Toward Integrated Active Management Based on SBDA

Koichi Miyazaki*, Kazuhiro Shimada

Department of Investment, Government Pension Investment Fund, Tokyo, Japan
Email: *f-research@gpif.go.jp

Abstract

Regarding the methodology for separating the investment performance of PE funds into a beta portion (investment performance of traditional assets), which is market performance, and an alpha portion (excess return), we presented it in comparison with the valuation method for interest rate swaps with credit risk in our JMF article dated August 25, 2023, "Spread-Based Direct Alpha (SBDA) as a Performance Measure for PE Funds". Once we separate the excess return from the benchmark return, we have only to estimate the risk of the excess return, namely tracking error, to evaluate the risk-return efficiency of the PE Fund relative to the benchmark. However, due to the complicated cashflows with capital calls and distributions, it is quite difficult to define and estimate the tracking error of the PE fund being different from the estimation of the tracking errors for the active funds of traditional assets. This study provides the methodology to estimate the tracking error for alternative assets by using the SBDA as a starting point and introducing various new concepts of excess returns. Utilizing the integrated active management, which is available with the estimated tracking error, pension fund managers are able to build the more risk-return (Information Ratio) efficient portfolio incorporating alternative assets in addition to the active funds of traditional assets.

Keywords

Alternative Assets, SBDA (Spread Based Direct Alpha), Trucking Error, Performance Evaluation Bond, Integrated Active Management

1. Introduction

The "integrated active management" mentioned in the title is intended to in-

corporate alternative assets into existing active management targeting traditional assets such as bonds and stocks, aiming for further excess returns and improving performance from a risk-return perspective. First, we review existing active management, mainly Ogishima and Yamamoto (2003) [1] and Kikukawa *et al.* (2017) [2] for Japanese bond active management, and Takehara (2012) [3] for equity active management.

[1] argues that the active management of Japanese bonds required under low interest rates is: 1) to shift from conventional interest rate selection (changes in duration and remaining maturity structure) to sector selection (changes in sector composition ratio) and issue selection (changes in issue composition ratio), and to improve investment efficiency by successfully combining sources of excess return, 2) to take advantage of market inefficiencies, and 3) to incorporate new products (CDS, ABS, etc.). Among these, [2] mainly incorporates (1) and (2) in the domestic bond active management.

[2] points out that bond active funds for pension management in Japan are homogeneous and mainly use credit factors as a source of excess returns, and since credit has the aspect of substituting for equity investments, there is a risk that returns will decline along with stocks when the economic environment deteriorates. On the other hand, carry rolldown factor has a high-risk premium and low correlation with credit factor and equities, so if this factor were incorporated 36.9% into the existing bond active fund, the tracking error relative to the BPI (NOMURA Bond Performance Index) would remain at 27.58 BP, while the average excess return would increase from 17.25 BP to 33.52 BP.

In order to give a deeper insight into equity investment, [3] asks the question, "Is it wrong to use the empirically rejected CAPM as a benchmark model, and is equity management and risk management using the Barra AEGIS inappropriate?" The discussion is essentially an active management vs. passive management debate. First, using portfolios on the efficient frontier, he derives the CAPM formula and shows that mean-variance efficiency and the CAPM formula are in fact equivalent. Secondly, the mean-variance efficiency of typical market value-weighted indices such as the S & P 500 in the U.S. and TOPIX in Japan is not satisfied because most of them are located slightly to the right of the minimum variance portfolio on the efficient frontier (hyperbola) composed by stocks belonging to the universe. Due to the equivalence of the efficiency of mean-variance and CAPM, there are returns that cannot be explained by the CAPM (*i.e.*, Jensen alpha). Therefore, the article states that there is no clear advantage of passive management, nor does it negate the potential of active management.

If the benchmark is not efficient, there will be significant variation in the fund's valuation ranking depending on what benchmark is used. This leads to a discussion of how to define alpha, *i.e.*, whether it is measured relative to a single benchmark or multiple benchmarks. [3] notes that for most practitioners, alpha is the excess return relative to the benchmark, and for researchers it is the

risk-adjusted return from a multi-beta model, and that such differences in definition also affect the results of equity valuation via the estimation of the cost of equity capital. In fact, from the researchers' perspective, Kubota and Takehara (2007) [4] point out that the mean-variance efficiency of the Fama-French three-factor model or style index is rejected as a long-term model and alpha is not zero even under the multi-beta model, whereas from the practitioners' perspective Arai and Yamada (2002) [5] find the rationale for investing in actively managed funds to be to allow for appropriate factor risk exposure consistent with investment objectives, since alpha is small under a multi-beta model. The differences of the opinions in the two articles support the discussion of how to define alpha in [3].

In addition to the articles introduced here, other studies on active management of Japanese bonds include Yamada (2000) [6], Takatsu and Yamazaki (2000) [7], Maeda and Koike (2002) [8], Nakatani (2010) [9], and Miyazaki, Abe, and Shimada (2021) [10], while those on active management of Japanese stocks include Oharazawa (1991) [11], Ito *et al.* (2009) [12], Komai and Oka (2012) [13], and Omori and Yano (2013) [14], among others, and the accumulation of research on these areas appears to be progressing steadily along with the progress of the asset management industry. On the other hand, to the best of our knowledge, there are not many studies on active management of alternative assets, much less integrated active management that incorporates alternative assets into existing active management.

The introduction of alternative assets into Japanese pension fund management is thought to have started in the early 2000s, and Miyai, Yamaguchi, Nakamura, Ishida, and Yamada (2005) [15] summarized the efforts of pension funds and pension consultants during this period in a vividly documented roundtable discussion memo. Although this memo was written about 20 years ago, the awareness of the issues that were raised about alternative assets at that time still remain as issues that need to be addressed today. The main issues are: 1) alternative assets are difficult to evaluate at market value; 2) although it is said that diversification effect can be obtained, how to measure the correlation coefficient; 3) whether diversification effect is really obtained under a large individuality impact of each fund; and 4) it is difficult to control the asset mix on an actual value basis because the amount of commitment and the actual investment are different.

In Miyazaki and Shimada (2023) [16], the authors reconfirmed (1) and (2) of their awareness of the problem: With regard to J-REITs, both the value of the total investment amount (investment units), which is the real estate price given by a real estate appraiser minus interest-bearing debt, and the market value (investment units), which has become a J-REIT after listing, are the values to the same real estate. Although both are values for the same real estate, the discrepancies between these values made us realize once again the difficulty of evaluation of fair value. In addition, real estate appraisals are priced with reference to

historical values, so returns based on appraisals are subject to smoothing. Therefore, when determining correlation coefficients between real estate returns and traditional assets, the correlation coefficients differ correspondingly depending on whether J-REIT returns, returns based on appraisal prices, or returns based on appraisal prices with de-smoothing are used as real estate returns. With these issues in mind, it is extremely difficult to establish a return or risk matrix that is absolutely correct, or to accurately estimate the diversification effects of including alternative assets in traditional assets. Therefore, it seems better to think of alternative assets as tools that aim to achieve excess returns over the returns of traditional assets, while fully acknowledging that their risk characteristics differ from those of traditional assets (under risk management based on tracking error).

In the roundtable discussion memo, it was pointed out that in the case of alternative assets (hedge funds), "there are many cases where the factors of investment performance are not explained in a state where they are properly broken down into beta and alpha. The fact that IRR, which expresses absolute returns, is currently the mainstream method for evaluating the performance of alternative assets may also be subject to this point of view. Attempting to maximize excess returns over the returns of traditional assets under risk constraints on tracking errors would help overcome the points raised here and the main issues (1) and (2).

SBDA (Spread Based Direct Alpha, hereafter "SBDA") proposed by Miyazaki and Shimada (2023) [17] enables us to divide the investment performance of a PE fund into a beta portion (investment performance of traditional assets), which is market performance, and an alpha portion (alpha for practitioners as pointed out by [3]), which expresses the pure investment ability of a PE fund, and SBDA is a convenient measure to evaluate the excess returns. The valuation method for SBDA relies on the valuation method for interest rate swaps with credit risk. Theoretical underpinning of SBDA is the principle that the present value of capital calls discounted back to the commitment time by benchmark rate as funding rate in funding side should be equal to the present value of the distributions discounted back to the commitment time by benchmark rate as funding rate plus SBDA as credit spread in investing side. The above definition of SBDA tells us that SBDA indicates the excess return (alpha portion) generated by PE fund manager's skill over benchmark rate (beta portion).

Once we separate the excess return from the benchmark return, we have only to estimate the risk of the excess return, namely tracking error, to evaluate the risk-return efficiency of the PE Fund relative to the benchmark. By the way, what is the tracking error of the PE fund? SBDA is the similar notion to credit spread and thus the excess alpha amount is derived by multiplying the SBDA and the present value of distributions at the commitment time, which is set equal to that of the capital calls. The risk amount required to get the excess alpha amount is assumed to be the fluctuation of the present value of distributions due only to the dynamics of the SBDA. Therefore, the tracking error of the PE fund

is thought to be the standard deviation of the fluctuation of the present value of distributions due only to the dynamics of the SBDA. Due to the complicated cashflows with capital calls and distributions, it is quite difficult to estimate the tracking error of the PE fund being different from the estimation of the tracking errors for the active funds of traditional assets.

This study provides the methodology to estimate the tracking error for alternative assets by using the SBDA as a starting point and introducing various new concepts of excess returns. To best of our knowledge, the SBDA is currently the only PME to have the advantage of possible derivation of the tracking errors for the alternative assets. Utilizing the integrated active management, which is available with the estimated tracking error, pension fund managers are able to build the more risk-return (Information Ratio) efficient portfolio incorporating alternative assets in addition to the active funds of traditional assets.

The structure of this paper is as follows. In Section 2, we introduce various new concepts regarding excess returns of alternative assets using SBDA as a starting point. In Section 3, we describe a framework for integrating active management based on excess return and tracking error, including alternative assets, using the new concepts introduced in Section 2. In Section 4, we confirm the mechanism of the new concepts with numerical examples. In the final section, a summary and future issues are added.

2. Various New Concepts of Excess Returns on Alternative Assets

2.1. Excess Return on Alternative Assets (SBDA)

The excess return of alternative assets over the performance evaluation period (N years) can be captured by the SBDA; see [17] for the detailed mechanism of the SBDA. In this section, we only describe the definition of SBDA. The time point i used in the definition is in months, where $i = 0$ represents the month in which the alternative investment was committed, and thereafter $i = 1, 2, \ldots,$ represents one month later, two months later, ... The point in time at which the SBDA is evaluated is n months after the month of commitment. Hereafter, symbols representing rates and returns are treated as monthly rates.

(Definition 1) SBDA

The SBDA at the valuation time n months from time 0, the point of commitment to the alternative asset, is that s_n which satisfies the following equality.

$$\sum_{i=1}^{n} \frac{Call(i)}{\left(1+r_i^B\right)^i} = \sum_{i=1}^{n} \frac{Dist(i)}{\left(1+r_i^B+s_n\right)^i} + \frac{NAV_n}{\left(1+r_n^B+s_n\right)^n} \tag{1}$$

where $Call(i)$, $Dist(i)$, and r_i^B denote the amount of capital call, the amount of distribution, and the benchmark rate (the monthly rate of the benchmark return from month 0 to month i) at month i, respectively, and NAV_n denotes the value of alternative assets under management at the valuation point n (Here, we assume that capital calls and distributions occur at the end of the month). In Equation (1), $Call(i) = Dist(i) = 0$ if there is no capital call or distribution at

time i months later.

Using the topmost schematic in **Figure 1**, we will review the information used to determine the SBDA s_{60} with the valuation point being 60 months later. Over the period from month 0 to month 60, we use monthly information for capital calls and distributions, and quarterly information for NAV (3 months later, 6 months later, …). From month 0 to month 6, we can see that capital calls occur after 1 month and 4 months, and distributions occur after 2 months and 5 months, etc. From month 59 to month 60, there are no capital calls and distributions occur after 59 months, etc. Since the time of valuation is after 60 months, NAV_n adopts NAV_{60}, which is shown after 60 months. The benchmark rate r_i^B is the one at the maturity of each month after the point $i = 1, \cdots, 60$.

2.2. Constant Maturity 5-Year SBDA

The constant maturity 5-year SBDA that we are newly introducing here is an SBDA that is based on information from 5 years back from the time of valuation at any future valuation point (which appears every 3 months after the 5-year period). We consider such SBDAs because we view SBDAs as analogous to credit spreads. For example, if a corporate bond with a maturity of 6 years is issued, its credit spread is the credit spread of the 6-year maturity, but after one year, this bond's credit spread becomes the credit spread of the 5-year maturity, after two years, this bond's credit spread becomes the credit spread of the 4-year maturity. In this way, when we observe changes in the credit spread of a corporate bond, we also include the effect of the term structure of the credit spread. To remove this effect, we often observe variation in the constant maturity 5-year credit spread, which is the spread between the generic constant maturity 5-year corporate bond yield and the generic constant maturity 5-year government bond yield. As a corresponding concept for alternative assets, we introduce the Constant Maturity 5-year SBDA.
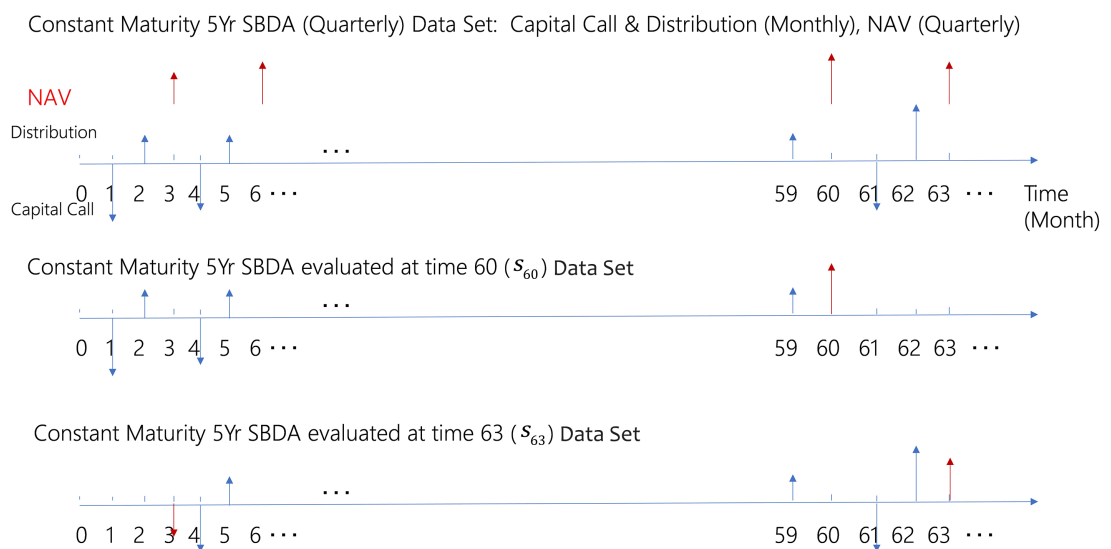
Figure 1. Schematic for constant maturity 5-year SBDA calculation. (Source: Prepared by the authors.)

The SBDA in (Definition 1) also has the same inherent observational problems as corporate bond credit spreads. If we follow SBDAs with valuation points of 4, 5, and 6 years (s_{48} , s_{60} , s_{72} , in that order), etc., when we observe changes in SBDAs, we also include the term structure of SBDAs in our observations. To remove this effect, we propose a Constant Maturity 5-year SBDA.

First, we obtain the constant maturity 5-year SBDA at the time 60 months after the 5-year elapsed from the time zero month. This is nothing other than obtaining the SBDA s_{60} shown in Section 2.1. In the following, we introduce $s_{j,60}$ to denote the Constant Maturity 5-year SBDA at the point j months further after the 5-year lapse, and denote this SBDA s_{60} newly as SBDA $s_{0,60}$.

Next, we obtain the Constant Maturity 5-year SBDA $s_{3,60}$ at the 63-month point in time, which is three more months after the 5-year lapse. This is the SBDA evaluated based on information from 3 months after the time point, which is 5 years back from 63 months after the time point. Therefore, in the cash flow in determining SBDA $s_{3,60}$, there are no capital calls or distributions from the middle of Figure 1 from the time point 0 month to 3 months later, and the amount equivalent to NAV_3 , the value of the NAV after 3 months, is considered to be newly capitalized. On the other hand, the capital call after 61 months and the distribution after 62 months will be added. In addition, NAV_{60} , which was at the time 60 months later, will be replaced by NAV_{63} , which is the NAV at that time 63 months later. Taking these factors into account, the cash flow in determining SBDA $s_{3,60}$ would be as shown in the lower part of Figure 1. To obtain the Constant Maturity 5-year SBDA $s_{3,60}$, we can use this cash flow to obtain $s_{j,60}$ defined by Equation (2) with $j = 3$.

(Definition 2) Constant Maturity 5-year SBDA

The Constant Maturity 5-year SBDA, which is measured at the point in time when 5 years plus j months have passed from the point in time 0, the point in time of commitment to the alternative asset, is $s_{j,60}$ satisfying the following equation. However, the valuation time points are the quarterly time points $j = 3$, 6, 9, …

$$\sum_{i=j}^{60+j} \frac{Call(i)}{\left(1+f_{j,i}^{B}\right)^{i-j}} = \sum_{i=j}^{60+j} \frac{Dist(i)}{\left(1+f_{j,i}^{B}+s_{j,60}\right)^{i}} + \frac{NAV_{j+60}}{\left(1+f_{j,j+60}^{B}+s_{j,60}\right)^{60}} \tag{2}$$

where $Call(i)$ and $Dist(i)$ represent the amount of capital call (but $Call(j) = NAV_j$) and the amount of distribution at the time i months later, respectively. $f_{j,i}^{B}$ represents the benchmark forward rate (the monthly rate of benchmark forward return from the time j months later to the time i months later). NAV_{j+60} represents the value of the alternative assets under management at the valuation point $j + 60$ months later. Equation (2) also assumes that $Call(i) = Dist(i) = 0$ if there are no capital calls or distributions at time point i months later.

Over time, information on the amount of capital calls, the amount of distributions, NAV, benchmark rates, and benchmark forward rates is accumulated, generating Constant Maturity 5-year SBDA $s_{j,60}$, $j = 3, 6, 9, \cdots$ every 3 months

starting after 5 years.

2.3. Two Constant Maturity 5-Year Performance Evaluation Bond Prices and Rate of Price Changes

The credit risk required to obtain the credit spread, which is the excess return on corporate bonds (here we assume high grade bonds that do not need to factor in bankruptcy risk), is considered to be the risk of price fluctuations of corporate bonds due to changes in the credit spread. In the case of corporate bonds, since the coupon and face value are fixed, the credit risk of corporate bonds can be captured by tracking the dynamics of the credit spread. For alternative assets, there is no equivalent to these asset prices (because the total investment amount to generate the NAV varies at each point in time), so it is necessary to calculate the price volatility risk of alternative assets by assuming an appropriate asset price. In this section, we introduce two types of constant maturity 5-year performance evaluation bond prices to derive the tracking error of alternative assets: one is the realized constant maturity 5-year performance evaluation bond price (Definition 3), and the other is the pre-Constant Maturity 5-year performance evaluation bond price (Definition 4).

(Definition 3) Realized Constant Maturity 5-Year Performance Evaluation Bond Price (P_j)

The realized constant maturity 5-year performance evaluation bond price (P_j) at the point in time j months back from the point in time when 5 years plus j months ($j = 3, 6, 9, \cdots$) have passed is defined as in Equation (3) as the right-hand side of Equation (2) with the constant maturity 5-year SBDA $s_{j,60}$ that satisfies Equation (2).

$$P_j = \sum_{i=j}^{60+j} \frac{Dist(i)}{\left(1 + f_{j,i}^B + s_{j,60}\right)^i} + \frac{NAV_{j+60}}{\left(1 + f_{j,j+60}^B + s_{j,60}\right)^{60}} \tag{3}$$

(Definition 4) Pre-Constant Maturity 5-Year Performance Evaluation Bond Price ($P_{j,pre}$).

The pre-constant maturity 5-year performance evaluation bond price ($P_{j,pre}$) at the point in time j months back from the point in time when 5 years plus j months ($j = 3, 6, 9, \cdots$) have passed is defined as in Equation (4), replacing the amount of real distributions $Dist(i)$ or NAV_{j+60} used in (Definition 3) with $\frac{Dist(i)\left(1 + f_{j,i}^B + s_{j-3,60}\right)^i}{\left(1 + f_{j,i}^B + s_{j,60}\right)^i}$ or $\frac{NAV_{j+60}\left(1 + f_{j,j+60}^B + s_{j-3,60}\right)^{60}}{\left(1 + f_{j,j+60}^B + s_{j,60}\right)^{60}}$ computed assuming that the fund manager's skill had remained at the SBDA $s_{j-3,60}$ measured three months earlier.

$$P_{j,pre} = \sum_{i=j}^{60+j} \frac{Dist(i)\left(1 + f_{j,i}^B + s_{j-3,60}\right)^i}{\left(1 + f_{j,i}^B + s_{j,60}\right)^{2i}} + \frac{NAV_{j+60}\left(1 + f_{j,j+60}^B + s_{j-3,60}\right)^{60}}{\left(1 + f_{j,j+60}^B + s_{j,60}\right)^{120}} \tag{4}$$

Using the two constant maturity 5-year performance evaluation bond prices, we define the rate of price change ($R_{j-3,j}^P$) in the constant-maturity 5-year per-

formance evaluation bond price due to the change only in the constant maturity 5-year SBDA (from $s_{j-3,60}$ to $s_{j,60}$) alone (eliminating changes in the benchmark return), as in Equation (5).

$$R^P_{j-3,j} = \frac{P_j - P_{j,pre}}{P_{j,pre}} \tag{5}$$

2.4. The Case of Alternative Assets Whose Prices Are Given as Index Values

Let the index value of the alternative asset at the point in time j months later, which is 5 years back from the point in time when 5 years plus j months ($j = 3, 6, 9, \cdots$) have passed, be (P_j) and the index value of the alternative asset at 5 years plus j months later be (P_{j+60}), the relationship between the two is shown in Equation (3), where $Dist(i) = 0$, $NAV_{j+60} = P_{j+60}$ in Equation (3). From Equation (6), SBDA $s_{j,60}$ is obtained.

$$P_j = \frac{P_{j+60}}{\left(1 + f^B_{j,j+60} + s_{j,60}\right)^{60}} \tag{6}$$

To get an intuitive understanding, we express Equation (6) as Equation (6)' on a continuously compounded basis.

$$P_j = P_{j+60} \cdot \exp\left(-\left(f^B_{j,j+60} + s_{j,60}\right) \cdot 60\right) \tag{6'}$$

In exactly the same way, if the index value of the alternative asset at the point j − 3 months after 5 years plus j − 3 months ($j = 3, 6, 9, \cdots$) and the index value of the alternative asset at 5 years plus j − 3 months after j − 3 months is (P_{j-3}), the relationship between the two satisfies Equation (7). From Equation (7), we obtain SBDA $s_{j-3,60}$.

$$P_{j-3} = P_{j-3+60} \cdot \exp\left(-\left(f^B_{j-3,j-3+60} + s_{j-3,60}\right) \cdot 60\right) \tag{7}$$

The pre-constant maturity 5-year performance evaluation bond ($P_{j,pre}$) is defined as the price that would have been assumed if the fund manager's skill had remained at the SBDA $s_{j-3,60}$ measured three months earlier in Equation (7), as in Equation (8).

$$P_{j,pre} = \exp\left(-\left(f^B_{j,j+60} + s_{j,60}\right) \cdot 120\right) P_{j+60} \cdot \exp\left(\left(f^B_{j,j+60} + s_{j-3,60}\right) \cdot 60\right) \tag{8}$$

The log change rate in the constant maturity 5-year performance evaluation bond price, $R^P_{j-3,j}$, from j − 3 months to j months due to the change in the constant maturity 5-year SBDA alone (removing changes in the benchmark return) is expressed by Equation (9). This is the change in SBDA multiplied by 60 (months) of duration.

$$
\begin{aligned}
R^P_{j-3,j} &= \ln \frac{P_j}{P_{j,pre}} \\
&= \ln \frac{P_{j+60} \cdot \exp\left(-\left(f^B_{j,j+60} + s_{j,60}\right) \cdot 60\right)}{\exp\left(-\left(f^B_{j,j+60} + s_{j,60}\right) \cdot 120\right) P_{j+60} \cdot \exp\left(\left(f^B_{j,j+60} + s_{j-3,60}\right) \cdot 60\right)} \\
&= \left(s_{j,60} - s_{j-3,60}\right) \cdot 60
\end{aligned} \tag{9}
$$

3. Integrated Active Management

3.1. Expected Excess Return, Tracking Error, and Expected Information Ratio for a Single Alternative Asset

We define the expected excess return (annualized), tracking error (annualized), and expected information ratio of the alternative assets using the tools prepared in Section 2.1 through Section 2.4.

(Expected excess return (annualized))

Expected value of the constant maturity 5-year SBDA $s_{j,60}$ ($j = 3,6,9,\cdots$).

$$E\left(s_{j,60}\cdot 12\right)$$

(Tracking error (annualized))

Annualized standard deviation of the rate of quarterly change in constant maturity 5-year performance evaluation bond price ($R_{j-3,j}^{P}$).

$$\sigma\left(R_{j-3,j}^{P}\right)\cdot 2$$

(Expected information ratio)

Expected excess return (annualized) divided by tracking error (annualized).

$$I.R. = E\left(s_{j,60}\cdot 12\right)\Big/\left(\sigma\left(R_{j-3,j}^{P}\right)\cdot 2\right)$$

3.2. Integrated Active Portfolio Optimization

Integrated active management refers to the management of an optimal portfolio from a risk-return perspective, covering not only various active strategies in bonds and stocks, but also investments in alternative assets as one active strategy, and covering all these active strategies. In order to use the notation introduced so far for multiple assets, the asset k is denoted by (k) in the upper right corner of the symbol. For example, the log price change in the constant maturity 5-year performance evaluation bond price from $j - 3$ months to j months ($R_{j-3,j}^{P}$) due only to changes in the constant maturity 5-year SBDA for alternative asset k (removing changes in the benchmark return) is written as $R_{j-3,j}^{P,(k)}$, etc. In addition, since the excess returns of various active strategies in bonds and stocks are the same as in the case of alternative assets, where the prices in Section 2.4 are given as index values, all notation here is unified with that of alternative assets. The optimization of the integrated portfolio requires the covariance between alternative asset k and alternative asset l in addition to the indices introduced in Section 3.1. The covariance (annualized) of the rate of quarterly price change of the constant maturity 5-year performance evaluation bond $R_{j-3,j}^{P,*}$ ($j = 3,6,9,\cdots$) with alternative asset k and alternative asset l is given by $Cov\left(R_{j-3,j}^{P,k},R_{j-3,j}^{P,l}\right)\cdot 4$.

Optimization of the integrated active portfolio, in which investment in alternative assets is one of the active strategies, is an attempt to maximize the expected excess return (Equation (10)) while keeping the tracking error of the integrated active portfolio within a given constraint (Equation (11)) and the amount allocated to each active strategy within a given constraint (Equation

(12)).

[Integrated Active Portfolio Optimization Model]

$$\max_{w_1,\cdots,w_n} \sum_{k=1}^{n} w_k \cdot E\left(s_{j,60}^{(k)} \cdot 12\right) \tag{10}$$

Constraints:

$$\boldsymbol{w}^{\mathrm{T}} \boldsymbol{COVw} < risk\ limit \tag{11}$$

$$0 \le w_k \le weight(k)limit(k=1,\cdots,n) \tag{12}$$

where $\boldsymbol{w}^{\mathrm{T}} = (w_1,\cdots,w_n)$ and $\boldsymbol{COV}$ is the variance-covariance matrix whose (k,l) component is $Cov\left(R_{j-3,j}^{P,k}, R_{j-3,j}^{P,l}\right) \cdot 4$.

(Remark)

The expected information ratio of an integrated active portfolio is the expected excess return (annualized) divided by the tracking error (annualized) and is given by Equation (13).

$$I.R. = \sum_{k=1}^{n} w_k \cdot E\left(s_{j,60}^{(k)} \cdot 12\right) \bigg/ \sqrt{\boldsymbol{w}^{\mathrm{T}} \boldsymbol{COVw}} \tag{13}$$

To find the optimal weights for maximizing the expected information ratio of the integrated active portfolio, Equation (13) is used as the objective function in (10) and Equation (11) can be removed from the constraints. Therefore, the concept of optimal weights here is the same as the concept of fund allocation according to the information ratio in corporate bonds active management as argued by Kasuga (2009) [18].

4. Numerical Examples for Various New Concepts

In this section, through numerical examples, we will examine the time series of the constant maturity (3- and 5-year) SBDA, the adjusted duration of the realized constant maturity (3- and 5-year) performance evaluation bonds, and the rate of the change $R_{j-3,j}^{P}$ in constant maturity (3- and 5-year) performance evaluation bond price due only to the change in SBDA (removing changes in the benchmark return) from $j - 3$ months to j months later. **Figure 2** shows the hypothetical cash flow (corresponding to the top row in the schematic diagram in **Figure 1**) as the fund accumulates, which is used to derive the numerical example corresponding to these concepts. In **Figure 2**, the downward-facing blue bar
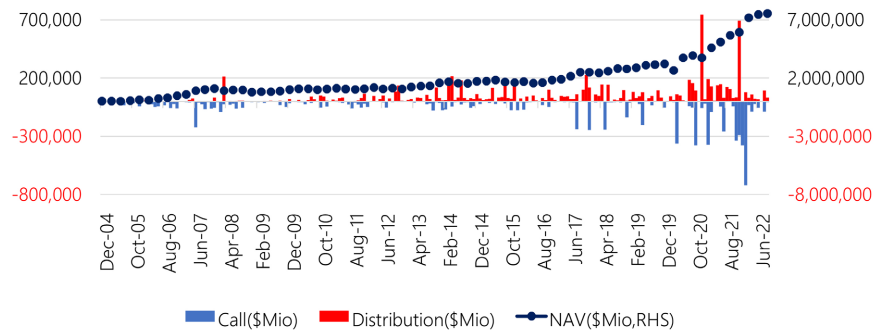

Figure 2. Cash flow and NAV. Source: Authors' calculations.

represents the amount of capital calls, the upward-facing red bar represents the amount of distributions, and the blue circle represents the amount of NAV, respectively. From **Figure 2**, it can be seen that the NAV rises immediately after a capital call and falls immediately after a distribution, and that the NAV gradually builds up as the fund accumulates.

The trends of the constant maturity (3-year and 5-year) SBDA are shown in **Figure 3**. (In order to obtain the (3- and 5-year) SBDA, data for the past 3 and 5 years, respectively, are required, so the starting points are March 2008 for the 3-year SBDA and March 2010, two years later, for the 5-year SBDA. The 3-year SBDA fluctuates between −5% and 18%, while the 5-year SBDA fluctuates between 0% and 15%. However, all SBDAs are confirmed to be stable as the measurement period becomes longer. In addition, prior to 2017, there were periods when the 3-year SBDA and 5-year SBDA diverged significantly, but after 2018, the divergence between the two is negligible.

The modified duration of the realized constant maturity (3-year and 5-year) performance evaluation bonds is shown in **Figure 4**. The modified duration is the percentage change in the price (P_j) of the realized constant maturity 5-year performance evaluation bond calculated by decreasing the constant maturity 5-year SBDA $s_{j,60}$ by 1 BP in Equation (3) from the original price. Looking at the entire period from **Figure 4**, the 3-year modified duration ranges from 1.3 to 2.9, while the 5-year modified duration ranges from 2.1 to 4.3, confirming that the modified duration fluctuates significantly as the measurement period becomes longer. This is because the modified duration tends to be smaller when there are more capital calls in the first half of the measurement period and more distributions in the second half, and larger when the opposite is true, as the measurement period becomes longer.

The rate of the change $R^P_{j-3,j}$ in constant maturity (3- and 5-year) performance evaluation bond price due only to the change in SBDA is shown in **Figure 5**. In the first half of the period, the variation in the rate of the change $R^P_{j-3,j}$ is greater for the 3-year than for the 5-year, reflecting the fact that the variation in the 3-year SBDA is greater than that in the 5-year SBDA, but in the second half of the period, the variation in the rates for 3- and 5-year are both stable at low levels.
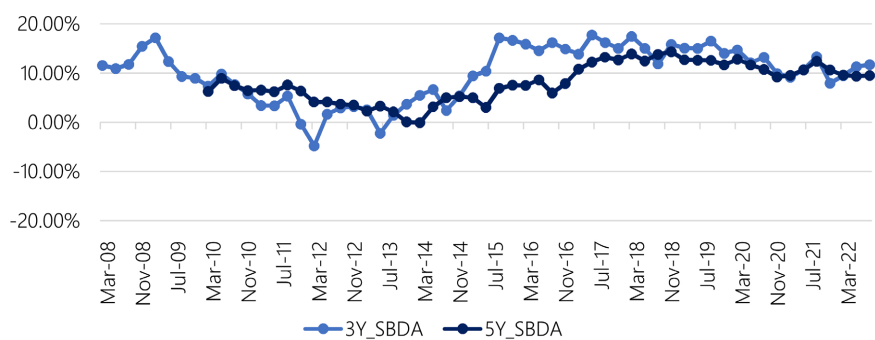
Figure 3. Changes in SBDA. Source: Authors' calculations.
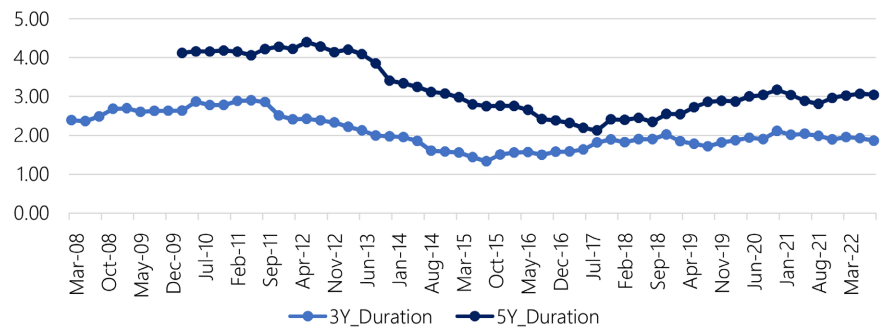
Figure 4. Duration. Source: Authors' calculations.
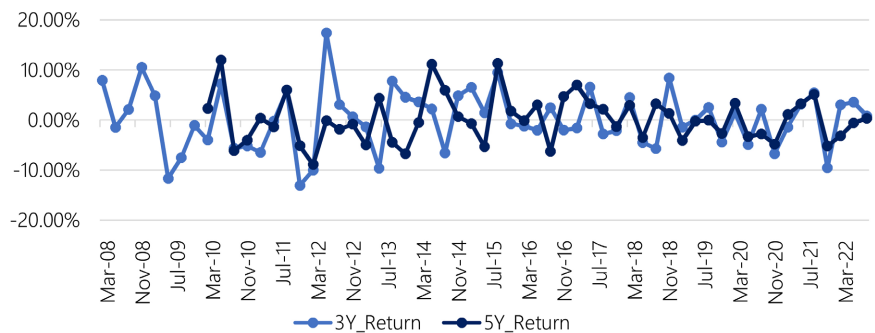
Figure 5. Returns. Source: Authors' calculations.

5. Summary and Future Issues

Based on our knowledge of the issues and analysis of alternative assets to date, we believe that when incorporating alternative assets into traditional assets, it is more cautious to try to get excess returns over traditional asset returns under some tracking error risk constraints, rather than to expect diversification effects that are difficult to measure. We have previously devised the SBDA as a measure to estimate excess returns on alternative assets, and in this paper, we propose a method to quantify tracking errors required for integrated active management by applying the SBDA approach. There are four main issues to be addressed in the future.

1) Based on the actual data for the specific alternative assets such as PE, real estate and infrastructure, we had better empirically examine the similarity or the difference in the characteristics of new concepts considered in Section 4 among the three alternative assets and also quantify their impact on the estimation of tracking errors and excess returns.

2) The challenge from finance theory is the decomposition of SBDA. In light of [3], SBDA is "alpha (return in excess of benchmark) for practitioners," but it is necessary to verify by some method how much alpha (risk-adjusted return by multi-beta model) for researchers remains in SBDA. The multi-beta model is a method to examine how much alpha (risk-adjusted return based on the multi-beta model) remains in the SBDA. Multi-beta here mainly includes liquidity risk and credit risk derived from credit facilities. Once this is possible, integrated active management including active strategies for other traditional assets as de-

scribed in "1. Introduction" will be possible.

3) The challenges from finance practice are, first, if the benchmark adopted for the PE funds is different from the policy benchmark, this part needs to be adjusted in the derivation of the integrated excess return and tracking error. Second, if the currency risk is to be neutral, the currency composition of the policy benchmark and the currency composition of the alternative asset should be adjusted by currency hedging or other means. It is also necessary to examine what value should be selected as N for the constant maturity N-year performance evaluation bond price to ensure that the estimated tracking error is consistent with the realized tracking error.

4) An issue that bridges finance theory and practice is the comparison of tracking errors in this study with tracking errors in risk models employed by asset management companies. Since risk models usually employ factor models, a detailed examination of tracking errors of risk models in a factor-dependent manner may provide a clue to solving the issue described in (1). This seems to be an interesting subject for analysis where theory and practice entangle.

Acknowledgements

Conflicts of Interest

The authors declare no conflicts of interest.

Disclaimer

This paper is a compilation of research results by GPIF staff, and the contents and opinions expressed in the text do not represent the official views of the GPIF.

References

[1] Ogishima, N. and Yamamoto, K. (2003) Domestic Bond Active Management under Low Interest Rates. *Securities Analyst Journal*, **41**, 48-63. (In Japanese)

[2] Kikukawa, T., Uchiyama, T., Motohiro, M. and Nishiuchi, S. (2017) Performance of Domestic Bond Active Management and Smart β Strategies. *Securities Analyst Journal*, **55**, 69-80. (In Japanese)

[3] Takehara, H. (2012) Equity Investment: A Reexamination from an Asset Pricing Perspective. *Securities Analyst Journal*, **50**, 22-36. (In Japanese)

[4] Kubota, K. and Takehara, H. (2007) Reexamination of the Validity of the Fama-French Factor Model. *Contemporary Finance*, **22**, 3-23. (In Japanese)

[5] Arai, T. and Yamada, H. (2002) Is It Rational to Invest in Actively Managed Funds? *Securities Analyst Journal*, **40**, 74-95. (In Japanese)

[6] Yamada, S. (2000) Risk Premium of Japanese Government Bonds and Its Application to Investment Strategies. *Securities Analyst Journal*, **38**, 32-62. (In Japanese)

[7] Takatsu, M. and Yamazaki, M. (2000) A Study of Bond Portfolio Management by

Yield Curve Analysis. *Securities Analyst Journal*, **38**, 16-31. (In Japanese)

[8]   Maeda, K. and Koike, T. (2002) On Yield Curve Tactics Using Market Overreaction. *Securities Analyst Journal*, **40**, 125-140. (In Japanese)

[9]   Nakatani, Y. (2010) The Relationship between the Curvature of the Yield Curve and Interest Rate Volatility: Verification by Measuring Butterfly Trade Returns. *Securities Analyst Journal*, **48**, 26-35. (In Japanese)

[10] Miyazaki, K., Abe, Y. and Shimada, K. (2021) Bond Investment Strategy Based on Factor Analysis: Revisited Application to Carry and Roll-down Strategy in the U.S. Treasury Market. GPIF Working Paper.

[11] Oharazawa, N. (1991) The Factor Model and Its Application to Active Strategies. *Securities Analyst Journal*, **29**, 36-55. (In Japanese)

[12] Ito, K., Ikeda, S. and Kido, T. (2009) Investment Style, Liquidity and Active Management. *Securities Analyst Journal*, **47**, 48-60. (In Japanese)

[13] Komai, H. and Oka, S. (2012) Japanese Equity Active Fund Performance: Diversification and Economies of Scale. *Securities Analyst Journal*, **50**, 38-46. (In Japanese)

[14] Omori, K. and Yano, M. (2013) Risk-Based Portfolio and Active Management. *Securities Analyst Journal*, **51**, 17-26. (In Japanese)

[15] Miyai, H., Nakamura, A., Yamaguchi, N., Ishida, H. and Yamada, M. (2005) Roundtable Discussion: Pension Management and Alternative Investments. *Securities Analysts Journal*, **43**, 77-102.(In Japanese)

[16] Miyazaki, K. and Shimada, K. (2023) Japanese Private Real Estate Models and Portfolio Selection. *Journal of Mathematical Finance*, **13**, 249-270.
https://doi.org/10.4236/jmf.2023.133016

[17] Miyazaki, K. and Shimada, K. (2023) Spread-Based Direct Alpha (SBDA) as a Performance Measure for PE Funds. *Journal of Mathematical Finance*, **13**, 380-393.
https://doi.org/10.4236/jmf.2023.133024

[18] Kasuga, K. (2009) Issues in Credit Management—Necessity of Risk Allocation commensurate with Information Ratio. *Securities Analyst Journal*, **47**, 71-80. (In Japanese)