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Constraints on Asymmetric Dark Matter in Quintessence Model

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Abstract

The modified cosmology like quintessence model with kination phase predicted the Hubble expansion rate of the universe before Big Bang Nucleosynthesis is different from the standard cosmological scenario. The modified expansion rate leaves its imprint on the relic density of asymmetric dark matter. In this work, we review the calculation of relic density of asymmetric WIMP dark matter in the standard cosmological scenario and quintessence model with kination phase. Then we use the Planck data to find constraints on the annihilation cross section and the mass of the asymmetric dark matter in those models.

Keywords

Asymmetric Dark Matter, Standard Model, Quintessence Model

1. Introduction

Astronomical and cosmological observations have demonstrated the existence of non-luminous matter. Measurements of the anisotropy of cosmic microwave background radiation have enabled accurate determinations of the total amount of dark matter. The energy density of dark matter and baryonic matter (visible matter, which participates in electromagnetic, strong and weak interactions) constituting the present-day universe have been determined from the observations by Planck satellite data [1],

$$\Omega_{DM}h^2 = 0.120 \pm 0.001, \Omega_B h^2 = 0.0224 \pm 0.001, \quad (1)$$

where Ω_{DM} represents the relic density of dark matter in units of the critical density, and h is the Hubble constant, $h = 0.7100 \pm 0.025$, measured in units of 100 kilometers per second per megaparsec, Ω_B is the relic density of baryonic matter.

Although we have an accurate measurement of the dark matter relic density, the physical properties of dark matter in the universe are still unknown. In order to detect the dark matter particles, many methods are used and ongoing, such as direct detection, indirect detection and particle collision experiments. In theory, candidate particles for dark matter are proposed beyond the Standard Model of particle physics. Weakly interacting massive particles (WIMPs) are the most interesting candidates for dark matter, and neutralino is the best motivated one which is appeared in supersymmetric models. Neutralinos are the Majorana particles which are their own anti-particles. Until now, we have no evidence that the dark matter particle should be Majorana particle. There is another option that the dark matter particles can be asymmetric. On the other hand, from Equation (1), we know $\Omega_{DM} \approx 5\Omega_B$. It motivates us to consider whether there is connection between the dark matter and the baryonic matter. There maybe similar origin which can explain the postulated asymmetry for the dark matter and baryon asymmetry.

Asymmetric dark matter has been widely studied in the literatures [2]-[13]. In many current models, asymmetry arises in one sector, either in the Standard Model sector or in the dark matter sector, and is subsequently transmitted via contact to the other sector. This situation often leads to similar baryon and dark matter number densities, with $n_{DM} \sim n_b$. The symmetric component of dark matter number density is typically eliminated in the annihilation, and the relic density of dark matter is determined mainly by the asymmetric components. Asymmetric dark matter abundance is obtained by solving the Boltzmann equations for particle and anti-particle which describes the time evolution of particle and anti-particle densities in the expanding universe. This kind of computation has been done for the standard cosmology and non-standard cosmological models in refs. [14] [15] [16] [17]. In the context of asymmetric, weakly interacting massive particles, the relic density of asymmetric dark matter depends on its thermal annihilation cross-section, mass, and the existed asymmetry. In ref. [14], the authors obtained the constraints on the mass and cross section of asymmetric dark matter in the standard cosmological scenarios.

The nonstandard cosmological models like quintessence model with kination phase, Scalar-Tensor Theories predicted the Hubble expansion rate before Big Bang Nucleosynthesis (BBN) is different from the standard cosmological scenarios [18]-[23]. In quintessence models based on tracking solutions for the scalar field, there is a kination phase. Kination is a period in which the energy density of universe is dominated by kinetic energy of the scalar field. Indeed the quintessence problems can be achieved in the framework of extended gravity [19] [20]. The universe grows faster than the standard cosmology in that era [18]. The Hubble expansion rate in that case is different from the standard one. The modified Hubble expansion rate leaves its imprint on the relic density of asymmetric dark matter. In our work, we review the calculation of the relic density of asymmetric WIMP dark matter in the standard cosmological scenario and quintessence model with kination phase. The modified Hubble parameter in quintessence model with kination phase affected the relic density of asymmetric dark matter. The detailed analysis has been done in refs. [16]. The authors found the relic density of asymmetric dark matter is increased due to the enhanced Hubble expansion rate in quintessence model. The enhanced Hubble expansion rate leads to the earlier decay of the asymmetric dark matter from the thermal equilibrium and there is increased relic density in the end. We use the observed abundance of dark matter to derive the constraints on the parameter spaces including the mass and cross sections for asymmetric WIMP dark matter inquintessence model. We plot the contour of the mass and cross section when the dark matter relic density satisfies the observational value. We find when the cross section is small, the relic density is insensitive for the varying mass. When the mass is increased to the maximum value, the final relic abundance of asymmetric dark matter is not much affected for the varying cross section in both standard and quintessence models. Because of the increased Hubble expansion rate in quintessence model, one needs larger annihilation cross section than the standard one in order the relic density falls in the observation range.

The structure of our paper is as follows. In Section 2, we briefly review the Boltzmann equations for asymmetric dark matter in the standard cosmological model. In Section 3, we discuss the Boltzmann equations with an altered Hubble parameter in the quintessence model for particle and anti-particle. In Section 4, we derive the constraints on the parameter spaces including the cross section and the mass. The conclusion and summaries are in the last section.

2. Boltzmann Equations for Asymmetric Dark Matter in the Standard Cosmological Model

In this section, we briefly review the Boltzmann equations for asymmetric dark matter in the standard cosmological scenario. We assume that the asymmetric dark matter particles and anti-particles were in thermal equilibrium in the early universe and they were decoupled when they are in non-relativistic case. The number densities of asymmetric dark matter in the expanding universe evolved over time are described by the Boltzmann equations. By solving the Boltzmann equations, we obtain the relic density of asymmetric dark matter for particles and anti-particles. Here we consider that only χ and $\bar{\chi}$ can be annihilated into the Standard Model particles, $\chi\chi$ and $\overline{\chi\chi}$ can not, then the general form of the Boltzmann equations for the particle and anti-particle is

$$\frac{\mathrm{d}n^{\pm}}{\mathrm{d}t}+3Hn^{\pm}=-\langle\sigma v\rangle\left(n^{+}n^{-}-n_{eq}^{+}n_{eq}^{-}\right), \tag{2}$$

where n^{+} is for particle and n^{-} for anti-particle, $\langle\sigma v\rangle$ is the thermal average of the annihilation cross section multiplied by the relative velocity of the two annihilating particles. During the radiation-dominated period, the Hubble parameter $H=\pi T^2\big/M_{Pl}\sqrt{g_*/90}$, where $M_{Pl}=2.4\times10^{18}\,\mathrm{GeV}$ is the reduced Planck mass and g_* the effective number of relativistic degrees of freedom. The number densities n_{eq}^{+} and n_{eq}^{-} of asymmetric dark matter in thermal equilibrium at non-relativistic limits can be written as

$$n_{eq}^{\pm} = g\left[mT/(2\pi)\right]^{3/2} \mathrm{e}^{-m/T \pm \xi}, \tag{3}$$

where g represents the number of internal degrees of freedom and m is the mass of the dark matter particle, $\xi = \mu/T$, here μ is the chemical potential. In the standard cosmological scenario, for $T \gg m$, asymmetric dark matter particles and anti-particles are in the thermal equilibrium state. In later time when $T \ll m$, the annihilation rate $\Gamma = n_{\chi}\langle\sigma v\rangle$ drops below the expansion rate H, therefore, the particles can not be effectively annihilated and the co-moving number densities become constant. The temperature at which particles decoupled from the hot bath is called the freeze out temperature T_F.

The Boltzmann Equation (2) is simplified by introducing the dimensionless quantities $Y^+ = n^+/s$, $Y^- = n^-/s$ and variable $x = m/T$, where the entropy density $s = 2\pi^2 g_*/45T^3$. Suppose that when the universe is in a state of adiabatic expansion, the entropy of each co-moving volume is conserved. The Boltzmann Equation (2) can be rewritten as

$$\frac{\mathrm{d}Y^{\pm}}{\mathrm{d}x} = -\frac{\lambda\langle\sigma v\rangle}{x}\left(Y^+Y^- - Y_{eq}^+Y_{eq}^-\right), \tag{4}$$

where $\lambda = 1.32 m M_{Pl}\sqrt{g_*}$. Subtracting equation for Y^- from the equation for Y^+, we obtain $\mathrm{d}Y^+/\mathrm{d}x - \mathrm{d}Y^-/\mathrm{d}x = 0$. It yields $Y^+ - Y^- = \eta$. Here η is considered as a primordial asymmetry between the particle and anti-particle. The dark matter asymmetry is expressed in units of the baryon asymmetry as $\eta = \epsilon\eta_B$, where $\eta_B = (0.870 \pm 0.004)\times 10^{-10}$ [1]. The Boltzmann Equation (4) can be rewritten as follows,

$$\frac{\mathrm{d}Y^{\pm}}{\mathrm{d}x} = -\frac{\lambda\langle\sigma v\rangle}{x}\left[\left(Y^{\pm}\right)^2 \mp \eta Y^{\pm} - Y_{eq}^2\right], \tag{5}$$

where $Y_{eq}^2 = Y_{eq}^+Y_{eq}^- = (0.145 g/g_*)^2 x^3 \mathrm{e}^{-2x}$. These equations can be solved through numerical and analytical methods.

Generally, the annihilation cross-section of WIMPs is expanded in the relative velocity v of the annihilating WIMP particles as

$$\langle\sigma v\rangle = a + b\langle v^2\rangle + O\left(\langle v^4\rangle\right) = a + 6bx^{-1} + O\left(x^{-2}\right). \tag{6}$$

Here a is s-wave contribution to σv while $b = 0$, and b is p-wave contribution to σv while $a = 0$.

3. Boltzmann Equations for Asymmetric Dark Matter in the Quintessence Model

In this section we briefly review the relic density computation for asymmetric dark matter in the quintessence model. It was originally appeared in [16]. The quintessence model with kination phase has been extensively studied as a class of scalar field models since the discovery of dark energy [18] [19] [20] [21] [22]. It assumes that the reason behind the current accelerated expansion of the universe is a scalar field that has minimal coupling to gravity. The ratio of the expansion rate in quintessence model with kination phase and the standard model is: $H^2/H_{std}^2 = 1 + \rho_\phi/\rho_r$, where ρ_ϕ is the scalar energy density. $\rho_\phi/\rho_r \approx k\left(T/T_0\right)^2$, and $\rho_r = \left(\pi^2/30\right) g_* T^4$ is the radiation energy density [18] [21] [22]. Here T_0 is the reference temperature which is close to the freezing out temperature, $k = \rho_\phi\left(T_0\right)/\rho_r\left(T_0\right)$. The expansion rate in the quintessence model can be written as:

$$H = A(T) H_{std}, \tag{7}$$

where $A(T)$ is expressed as $A(T) = \sqrt{1 + k\left(T/T_0\right)^2}$.

Using the modified expression for the expansion rate, we rewrite the Boltzmann Equation (5) in quintessence model with kination phase as:

$$\frac{\mathrm{d}Y^\pm}{\mathrm{d}x} = -\frac{\lambda\langle\sigma v\rangle}{x^2 A(x)}\left[\left(Y^\pm\right)^2 \mp \eta Y^\pm - Y_{eq}^2\right], \tag{8}$$

we first solve the Boltzmann equation for anti-particle in Equation (8) for Y^-, then Y^+ is obtained using the relation $Y^+ - Y^- = \eta$. For *s*-wave annihilation, the Boltzmann equation for Y^- is

$$\frac{\mathrm{d}Y^-}{\mathrm{d}x} = -\frac{\lambda a}{x^2\sqrt{1 + k\left(x_0/x\right)^2}}\left[\left(Y^-\right)^2 + \eta Y^- - Y_{eq}^2\right]. \tag{9}$$

In the case of *p*-wave annihilation, the Boltzmann equation for Y^- becomes,

$$\frac{\mathrm{d}Y^-}{\mathrm{d}x} = -\frac{\lambda\left(6b/x\right)}{x^2\sqrt{1 + k\left(x_0/x\right)^2}}\left[\left(Y^-\right)^2 + \eta Y^- - Y_{eq}^2\right]. \tag{10}$$

We obtain the analytic solution of the relic density for asymmetric dark matter in quintessence model. We introduce the quantity $\Delta_- = Y^- - Y_{eq}^-$. In terms of Δ_-, Boltzmann Equation (8) can be rewritten as:

$$\frac{\mathrm{d}\Delta_-}{\mathrm{d}x} = -\frac{\mathrm{d}Y_{eq}^-}{\mathrm{d}x} - \frac{\lambda\langle\sigma v\rangle}{x^2 A(x)}\left[\Delta_-\left(\Delta_- + 2Y_{eq}^-\right) + \eta\Delta_-\right]. \tag{11}$$

For high temperature, the solution for Boltzmann Equation (11) is

$$\Delta_- \simeq \frac{2x^2 A(x) Y_{eq}^2}{\lambda\langle\sigma v\rangle\left(\eta^2 + 4Y_{eq}^2\right)}. \tag{12}$$

Detailed analysis is found in [16]. This solution is used to fix the freezing out temperature. When the temperature is low enough, for $x > \bar{x}_F$, the term which is proportional to Y_{eq}^- can be ignored in Equation (11), therefore

$$\frac{\mathrm{d}\Delta_-}{\mathrm{d}x} = -\frac{\lambda\langle\sigma v\rangle}{x^2 A(x)}\left(\Delta_-^2 + \eta\Delta_-\right). \tag{13}$$

We may assume $\Delta_-\left(\bar{x}_F\right) \gg \Delta_-(\infty)$. Integrating Equation (13) from $\bar{x}_F$ to ∞, then

$$Y^-\left(x \to \infty\right) = \frac{\eta}{\exp\left[\eta\left(4\pi/\sqrt{90}\right) m M_{Pl}\sqrt{g_*} I\left(\bar{x}_F\right)\right] - 1}. \tag{14}$$

where

$$I(\bar{x}_F) = \int_{\bar{x}_F}^{\infty} \frac{\langle \sigma v \rangle}{x^2 A(x)} \mathrm{d}x = \frac{a}{\sqrt{k} x_0} \ln\left(\sqrt{k} \frac{x_0}{\bar{x}_F} + \sqrt{1 + k \frac{x_0^2}{\bar{x}_F^2}} \right) + \frac{6b}{k x_0^2} \left(\sqrt{1 + k \frac{x_0^2}{\bar{x}_F^2}} - 1 \right). \tag{15}$$

Using $\eta = Y^+ - Y^-$, we obtain the relic density for χ particles. The result is

$$Y^+(x \to \infty) = \frac{\eta}{1 - \exp\left[-\eta \left(4\pi/\sqrt{90} \right) m M_{Pl} \sqrt{g_*} I(x_F) \right]}. \tag{16}$$

where

$$I(x_F) = \int_{x_F}^{\infty} \frac{\langle \sigma v \rangle}{x^2 A(x)} \mathrm{d}x = \frac{a}{\sqrt{k} x_0} \ln\left(\sqrt{k} \frac{x_0}{x_F} + \sqrt{1 + k \frac{x_0^2}{x_F^2}} \right) + \frac{6b}{k x_0^2} \left(\sqrt{1 + k \frac{x_0^2}{x_F^2}} - 1 \right). \tag{17}$$

Equation (14) and Equation (16) are consistent with the constraint $\eta = Y^+ - Y^-$ if $x_F = \bar{x}_F$. We express the final abundance as

$$\Omega_{DM} h^2 = \frac{2.76 \times 10^8 m \eta}{\exp\left[\eta \left(4\pi/\sqrt{90} \right) m M_{Pl} \sqrt{g_*} I(\bar{x}_F) \right] - 1} + \frac{2.76 \times 10^8 m \eta}{1 - \exp\left[-\eta \left(4\pi/\sqrt{90} \right) m M_{Pl} \sqrt{g_*} I(x_F) \right]}. \tag{18}$$

Using the fact that when the deviation Δ_- is of the same order of the equilibrium value of Y^-:

$$\zeta Y_{eq}^-\left(\bar{x}_{F_0}\right) = \Delta_-\left(\bar{x}_{F_0}\right). \tag{19}$$

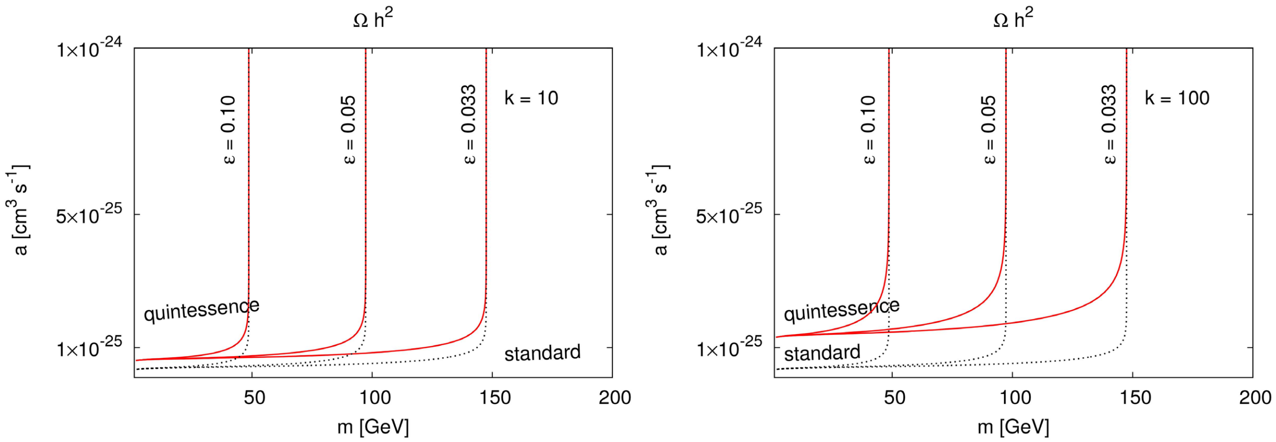
The freeze out temperature $\bar{x}_F$ is determined as

$$\bar{x}_F = \bar{x}_{F_0} \left[1 + \frac{\lambda \eta}{A\left(\bar{x}_{F_0}\right)} \left(\frac{0.285 a}{\left(\bar{x}_{F_0}\right)^3} + \frac{1.350 b}{\left(\bar{x}_{F_0}\right)^4} \right) \right]. \tag{20}$$

Here $\zeta = \sqrt{2} - 1$ [24].

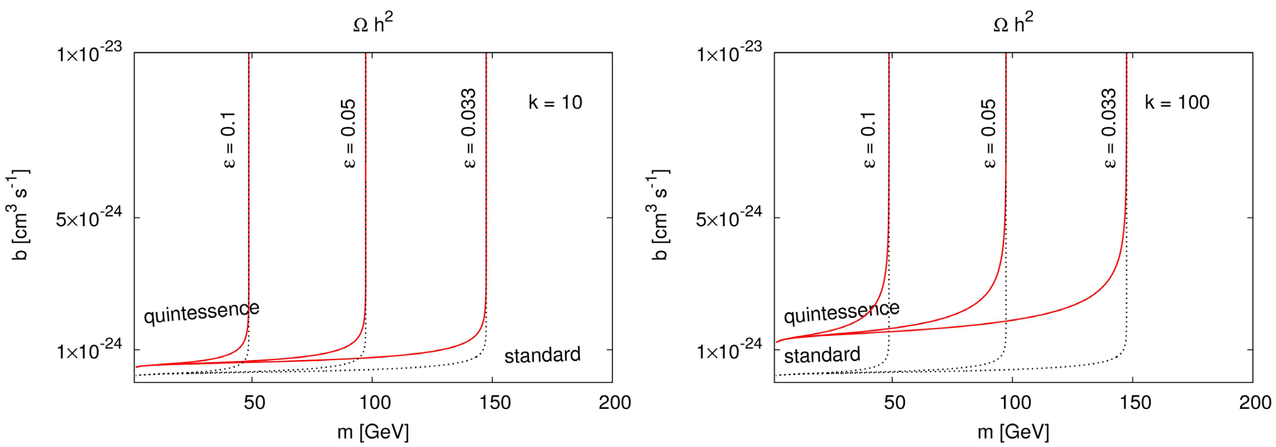
4. Constraints on the Parameter Spaces

Figure 1 and **Figure 2** are obtained by the numerical solution of Equation (8). We plot the parameter spaces allowed in *m*-*a*(*b*) plane which satisfies the observed value of dark matter relic density Equation (1) in the standard cosmological scenario and quintessence model with kination phase for different parameters ϵ. We take $\epsilon = 0.033, 0.05, 0.1$. In **Figure 1**, panels (a) and (b) refer to the *s*-wave processes for $k = 10$ and $k = 100$. The solid (red) lines are for the quintessence model and the dotted (black) lines are for the standard model. Panels (a) and (b) in **Figure 2** are for *p*-wave processes. We find that for the different asymmetry ϵ, the allowed region is bounded by the cross section a(b) from below and by the maximum value of the mass *m* from the right.



(a) (b)

Figure 1. Contour plots of the annihilation cross section a and mass m for different asymmetry when $\Omega_{DM} = 0.120$. Here $\eta = \epsilon \eta_B$, where $\eta_B = 0.87 \times 10^{-10}$, $b = 0$, $g_* = 90$, $g = 2$, $x_0 = 20$.



(a) (b)

Figure 2. Contour plots of the annihilation cross section b and mass m for different asymmetry when $\Omega_{DM} = 0.120$. Here $\eta = \epsilon \eta_B$, where $\eta_B = 0.87 \times 10^{-10}$, $a = 0$, $g_* = 90$, $g = 2$, $x_0 = 20$.

For small values of cross sections, the curves are almost flat. When the cross section is small, the symmetric dark matter case is recovered and the mass is irrelevant in that case. This is the reason why the abundance is not sensitive to the varying mass for the small cross section. On the other hand, when the cross section is increased, the abundance is bounded by the maximum value of the mass from the right. The mass limits are increased when the asymmetry is decreased. For smaller asymmetry factor the mass bound is larger. For the quintessence model, the lower bound of the cross section is larger than the standard cosmology for the same mass value. The reason is that the Hubble expansion rate is increased in quintessence model. This leads to the larger relic density in the end. Therefore, the cross section should be larger in order to the relic density falls in the observed value of dark matter abundance. The same rule is applied for the case of p-wave annihilation. For p-wave annihilation, the cross section limit is one order larger than the s-wave annihilation.

5. Summary and Conclusions

We review the relic density of asymmetric dark matter in the standard cosmological scenario and quintessence model with kination phase. We use the observed value of dark matter abundance to find constraints on the parameter spaces including cross section and the mass. We found for the smaller cross section, the symmetric case is recovered, therefore, the abundance is not sensitive to the varying mass. When the cross section is increased, the asymmetric dark matter abundance is decreased and this can be compensated by increasing m. The dark matter abundance is determined almost by the maximum value of the mass and it is not sensitive to the varying cross section. In the case of quintessence model, the cross section is larger than the standard cosmological scenario due to the increased Hubble expansion rate. The increased Hubble expansion rate leads to the larger relic density. The cross section should be large in order to the dark matter relic density falls in the observed range. The mass limit is increased for smaller asymmetry factor ϵ. Our result is important for the asymmetric dark matter models. The constraints on the parameter spaces like cross section and mass provide the theoretical reference values for the asymmetric dark matter detection experiments.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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