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Dependence Structure of the US Dollar Index and Crude Oil Prices: A Regime-Switching Copula Approach

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Abstract

In the past few years, the frequent crises have deeply affected the relationship between the US dollar index and crude oil, and it has become imperative to rethink the dependency structure between the two. This paper examines the structure of the dependence between the US dollar index and crude oil prices in 2018-2023 using a Gaussian regime-switching copula model. The model results reveal the existence of two distinct dependency structures: one characterized by positive correlation and the other by negative correlation and these two dependency structures are mutually convertible, with this conversion process exhibiting Markovian properties. The result suggests that there is not a single negative dependence structure between the US dollar index and crude oil but a more complex multivariate structure. The finding enriches the theoretical knowledge of the dependence on the US dollar index and crude oil. In addition, the findings facilitate better choices for policymakers and investors in decision-making.

Keywords

Dependence Structure, US Dollar Index, Crude Oil, Regime-Switching Copula

1. Introduction

The strong link between crude oil, one of the most important commodities in the world economy, and the US dollar, the main currency of world trade, dates back to the establishment of the US dollar oil system in the 1970s. Changes in their relationship have deeply affected the development and direction of the world economy, and therefore the structure of their dependence remains an important issue in today's economy.

Throughout history, liquid fuel, predominantly consisting of crude oil, has consistently stood as the leading source of primary energy worldwide. This pivotal role of liquid fuel establishes crude oil prices as a critical gauge for the progress of the global energy market. As energy commodities derived from crude oil continue to assume an ever more significant role in human economic endeavors, the oscillations in crude oil prices wield a profound influence over global economic activities (see [1] [2] [3] [4]). The US dollar stands as the central currency within the international monetary system (see [5] [6] [7]), with the role of transmitting and regulating price changes in the global economy and markets. Fluctuations in the value of the US dollar directly impact international trade, thereby exerting a consequential influence on the global economy (see [8]). The exchange rate of the US dollar reflects the value of the US dollar. However, individual exchange rates do not fully represent the international value of the US dollar. The US dollar index (USDX) amalgamates the US dollar's exchange rate within the international foreign exchange market, providing a quantitative representation of the US dollar's value. Utilizing the USDX as a surrogate for the US dollar in practical research is a more rational approach.

When the US dollar oil system was established in the 1970s, the US dollar became the sole currency for oil settlements, which solidified the connection between the USDX and crude oil prices. The US dollar is the currency in which international crude oil is denominated, which makes fluctuations in the USDX also considered to be the basis for crude oil price fluctuations (see [9]). The intimate connection between the US dollar and crude oil has mutually reinforced their positions and impact on global economic activity. Delving into their interdependence constitutes a significant subject within the realm of economics.

Initially, scholars were inclined towards examining their correlation from an empirical standpoint. For example, [10] reviews some of the classic economic arguments for rising oil prices and analyze them. The results suggest that oil market turbulence may have less of an impact on the performance of the U.S. economy than is commonly believed. However, the empirical analysis method possesses inherent limitations and may fall short in delivering precise and all-encompassing conclusions when confronted with the dependency between them. Consequently, an increasing number of scholars are gravitating toward dissecting the relationship from a modeling perspective. For example, [11] examines the time-varying relationship between crude oil prices and the USDX using a DCC-GARCH model and suggests the existence of key mediating factors that make the relationship time-varying.

As global crises continue to mount, certain scholars contend that these crises have changed the structure of dependence among financial commodities. One intriguing inquiry arises: Has the dependency framework between the US dollar and crude oil, both pivotal financial assets, undergone any transformations? This question warrants further investigation. However, classical time series models have limitations in describing complex dependency structures among financial data, for example, classical models do not capture nonlinear and asymmetric structures well. The adoption of copula models presents a more robust solution to address these challenges. Nevertheless, the plain-vanilla copula model lacks sensitivity in the face of emergencies. [12] indicates that the Markov regime-switching (MRS)[1] copula model outperforms the plain-vanilla copula model.

In this paper, we propose an MRS copula model to investigate the relationship between the USDX and crude oil prices. The MRS copula model combines the versatility of a plain-vanilla copula with heightened sensitivity to fluctuations in variable interactions. This combination enhances the precision and comprehensiveness of studying the interdependence structure, offering a more nuanced exploration of the relationships between variables.

Currently, there are relatively limited literatures on the study of changing relationships between financial markets using the MRS copula model. Our paper contributes to this area. Additionally, our work offers a distinctive case study on the relationship between the US dollar and crude oil. Our findings indicate that the relationship between them is no longer inherently negative these years. This observation aligns with the ongoing global trend of de-dollarization and the enduring trajectory toward economic diversification. This change in the relationship has certain warnings for managers of countries whose economies are highly dependent on the US dollar and for investors who have a fervent investment sentiment for the US dollar. Lastly, our paper provides evidence supporting the conclusion that the MRS copula model outperforms the plain-vanilla copula model, which facilitates the generalization of the MRS copula model. This promotion fosters researchers to attain a deeper comprehension of the transformations induced by the crisis in the interdependence structure among financial markets. Consequently, it enables them to formulate more precise evaluations, which have significant implications for both investors and policymakers.

The remainder of this paper is organized as follows. Section 2 gives a survey of extant literature. Section 3 describes the data. Section 4 presents the methodology used. Section 5 provides empirical results and analysis. Conclusion and interpretations are put forward in Section 6.

2. Literature Review

While a significant link between the US dollar and crude oil exists, it is important to note that this relationship is not necessarily straightforward or simple (see [14] [15]).

[16] verifies the existence of causality and cointegration between them and indicate that the non-stationary behavior of the real dollar exchange rate in the post-Bretton Woods era is due to the non-stationary behavior of real oil prices. [17] uses a currency model with an oil price factor to forecast the movement of the US dollar's foreign exchange rate, and the results show that the results of the

[1]The MRS model was first proposed in [13].

long-term forecast are the same as the movement from exchange rates of oil prices. [18] delves into the examination of the long-term association between the USDX and crude oil prices, proposing the existence of a stable and enduring relationship between the two.

However, some scholars declare that the relationship between the US dollar and crude oil is not consistent in the short term and long term (see [19] [20] [21]). [22] examines the dynamic link between crude oil and the US dollar at multiscale frequencies. The results show that the overall and long-term trend correlations between crude oil prices and the US dollar exhibit similar patterns, with negative correlations in the high-frequency intrinsic mode functions being weaker and showing time-varying characteristics over a short high-frequency intrinsic mode functions time horizon, and negative correlations in the low-frequency intrinsic mode functions being stronger and more static over a longer time horizon. [23] presents a structural vector autoregressive model to examine the correlation among oil prices, the US dollar rate, and emerging market stock prices. The results reveal that positive oil price shocks in the short run always inhibit stock prices and the dollar rate. [24] employs a time-series network model to scrutinize the dynamics between the USDX and crude oil prices. The research reveals that the inverse correlation between the USDX and high-frequency oil prices weakens concomitantly with the diminishing prominence of the US as a global economic leader.

Financial data are mostly characterized by nonlinearity, thick tails, spikes, and high volatility, and a single traditional time series model often does not perform well in the face of these problems. Copula function is a new class of models widely used in the financial field in recent years, with flexible and versatile construction methods that can effectively capture nonlinearities and tail relations. [25] investigates the conditional dependence structure between crude oil prices and the US dollar exchange rate using the copula approach, including fitting the potential dependence structure of the rising and falling market phases using various copula function models such as the elliptic family and the Archimedean family. The results of the study indicate that the dependence structure between crude oil and the US dollar exchange rate has significant symmetry during 2000-2011 and that crude oil price increase is relevant to the weaker US dollar. In addition, the study also demonstrates that the *t*-copula model fits best. [26] uses a dynamic copula model to discuss the dependence between crude oil prices and the USDX. The results show that dynamic strategies based on *t*-copula models have greater economic efficiency than static and other dynamic strategy models. In addition, the study indicates that positive feedback trading activity in the oil market is statistically significant. [27] examines the correlation structure and relationship between crude oil prices and USDX using the copula model and the CARR model. The results reveal that the variation points are intimately involved with actual economic occurrences. [28] examines the dynamic dependence between crude oil and the exchange rates of the US and China with the time-varying copula method. The results indicate that there are structural dependence breakpoints between daily or weekly crude oil prices and the USDX. Crude oil prices and the USDX exhibit a significant negative correlation with upper and lower tail dependence.

Today's economic developments are more complex and face more difficulties than in the past, thus more scholars have started to focus on the impact of the crisis on the dependency structure of the US dollar and crude oil. [29] investigates the dynamic linkages between gold, the US dollar, and crude oil. It is found that there is a significant long-run dependence between these markets, and the dynamic relationship between oil and the US dollar is always negative. Positive nonlinear causality exists between the US dollar and crude oil in the post-crisis period. [30] explores the relationship between oil prices and the US dollar exchange rate employing the use of copula and DCC--MGARCH models. The results demonstrate that oil prices and exchange rates were separated before the onset of the crisis, they interacted with each other in the event of a crisis. [31] investigates the dependence structure of oil, gold, and dollar rates by using a GARCH model based on a nested copula. The results present that the interdependence between the three is stronger in crisis than in non-crisis.

After a comprehensive review of the literature on crude oil and the US dollar, we find that the economic boom and the increase in global crisis events have complicated the nexus that exists between the US dollar and crude oil. The existing classical research methods do not cope well with this situation. For example, traditional time series methods mostly consider linear relationships, whereas there is a non-linear relationship between the US dollar and crude oil. Additionally, cointegration methods mostly focus on the long-term relationship between the two and are insensitive to short-term changes. The general static copula and time-varying copula can better characterize the changes in their relationship, however, each global crisis has the potential to bring new changes to the relationship between them. The static copula and time-varying copula are not sensitive to the occurrence of anomalies. The MRS copula model is a dynamic mixed copula model, integrating the strengths of both the MRS model and the copula model. It can mix different copula according to different research problems, which is extremely flexible (see [32]-[37]). [38] points out that in contrast to conventional time series analysis methods, the MRS copula excels in capturing nonlinear, asymmetric, and tail-dependent characteristics among variables. It is also more flexible than the plain-vanilla copula.

3. Data Interpretation and Preliminary Analysis

Drawing from the inference that global crisis events can alter the dynamics among financial commodities, we have opted to analyze data concerning the USDX and crude oil prices within the 2018-2023 period, characterized by a heightened occurrence of crisis events.

Figure 1 shows the time series of the closing price of the USDX and the closing price of crude oil in 2018-2023. We can clearly see that the USDX and crude oil prices show both negative and positive trends.

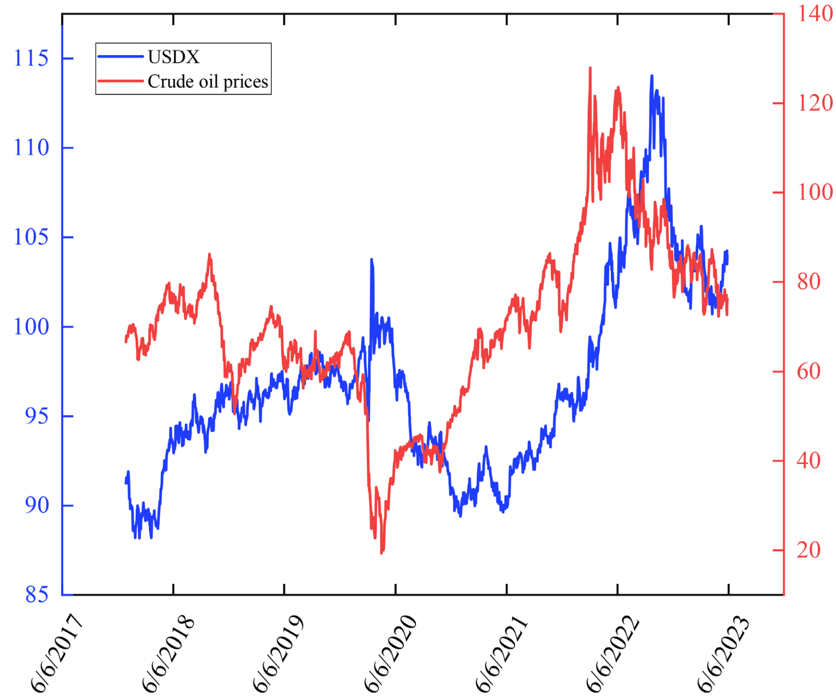

Figure 1. Time-series chart of closing prices.

In specific experiments, the logarithmic return series of the data is often taken for the actual study, and logarithmic return refers to the logarithmic differential treatment of the closing price, which can be written as:

$$r_t = \ln p_t - \ln p_{t-1}. \tag{1}$$

Table 1 shows some of the statistical properties of the data used for the study, while **Figure 2** illustrates a graph of the USDX and crude oil return series. The results of the chart and table show that the volatility cycles of crude oil and the USDX are close and mostly opposite, that is, crude oil stays in a downtrend while the dollar stays in an uptrend. Both become significantly more volatile around 2020.

To more visually compare the change in correlation between the two, we create a CCF plot of the USDX and crude oil prices return series in **Figure 3**. The results show that the USDX is significantly positively and negatively correlated with crude oil prices.

Data analysis shows that the relationship between the USDX and crude oil may have changed. However, the linear correlation coefficient between them is −0.07, which implies that the traditional analytical method lacks sensitivity and a more robust model is needed to analyze the dependency structure between them.

4. Methodology

One of the advantages of the copula function is that the marginal distribution and the dependency structure of the data can be studied separately, which simplifies the modeling complexity and improves the modeling accuracy, but it also

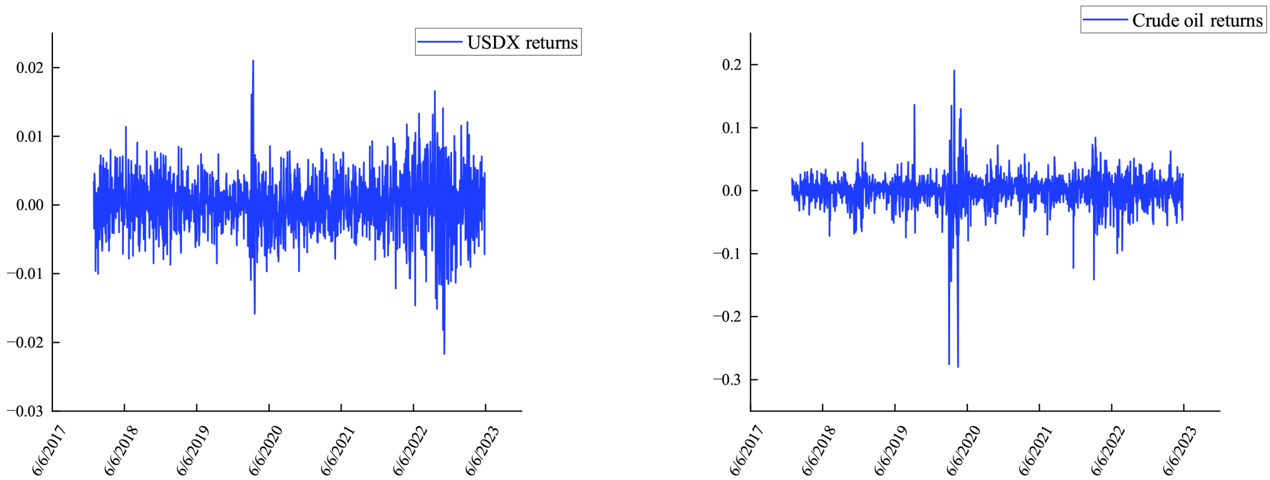

Figure 2. The series of USDX and crude oil return.

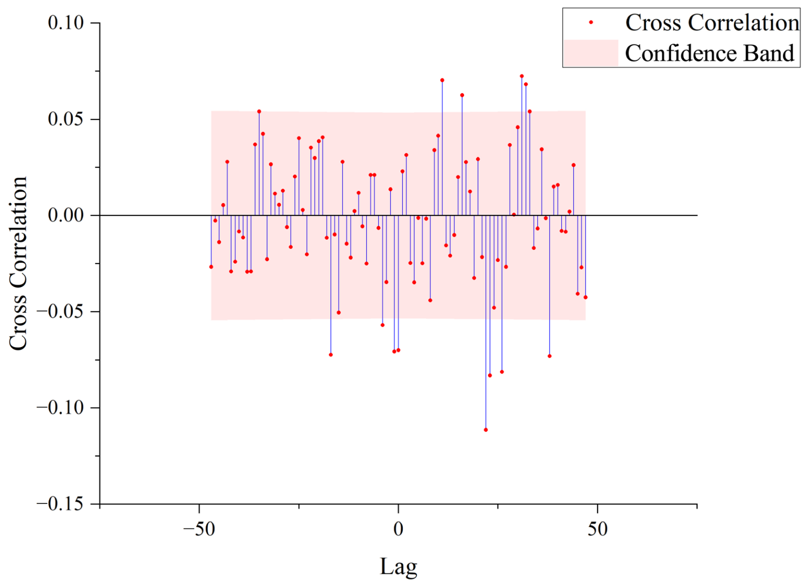

Figure 3. Inter-correlogram of the return series over time.

Table 1. Descriptive statistics of return series.

	USDX	Crude oil
Mean Value	0.000093	0.000097
Standard Deviation	0.004280	0.027700
Variance	0.000018	0.000768
Min Value	−0.021670	−0.279760
Median Value	0.000000	0.002340
Max Value	0.021000	0.190770
Skewness	−0.013960	−1.486530
Kurtosis	1.939780	16.690570

means that two steps are needed in constructing the copula model, the first step is to construct a suitable marginal model for the data, and the second step is to construct a suitable copula model.

4.1. Specification of Marginal Models

The GARCH family model is a classical method in financial time series analysis, this paper will select the most suitable GARCH function from several classical GARCH family models based on data characteristics to build the marginal model.

The GARCH model is one of the most widely used models in the GARCH family model. Its general expression is as follows:

$$\begin{cases} \varepsilon_t = \sqrt{h_t} v_t \\ h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i}, \end{cases} \tag{2}$$

where $v_t \sim IID(0,1)$; $\alpha_0 > 0$; $\alpha_i \ge 0, i = 1,2,3,\cdots,q-1$, $\alpha_q > 0$; $\beta_i \ge 0, i = 1,2,3,\cdots,p-1$, $\beta_p > 0$; $\sum_{i=1}^{q} \alpha_i + \sum_{i=1}^{p} \beta_i < 1$.

4.2. Specification of MRS Copula Models

Sklar's theorem is the basis of the whole copula theory. Let X and Y be the returns of the USDX and crude oil prices, respectively. According to Sklar's theorem, there exists a copula function C such that

$$H(x_1, x_2) = C(F_1(x_1), F_2(x_2)), \tag{3}$$

where $x_1 \in X$ and $x_2 \in Y$, H is the joint distribution of the marginal distributions F_1 and F_2 of X and Y. Equation (3) can also be written as:

$$C(u, v) = H(F_1^{-1}(u), F_2^{-1}(v)), \tag{4}$$

where $u = F_1(x_1)$, $v = F_2(x_2)$, u and v follow a uniform distribution.

In this paper, the MRS copula we construct is a mixture of two static copula, and the dynamic properties of the MRS copula are completely controlled by the state variables. When the state variable changes, the copula function also changes. Markov transition models with two states are sufficient for most variations of financial data. We assume that $S_t \in \{0,1\}$ obeys a two-state Markov process[2]. Its transfer probability can be expressed as:

$$p_{ij} = \mathbb{P}(\theta_t = j \mid \theta_{t+1} = i), \quad i, j = 0,1.$$

Its transfer probability matrix can be expressed as:

$$P = \begin{pmatrix} p_{00} & 1 - p_{00} \\ 1 - p_{11} & p_{11} \end{pmatrix},$$

where p_{ij} is the transition probability from state i at time t to state j at time $t+1$.

[2]A stochastic process is considered Markovian when the conditional probability distribution of its future state, given the present state and all past states, depends only on the current state.

Based on the additivity of the copula function, the MRS copula model can be written as follows:

$$C_t\left(u_t, v_t; \alpha, \beta, S_t\right) = S_t C_t^1\left(u_t, v_t; \alpha\right) + \left(1 - S_t\right) C_t^2\left(u_t, v_t; \beta\right). \tag{5}$$

In constructing the MRS copula, we opt for selecting the suitable copula from a range of options, including the Gaussian copula, *t*-copula, Clayton copula, and Gumbel copula. To save space, this section only shows the expression of the copula used in the final modeling, and the rest of the copulas can be found in [39].

Gaussian copula is a simple and easy-to-understand copula model with good symmetry and independence characteristics. The expression of the binary Gaussian copula distribution function is as follows:

$$C\left(u, v; \rho\right) = \int_{-\infty}^{\Phi^{-1}(u)} \int_{-\infty}^{\Phi^{-1}(v)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(\frac{-\left(r^2 + s^2 - 2\rho rs\right)}{2\left(1-\rho^2\right)}\right) \mathrm{d}r\mathrm{d}s. \tag{6}$$

The expression of its density function is as follows:

$$\begin{aligned} c\left(u, v; \rho\right) = & \frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{\Phi^{-1}\left(u\right)^2 + \Phi^{-1}\left(v\right)^2 - 2\rho\Phi^{-1}\left(u\right)\Phi^{-1}\left(v\right)}{2\left(1-\rho^2\right)}\right) \\ & \times \exp\left(-\frac{\Phi^{-1}\left(u\right)^2 \cdot \Phi^{-1}\left(v\right)^2}{2}\right). \end{aligned} \tag{7}$$

4.3. Parameter Estimation

Maximum likelihood (ML) is the most commonly used approach for copula model parameter estimation, and in this paper, we still use ML to estimate the parameters of MRS copula. Taking the logarithm of both sides of Equation (5), we can obtain the expression of the log-likelihood function as follows:

$$\begin{aligned} & \log h\left(x_{1t}, x_{2t} \mid \Omega_{t-1}; \theta\right) \\ & = \log f_1\left(x_{1t} \mid \Omega_{t-1}; \theta_1\right) + \log f_2\left(x_{2t} \mid \Omega_{t-1}; \theta_2\right) + \log c\left(u_t, v_t \mid \Omega_{t-1}; \theta_c\right), \end{aligned} \tag{8}$$

where h is the joint density of X_{1t} and X_{2t}, f_i is the marginal density of X_{it}, c is the density of copula, Ω_{t-1} the information set available at the end of time $t-1$, $u_t = F_1\left(x_{1t}\right)$, $v_t = F_1\left(x_{2t}\right)$.

According to the characteristics of the copula function, this paper adopts a two-step method to estimate the parameters of the MRS copula ARMA-GARCH model. Specifically, the parameters of the marginal distribution are estimated in the first step, and the parameters of the copula are estimated in the second step after substituting the estimated marginal distribution parameters.

In the first stage, the parameters of the marginal densities are estimated by:

$$\hat{\theta}_1 = \arg\max_{\theta_1} \sum_{t=1}^{T} \log f_1\left(x_{1t} \mid \Omega_{t-1}; \theta_1\right), \tag{9}$$

$$\hat{\theta}_2 = \arg\max_{\theta_2} \sum_{t=1}^{T} \log f_2\left(x_{2t} \mid \Omega_{t-1}; \theta_2\right). \tag{10}$$

In the second stage, the parameters of the copula are estimated by:

$$\hat{\theta}_c = \arg\max_{\theta_c} \sum_{t=1}^{T} \log c\left(F_1\left(x_1;\hat{\theta}_1\right), F_2\left(x_2;\hat{\theta}_2\right) \mid \Omega_{t-1};\theta_c\right). \tag{11}$$

Combining Equation (5), we can transform the $\log c$ part in Equation (8) as follows:

$$\begin{aligned} c_t\left(u_t, v_t \mid \Omega_{t-1};\theta_c\right) &= \mathbb{P}\left(S_t = 0 \mid \Omega_{t-1}\right) \times c_t^1\left(u_t, v_t \mid S_t = 0, \Omega_{t-1};\theta_c^1\right) \\ &\quad + \mathbb{P}\left(S_t = 1 \mid \Omega_{t-1}\right) \times c_t^2\left(u_t, v_t \mid S_t = 1, \Omega_{t-1};\theta_c^2\right) \\ &= \xi_{t|t-1}' \cdot \eta_t, \end{aligned} \tag{12}$$

where $\mathbb{P}\left(S_t = 0\right)$ represents the probability that S_t is a 0 state at time *t*. c_t^i is the density function of the copula in different regimes at time t ($i = 1, 2$). $\xi_{t|t-1} = \left[\mathbb{P}\left(S_t = 0 \mid \Omega_{t-1}\right), \mathbb{P}\left(S_t = 1 \mid \Omega_{t-1}\right)\right]'$, $\eta_t = \left[c_t^1\left(u_t, v_t \mid S_t = 0, \Omega_{t-1};\theta_c^1\right), c_t^2\left(u_t, v_t \mid S_t = 1, \Omega_{t-1};\theta_c^2\right)\right]'$. Based on [40], we can use the Hamilton's filter proposed to calculate $\xi_{t|t-1}$ for $t = 1, \cdots, T$ as follows:

$$\xi_{t|t} = \frac{\xi_{t|t-1} \odot \eta_t}{\xi_{t|t-1}' \cdot \eta_t}, \tag{13}$$

$$\xi_{t+1|t} = P \cdot \xi_{t|t}, \tag{14}$$

where the symbol $\odot$ denotes element-by-element multiplication. Equations (12) - (14) are then used to maximize Equation (11).

Then, we can use these conditional probabilities to gain an even better gauge of the latent state variable. Starting with $\xi_{T|T}$, we can iterate backwards from T to 1 to calculate the smoothed probabilities $\xi_{t|T}$ by iterating on:

$$\xi_{t|T} = \xi_{t|t} \odot \left\{P' \cdot \left[\xi_{t+1|T} \left(\div\right) \xi_{t+1|t}\right]\right\}, \tag{15}$$

where the sign $\left(\div\right)$ denotes element-by-element division.

4.4. Kendall's Rank Correlation Coefficient

For two random observations $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ of random variables $\left(X, Y\right)$, $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ are said to be consistent if $\left(x_1 - x_2\right)\left(y_1 - y_2\right) > 0$. If $\left(x_1 - x_2\right)\left(y_1 - y_2\right) < 0$, then $\left(x_1, y_1\right)$ and $\left(x_2, y_2\right)$ are said to be inconsistent. Combined $\operatorname{sign}\left(x_1 - x_2\right)$. We reference the definition of the consistency correlation measure Kendall's rank correlation coefficient proposed by [41] and [42] and give the following deformation:

$$\tau = \mathbb{P}\left[\left(x_1 - x_2\right)\left(y_1 - y_2\right) > 0\right] - \mathbb{P}\left[\left(x_1 - x_2\right)\left(y_1 - y_2\right) < 0\right], \tag{16}$$

It is further obtained:

$$\tau = 2\mathbb{P}\left[\left(x_1 - x_2\right)\left(y_1 - y_2\right) > 0\right] - 1. \tag{17}$$

Measuring the random variables X and Y with τ shows that when $\tau \in \left(0, 1\right]$, X and Y change in the same way and X and Y are positively correlated; when $\tau \in \left[-1, 0\right)$, X and Y change in the opposite way and X and Y are negatively correlated, when $\tau = 0$, it is impossible to determine whether X and Y are correlated.

5. Empirical Analysis

5.1. Results for Marginal Models

This paper specifies the range of p, q in ARIMA (p, q) between [0, 3] and selects the optimal order based on the AIC information criterion. The final mean portion of USDX returns is ARIMA (0, 1) and the mean portion of crude oil price returns is ARIMA (2, 3). Next, according to the information in Table 1, we consider five GARCH family models (GARCH, EGARCH, IGARCH, GJR-GARCH and APARCH)[3] and three distributions for innovation (skewed normal, Student's *t* and skewed Student's *t* distributions).

Then, follow these steps to select the optimal marginal model from the alternatives. Firstly, based on the AIC information criteria, the model with the lowest AIC values is chosen to advance to the second step. Secondly, the autocorrelation and heteroskedasticity of these models were assessed. This evaluation needs conducting tests, specifically applying the weighted Ljung-Box test and weighted ARCH LM test to the residuals[4]. Thirdly, a probability integral transformation is applied to the residuals of the models that pass the test, and the Kolmogorov-Smirnov (KS) test is used to verify that the probabilistic integral transformed sequences obey an independent uniform standard distribution.

If all the conducted tests yield satisfactory results, the marginal model that successfully passes the tests is considered optimal. However, if the tests in the second or third step do not pass, the process reverts back to the first step to begin anew. This iterative approach ensures the selection of the most appropriate model.

Table 2 and Table 3 report the estimation results of the margin model[5]. Eventually, Student's *t* distribution ARMA (0, 1) - GARCH (1, 1) margin model is selected for the USDX returns. Skewed Student's *t* distribution ARMA (2, 3) - APARCH (1, 1) margin model is assigned to the crude oil price returns.

5.2. Results for MRS Copula Models

We then model the MRS copula based on the observations obtained from the marginal model. In this paper, we construct the MRS copula model following these steps: Firstly, 16 different MRS copula models were established (Gaussian-Gaussian copula, Gaussian-*t* copula, Gaussian-Clayton copula, Gaussian-Gumbel copula, *t*-Gaussian copula, *t*-*t* copula, *t*-Clayton copula, *t*-Gumbel copula, Clayton-Gaussian copula, Clayton-*t* copula, Clayton-Gumbel copula, Gumbel-Gaussian copula, Gumbel-*t* copula, Gumbel-Clayton copula and Gumbel-Gumbel copula). Subsequently, the significance of the estimated parameters of alternative models is tested. Finally, the optimal model is selected from the models that pass the significance test according to AIC information criteria.

[3]The GARCH model not mentioned in Section 4.1 can be found in [43].

[4]In contrast to the Ljung-Box and ARCH-LM tests, [44] proposes weighted Ljung-Box and weighted ARCH-LM tests, which better explain the distribution of the statistics of the median values in the estimated model.

[5]To save space, we omit the estimation results for the ARIMA and GARCH coefficients from the report. These results are available from the authors upon request.

Table 2. Marginal model estimation results for USDX returns.

Skewed normal distribution	AIC	WLB test	WLM test	KS test
GARCH (1, 1)	−8.228	0.980	0.348	1.000
EGARCH (1, 1)	−8.221	0.970	0.215	1.000
IGARCH (1, 1)	−8.226	0.970	0.408	1.000
GJR-GARCH (1, 1)	−8.227	0.977	0.407	1.000
APARCH (1, 1)	−8.226	0.984	0.504	1.000
Student's *t* distribution	AIC	WLB test	WLM test	KS test
GARCH (1, 1)	−8.229	0.981	0.416	1.000
EGARCH (1, 1)	−8.221	0.971	0.166	1.000
IGARCH (1, 1)	−8.227	0.976	0.322	1.000
GJR-GARCH (1, 1)	−8.227	0.981	0.378	1.000
APARCH (1, 1)	−8.208	0.979	0.714	1.000
Skewed *t* distribution	AIC	WLB test	WLM test	KS test
GARCH (1, 1)	−8.228	0.981	0.331	1.000
EGARCH (1, 1)	−8.221	0.971	0.160	1.000
IGARCH (1, 1)	−8.226	0.937	0.324	1.000
GJR-GARCH (1, 1)	−8.226	0.980	0.316	1.000
APARCH (1, 1)	−8.210	0.985	0.675	1.000

Note: The WLB test is the weighted Ljung-Box test and the WLM test is the weighted LM test. The original hypothesis of the weighted Ljung-Box test is that there is no autocorrelation. The original hypothesis of the weighted LM test is that there is no heteroskedasticity effect.

Table 3. Skewed student-*t* distribution marginal model estimation results for crude oil returns.

Skewed normal distribution	AIC	WLB test	WLM test	KS test
GARCH (1, 1)	−4.740	0.944	0.655	1.000
EGARCH (1, 1)	−4.755	0.817	0.536	1.000
IGARCH (1, 1)	−4.738	0.915	0.577	1.000
GJR-GARCH (1, 1)	−4.747	0.989	0.605	1.000
APARCH (1, 1)	−4.772	0.597	0.348	1.000
Student's *t* distribution	AIC	WLB test	WLM test	KS test
GARCH (1, 1)	−4.845	0.902	0.667	1.000
EGARCH (1, 1)	−4.851	0.913	0.538	1.000
IGARCH (1, 1)	−4.846	0.891	0.641	1.000
GJR-GARCH (1, 1)	−4.848	0.984	0.574	1.000
APARCH (1, 1)	−4.857	0.958	0.358	1.000

Continued

Skewed *t* distribution	AIC	WLB test	WLM test	KS test
GARCH (1, 1)	−4.868	0.534	0.250	1.000
EGARCH (1, 1)	−4.875	0.732	0.561	1.000
IGARCH (1, 1)	−4.868	0.362	0.639	1.000
GJR-GARCH (1, 1)	−4.871	0.912	0.598	1.000
APARCH (1, 1)	−4.884	0.956	0.235	1.000

Table 4 reports the results of parameter significance tests and the values of AIC and BIC. According to the above modeling steps, the Gaussian-Gaussian MRS copula model is used to analyze the dependence structure between USDX and crude oil price. Table 5 and Figure 4 report the results of the Gaussian-Gaussian MRS copula model estimation. In Table 5, θ is the correlation parameter of the copula, which reflects the correlation between the two variables connected by the copula. τ is the Kendall coefficient. Figure 4 illustrates the probability of the emergence of different regimes, in which the negatively correlated regime is called the calm regime, and the positively correlated regime is called the extreme regime. The results show that the emergence probability of the calm regime is generally higher than that of the extreme regime, but the emergence probability of the extreme regime is obviously higher than that of the calm regime before and after some events.

5.3. Anaylsis of Dependence Structure

With the US dollar as the designated international trading currency for crude oil, it has become the prevailing view that there is a static or time-varying negative correlation structure between the USDX and crude oil. Nonetheless, the results of this study indicate the presence of two distinct dependence structures, one positive and one negative, between the USDX and crude oil prices, with transitions occurring between these different dependence structures. The conclusions reached in this paper are not difficult to understand.

From the perspective of the US dollar, on the one hand, although the US dollar is designated as the sole trading currency for crude oil in international trade, with the development of a more diverse currency system, some other currencies are also used for settling crude oil transactions[6]. This poses a substantial threat to the unbreakable relationship between the dollar and crude oil. On the other hand, the aggressive interest rate increases by the United States dollar have had a dual effect: they have temporarily stabilized the negative correlation structure between USDX and crude oil prices and, concurrently, accelerated the global de-dollarization process. Consequently, sharp interest rate hikes by the United States dollar can only offer short-term stability to the negative correlation between the USDX and crude oil prices. In the long term, they might create conditions for the emergence of alternative dependence structures between the two.

[6]In 2022 at the summit of China and the Cooperation Council for the Arab States of the Gulf, China and the GCC signed an agreement to use the yuan to settle oil.

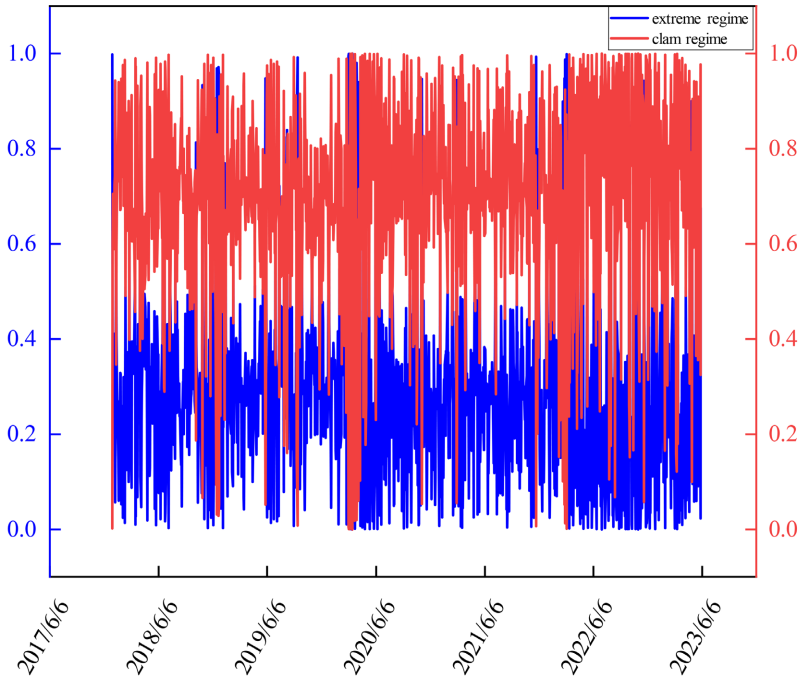

Figure 4. Probability of regimes transition for the 2018-2023.

Table 4. Tests and selection of MRS copula models.

Copula	θ	Dof	Copula	θ	Dof	AIC
Gaussian	0.565*** (0.000)	/	Gaussian	−0.447*** (0.000)	/	−79.728
Gaussian	0.990*** (0.000)	/	*t*	−0.139* (0.034)	4.352** (0.008)	−76.982
Gaussian	−0.488*** (0.000)	/	Clayton	0.591* (0.018)	/	−73.848
Gaussian	−0.591*** (0.000)	/	Gumbel	1.131*** (0.000)	/	−61.416
t	−0.010 (0.675)	2.745** (0.010)	Gaussian	−0.262** (0.007)	/	−89.191
t	0.028 (0.524)	2.708* (0.014)	*t*	−0.252** (0.009)	14.698 (0.243)	−88.848
t	−0.214* (0.015)	5.166* (0.027)	Clayton	0.789 (0.074)	/	−77.264
t	−0.137* (0.033)	4.315** (0.008)	Gumbel	8.376 (0.246)	/	−76.836
Clayton	0.591* (0.018)	/	Gaussian	−0.488*** (0.000)	/	−73.840
Clayton	0.789 (0.074)	/	*t*	−0.214* (0.015)	5.167* (0.027)	−77.258
Clayton	0.714 (0.146)	/	Clayton	0.010 (0.613)	/	8.189
Clayton	0.726 (0.147)	/	Gumbel	1.010*** (0.000)	/	7.623
Gumbel	1.131*** (0.000)	/	Gaussian	−0.591*** (0.000)	/	−61.408
Gumbel	8.339 (0.249)	/	*t*	−0.139* (0.032)	4.337** (0.008)	−76.845
Gumbel	1.151*** (0.000)	/	Clayton	0.010 (0.616)	/	6.036
Gumbel	1.167** (0.002)	/	Gumbel	1.010*** (0.000)	/	9.051

Note: * represents significance at the 5% level, ** represents significance at the 1% level, *** represents significance at the 0.1% level, and the p-values are in parentheses.

Table 5. Results of the optimal MRS copula.

Copula	θ	τ	Copula	θ	τ
Gaussian	0.565	0.383	Gaussian	−0.447	−0.295

From the perspective of crude oil, geopolitical risks such as the Russian-Ukrainian conflict, the Iranian nuclear issue and the Israeli-Palestinian conflict, which are taking place between crude oil-exporting countries, have a direct impact on the production as well as the export of crude oil. This has caused international crude oil prices to fluctuate, leading to a series of price problems.

6. Conclusions and Suggestions

In this paper, an MRS copula model is developed with the aim of investigating the dependence structure between USDX and crude oil prices over the period from 2018 to 2023. The results show that there is a dynamic dependence structure with positive and negative correlations switching with each other between USDX and crude oil prices during this period, and this switching process is Markovian.

[28] shows that there is a tail dependence between USDX and crude oil prices in the 2011-2017 period, the empirical results in this paper show that there is tail independence between USDX and crude oil prices in the 2018-2023 period. Nevertheless, crude oil is not a safe-haven investment for the US dollar. The conclusion suggests that in the face of a crisis, the probability of a positive correlation between the USDX and crude oil prices tends to be greater than the probability of a negative correlation.

Some past experience suggests that both gold and crude oil can be used as safe-haven investments in times of crisis, however the conclusions of this paper demonstrate that crude oil cannot be a safe-haven investment for the U.S. dollar in times of crisis. This has important implications for investors. The results are also enlightening for policymakers. In the new wave of de-dollarization, policymakers should reconsider the monetary policy for international trade, diversify foreign exchange and improve their ability to withstand risks.

The MRS copula model has good results in fitting the dependence structure of crude oil prices and the USDX in unexpected (often short-term) situations. Nevertheless, it also has some limitations. To enhance the model's practical utility, certain scholars have refined the copula within the MRS framework, while others have augmented its applicability through the inclusion of additional research variables [36] [45] [46]. In addition, on the basis of this conclusion, further research on risk management, risk contagion and risk prediction can be carried out. This gives us some directions for further research in the future.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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