the use of the water call option, water resources can be transferred from agricultural users with low value in the water diversion area to the municipal and industrial users with high value in the water receiving area, and especially in the dry season, the water demand in the water receiving area is guaranteed to a certain extent.

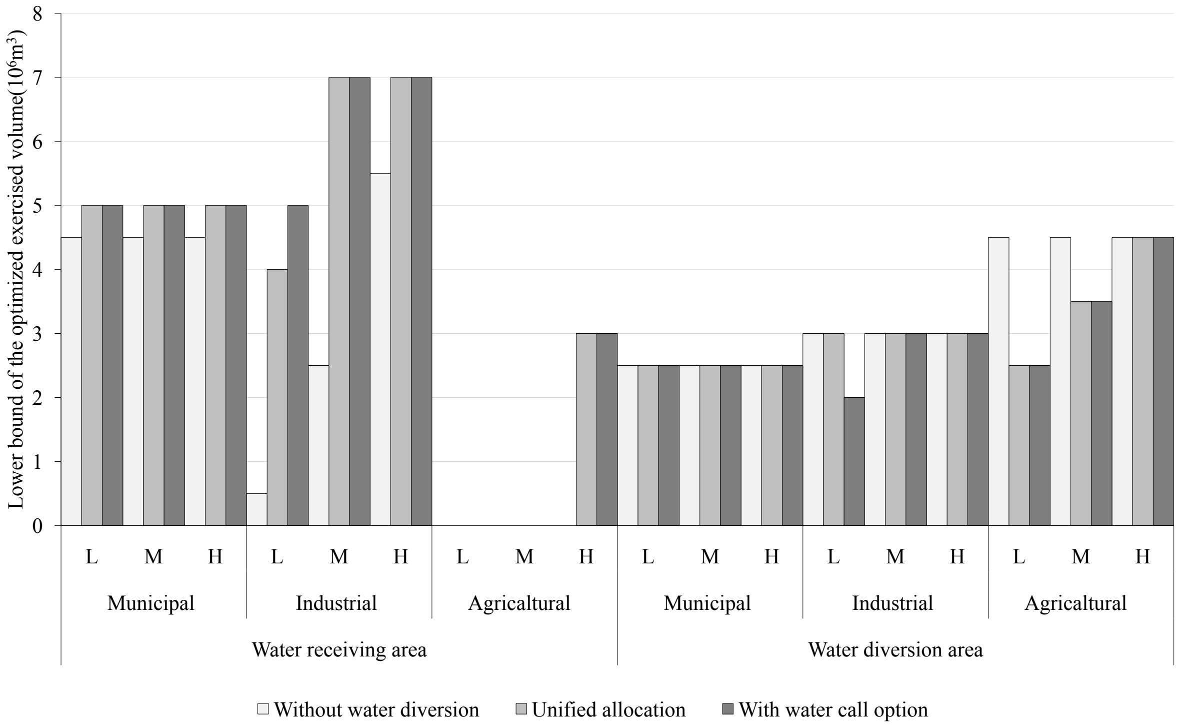

Figure 5. Lower bounds of the optimal water allocation under different inter-basin water allocation models.

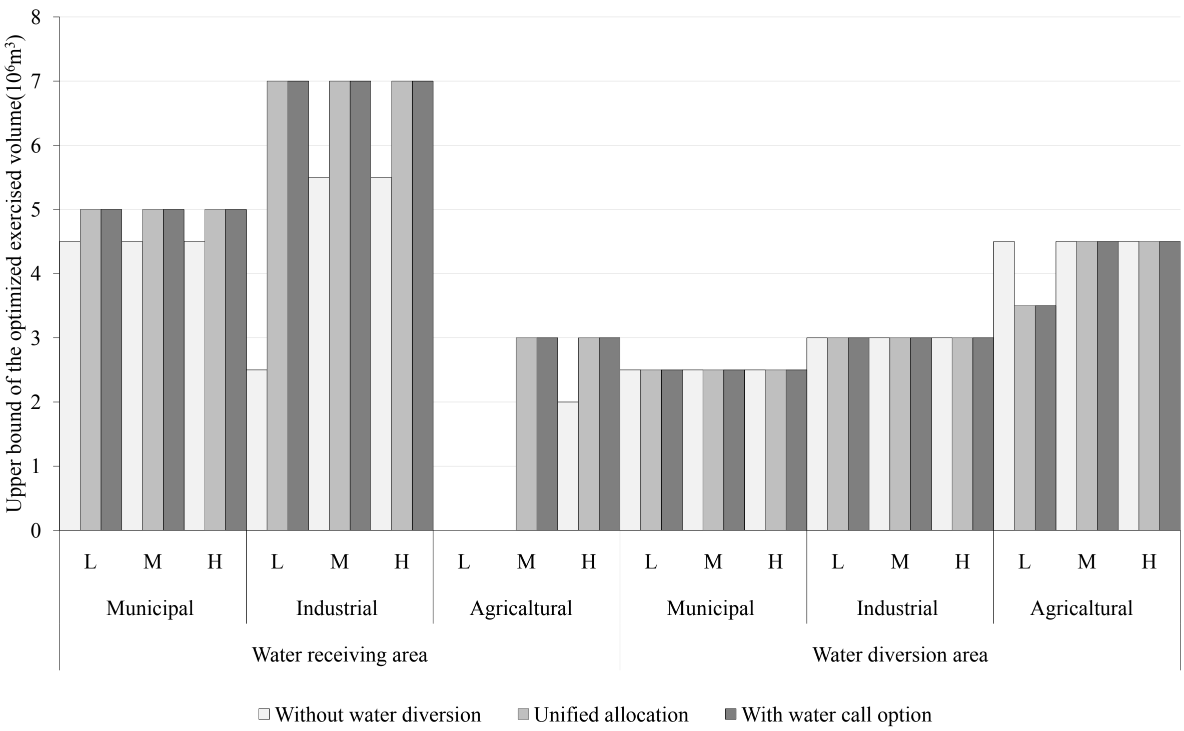

Figure 6. Upper bounds of the optimal water allocation under different inter-basin water allocation models.

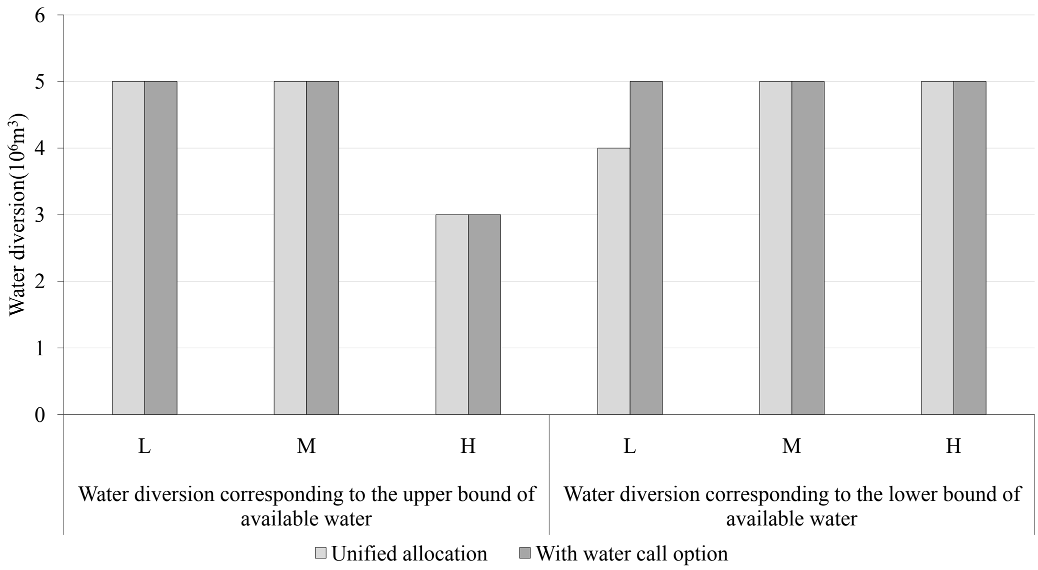

Figure 7. Water diversion under different inter-basin water allocation models.

Figure 7 shows the amount of water diversion under different inter-basin water allocation models. There is little difference between the two models in the wet and normal seasons, and the difference is mainly reflected in the dry season. In the wet season, the water diversion corresponding to the upper bound of the available water is 3.0×10^6 m^3 and that corresponding to the lower bound of the available water is 5.0×10^6 m^3. This is because there is more local water available during the wet season, so it is only needed to buy less water by exercising the option. In the dry season, when the available water reaches the lower bound, the amount of water bought by exercising the water call option is 5.0×10^6 m^3, which is more than the amount 4.0×10^6 m^3 under Model 2. This is because the water receiving area, as the holder of the water call option, has the right to purchase a certain amount of water by the exercise of the option, while the water transfer area, as the seller of the option, has the obligation to sell the corresponding water to the water receiving area even in the face of local water shortage.

4. Conclusion

In this paper, an interval two-stage stochastic multi-objective mixed integer programming (ITSMMIP) model has been established to solve the problem of optimal allocation of water resources in inter-basin water transfer based on the water call option. This model uses the methods of interval programming, two-stage stochastic programming and multi-objective stochastic programming, which can reflect the economic benefit of the water diversion and receiving areas comprehensively and give the optimal water allocation scheme at the same time. The proposed model has been applied to a case study of inter-basin water resource allocation. By comparing with other inter-basin water resource management methods, the amount of water transferred by using the water call option in the dry season is greater, and the water diversion area obtains economic compensation. Therefore, it is shown that the water call option is a valuable, flexible and low-cost instrument for inter-basin water resource management.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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