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Averting Disaster: Leverage Limits for Single-Stock Leveraged ETFs

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Abstract

Despite some major successes, Leveraged ETFs (LETFs) have resulted in several striking failures that we believe could dramatically increase with the recent introduction of single-stock LETFs in the U.S. Arguing for an urgent need to regulate leverage, we seek to educate regulators, investors, and LETF sponsors as to which single-stock LETFs are likely to be viable and which should be avoided. LETFs suffer from the effects of volatility drag and tail risk. With individual stocks tending to be much more volatile than stock indexes, these two negative factors are considerably amplified in single-stock LETFs and are why so many European single-stock leveraged products have failed. We derive a volatility-based formula for regulating leverage to prevent the toxic mix of high leverage and extreme volatility. We calibrate the parameters of our formula through historical analysis of leverage applied to stocks in the S&P 500. We then demonstrate its effectiveness when applied to the stocks that induced failed European leveraged products. Finally, we apply our framework on a forward-looking basis to stocks in the S&P 500 with comparisons to single-stock LETFs that already trade in the U.S.

Keywords

Financial Regulation, Leveraged ETF, Volatility Drag, Tail Risk, Volatility Decay

1. Introduction

Leveraged exchanged traded funds (LETFs) provide investors a leveraged exposure of k times to a market index (before expenses), with k typically ranging from −3 to 3. For example, $k = 2$ corresponds to a 2x LETF: On a day when the index increases by 1%, the 2x LETF rises by 2%. The case $k = -1$ corresponds to an inverse fund that benefits as the index declines: On a day when the index decreases by 1%, the inverse LETF increases by 1%. Recently, single-stock leveraged ETFs have been launched in the U.S., already having been introduced in Europe [1].

A well-known drawback of LETFs is volatility drag or volatility decay, the tendency of volatility to impair investment returns over extended periods [2]. Most LETFs rebalance leverage daily and have to "buy high" and "sell low" each day to keep their leverage ratios fixed, which leads to poor performance in choppy markets.

A striking example of volatility drag is provided by the Direxion Daily Gold Miners Index Bull and Bear 2x Shares, tickers NUGT and DUST, respectively. NUGT is the 2x bull ($k = 2$) Gold Miners LETF, and DUST is the 2x bear ($k = -2$) Gold Miners LETF. Together they are some of the worst-performing ETFs over the ten years through June 30, 2022, with NUGT having an average annual return of −47.5% and DUST an average annual return of −51.4%.[1] We suggest this dreadful contemporaneous performance reflects failed products: Investors in these LETFs have lost tremendous value and would have fared better using alternative speculative vehicles such as options.

Tail risk is a lesser-known but more striking risk of LETFs and the broader class of leveraged exchanged traded products (LETPs) that includes both LETFs and leveraged exchange-traded notes. In [3], it was suggested that the LETPs of certain narrow and volatile investment indexes, such as mortgage REITs, had significant long-term liquidation risk. Several such liquidations occurred in 2020 during the Covid-19 crisis [4].

European single-stock LETPs provide even more striking examples of tail risk. The U.K. Leverage Shares offers single-stock LETPs listed on various European stock exchanges. (The U.K. company Granite Shares has similar products.) As shown in **Table 1**, Leverage Shares has several single-stock 3x LETPs trading with returns of −95% or worse year-to-date through 8/16/2022. Many of these LETPs have prices of only a few pennies.

Table 1. Price and cumulative year-to-date performance of 3x LETPs offered by Leverage Shares. Tail risk explains much of this behavior, with one-day stock price declines ranging from −15.6% to −35.1% translating into theoretical 3x LETP one-day losses ranging from −46.8% to −100%. (Actual one-day LETP losses can vary from theoretical losses due to emergency leverage reset mechanisms.)

3x LETP	COIN	META	NFLX	NIO	PLTR	PYPL	ROKU	SHOP	SQ
Price as of 8/16/2022	\$0.01	\$1.62	\$0.17	\$0.19	\$0.04	\$0.07	\$0.00	\$0.01	\$0.02
Cumulative Return 12/31/21 to 8/16/22	−99.8%	−95.3%	−99.4%	−95.4%	−96.6%	−96.0%	−99.7%	−99.8%	−97.6%
Max One-Day Stock Decline	−26.4%	−26.4%	−35.1%	−15.2%	−21.3%	−24.6%	−23.1%	−16.0%	−15.6%
Theoretical Max One-Day 3x LETP Decline	−79.2%	−79.2%	−100.0%	−45.5%	−63.9%	−73.8%	−69.2%	−48.1%	−46.8%

Source: Leverage Shares website for LETP prices and returns.

[1]Just to be clear, these returns are −47.5% and −51.4% per year(!), not cumulative returns for the entire period. The source of these returns is the 6/30/2022 Fact Sheet provided by Direxion.

Tail risk accounts for much of the dramatic cumulative losses. For example, Netflix (Ticker: NFLX in **Table 1**) had a −35.1% return on April 20, 2022, which would translate to a −100% return (3x −35.1% = −105.3% < −100%), in theory completely wiping out the 3x LETP. The Netflix 3x LETP price is not zero due to an "unscheduled rebalance" mechanism that preserves a modicum of value during extreme price moves. Nevertheless, the product has lost all of its value for practical purposes.

Tail risk doesn't have to result in complete liquidation to be detrimental. For example, if a stock has a 25% one-day decline (several stocks in **Table 1** have such a decline) then $1 invested in a 3x LETP will decrease 75% to $0.25. This loss requires a much higher positive return of 300% in the 3x LETP to get back to even, exemplifying how even non-liquidation tail events are damaging and bleed into volatility drag over longer time intervals.

Without proper regulation, single-stock LETFs could lead to a host of failed LETFs like DUST, NUGT, and the European single-stock LETP examples. That said, not all LETFs are flawed products. As noted in [5], several long index LETFs such as the S&P 500 have outperformed their stated multiples on a cumulative-return basis suggesting that investors, in the aggregate, have benefited from those products. Through July 31, 2020, ProShares UltraPro QQQ (TQQQ) had been the single-best performing mutual fund or ETF through the prior decade [6].

These examples and counterexamples are one reason why LETFs have been so hotly debated, with some investors and academics touting them and others decrying them. We take the middle view. Rather than proposing a complete ban on single-stock LETFs, we suggest regulating and optimizing these products to allow viable products and avoid destructive ones.

Unfortunately, the analysis of LETFs has been so theoretically complex as to be poorly understood by investors and regulators. We provide background on the volatility drag and tail risk phenomena to help cut through this complexity. Although most prior research on LETFs focuses on cumulative returns, cumulative returns correspond to nonstationary processes that are difficult to characterize. We instead analyze expected compound returns (which are stationary and easily analyzed) rather than cumulative returns, providing the critical formulas driving volatility drag and illustrative tables demonstrating its impact.

We then develop an analytic framework for capping the leverage levels of single-stock LETFs. We propose rejecting any leveraged product that converts an investment with a positive expected return (e.g., a stock) into an investment with a negative expected return (e.g., a long single-stock LETF). This proposal aims to prevent LETFs that will lose money for investors (in the aggregate) without offering any useful purpose, such as hedging risk or improving market liquidity.

We know beforehand that stocks with high measured volatility will likely have negative returns when high leverage is applied. High underlying stock returns can offset the negative impact of volatility, but individual stock returns are nearly impossible to predict. We instead analyze empirical data to set a general expected return level, applied to all stocks, that will tend to prevent failed products while preserving viable ones. The resulting formula provides a simple analytic framework for setting regulatory leverage caps as a function of expected return and volatility, with more volatile stocks having lower leverage caps. We also justify applying the same derived caps to inverse LETFs, resulting in a comprehensive framework for regulating LETF leverage.

We optimize the parameters of this approach by analyzing leverage constraints of single-stock LETFs for stocks in the S&P 500. We demonstrate the effectiveness of our solution when applied to the stocks that induced failed European leveraged products. We then provide forward-looking leverage analysis, offering tables of leverage caps for the top twenty companies in the S&P 500 and comparisons with current single-stock LETFs already trading in the U.S.

The remainder of this paper is organized as follows. Section 2 discusses the mathematics of volatility drag, including background on the volatility and return calculations that we will apply. In Section 3, we justify and derive our formula for capping leverage and discuss the application of leverage to inverse funds. Section 4 explains our technique for setting the formula parameters of volatility and return. In Section 5, we optimize the return parameter using historical data on stocks in the S&P 500 and then apply our approach to stocks that induced failed European leveraged products. In Section 6, we provide proposed leverage limits for the top twenty stocks in the S&P 500 and compare our leverage limits with single-stock LETFs already trading in the U.S. Section 7 provides our conclusions and ideas for future research.

2. Mathematics of Volatility Drag

Geometric average returns, corresponding to compound returns, are always less than or equal to arithmetic average returns, with equality occurring only in the case of non-varying (*i.e.*, constant) returns. Under standard market assumptions of the Black-Scholes Merton model, we can statistically relate the geometric mean (compound return) to the arithmetic mean (average one-period return) with the following formula [7]:

$$\mu = r - \frac{\sigma^2}{2}, \tag{1}$$

with r the expected return over one period (arithmetic average), σ the standard deviation of the return over one period (volatility), and μ the expected growth compounding rate over time (geometric average). Equation (1) is the source of the term volatility drag, also known as volatility decay.

We present a 2nd-order approximation (derived in [3]) for volatility drag for an LETF with leverage k applied to an investment over T periods with the return series $r_1, r_2, \cdots, r_T$, whose arithmetic average is $\bar{r} = \frac{1}{T}\sum_{t=1}^{T} r_t$:

$$\hat{r}(k) \approx k\bar{r} - \frac{k^2\sigma^2}{2(1+k\bar{r})} \tag{2}$$

with $\hat{r}(k)$ the geometric average (compound) return of the k-times LETF. Unlike Equation (1), Equation (2) makes no assumptions about the probability distribution of investment returns and was first derived in [8].

We have found that Equation (2) is generally accurate when applied to index LETFs that rebalance leverage daily, with inaccuracies starting to creep in when $k\sigma$ is large or when applied to LETFs that rebalance monthly. These errors are corrected by incorporating the full Taylor Series expansion and including skew and kurtosis terms [3].

Moreover, the errors in Equation (2) tend to underestimate volatility drag (*i.e.*, overestimate compound returns) by not factoring in the impact of extreme negative returns that can occur in actual market data. In other words, Equation (2) does not fully incorporate tail risk, the risk of a dramatic one-day loss greatly diminishing the compound return [3].

With justification to follow, we will apply the 2nd-order approximation of Equation (2) for our analysis and even simplify it further. We note that the factor $1+k\overline{r}$ is small when applied on a daily time scale. For example, $k\overline{r}$ is roughly 0.001 for a leverage factor of $k = 3$ and a daily average return corresponding to an average annual return of 11%. We thus will omit the term $1+k\overline{r}$, resulting in the equation

$$\hat{r}(k) \approx k\overline{r} - \frac{k^2\sigma^2}{2} \tag{3}$$

The advantage of Equation (3) is that it results in a simple solution for setting leverage caps. Note that plugging in $k = 1$ and rearranging terms gives us the relation between compound return and arithmetic average return for the unleveraged investment $\overline{r} = \hat{r}(1) + \frac{\sigma^2}{2}$, a relation that we will use in future calculations.

The astute reader may question our use of Equations (2) and (3) given that they don't incorporate tail risk even as single-stock LETFs have much more tail risk than index LETFs. Ideally, we would factor in tail risk through skew and kurtosis such as in [3]. This criticism is valid, but we offer several counterarguments.

First, adding skew and kurtosis factors would preclude us from finding a closed-form solution for setting leverage caps. Also, skew and kurtosis are challenging to forecast accurately for individual stocks. Finally, the formula we derive for capping leverage significantly reduces tail risk and its impact on return calculations. In other words, our solution purposefully mitigates the very phenomenon that would make our equations inaccurate by severely limiting k when volatility σ is large.

Note that volatility drag in Equation (3) is quadratic in both volatility σ and leverage factor k, which explains why it is essential to strictly limit leverage k when stock volatility σ is high in order to prevent failed LETFs.

Background on Volatility and Return Calculations

We next explain our volatility and return calculations. Although this background appears to be elementary, we believe it is necessary so that others can accurately apply our approach. In particular, we want financial regulators such as the SEC to be able to implement our analysis. To begin, Equation (3) applies to volatility calculated using "simple" daily returns, whereas volatilities used in options models are computed using logarithmic daily returns. The two measures result in similar values when applied to daily price changes but do differ slightly.

We quote returns and volatilities on an annualized basis since finance professionals think in annualized terms. However, applying Equation (3) and related formulas using annualized returns and volatilities would be a mistake. Computations must be performed using daily returns and volatility because the leverage compounding period is daily. We thus provide formulas to convert between the daily stock returns and volatilities we use and the annualized stock returns and volatilities we quote.

We use the standard textbook conversion formula [7] for volatility $\sigma_a = \sigma_d \sqrt{252}$, with σ_a the annualized volatility and σ_d the daily volatility 1 (assuming 252 trading days in a year). For example, if we quote an annualized volatility of 30% that implies a daily volatility of roughly $1.8898\% \approx 30\% / \sqrt{252}$. Note that this formula is only theoretically valid for converting volatilities measured using logarithmic returns. However, the conversion provides intuition for finance professionals accustomed to annualized volatilities.

The conversion between daily compound returns and annual compound returns is found by compounding daily returns through the course of one year according to the formula $\hat{r}_a = (1+\hat{r}_d)^{252} - 1$, with $\hat{r}_a$ the average annual compound return and $\hat{r}_d$ the average daily compound return of an investment. Thus, an annual compound return of 10% corresponds to a daily compound return of roughly 0.03783%.

3. Justifying and Deriving the Formula to Cap Leverage

We propose that a long (*i.e.*, $k > 0$) LETF should not exist if it converts an investment with a positive return into one with a negative return. Such an LETF will lose money for investors over time when these investors would have, on average, profited from just owning the underlying stock or index. Moreover, investment vehicles such as options and futures allow leveraged speculation without the drawback of volatility drag. Investors who intend on using leverage are better served using these alternative speculative vehicles.

We grant that not all products need have a positive return to have a reason to exist. Products that reduce risk by hedging are valuable and necessary. For example, inverse ETFs that short major stock indexes lose money in the long run yet have a valid purpose of hedging risk. No such risk-reduction purpose exists for long LETFs.

Another justification for speculative products, such as futures, is that they enhance market liquidity and decrease volatility over the long run. Intuitively, this does not apply to LETFs given that LETFs buy high and sell low with trading that tends to exacerbate extreme market moves. Some researchers have shown that LETFs increased volatility during the financial crisis [9]. Although other researchers have found that LETFs don't meaningfully increase volatility [10], it is clear that LETFs don't reduce market volatility—they either increase volatility or have no meaningful impact. Thus, we argue that single-stock LETFs should not exist if they cause investors to consistently lose money (versus an investment in the stock) while neither serving as hedging vehicles nor acting to reduce market volatility.

Converting the above arguments into math, we propose regulatory caps k_{reg} for the leverage k by setting the expected LETF compound return $\hat{r}(k)=0$ in Equation (3) and solving for k, which results in the formula

$$k_{reg}=\frac{2\overline{r}}{\sigma^2} \tag{4}$$

Interestingly, our proposed regulatory leverage of Equation (4) is twice the return-optimizing leverage derived in [8]. Recalling from Equation (3) that $\overline{r}=\hat{r}(1)+\frac{\sigma^2}{2}$, we can express Equation (4) as

$$k_{reg}=\frac{2\left(\hat{r}(1)+\frac{\sigma^2}{2}\right)}{\sigma^2}=1+\frac{2\hat{r}(1)}{\sigma^2}, \tag{5}$$

with $\hat{r}(1)$ the average daily compound return and σ the daily volatility of the underlying stock.

Table 2 shows the suggested leverage cap k_{reg} of Equation (5) for various combinations of stock returns and volatilities. Stocks with low volatility can have leverage caps of several multiples, while stocks with high volatility have leverage caps close to one.

Table 2. Setting leverage caps k_{reg} as a function of stock volatility and return. Caps are highly dependent on stock volatility, with highly volatile stocks having caps close to one. Independent of this table, we suggest that leverage should also be capped at a fixed ratio of three or lower.

Table of k_{reg}		Annualized Stock Volatility								
		20%	**30%**	**40%**	**50%**	**60%**	**70%**	**80%**	**90%**	**100%**
Average Annual Compound Return	**1%**	1.50	1.22	1.12	1.08	1.06	1.04	1.03	1.02	1.02
	2%	1.99	1.44	1.25	1.16	1.11	1.08	1.06	1.05	1.04
	3%	2.48	1.66	1.37	1.24	1.16	1.12	1.09	1.07	1.06
	4%	2.96	1.87	1.49	1.31	1.22	1.16	1.12	1.10	1.08
	5%	3.44	2.08	1.61	1.39	1.27	1.20	1.15	1.12	1.10
	6%	3.91	2.30	1.73	1.47	1.32	1.24	1.18	1.14	1.12
	7%	4.38	2.50	1.85	1.54	1.38	1.28	1.21	1.17	1.14
	8%	4.85	2.71	1.96	1.62	1.43	1.31	1.24	1.19	1.15
	9%	5.31	2.92	2.08	1.69	1.48	1.35	1.27	1.21	1.17
	10%	5.77	3.12	2.19	1.76	1.53	1.39	1.30	1.24	1.19

Single-stock LETFs are riskier than index LETFs which, with a few exceptions, have leverage of at most three. Common sense dictates that riskier single-stock LETFs should have a similar leverage restriction. Moreover, the number of failed European LETPs suggests that a fixed leverage cap below three would be more appropriate. In this paper, we will apply a fixed cap $k_{reg} \le 3$ to show a fuller range of how k_{reg} varies with volatility. However, we suggest that a lower cap level (e.g., $k_{reg} \le 2$ or $k_{reg} \le 2.5$) makes more sense from a regulatory perspective. Note that this dual cap approach is comparable to how banks are regulated using both a model-based tier 1 capital adequacy ratio (akin to Table 2) and a non-model-based leverage ratio (akin to a fixed leverage cap).

Leverage Limits for Inverse Funds

Let us also consider inverse funds ($k < 0$) and the impact of volatility on them. We define **leveraged volatility drag** as $\hat{r}(k) - k \cdot \hat{r}(1)$, the difference between the compound return of a k–times LETF and k times the unleveraged index's compound return. Leveraged volatility drag measures how much, on a compound return basis, a k–times LETF will underperform its stated performance. We can calculate leveraged volatility drag by applying Equation (3) and substituting $\bar{r} = \hat{r}(1) + \frac{\sigma^2}{2}$ to get

$$\hat{r}(k) - k \cdot \hat{r}(1) \approx \frac{-\sigma^2 k(k-1)}{2} \tag{6}$$

We provide this formula to show that an inverse LETF with $k = -1$ has the same leveraged volatility drag as a 2-times LETF with $k = 2$. A double inverse LETF with $k = -2$ has the same amount of leveraged volatility drag as a 3-times LETF with $k = 3$. In other words, inverse LETFs have similar performance drags to long LETFs with an additional turn of leverage, a dynamic which has also been observed in [2].

We argue that inverse LETFs should be capped with at most the same amount of leverage as long LETFs given that inverse LETFs have heightened performance drag (in addition to the inherent risk in shorting stocks). In other words, we recommend that leverage be restricted to an amount ranging from $-k_{reg}$ to $+k_{reg}$. Equation (6) even suggests capping inverse LETFs with lower magnitudes than long LETFs. For example, a leverage range of −2.0 to 2.5 seems more defensible than a range of −2.5 to 2.5. However, this is not how most LETFs are currently structured, with index LETF sponsors typically offering long LETFs and inverse LETFs with the same amount of leverage (e.g., $k = \pm 3$ LETFs).

4. Determining the Regulatory Parameters

Setting the return and volatility parameters of Equation (5) for an individual stock requires a reasoned analysis. Given that the past volatility of a stock is a good predictor of future volatility, we can use a stock's historical volatility as an estimate of the stock's future volatility. However, a stock's past returns are a poor predictor of future returns, so we will instead set return as a single parameter applied uniformly across all stocks.

4.1. Setting the Volatility Parameter

To estimate future volatility, we propose taking the maximum of 5-year and 10-year historical volatilities. We use these extended periods because historical volatility measurements can change dramatically from year to year when measured using shorter periods such as one year. This would result in our k_{reg} cap changing dramatically from year to year, whereas any regulatory limit should be stable and consistent.

Ideally, the volatility measurement period should include a complete business cycle with at least one downturn, suggesting the use of the 10-year volatility as used in other economic measures such as the Cyclically Adjusted PE (CAPE) Ratio invented by Campbell and Shiller [11]. Otherwise, the measurement may miss the heightened volatility that stocks experience during an economic downturn. However, shorter time frames for historical volatility tend to offer more accurate predictors for near-term future volatility. Thus, we use the 5-year historical volatility to capture more recent volatility trends. We apply the maximum of these two volatility measures to decrease the possibility of underestimating future volatility, as such underestimation could significantly increase the likelihood of tail events and failed products.

When finance professionals measure volatility over extended periods (e.g., ≥5 years), they typically use weekly or monthly price returns. However, we stress the importance of measuring volatility using daily price returns. For most LETFs, including all single-stock LETFs, leverage is rebalanced daily, and thus daily volatility determines the amount of volatility drag an LETF experiences. Moreover, volatility measured using monthly price returns tends to be significantly lower than volatility measured using daily price returns [3] and thus underestimates the impact of volatility drag for LETFs that rebalance leverage daily.

4.2. Setting the Return Parameter

Setting the compound return $\hat{r}(1)$ parameter for a given stock is more challenging. First, historical returns of an individual stock are not a good predictor of future returns of that stock. Instead, we set the return parameter by estimating the expected return prospects of the market as a whole. We will now discuss several alternatives for this calculation.

The first alternative is to use long-term historical averages. According to data from the Center for Research in Security Prices (CRSP), the annual compound return of the S&P 500 from 1928 to 2021 was 10.21% per year. Most finance professionals would agree that 10.21% is too high a return expectation given various factors (such as the risk-free rate, stock yields, and expected growth) used to derive forward-looking returns [12].

Damodaran has compared various approaches for estimating the equity risk premium (ERP), finding that the implied ERP has the highest correlation with future returns [12]. As of August 1, 2022, Damodaran's implied market risk premium results in an expected return of 8.07%, as shown on his website [13]. Thus, we could set $\hat{r}(1)$ using the annual compound return of 8.07% converted to a daily compound return of roughly 0.03080%. This would be a reasonable approach to setting the return parameter if we were engaged in a purely academic exercise.

However, we are not purely engaged in an academic exercise. We are proposing regulatory limits to prevent failed LETF products while allowing successful LETF products. Moreover, "typical" stock returns often underperform the market return [14]. This fact suggests that the expected return parameter for regulating a typical single-stock LETF should be lower than the expected market return, as a lower parameter value would be more effective for a broader set of stocks.

Thus, we apply a data-driven approach instead of setting the return parameter through a theoretical estimate. Using historical returns of S&P 500 stocks, we optimize the return parameter by choosing which parameter would work best for preventing failed LETF products while preserving viable products.

5. Application to S&P 500 Data

For our analysis, we collected daily return data on the constituents of the S&P 500 as of July 31, 2022. We sought to test the effectiveness of leverage caps for hypothetical LETFs over the ten years from July 31, 2012, to July 31, 2022. We chose a 10-year testing period to characterize volatility drag effectively and have a sufficient sample size to capture low-probability tail events that cause liquidations.

We used data from the prior ten years, July 31, 2002, to July 31, 2012, to calculate the volatility used in setting the leverage caps for each stock of the S&P 500, using the maximum of both a 10-year and 5-year historical volatility measure. Note that some stocks did not have sufficient historical data for a full 5-year or 10-year volatility measure. In this case, we calculated volatility using the available historical data subject to the constraint that at least one year of return data was available. We thus excluded stocks with less than one year of historical data for the period ending July 31, 2012, resulting in a sample size of 454 stocks.

With the volatility parameters in hand, we then uniformly varied the compound return parameter in the range of 1% to 10%, applying Equation (5) to set leverage caps k_{reg} for each stock. We then classified "errors" into three types:

1) Liquidation Event: If a hypothetical LETF had a negative "tail event" that would wipe out the LETF (e.g., a 1/3 drop for 3x leveraged LETF), we classified it as a liquidation error. In this case, LETF investors would have seen most to all of their value destroyed in a single day.

2) Failed Product: If a hypothetical LETF had a negative return while the unleveraged return was positive, we classified this as a failed product error. In this case, LETF investors, on the aggregate, would have lost money even as the stock had a positive return over the period.

3) Higher Viable Leverage: If a hypothetical LETF had a greater positive compound return than the unleveraged stock return, we classified this as a higher viable leverage error. In this case, a higher leverage level could have been employed without significant negative consequences.

Our goal was to find the return parameter that minimized a weighted average of the three types of errors. Note that error type 3 counterbalanced the other errors and ensured that the optimization didn't select the lowest possible return parameter. Error weightings were chosen with a type 1 error weighting of eight, a type 2 error weighting of two, and a type 3 error weighting of one. Intuitively, type 1 errors are more consequential than type 2 errors which are more significant than type 3 errors. The exact value of weightings was subjective but resulted in an optimal return parameter within a reasonable range.

One may contend that our data optimization has a survivorship bias since we selected current S&P 500 members rather than S&P members as of July 31, 2012. Thus, our data exclude prior S&P 500 members that have gone bankrupt during the period. We claim that no long LETFs, regardless of leverage amount, can be successful for companies that go bankrupt. Therefore, optimizing parameters based on those companies is questionable, as they typically have highly volatile returns when it is clear that the companies will likely fail.

Allowing for a return range of 1% to 10%, we found that an annual compound return parameter of 5.9% (daily compound return of 0.02275%) worked best at minimizing weighted errors. The interpretation of our model with the 5.9% return parameter is as follows: Our model targets leverage caps such that single-stock LETFs will have positive average annual returns if their underlying stocks have average annual returns greater than 5.9%.

In our view, 5.9% is an intuitively reasonable parameter choice, as it is lower than the S&P 500 return expectation of 8.07% per our prior arguments but also bounded away from 1%, which is overly restrictive compared to viable index LETFs that are already outstanding. For example, the S&P 500 index has had a volatility of just under 20% over the past 20 years. From **Table 2**, an investment with a volatility of 20% is restricted to leverage under 2.50x for a return parameter in the range of 1% to 3%. However, 3x S&P 500 LETFs have been successful products over the past ten years. For instance, the ProShares 3x S&P 500 (ticker: UPRO) had an average annual NAV return of 27.35% per year versus 21.27% for the ProShares 2x S&P 500 (ticker: SSO) and 12.96% per year for the S&P 500 for the ten years ending June 30, 2022.[2]

Table 3 provides the error types (including the number and percentages of errors) and leverages statistics of our optimization in contrast to various fixed leverage caps. We include the cap $k_{reg} = 1.82$ as a point of comparison versus our optimization, which has an average $k_{reg} = 1.82$.

[2]The sources of these returns are the June 30, 2022 Fact Sheets from the ProShares website.

Table 3. Comparison of the performance of a volatility-based leverage cap with a fixed leveraged cap for companies in the S&P 500. Liquidation events and failed products significantly increase as leverage is ramped up to 3x. Our volatility-based cap has far fewer failed products (20 versus 32) than a fixed cap for the same average degree of leverage ($k_{reg} = 1.82$).

	Error Types			Statistics of k_{reg}		
	Liquidation Event	Failed Product	Higher Viable Leverage	Avg k_{reg}	Min k_{reg}	Max k_{reg}
Volatility-based k_{reg} with return = 5.9%	0 (0.00%)	20 (4.41%)	370 (81.50%)	1.82	1.15	3.00
k_{reg} = 1.50	0 (0.00%)	20 (4.41%)	378 (83.26%)	1.50	1.50	1.50
k_{reg} = 1.82	0 (0.00%)	32 (7.05%)	357 (78.63%)	1.82	1.82	1.82
k_{reg} = 2.00	2 (0.44%)	35 (7.71%)	340 (74.89%)	2.00	2.00	2.00
k_{reg} = 2.50	4 (0.88%)	60 (13.22%)	297 (65.42%)	2.50	2.50	2.50
k_{reg} = 3.00	13 (2.86%)	97 (21.37%)	259 (57.05%)	3.00	3.00	3.00

The high frequency of liquidation events and product failure support our proposal for a common-sense threshold of $k_{reg} = 3.00$ or lower. In Table 3, we see that employing a flat $k_{reg} = 3.00$ results in 2.86% liquidation events and 21.37% failed products. These percentages comport with the number of liquidations and failed products of Table 1 given that LETFs are more likely to be introduced for volatile stocks (such as volatile tech stocks instead of sedate utility stocks), some of which are not in the S&P 500.

Comparing the volatility-based cap with a fixed leverage cap of $k_{reg} = 1.50$, we see that the volatility-based cap has the same number of liquidations (0) and failed products (20) but with a higher average leverage amount of $k_{reg} = 1.82$. Comparing the volatility-based cap with a fixed leverage cap of $k_{reg} = 1.82$, we see that the volatility-based cap has many fewer failed products (20 versus 32 for the fixed cap) for the same amount of average leverage. Together these results confirm our claim that volatility-based caps are superior to fixed leverage ratio caps.

Effectiveness for 2022 Technology Sell-off

As shown in Table 4, we applied our leverage caps to evaluate how they would perform relative to the failed Leverage Shares 3x LETPs shown in Table 1 over the period 12/31/2021 to 8/16/2022. To avoid hindsight bias, we measured volatility through 12/31/2021 to set leverage caps going into 2022, relaxing our one-year data requirement for COIN (7.5 months were available) to ensure a complete comparison.

Table 4. Effectiveness of our volatility-based caps for regulating failed European 3x LETPs during the 2022 technology sell-off. With an average leverage k_{reg} = 1.43, our hypothetical LETFs improve tail risk and cumulative return metrics despite historical volatility underestimating actual in-period volatility. Surprisingly, the returns of inverse LETFs, shown at the bottom of the table, show dramatic improvements with leverage caps.

YTD Thru 8/16/2022	COIN	META	NFLX	NIO	PLTR	PYPL	ROKU	SHOP	SQ	Avg
Preceding Historical Volatility	58.9%	36.7%	47.5%	101.9%	77.6%	35.7%	78.9%	52.6%	54.0%	**60.4%**
Actual In-Period Volatility	116.2%	64.5%	76.4%	94.0%	80.9%	65.9%	99.4%	103.8%	97.1%	**88.7%**
k_{reg}	1.33	1.85	1.51	1.11	1.19	1.90	1.18	1.42	1.39	**1.43**
3x Max 1-Day Loss	−79.2%	−79.2%	−100.0%	−45.5%	−63.9%	−73.8%	−69.2%	−48.1%	−46.8%	**−67.3%**
k_{reg} **Max 1-Day Loss**	−35.1%	−48.9%	−53.0%	−16.8%	−25.4%	−46.7%	−27.3%	−22.7%	−21.7%	**−33.1%**
3x Cum Rtn	−99.8%	−95.5%	−100.0%	−94.2%	−96.4%	−94.8%	−99.5%	−99.7%	−97.4%	**−97.5%**
k_{reg} **Cum Rtn**	−79.1%	−75.6%	−78.9%	−39.0%	−54.8%	−76.2%	−71.5%	−86.0%	−64.5%	**−69.5%**
−3x Cum Rtn	−87.7%	41.3%	100.3%	−92.4%	−42.7%	28.5%	−48.8%	−33.6%	−87.8%	**−24.8%**
$-k_{reg}$ **Cum Rtn**	6.6%	62.5%	96.3%	−16.8%	24.1%	53.9%	47.4%	84.8%	−10.4%	**38.7%**

Note in Table 4 that our historical volatility calculations going into the period (averaging 60.4%) were substantially below the actual volatility in the period (averaging 88.7%). The period 2016-2021 was relatively quiescent for technology stocks, while 2022 was highly volatile in the face of Fed rate hikes and a sell-off in technology stocks. This type of unforeseen volatility spike is not uncommon and provides a robust test for our method.

To keep an apples-to-apples comparison, we evaluated the hypothetical performance of 3x LETFs with our hypothetical LETFs capped at k_{reg}. At the bottom of Table 4, we also compared the hypothetical performance of −3x inverse LETFs to our hypothetical inverse LETFs capped at $-k_{reg}$. Note that the actual 3x LETPs include fees and emergency leverage-reset mechanisms that we do not attempt to model. However, a comparison of the hypothetical returns in Table 4 with the actual Leverages Shares 3x LETP returns in Table 1 shows that our hypothetical 3x LETF returns are very close to those of the actual 3x LETP products.

Despite using underestimated volatility, our proposed k_{reg} caps were successful across many fronts. First, we avoided the Netflix liquidation event, with a −53% max loss compared to a −100% max loss for the theoretical 3x LETF. More generally, the maximum one-day losses were, on average, more than halved from −67.3% using 3x leverage to −33.1% using k_{reg}. In addition, cumulative returns averaged −69.5% using k_{reg} versus −97.5% using 3x leverage. Although this difference may not seem substantial, a 97.5% loss requires a 39.7x increase in capital to offset the loss versus a 3.28x increase in capital to offset a −69.5% loss. In other words, the −97.5% loss is catastrophic, whereas the −69.5% loss, while substantial, is recoverable.

The performance of hypothetical inverse LETFs at the bottom of the table is even more striking. Limiting leverage to $-k_{reg}$ substantially improved the performance of hypothetical inverse LETFs by a staggering amount of 63.5%

(!), with a −24.8% average cumulative return for the −3x inverse LETFs versus a +38.7% average cumulative return for inverse LETFs capped at $-k_{reg}$. This result is surprising given that this period was a bear market in which one would expect highly leveraged inverse LETFs to outperform. Except for Netflix, using lower leverage at $-k_{reg}$ significantly outperformed for all the inverse LETFs.

These results demonstrate the power of tail risk and volatility drag – the huge volatilities experienced during this period meant that highly leveraged LETFs were likely to underperform whether they were long or short. The implication for leverage regulation is clear: ±3x leverage is too much for this set of highly volatile stocks.

6. Suggested Leverage Limits Going Forward

In Table 5, we provide leverage caps on a go-forward basis for the top twenty stocks in the S&P 500 using our 5.9% annual return parameter, with volatility the maximum of the 5-year and 10-year historical volatilities (applied to daily price returns) measured through July 31, 2022. For this data, the 5-year volatilities are consistently higher than the 10-year volatilities, thus providing the primary limiting factor for k_{reg}. As in our optimization, we use the longest period available for stocks with a price history of fewer than five years, subject to the constraint that at least one year of history is available.

In Table 6, we compare our leverage caps with leverage levels for stocks that already have single-stock LETFs trading in the U.S. We use the same methodology as described for Table 5.

Overall, we find that current single-stock LETFs in the U.S., for the most part, fit within our proposed guidelines. We note that the LETFs of AXS Investments adhere to our recommendations and appear to incorporate volatility analysis into their leverage amounts. The two exceptions to our guidelines are the Coinbase (Ticker: COIN) LETF of GraniteShares and the Tesla (Ticker: TSLA) LETF of Direxion, which at leverage ratios of $k = 1.50$ each exceed our recommendations of $k_{reg} = 1.15$ for Coinbase and $k_{reg} = 1.28$ for Tesla.

Table 5. Proposed leverage caps for single-stock LETFs for the top 20 stocks in the S&P 500. The caps vary significantly based on stock volatility, with Tesla having the lowest cap at $k_{reg} = 1.28$. We have applied a hard cap on leverage of $k_{reg} = 3.00$, but arguments can be made for a lower hard cap level (e.g., $k_{reg} = 2.00$ or 2.50).

Ticker	k_{reg}	Ticker	k_{reg}	Ticker	k_{reg}	Ticker	k_{reg}
AAPL	2.11	CVX	1.95	MA	2.09	PG	3.00
ABBV	2.40	GOOGL	2.30	META	1.73	TSLA	1.28
AMZN	1.98	HD	2.51	MSFT	2.30	UNH	2.32
BAC	1.95	JNJ	3.00	NVDA	1.46	V	2.39
BRK.B	3.00	JPM	2.17	PFE	2.77	XOM	2.07

Table 6. Comparison of proposed leverage caps versus the leverage levels of single-stock LETFs trading in the U.S. The leverage levels of most LETFs are within our guidelines except for the two stocks highlighted in orange – Coinbase and Tesla.

	Proposed Caps		AXS Investments		GraniteShares		Direxion	
Ticker	**Inverse**	**Long**	**Inverse**	**Long**	**Inverse**	**Long**	**Inverse**	**Long**
AAPL	−2.11	2.11				1.75		
COIN	−1.15	1.15				1.50		
NKE	−2.18	2.18	−2.00	2.00				
NVDA	−1.46	1.46	−1.25					
PFE	−2.77	2.77	−2.00	2.00				
PYPL	−1.66	1.66	−1.50	1.50				
TSLA	−1.28	1.28	−1.00		−1.00	1.25	−1.00	1.50

In our view, Coinbase is currently too volatile to have a successful long LETF with any significant leverage due to a high historical volatility of 87.25% measured using data starting from Coinbase's 4/14/2021 IPO. Given that our proposed leverage limit of 1.15 is likely not high enough to attract speculators, we suggest that sponsors avoid long LETFs for Coinbase until there is a noticeable and persistent decline in stock volatility.

For Tesla, we measure a volatility of 63.81%, resulting in a leverage cap of k_{reg} = 1.28. Although the leverage of k_{reg} = 1.50 does not deviate as much from our cap as does the Coinbase LETF, we believe that the Direxion Tesla LETF is also at risk of becoming a failed product over the long term.

7. Conclusions and Policy Implications

We have provided a simple, intuitive framework for regulating LETF leverage for what has been a controversial and theoretically complex topic. We have shown why single-stock LETFs should have regulatory leverage caps and why volatility should be incorporated into those caps. We have derived how to set leverage caps and, through a simple optimization, have set parameters for calculating those caps in the future. One may quibble with the exact value of our parameters, but we believe that our choices are intuitive and adhere to common sense.

Applying our approach to failed European single-stock 3x LETPs during the 2022 sell-off, we find that our proposed regulation of leverage substantially improves both long and inverse LETF performance. This result demonstrates that 3x leverage is too high for this set of volatile stocks and that our simple regulation could significantly improve single-stock LETF products.

Although the current single-stock LETFs in the U.S. mostly follow our leverage guidelines, we expect LETF sponsors to "push the envelope" as these products become popular. We do not claim that our cap levels are perfect but do suggest that any LETFs with leverage significantly higher than our proposed caps should be rejected or at least closely scrutinized. If not, the U.S. will risk having a graveyard of failed products similar to Europe.

Philosophically, one could argue for free markets such that speculators employing leverage who are wiped out "get what they deserve". We offer two counterarguments. First, many U.S. LETF investors are retail investors that don't fully comprehend the risks inherent in single-stock LETF products. Second, we can identify ahead of time which single-stock LETF products are flawed (*i.e.*, highly leveraged LETFs of stocks with elevated volatilities). Rather than allow these potentially defective products, we should improve them through prudent regulation and thoughtful product design, as we have discussed in this article. We thus hope the SEC will incorporate our analysis as they consider regulating single-stock LETFs.

For future research, we recommend using this method to regulate leverage levels for volatile foreign and sector LETFs. For example, there is strong investor demand for LETFs in China, with the Chinese-A market tending to exhibit greater volatility than the U.S. market [15], which suggests that our volatility-based approach could lead to regulatory guidelines for the Chinese market. Even sector index LETFs in the U.S. could benefit from this approach—if not from strict regulation, then from improved product design. The expected return parameter should be tailored to the particulars of each market or index, but the general framework still applies.

In addition, more in-depth volatility and extreme-value analysis could lead to improved volatility forecasting and further insights on tail risk [16], with the trade-off that a more rigorous analytical framework could be more abstruse and difficult to replicate as a regulatory guideline. That said, we welcome any attempts to improve upon our methods, as we anticipate this work to generate improved understanding and further research on this crucial subject.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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